

EXHIBIT 7

Rebuttal Report

Moon Duchin
Professor of Mathematics, Tufts University

July 31, 2024

1 Background

I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University. In Fall 2024, I will begin a position as Professor of Mathematics and Public Policy at Cornell University. Besides the current case, I have previously testified in court or deposition in *NC League of Conservation Voters, et al. v. Hall, et al.* No. 21-cvs-500085 (Wake Cnty. Sup. Ct. 2021); *Carter v. Chapman*, No. 7 MM 2022, 2022 WL 702894 (Pa. Mar. 9, 2022); *O'Shea v. Boston City Council*, Case No. 1:22-cv-12048 (D. Mass. 2023); and *SC NAACP et al. v. Alexander, et al.*, Case No. 3-21-cv-03302-MBS-TJH-RMG (D.S.C.) (three-judge ct.).

1.1 Summary

This rebuttal mainly addresses the expert report of Dr. Sean Trende dated June 28, 2024. I also reviewed portions of the expert report of Dr. Wilfred Reilly (also dated June 28).

- Dr. Trende shows limited familiarity with metrics of compactness.
 - Cut edges ("removed edges") is mentioned but not scored. Demonstrative plans score well on this measure.
 - Moment of inertia ("MOI") is discussed but not scored. Indeed, using this metric would highlight the lack of a serious argument about population compactness.
- Dr. Trende's qualitative and quantitative comparisons are selective and exaggerated.
 - As noted above, several scores are favorably discussed but are not reported. This opens Trende's quantitative comparisons to a reasonable criticism of cherry-picking.
 - Despite issuing multiple warnings that it is unwise to make comparisons across time and place, just such comparisons are offered, and without adequate discussion of data preparation.
 - Predominance of race is alleged in discussing some of the boundary lines in demonstration maps, while clear evidence that race did not predominate is passed over.
- The Black Belt of Alabama is a recognized community of interest, but several of Trende's critiques amount to penalizing adherence to the state's own redistricting guidelines, which instruct mapmakers that the Black Belt should be "kept together to the fullest extent possible" [7].
 - Every claim about lack of "population compactness" in demonstration maps simply reduces to a claim that rural population should not be kept together.

Mapmaking in the presence of multiple complex criteria is an exercise in tradeoffs. In fact, Dr. Trende's report serves only to highlight that certain criteria can be pursued with a cost to others, a point that is already present in all earlier reports and testimony, including my own.

2 Scores of compactness

The measurement of shape, including scores of compactness in redistricting, is a research specialty of mine; this is reflected in a range of peer-reviewed publications in journals such as *Experimental Mathematics*, *Contemporary Mathematics*, *Geometric and Functional Analysis*, and *Political Geography*, as well as a short "explainer" chapter specific to redistricting metrics in the edited volume *Political Geometry*. In particular, though in the redistricting world it bears the names of authors from the 1990s, the Polsby-Popper score is just a standard isoperimetric ratio, whose use goes back to antiquity; in my pure mathematics research, I have studied isoperimetric problems in publications such as *The Sprawl Conjecture in Convex Bodies* [6].

One reason I have chosen to publish in this area in the last few years is that the literature on district shape metrics by non-mathematicians is notoriously weak. In the PI hearing for this case, defense expert Thomas Bryan relied on screenshots from a student homework project to ground his opinions on compactness. Dr. Trende similarly cites an online explainer¹ that is not published and has no author listed. This explainer includes several discrete measures for which my collaborators and I are the chief developers and proponents [3, 5, 4, 1], though it gives no attribution.

Dr. Trende writes: "In practical terms, Reock scores measure how distended a district is. Elongated districts have low Reock scores, while districts with high Reock scores tend to be, for lack of a better word, 'stocky.'" This misunderstands a fairly simple score. Reock does not measure distendedness per se, but is nothing more or less than how well the district fills out a circle. A district with arms and tentacles (as in Figure 1) can have a Reock score approaching a perfect 1.00, far better than a "stocky" triangle, whose Reock score is about 0.41.

