

# EI Extended Model and the Fear of Ecological Fallacy

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Most existing models of ecological inference are based on the assumption that there is no aggregation bias. Few studies have focused on how to correct/model aggregation bias. This article takes advantage of a unique opportunity to compare the controversial ecological inference methods by using aggregate as well as individual-level data from an actual election. Furthermore, the true quantities of interest are also available, which guarantees the accuracy of the empirical tests. Our mean squared error analyses show that King's Ecological Inference (EI) basic model does not always outperform the traditional ecological regression and neighborhood methods when aggregation bias does exist. However, using an appropriate covariate in the King's EI extended model to correct the aggregation bias problem can drastically improve the estimation accuracy at both the precinct and district levels. This article also makes suggestions on how to use a covariate in a King's extended model.

**Keywords:** *EI; ecological fallacy; ecological inferences; aggregation bias; contextual effect*

When the goal of a quantitative study is to explain individual-level political behaviors such as voting, one has two choices in data collection: using individual-level social surveys or adopting aggregate-level data such as election returns based on a certain geographic unit. The fear of ecological fallacy has convinced many scholars to pursue an individual-level approach since William Robinson published his famous article in 1950 to warn scholars never to adopt aggregate-level data to infer individual-level relationships.<sup>1</sup> Perhaps the so-called methodological individualism best indicates the influence of this approach in contemporary social sciences. The award-winning Ecological Inference (King's EI) procedure recently developed by Gary King (1997), however, has invited a new wave of research endeavors that take advantage of aggregate data

(see published works using King's EI, for example, Burden and Kimball 1998, 2002; Gay 2001; Gimpel and Schuknecht 2004; Liu 2001a, 2001b; Tolbert and Hero 2001; Voss and Lublin 2001).

Most scholars who used King's EI in their quantitative research or federal voting rights litigations are familiar with the basic version of King's EI procedure. Very few studies, however, examined the effectiveness of King's extended EI model, which is supposed to deal with the violation of EI basic model's assumptions, especially aggregation bias. Furthermore, there has been a lack of verification studies that used actual election data sets to compare survey data and aggregate-level data and, more important, to test the accuracies of estimations through all major ecological methods used in social research and voting rights cases. One reason for this lack of examination of King's EI extended model and rigorous verification studies of all major ecological inference methods is because most existing models of ecological inference, including King's EI basic model, assume that there is no aggregation bias, or no contextual effect produced by the dependence between the observed variable and the quantities of interest, which are unobservable. Modeling contextual effect, which the EI extended model intends to do, is still in its infant stage. Moreover, scholars often have no way to know how individuals actually behaved. For example, voting takes place in private, and this makes comparing statistical methods especially difficult, if not impossible. Many of the previous comparative studies, thus, were based on simulated data or exit polls (see, e.g., Freedman et al. 1991; Zax 2005). Survey data, however, as demonstrated later, may be especially misleading when polls are concerned with racial issues.

This research is designed to examine King's EI basic and extended models and compare the performances of King's EI with those of other major different statistical methods by applying them into actual data sets. The empirical test presented here has three very important advantages. First, both survey data and aggregate data are employed for a same election. Moreover, the truths regarding the quantities of interests are known (discussed later). Thus the results of the test can help answer the questions, "Are survey data more reliable than aggregate data?" "Is survey approach a solution to ecological fallacy?" Second, three previously used ecological methods (Goodman regression, double-regression, and neighborhood model) will be compared with King's EI to find the answer to the questions, "Does King's EI represent a major improvement over previous methods?" "Is it true that the double-regression method has 'no scientific justification of continued application' (Zax 2002:85)?"<sup>2</sup> Finally, the data sets adopted here have not

only the major variables at the precinct level needed to do traditional ecological regressions, neighborhood models, and King's EI procedures, but they also have other important contextual information that can serve as the covariates in the extended King's EI model.<sup>3</sup> Therefore, we can also test the validity of the suggestion that only King's EI extended model may avoid the problem of ecological inconsistency in a second-stage analysis (Herron and Shotts 2004).

