Exhibit 10
1. My name is Gregory Herschlag. I received a Ph.D. in Applied Mathematics from the University of North Carolina at Chapel Hill in 2013. I am currently a Visiting Assistant Professor in Mathematics and Biomedical Engineering at Duke University.

2. My curriculum vitae is attached as Exhibit 1.

3. Counsel for Plaintiffs requested that I provide a report with my initial analysis of the 2017 NC House and Senate districts in Wake, Mecklenburg, Cumberland and Guilford Counties. This report is part of an ongoing research program that analyzes political districts by comparing the partisan outcome of elections in a given district map to the partisan outcome of simulated elections using an ensemble of maps. The ensemble is generated with established mathematical and methodological principles [1,2]. The counsel specifically requested that I provide an analysis of the racial makeup of these districts in addition to their partisan composition.

4. I will be compensated for my work at the rate of $200 per hour. My colleague, Dr. Jonathan Mattingly, Chair of the Math Department at Duke has worked with me on this project. Professor Mattingly has reported on corresponding work in Common Cause v Rucho and Whitford v Gill. All of these reports summarize aspects of the larger, ongoing, research program mentioned above.

5. My work on the 2017 NC districts began soon after the districts were enacted on August 31, 2017. In the two-week period since then, I have completed an initial analysis of Senate districts in Wake, Mecklenburg and Cumberland Counties but have not had time to undertake any analysis of the Senate districts in Guilford or any analysis of House districts.
6. The traditional redistricting criteria I use to generate simulated districts are equal population (plus or minus 5%), compactness (as measured by the Polsby-Popper test), preservation of precincts, and contiguity. I use 2010 census data to determine equal population for, and racial composition of, simulated districts. To determine the partisan performance of the simulated and enacted districts, I use the NC Senate election results in 2012 and 2014, corrected for unopposed races. 2016 Senate and 2016 US House of Representative election results are not used because these results were reported without partisan information on the NC State Board of Elections website [3], and I have not had time to process these files. To ensure the robustness of the partisan results, I also consider all Presidential, US Senate, and NC Governor election results occurring in 2012, 2014, and 2016, along with US House of Representatives election results occurring in 2012 and 2014.

7. To date we have generated an ensemble of 7,606 simulated district plans in Wake, 6,044 district plans in Mecklenburg, and 78,485 district plans in Cumberland. The methods used to produce this ensemble have been shown to accurately sample the distribution on possible redistricting plans respecting the above redistricting criteria [1,2].

8. The racial composition of the 2017 districts in Cumberland County is highly atypical. One of the two enacted 2017 districts in Cumberland contains a significantly higher percentage of the population that is African American (46.5%) than any district in the 78,485 simulated districting plans for the county (see Figure 1; right). In Cumberland, the average percentage of the population that is African American over all simulated districts is 36.3%.

9. In Mecklenburg County, the racial composition of the 2017 districts is also highly atypical. The 2017 enacted districts and the 6,044 simulated districts each contain 5 districts in Mecklenburg. The 5 districts are ordered from least to most in terms of the percentage of the population that is African American in the 2017 enacted districts and in each district map of the ensemble. I find that the middle ordered district of the 2017 enacted plan has a significantly higher minority population than is expected; only 3 of the 6,044 simulated districting plans have a higher percentage of African Americans. In the 2017 enacted plan, the population of the middle ordered district is 39.6% African American – higher than the 30.2% average of the simulated plans (see Figure 1; middle).

10. In contrast, the 2017 enacted Senate districts in Wake County are within the expected racial composition range (see Figure 1; left)

11. For a presentation of racial statistics see Figure 1 below.

12. To compare the partisan composition of the 2017 Senate districts with the ensemble, we simulate elections in all three counties by using the actual votes cast in the
races for the NC Senate, President, US Senate, US House of Representatives, and NC Governor in elections between 2012 and 2016; as described above, all elections but the 2016 NC Senate and 2016 US House elections are omitted. I plot histograms of elected Democrats for the ensemble in each simulated election with respect to the overall fraction of the Democratic vote (see Figure 2). I compare the histograms results to the simulated elections in the 2017 enacted Senate districts (see Figure 2). Only the votes cast in the 2012 NC Governor election produce an expected partisan composition for the 2017 enacted districts when compared with the ensemble. In using the votes cast in the 2012 and 2016 Presidential elections, the 2014 and 2016 US Senate elections, the 2012 and 2014 US House of Representatives elections, and the 2016 Governor election, the partisan compositions based on the 2017 districts occur in less than 0.1% of the ensemble. In the 2012 and 2014 NC Senate simulated elections, the partisan outcomes based on the 2017 districts occur in 7.14% and 3.50% of the ensemble, respectively. With the exception of the simulated election based on the 2012 race for the NC Governor, the number of Democrats elected with the enacted 2017 maps is consistently smaller than median number of Democrats elected in the ensemble by 1 to 3 seats. In nearly every examined simulated election, the partisan outcome is both unlikely and favors the Republican party.