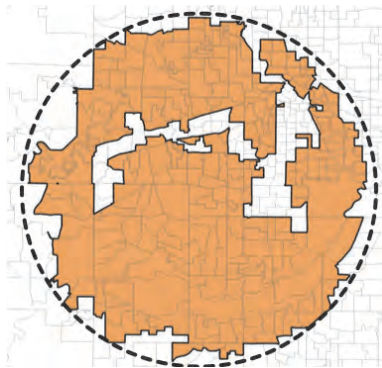


Figure 1: This district, made from real precincts, has "distended" arms, but a nearly perfect Reock score. (Reprinted from [5]).

Dr. Trende is right about one element here—that elongated shapes with high eccentricity, in particular those resembling a rectangle with length significantly greater than width, will always have a relatively low Reock score. We may note that the Black Belt has just such a shape, and so following the instructions in the legislative guidelines to keep the Black Belt together will cause CD 1 (and CD 2 to its south) to have lower Reock scores than a mapmaker could produce if ignoring that guidance.

In addition, Dr. Trende pulls from a questionable authority on redistricting, *Webster's Dictionary*, to explain his interpretation of compactness as "solid, dense" and "arranged neatly in a small space" (p7-8). This is patently inappropriate to its technical meaning in redistricting. Those who draw the lines cannot control the patterns of human geography and cause rural population to become denser or more neatly arranged. Mainly rural districts will always be spatially larger due to the primacy of equalizing population. Thus this passage gives us more evidence of Dr. Trende's limited expertise with compactness.

¹<https://alarm-redist.org/redistmetrics/articles/compactness.html>

3 Selective or erroneous comparisons

The threshold matter of compactness. Dr. Trende summarizes his compactness comparison by saying that, "As a threshold matter, the Illustrative Maps are all less compact than the Enacted Map" (Trende p25). However, Table 1 shows evidence to the contrary in a direct comparison of SB-5 (Dr. Trende's "Enacted Map") with my Plan B.

	block cut edges (lower is better)	avg Polsby-Popper (higher is better)	avg Reock (higher is better)
SB-5	3246	0.282	0.411
Plan B	3127	0.282	0.365
more compact	<i>Plan B</i>	<i>equal up to rounding</i>	<i>SB-5</i>

Table 1: A comparison of compactness scores shows that neither SB-5 nor Plan B is clearly more compact than the other.

Plan B is equally compact (to three decimal places) to SB-5 on the single most common measure of plan compactness, and superior on another measure which is common enough to be pre-programmed in Maptitude commercial redistricting software and used in expert reports in multiple states, including the recent Special Master's report in Alabama. Dr. Trende's stated reason to omit the cut edge score, despite the fact that it addresses his concerns about whether it is proper to average district-based scores, is that its plan-wide construction "might lead a court to disfavor it" (p7). This does not justify omitting it from the comparison rather than letting courts take that decision on their own.²

Cross-state comparisons. At a high level, the Trende report gives a multitude of reasons *not* to do cross-state quantitative comparisons, then does it anyway—and gives no evidence that it was done systematically. His reasons to cast doubt on comparisons across state and time include the following.

- "Many of these districts are draawn [sic] under severe geographic limitations" (p15);
- "using "all maps" passed in America as the benchmark ignores the fact that many maps that are purposely not reasonably configured" (p15);
- Water is handled inconsistently (p15);
- Boundaries that follow natural geography impose a high cost on certain scores (p11).

These are all valid points, and the list of reasons not to uncritically rely on comparisons across state and time goes on: other drawbacks include differing numbers of districts and tradeoffs imposed by respecting communities of interest recognized by courts, such as American Indian territorial land and the Black Belt of Alabama. (See [4] for more discussion.)