## Ecological Inference Problem and the Ecological Inference Methods

To illustrate an ecological inference problem, suppose that the goal of an empirical study is to find whether White and Black voters voted differently in a biracial election that involves both a White candidate and a Black candidate. This research question is very common not only in voting literature but also in voting rights lawsuits concerning racial polarization. Very often, the key is to find if there is a high level of racial polarization for the Black candidate. To be more specific, the quantities of interests are the proportions of registered Black voters and White voters who voted ( $\beta^b$  and  $\beta^w$ ) and voted for the Black candidate ( $\lambda^b$  and  $\lambda^w$ ). To reach this goal, one, however, only knows the following aggregate-level variables for each precinct  $i$  of the electoral district, assuming there are only two racial groups, Blacks and Whites:

$$\begin{aligned} n_i &= \text{total number of registered voters} \\ n_i^b &= \text{number of Black registered voters} \\ n_i^B &= \text{number of votes cast for the Black candidate} \\ n_i^W &= \text{number of votes cast for the White candidate} \end{aligned}$$

As Robinson (1950) correctly pointed out, aggregate statistical findings may not necessarily mirror the relationships at the individual level, and the problem of ecological inference is “statistical underidentification” (Richmond 1976). Achen and Shively (1995) further explained this problem as

data gathered at the macrolevel do not allow us to definitely determine the process at work among the same variables at the microlevel unless—as is almost never the case—we have complete knowledge of the process of aggregation itself. As is usual in cases of underidentification, the solution is to add additional assumptions or external information to effect closure. (pp. 3-4).<sup>4</sup>

We discuss four major ecological methods that are used in the social sciences and voting rights litigations.

The first method of estimating White crossover vote for a Black candidate in an election is Goodman regression, or the single-regression method. The unit of analysis is the voting precinct ( $i$ ). The independent variable is the proportion of registered voters that are Black in the precinct ( $X_i$ ), and the dependent variable ( $Y_i$ ) is the proportion of votes cast for the Black candidate ( $v_i$ ).

$$Y_i = a + bX_i + \varepsilon. \quad (1)$$

The quantity of interest, the proportion of Black vote for the Black candidate in the election ( $\lambda^b$ ), is the sum of intercept and the regression coefficient.<sup>5</sup> Thus

$$\lambda^b = a + b. \quad (2)$$

The proportion of White vote for the Black candidate is:

$$\lambda^w = a. \quad (3)$$

In an effort to take racial turnout rates into account, one alternative method has been to use “double regression.”<sup>6</sup> The unit of analysis in double regression is also the voting precinct. In the first regression, the independent variable is the number of Black registered voters divided by the total number of registered voters ( $X_i$ ), the dependent variable ( $Y^*$ ) is the number of votes for a Black candidate ( $n_i^B$ ) divided by the number of registered voters ( $n_i$ ). The value of the unstandardized regression coefficient plus the intercept is the estimate of the proportion of Black registered voters that cast a vote for Black candidates (A).

The second regression uses the White candidate vote proportion ( $n_i^W / n_i$ ) as the dependent variable ( $Y^{**}$ ) to get the estimate of the percentage of Black registered voters that cast a vote for a White candidate (B). The sum of A and B is the estimate of Black participation in the given election ( $\beta^b$ ). Finally, by dividing the percentage of Black registered voters that cast a vote for Black candidates (A) by the Black participation ( $A + B = \beta^b$ ), the quantity of interest, the proportion of Blacks that cast ballots in a particular biracial election for the Black candidate ( $\lambda^b$ ), is estimated.