13. For a presentation of the partisan composition of the three combined county districts, see Figure 2.

Pursuant to 28 U.S.C. § 1746, I declare under penalty of perjury that the foregoing is true and accurate.

Executed on September 14, 2017.

[Signature]
Gregory Henschel

Figures
Figure 1. Districts in each county are ordered from least to most percentage of the population that is African American. Marginal distributions of the ensemble are presented with a standard violin plot (green) – thicker horizontal regions are more likely to occur in the ensemble. The 2011 and 2017 NC Senate districts are also ordered from least to most percentage of the population that is African American and compared with the marginal distributions. In Cumberland, not a single generated districting plan packs as many African Americans into a single district as the 2011 and 2017 NC Senate districts. In Mecklenburg, African Americans are over represented in the district with the third highest percentage of African Americans in the 2017 NC Senate districts.
Figure 2. I present histograms of the simulated election results over the ensemble of districting plans and plot them with respect to the fraction of the vote that went to Democrats. I compare the histograms with simulated elections using the 2011 and 2017 enacted Senate districts. I present simulated elections based on the 2016 Presidential (PRE16), 2014 NC State Senate (NCSS14), 2014 US Senate (USS14), 2012 Presidential (PRE12), 2014 US House of Representatives (USH14) and 2012 NC Governor (GOV12) elections. Other reported elections are omitted for clarity but provide qualitatively identical results.

References

Gregory Herschlag - Curriculum Vitae

Contact Information
Duke University
Durham, NC 27708-0320
E-mail: gjh@math.duke.edu
https://www.math.duke.edu/~gjh/

Education
- Bachelors of Science. October 2003 to June 2007. Mathematics major with honors at University of Chicago, Chicago, Il, USA.
- High School degree. September 1999 to June 2003. Taylor Allderdice High School, Pittsburgh, PA, USA.

Research Interests
- Fluid dynamics
- High performance computing with GPUs
- Gerrymandering and redistricting

Employment
- 2016-present. Visiting Assistant Professor, mathematics and biomedical engineering departments, Duke University.
- 2013-2016. Visiting Assistant Professor, mathematics department, Duke University.
- 2007-2013. Research/teaching assistant at the mathematics department at the University of North Carolina at Chapel Hill.

Reports

Papers undergoing peer review

Publications

Exhibit 1


**Teaching Experience**

• Fall 2017. Ordinary and partial differential equations (MATH 353, two sections), mathematics department, *Duke University.*

• Fall 2016. Ordinary and partial differential equations (MATH 353, two sections), mathematics department, *Duke University.*


• Fall 2015. Multivariable calculus (MATH 212), mathematics department, *Duke University.*

• Spring 2015. Advanced calculus (MATH 431), mathematics department, *Duke University.*

• Fall 2014. Ordinary and partial differential equations (MATH 353), mathematics department, *Duke University.*

• Fall 2013. Ordinary and partial differential equations (MATH 353), mathematics department, *Duke University.*

• Fall 2011. Mathematical modeling (MATH 119), mathematics department, *University of North Carolina at Chapel Hill.*

• Fall 2009. Calculus 1 (MATH 231), mathematics department, *University of North Carolina at Chapel Hill.*

**Honors**

• Nominated for the 2012 J. Bruton Linker Award for excellence in teaching. University of North Carolina at Chapel Hill, Chapel Hill, NC. 2011

• Inducted to Sigma Xi, 2007.

**Computational background**

Operating systems: Linux, OSX.

Programming: C, C++, Python, PyCUDA, CUDA, Java, Matlab.

Scientific software: *\LaTeX*, Matlab, Mathematica, DataTank.