²In addition to the omissions of cut edge scores and moment of inertia scores, there are minor errors throughout the report that suggest limited expertise with shape metrics. For instance, on page 8, Dr. Trende claims a point would have a Reock score of zero. In fact the more proper answer is that it is undefined, though an argument could be made for the convention that a point, as a degenerate circle, earns a perfect Reock score. On page 12, a simple calculation refers to " $2/\pi$ percent" instead of share. More significantly, Dr. Trende repeatedly takes ratios of scores in a way that leads to specious comparisons. For instance, the Polsby-Popper score itself is already difficult to interpret because it is unitless and dimensionless—its ratio of $4\pi A/P^2$ is not in units of miles or census blocks, making it hard to know whether certain scores are meaningfully higher or lower than others. This issue is compounded when taking a ratio of these already unitless ratios, as on page 49 where he writes of "a map whose average Polsby-Popper score is half that of the Enacted Map." It would be very challenging to phrase what this means in plain English without simply calling it half as Polsbyish—it is just not accurate to call it half as compact. Making such a comparison again on page 35, in what is presumably a careless error but fits into a pattern of exaggeration, one score is said to be "almost five times higher" than another when the factor is actually closer to three—and ratios are not terribly meaningful in the first place.

After all of these caveats against these comparisons, it is surprising to then find that Section 6.2.3 of the Trende report is devoted to just that, unfavorably comparing plaintiffs' demonstration plans to plans from Illinois, Texas, Maryland, and beyond. Thus districting in Texas, where county preservation is a strong priority and many counties are near-perfect squares, is put on equal numerical footing with districting in Maryland, where the coastline is so long and complicated that several districts must have an essentially fractal boundary.

Even if Dr. Trende had not already made clear the weakness of this method, the details of its implementation, such as standardizing map projections and water clipping, would require a great deal more care and description.³ That leaves this section of the report deeply flawed, both conceptually and methodologically.

Racial predominance in line-drawing. On pages 71-72, Dr. Trende writes of my Plan E: "Recall that the boundary between the districts here is comprised largely of split precincts. The fact that you can still make out a racial boundary along the district lines means that she has not only divvied up the districts by BVAP, but has also split precincts by BVAP." This not only passes over a glaring fact in the very figures he presented to support this claim, but also makes an error of interpretation.

First, simply comparing Trende's own Figures 51 and 52, which show my Plans C, D, and E in Jefferson County, makes it extremely clear that Plan E has had tens of thousands of White Alabamians added to CD 7 relative to the earlier maps—the entire area encircling Birmingham to the North and Northeast—which correctly suggests that I was trying to unite more of the municipality in CD 7, in balance with compactness and county integrity. Indeed, several of the splits cut off zero-population pieces from oddly shaped precincts. This district is still comfortably majority-Black, having 53% Black by voting age population in Plan E. The claim that precincts were split in a race-based way in order to manipulate demographics on the level of a few hundred people does not stand up to basic scrutiny. Indeed, Dr. Trende himself produces the correct reason elsewhere in the report: I made these splits to improve compactness, which was a high priority of mine in map-drawing.

Though the details of the choropleth construction are not provided, there also seems to be an error of interpretation. A *choropleth* is the name geographers give to a map whose units are colored or shaded by the levels of some attribute. In this case, Dr. Trende's map appears to be colored on the precinct level, with a yellow-to-blue color scale for the Black voting age population share. Even if the low resolution of these figures were improved, a map colored by precinct is fundamentally incapable of showing anything at all below the precinct level—the same color will be shown on both sides of a line that splits a precinct, by construction. This means that this visual could not even in principle support a claim about race-conscious division of precincts.

Obfuscating tradeoffs. On page 25, Dr. Trende complains that district boundaries in a demonstration map are not smooth enough; on page 35, he complains that precincts have been split to make smoother boundaries. This is one of quite a few examples that ignore the clear fact that compactness, which I have repeatedly described as a high priority, trades off with other priorities.