Thus the two regressions are as follows:

$$Y^* = c + d(X_i) + \varepsilon \quad (4)$$

$$Y^{**} = e + f(X_i) + \varepsilon. \quad (5)$$

Then, to calculate the quantities of interests, one first computes

$$A = c + d \quad (6)$$

$$B = e + f \quad (7)$$

$$\lambda^b = A/(A + B) \quad (8)$$

$$\lambda^w = c/(c + e). \quad (9)$$

In addition to impossible and unrealistic estimates that may be calculated (e.g., White crossover voting at -5 percent), the most serious problem with single and double regression methods in contextual research is the assumption that the parameters of interests (e.g., White crossover voting) are constant across the observations (e.g., precincts) regardless of context. There can be a massive level of heteroskedasticity, however, in aggregate data concerning voting (King 1997:58, 66). This assumption is relaxed by the neighborhood model, which assumes that Black and White groups vote in an identical way in a same neighborhood. According to Freedman et al. (1991), there are two versions of the neighborhood model. The first one is a nonlinear version, which can be simply described as the following equation for a precinct  $i$ :

$$\lambda_i^b = \lambda_i^w = v_i. \quad (10)$$

For the second, linear, version of the neighborhood model, like the Goodman regression, one first runs equation (1):

$$Y_i = a + bX_i + \varepsilon.$$

Then, based on the intercept and the regression coefficient, one calculates the following to get estimates for precinct  $i$  quantities of interests:

$$\lambda_i^b = \lambda_i^w = a + (b \times X_i). \quad (11)$$

Finally, to calculate the district-level quantities of interests ( $\lambda^b$  and  $\lambda^w$ ), one simply multiplies each precinct's percentage of Black candidate vote ( $v_i$ ) by the number of registered Blacks ( $n_i^b$ ) and add up the results.

## King's EI Basic Model and a Quick Comparison of the Four Methods

King's EI method incorporates the deterministic method of bounds with maximum likelihood probabilities. One advantage of King's EI over all previous methods is that it provides point estimates and their standard errors at both precinct and election-unit levels. To receive an estimate of

White crossover vote for a Black candidate, two-stage operations are necessary in King's EI basic procedure. The first stage uses three precinct-level variables, that is, the proportion of registered voters that are White ( $x_i$ ), total number of votes cast ( $n_i$ ), and the proportion of registered voters that voted ( $T_i$ ), to estimate White and Black turnout rates at both the precinct level ( $\beta_i^b$  and  $\beta_i^w$ ) and the district level ( $\beta^b$  and  $\beta^w$ ). The second stage then adds one more variable, also at the precinct level, the proportion of votes cast for the Black candidate ( $v_i$ ). These procedures then finally provide point estimates and the standard errors of racial voting for the candidates at the precinct level ( $\lambda_i^b$  and  $\lambda_i^w$ ) and the district level ( $\lambda^b$  and  $\lambda^w$ ).<sup>7</sup>

One way to compare King's method with previous ones is to see how much more information King's EI estimation can produce than can those previous methods—such as statistical and visual outputs, precinct and district point estimates. Table 1 is used to make these comparisons. It is reasonable to suggest that King's EI method is more developed than the traditional single- and double-regression methods, as Table 1 shows that there are five yeses for King's EI while the regression methods have only three yeses. However, because of the nature of ecological inferences, which in most circumstances does not allow the researchers to know the real quantities of interests, none of the methods may guarantee reliable estimations.

The most critical scrutiny of King's EI came from those scholars who directly questioned the model assumptions of King's EI and the consequences of violations of these assumptions (Anselin and Cho 2002; Cho 1998; Freedman et al. 1991).<sup>8</sup> Based on the suggestions of these scholars, one should avoid King's EI almost completely because King's EI assumptions “are often inappropriate for instances of aggregate data” (Cho 1998:144); furthermore, more pessimistically “in mathematical terms, the ecological fallacy problem is exactly an ill-posed inverse problem. The definition of an ill-posed inverse problem posits that no unique inverse or solution exists” (Cho and Yoon 2001:253).<sup>9</sup>