**Conferences, talks, workshops**

• Memory access patterns for Lattice Boltzmann methods on GPUs (Poster at the Duke Research Computing Symposium 2017)

• The Lattice Boltzmann Method on Graphics Processing Units. Duke, GradFrac 2016 (invited talk)

• Optimal pumping strategies for material extraction at low Reynolds number. Mathematical Biology Colloquium 2016, Duke (invited talk)

• Transmembrane channel workshop at MBI (Poster on stochastic boundary conditions, 2015)

• A consistent hierarchy of generalized kinetic equation approximations to the chemical master equation. Tulane University 2015 (invited talk)

• A consistent hierarchy of generalized kinetic equation approximations to the chemical master equation. NCSU 2015 (invited talk)
• Enhanced material extraction with pumping through channels. Society of Mathematical Biology (talk)
• Non-periodic boundary conditions in molecular dynamic simulations. SIAM Dynamical Systems 2015 (invited talk)
• Enhanced material extraction with pumping through channels. University of Utah, Computational Biofluids in Physiology 2015 (poster)
• Material transport through and across channels at low Reynolds number. NCSU, Diff. Eq. Seminar 2015 (invited talk)
• Material transport through and across channels at low Reynolds number. Purdue, CAMS seminar 2015 (invited talk)
• Material transport through and across channels at low Reynolds number. Duke, Grad-Frac 2015 (invited talk)
• Material transport through and across channels at low Reynolds number. Notre Dame, ACMS seminar 2015 (invited talk)
• An exact solution for Stokes flow in a channel with arbitrarily large wall permeability. APS DFD 2014 (talk)
• An efficient hierarchy of generalized kinetic equation approximations to the chemical master equation. SIAM Annual 2014 (talk)
• Flow through permeable channels. Grad-Fac 2014, Duke University (talk)
• Tiling structures to approximate the chemical master equation with applications to surface catalysis. Analysis and applied mathematics seminar 2014, Duke University (talk)
• A mechanochemical model for auto-regulation of hydration on brachial passageways. Mathematical Biology Colloquium 2013, Duke (invited talk)
• Boundary conditions for molecular dynamic simulations. Cha Cha Days 2013 (invited talk)
• A method to approximate the chemical master equation. Computational Science and Engineering, Boston, 2013 (talk)
• Simulation of Solidification by Coupling of Phase Field and Microscopic Computations. ICIAM 2011 (talk)
• Reynolds number limits for jet propulsion. Cha Cha Days 2009 (poster)
• Leaf roll-up and aquaplaning in strong winds and floods. American Physics Society. Division of Fluid Dynamics 2008 (movie)
• Cha Cha Days 2008 (attended)

Mentorship roles

• Data+ Mentor. Gerrymandering project. Summer 2016
• Mathematical biology workshop at Duke. The theory of entropy applied to biological experiments. Summer 2014.

Organizational roles

• Leading seminars on journal articles in mathematical biology at Duke; about nine per year. Fall 2013-2016
• Organizing and running the mathematical biology colloquium at Duke. Fall 2014
• Director of the mathematical biology workshop at Duke. Summer 2014.

Fellowships

• 2008-2009. Bioinformatics and computational biology fellowship as part of the certificate program at the University of North Carolina at Chapel Hill.
References

- Jonathan Mattingly, email: jonm@math.duke.edu. Duke University, Mathematics Department. Phone: 919-660-6978
- Amanda Randles, email: amanda.randles@duke.edu, Duke University, Biomedical Engineering Department. Phone: 919-660-5425
- Jian-Guo Liu, email: jliu@math.duke.edu. Duke University: Physics and Mathematics Departments. Phone: 919-660-2500
- Michael Reed, email: reed@math.duke.edu. Duke University: Mathematics Department. Phone: 919-660-2810
- Anita Layton, email: alayton@math.duke.edu. Duke University: Mathematics Department. Phone: 919-660-6971
- Sorin Mitran, email: mitran@unc.edu. UNC Chapel Hill; Department of Mathematics. Phone: 919-843-8901 Thesis advisor
- M. Gregory Forest, email: forest@unc.edu. UNC Chapel Hill: Department of Mathematics. Phone: 919-962-9606
- Laura Miller, email: lam9@unc.edu. UNC Chapel Hill: Department of Mathematics. Phone: 919-843-4557