Communities of interest (COIs) in Alabama receive no mention in the Trende report, though Dr. Trende mentions that COIs were the topic of a chapter in his recent dissertation. The Black Belt itself is only mentioned in passing in the section about historical Board of Education maps. But the district court in this case identified the Black Belt as a salient community of interest,

³DeFord et al. conducted a much more systematic project to compare district shapes from the 113th Congress (Jan 2013–Jan 2015) [2], with methodology carefully described in the [README](#) and well-commented replication code. Fully 41 Congressional districts, spanning 17 states, had Polsby-Popper scores below 0.100, while Dr. Trende reports that the lowest score for any single district in any of my illustrative maps is 0.129. Indeed, the median Polsby-Popper score across all 435 districts in that Congress is 0.226 and the mean is 0.249. My Plans A-E for Alabama, with overall scores from 0.249 to 0.282, are therefore clearly reasonably compact according to a better execution of the Trende method. Dr. Trende has had to present numbers very selectively to imply otherwise.

and this was echoed and codified by the legislature in the new redistricting guidelines passed as part of Senate Bill 5, which list the 18-county Black Belt region as one of only three officially recognized COIs.⁴

Indeed, as noted in my report of May 17, 2024, the core Black Belt counties have a combined population of 562,358 compared to ideal congressional district population of 717,754; this and the rest of the state's geography means that the Black Belt can anchor a district—the region can be kept mostly whole as the heart of a single district. Of course this will require some other tradeoffs in metrics, a fact not acknowledged in the Trende report.

It is also worth noting that the measure of respect for a community of interest is not simply the number of districts that it touches, but whether it forms a sizeable portion of those districts so that community residents have a strong voice with their representatives. This explains that not all two-way splits are created equal, and a two-way split can still be regarded as "cracking," depending on context. This is arguably what occurred in the SB-5 map—9 out of 18 Black Belt counties are submerged in a CD 2 that has below 40% BVAP, potentially diluting the power of Black voters in those counties.

4 "Population compactness" and rural communities

Moment of inertia metrics measure the dispersion of matter about a basepoint x_0 , typically the center of mass, by summing (or integrating) the distance from x to x_0 over points x in the region. This concept has been used in physics for centuries and has been suggested in redistricting for decades. Dr. Trende suggests that MOI metrics fell out of favor in redistricting because other metrics became easier to compute, but I know no evidence for that assertion. Rather, my understanding is that MOI metrics are hard to standardize on a scale that is meaningful in redistricting, partly because there is no consensus on which kind of distance between people to employ—straight-line distance? road network distance? travel time? Overall, MOI scores would require so many user choices and difficult data sourcing that it would be hard to build consensus around how to compute and interpret them.⁵ Indeed, Dr. Trende does not compute any MOI scores in his report, which means he has not had to contend with any of those choices.

But beyond this, there is an arguably more fundamental problem: moment of inertia scores would have to be carefully calibrated in order to give useful information about choices made in *districting plans themselves*, and not just the residential patterns of human geography in the region under consideration.

To see this, consider Figures 57–69 in the Trende report, which show the same dot density pattern in every instance: simply the distribution of Black adults in Alabama. Compare Figure 2 below, which also appears as Figure 2 in my report of September 11, 2023 in this case. My dot density visualization puts Black demographic distribution in context by juxtaposing White demographic distribution. This makes it clear that the Black Belt is coherently identifiable as a low-density strip with a high share of Black voters spread in a fairly uniform manner throughout.

On p75, Dr. Trende writes of population in a demonstration map that "these clusters [of Black population] are spread across an already-sprawling district." However, rural areas have lower population density—this is tantamount to a definition of "rural"—so a district that holds intact a large and mostly rural community is necessarily "sprawling." Furthermore, if the clusters are indeed "spread across" the district, this strengthens the community justification for the district, in contrast to a situation where remote clusters of Black population anchor opposite poles of a district with White counties in between.

⁴"The Legislature declares that at least the three following regions are communities of interest that shall be kept together to the fullest extent possible in this congressional redistricting plan: the Black Belt, the Gulf Coast, and the Wiregrass." See <https://alison.legislature.state.al.us/files/pdf/SearchableInstruments/2023SS2/SB5-eng.pdf>.