More recently, some scholars acknowledged the improvement of King's EI over previous methods, yet they are more concerned with the applications of King's EI. Johnston and Pattie (2000) discussed an entropy-maximizing procedure to include the use of survey data as “an alternative method of estimating matrix values. . . a byproduct of King's method” (p. 344). But their method is a mathematic solution rather than statistically defined ecological inferences. Some scholars suggested that combining King's EI with other statistical techniques, such as a geographically weighted approach, may help resolve some of the assumption

**Table 1**  
**Four Ecological Inference Methods Compared**

	District Point Estimates	Precinct Point Estimates	Visual Presentation of Data and Findings	Always Producing Realistic Estimates	Allowing Racial Turnout Differences	Always Producing Results
Single regression	Yes	No <sup>a</sup>	Yes (scatterplot)	No	No	Yes
Double regression	Yes	No <sup>a</sup>	No	No	Yes	Yes
Neighborhood Model	Yes	Yes	No	Yes	No	Yes
Ecological Inference King's EI	Yes	Yes	Yes (XT-fit, tomographic etc.)	Yes	Yes	No <sup>b</sup>

a. One may also argue that single- and double-regression methods also produce precinct estimates, which are the same for all precincts (i.e., the consistency assumption).

b. Under most circumstances, King's EI produces estimates. Sometimes, perhaps because of presence of collinear covariance or software failure, King's EI does not produce results. However, failure to generate a result that is wrong is a feature, not a bug.

violation problems, especially spatial aggregation bias (see Calvo and Escolar 2003). Others indicated that using King's EI estimates blindly in a second-stage regression analysis may cause a serious logical inconsistency problem (Herron and Shotts 2004; see below for a discussion of this problem). It is interesting that the King's EI extended model, rather than its basic model, which King himself argues as "robust" and has been widely used in academic research and voting rights litigations, may actually provide the best hope to save King's EI from the violations of model assumptions, especially the existence of contextual effect and logical inconsistency (Herron and Shotts 2004).

## Data

To compare the statistical methods introduced above as well as the survey approach, this empirical study focuses on the 2002 New Orleans mayoral election. This open-seat election attracted as many as 15 candidates, including a famous White professor and an influential Black female politician. The race was very competitive and expensive, and 2 Black candidates entered into the runoff. Ray Nagin, the general manager/vice president of Cox Communications and co-owner of the Brass hockey team, finally defeated



Richard J. Pennington, the popular police superintendent. Many believe Nagin's success was largely because of the White support, although Pennington was more popular among Black voters (Perry 2003).

The precinct-level racial registration data are available. These data were matched with the precinct-level election return data to form the first data set. The second data set came from a citywide survey conducted in New Orleans by the Survey Research Center of the University of New Orleans between February 14 and 19 of 2002. A random digit dialing procedure with a screen for registered voters was used, and the survey was based on telephone interviews of a random sample of 472 registered voters. The racial distribution of the survey respondents—30.9 percent White and 63.1 Black—closely reflects the city's registration characteristics, which was 31.9 percent White and 62.6 percent Black at the time of 2002 mayoral primary. The respondents were asked how they had voted in the primary and, furthermore, how they would vote in the upcoming mayoral runoff election.

### **Comparing Estimates of Racial Voting Through King's Basic EI and Other Ecological Inference Methods**

Table 2 reports the first set of quantities of interest, that is, the estimates of Ray Nagin's racial votes ( $\lambda^b$  and  $\lambda^w$ ). The University of New Orleans (UNO) survey shows that Nagin would receive 48.5 percent of the Black votes. Among the four main ecological inference methods, only King's EI and neighborhood model estimated that Nagin's Black vote was greater than 40 percent. As for Nagin's White vote, UNO survey indicated that 85.7 percent of the White voters supported him, which is only 0.53 percent over the King's EI estimate, while Goodman and Double Regression estimates were at least 3 percentage points greater than the UNO survey result.

If one only uses the survey approach as the most accurate estimates of Nagin's racial votes, King's EI clearly outperformed the traditional Goodman and double-regression methods. However, one cannot know the real level of White crossover vote for Nagin because voters cast their vote secretly. In contrast, this study can compare the accuracy of estimates empirically because New Orleans provided a very valuable postelection data set—the voters' sign-in data by race and by precinct. In other words, the data set collected by the government because of the voting-rights law requirement can allow us to precisely compute the true levels of White and Black turnout rates at both the city and precinct levels, and these truths can be compared with the survey results and the estimated group turnouts through the three methods discussed previously.<sup>10</sup>



**Table 2**  
**White and Black Votes for Nagin, 2002 New Orleans Mayoral Runoff**

	$\lambda^b$ : Black Vote %	$\lambda^w$ : White Vote %
Goodman	37.78	90.86
Goodman weighted <sup>a</sup>	39.80	90.66
Double regression	37.85	89.28
Double-regression weighted <sup>b</sup>	38.53	89.44
Neighborhood nonlinear	47.16	73.60
Neighborhood linear	46.80	73.26
King's basic model	40.20 ( <i>SE</i> .19)	86.26 ( <i>SE</i> .23)
UNO survey	48.50	85.70

Note: UNO = University of New Orleans.

a. Goodman weighted uses total number of vote of the precinct/mean total number of vote as the weight in the regression.

b. Double-regression weighted uses total number of registered voters of the precinct/mean total number of registered voters as the weight in the regression.

### Comparing Estimates of Racial Turnout Rates

Table 3 provides the true Black and White turnout rates, compared with the estimates through ecological inferences and survey methods. The true Black and White turnouts were 44.11 percent and 47.79 percent, respectively. The most surprising finding is that the survey shows that the turnout rates would be 84.2 percent and 80.7 percent, respectively, which were more than 32 percent above the true turnout rates. These great disparities might have something to do with the fact that the survey was conducted shortly before the runoff was held. Perhaps some voters could not vote because of their personal circumstances on election day, although they had intended to vote before the election. At any rate, the great disparities between the survey results and the true turnout figures confirm the findings in the literature that Americans tend to overreport their voting participation in surveys (Kuklinski, Cobb, and Gilens 1997).<sup>11</sup> As Burden and Kimball (2002) summarized, previous verification studies show that social surveys, even including the respected National Election Study, may overreport support for winning congressional candidates and incumbents. The possible causes for inaccurate estimates of voting behavior may include “forgetfulness,” “social desirability pressures,” and “question wording artifact” (p. 42). In short, this article also shows that one certainly should not blindly assume that surveys were the only way to avoid the ecological fallacy problem.

**Table 3**  
**White and Black Turnout Estimates for the 2002**  
**New Orleans Mayoral Runoff**

	$\beta^b$ : Black %	$\beta^b$ Bias %	$\beta^w$ : White %	$\beta^w$ Bias %
Goodman	40.76	-3.35	50.10	2.31
Goodman weighted	42.74	-1.37	51.81	4.02
Double regression	40.79	-3.32	50.10	2.31
Double-regression weighted	40.33	-3.78	49.38	1.59
Neighborhood nonlinear	41.71	-2.4	46.27	-1.52
Neighborhood linear	42.23	-1.88	46.53	-1.26
King's basic <sup>a</sup>	42.83	-1.28	51.75	3.96
	(SE .3)		(SE .48)	
UNO survey results	84.20	40.09	80.70	32.91
Actual turnout	44.11		47.79	

Note: All turnout estimates are based on percentage of registered voters rather than voting age population. UNO = University of New Orleans.

a. King's extended model provided a much more accurate set of estimates:  $\beta^b = 44.59$  percent with  $SE = .48$  percent, and  $\beta^w = 48.14$  percent with  $SE = .8$  percent. These estimates are the best with the smallest bias values .48 percent and .35 percent.