⁵See [4] for a further discussion of why MOI-style scores are "challenging to execute."

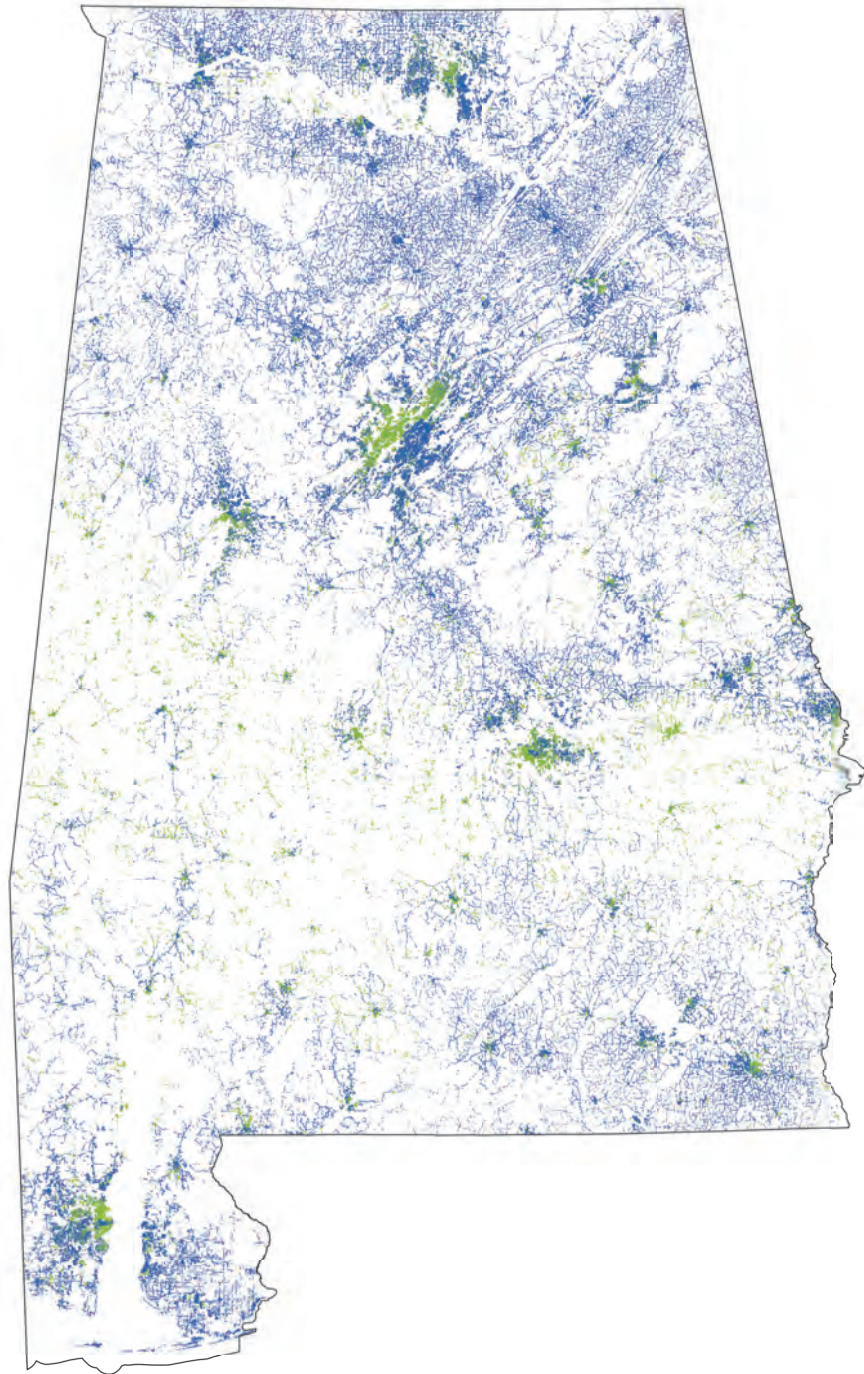


Figure 2: A dot density plot of address locations of registered voters from the July 18, 2023 voter file, with those self-identified as Black and those self-identified as White shown in green and blue, respectively. (Reprinted from Duchin report of September 11, 2023.)

In the end, Dr. Trende faults plans merely for trying to keep together "lightly populated counties in the surrounding countryside"—despite the fact that the enacted legislative guidelines mandate doing just that. This complaint about "lightly populated" areas is leveled four times, twice on page 75 and twice on page 90. This theme reaches its clearest articulation on page 82, where Dr. Trende opines that "In short, Mr. Cooper's maps consist of districts where supermajorities of Black residents are concentrated in two geographically distant cities that have never been in a Congressional district together before in the state's history. The remainder of the Black population is scattered across multiple counties and small towns that dot the countryside." Under cover of a worry about compactness, this clearly describes map-making that faithfully follows the legislative guidance that the Black Belt counties "shall be kept together to the fullest extent possible in a congressional redistricting plan."⁶

By failing to present alternatives with lower (and thus presumably better) MOI scores, Dr. Trende elides the fact that the only way to reduce a high moment of inertia is to chop rural areas into more pieces, combining them with more urban populations whether or not this is justified by shared interests. It is possible that some researcher in the future will devise a framework that uses the MOI concept to make a meaningful summary score for redistricting. For now, its use to diagnose population compactness remains decisively "under-developed," as acknowledged by Dr. Trende himself (p22).

5 Conclusion

Mapmaking is an exercise in tradeoffs. Plaintiffs' experts have introduced over a dozen maps in this case so far, clearly explaining that the variety of options is useful to understand tradeoffs between traditional districting criteria. The Trende report should help clarify that meeting the requirements of Gingles 1 and treating the Black Belt as a community of interest can indeed be successfully balanced with other criteria, producing demonstration maps that are highly reasonable through the lens of the traditional districting principles, including compactness.

References

- [1] Sarah Cannon, Moon Duchin, Dana Randall, and Parker Rule, *Spanning tree methods for sampling graph partitions*. In submission. Available at <https://arxiv.org/abs/2210.01401>.
- [2] Daryl DeFord et al., DiscreteCompactness GitHub Repository, <https://github.com/mggg/DiscreteCompactness>
- [3] Daryl DeFord, Moon Duchin, and Justin Solomon, *Recombination: A family of Markov chains for redistricting*. Harvard Data Science Review, Issue 3.1, Winter 2021.
- [4] Moon Duchin and Bridget Tenner, *Discrete geometry for electoral geography*. Political Geography, Volume 109, March 2024.
- [5] Moon Duchin, *Compactness, by the numbers*. In Political Geometry: Rethinking Redistricting in the U.S. with Math, Law, and Everything In Between. (eds. Moon Duchin, Olivia Walch). 25 chapters, 475 pages. Birkhäuser Books 2022.
- [6] Moon Duchin, Samuel Lelièvre, and Christopher Mooney, *The sprawl conjecture for convex bodies*. Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122.
- [7] Legislative redistricting guidelines in Senate Bill 5, passed July 19, 2023. <https://alison.legislature.state.al.us/files/pdf/SearchableInstruments/2023SS2/SB5-eng.pdf>

⁶Though this report focuses on rebutting the Trende report, it is worth mentioning that the expert report of Dr. Wilfred Reilly treads some of the same ground. The Reilly report devotes a major portion of content—pages 5-16—to a section called "Comparing and Contrasting the Counties" that points out that many Black Belt counties are rural, while Mobile, Baldwin and Montgomery counties contain urban/suburban areas. This is acknowledged by all observers.

I reserve the right to continue to supplement my report in light of additional facts, testimony and/or materials that may come to light. Pursuant to 28 U.S.C. 1746, I declare under penalty of perjury that the foregoing is true and correct according to the best of my knowledge, information, and belief.

Executed this 31st day of July, 2024.

A handwritten signature in black ink, appearing to read 'Moon Duchin', written over a horizontal line.

Moon Duchin