In fact, Table 3 reveals that all four ecological inference methods produced the estimates that were much closer to the truth than were the survey results. For the Black turnout estimate, King's EI method was again the best. King's basic EI estimated it at the 42.83 percent level, which was only 1.28 percent less than the true Black turnout rate. It also should be noted that the weighted Goodman regression provided a better estimate than the double-regression method. Overall, none of the ecological inference methods had an error of more than 4 percent for Black turnout estimation.

With respect to the White turnout estimate, however, King's EI basic model does not provide the best estimate. According to King's EI basic model, the White turnout was 51.75 percent, whereas the true White turnout was only 47.79 percent. Thus King's method overestimated the White turnout by 3.96 percent, while the traditional Goodman and double-regression methods had an error of less than 2.5 percent. Moreover, the King's basic EI standard error estimate, at 0.48 percent, was also clearly too conservative.<sup>12</sup>

These findings also seem inconsistent with the claim that there is no use in the double-regression approach and King's EI should completely replace the traditional ecological inference methods, especially the double-regression approach (Zax 2002, 2005).<sup>13</sup> In fact, based on King's EI basic

method, one would conclude that the majority of White registered voters turned out to vote in the 2002 New Orleans runoff election, when the majority of them actually did not. Only the weighted double-regression method and the neighborhood models would allow researchers to make a conclusion that the majority of White voters did not turn out to vote. As for this research, the remaining questions are why King's basic EI method overestimates White turnout rate at the city level? Can King's EI precinct-level analysis provide a better picture of turnout activities? More important, how can one improve King's EI estimation? Can King's EI extended model help?

It should be noted that the graphic diagnoses recommended by King did reveal there was a sign of violation of King's EI basic assumptions in the data. For example, the "boundx" graph, which plots  $X_i$  by the bounds of parameters, suggests that the parameter is positively related to  $X_i$ , which suggests that "aggregation bias is confirmed on the basis of aggregate data alone" (see King 1997:237).<sup>14</sup> Thus, when a model assumption is violated, naively using King's EI basic method indeed may produce inaccurate estimates (in this case, it overestimates White turnouts at both precinct and city levels; see Cho and Gaines 2004 for a critique of naïve use of King's EI). Nevertheless our following analysis also shows that aggregation bias may be greatly corrected by adopting a right covariate in the extended King's EI model. Before we report the effectiveness of King's EI extended model, we introduce a danger of using basic King's EI estimates in a second-stage regression analysis when the problem of logical inconsistency exists (Herron and Shotts 2004).<sup>15</sup>

### **What Can Go Wrong With King's EI Basic Model?**

One of the most important improvements of King's EI over the traditional Goodman and double-regression approaches is that only King's EI produces precinct-level estimates that are not assumed to be a constant. These precinct estimates not only help compute the city-level quantities of interests but also may be applied to a second-stage analysis. Table 4 lists two equations that use King's EI precinct-level estimates of White crossover voting for Nagin as the dependent variables. Equation (1) is a simple bivariate regression with precinct-level Black density (i.e., the proportion of registered voters that are Black) as the independent variable. The regression coefficient is not statistically significant, which suggests that there is no relationship between Black density and White crossover voting at the precinct level.

**Table 4**  
**White Crossover Voting for Nagin in 2002:**  
**What Can Go Wrong With King's EI Basic Model?**

	$\lambda_i^w$		$\lambda_i^w$	
	Nagin White Support		Nagin White Support	
	B	SE	B	SE
% Black registered voters	.015	.008	.146	.015***
% White Republican			.534	.052***
Intercept		.828		.706
$R^2$		.008		.201
Adjusted $R^2$		.005		.198
$N$		442		442

Note: The correlation coefficient  $r$  between  $X_i$  and % White Republican = .936 at .001 level. Clearly, EI Basic Model's assumption of no aggregation bias is violated, and there is a logical inconsistency in this second stage regression analysis (see, Herron and Shotts 2004).

\*\*\* $p < .01$ .

It is reasonable to expect a higher level of support for the leading Black candidate from White Democrats than from White Republicans (see the political incorporation theory stated in Browning, Marshall, and Tabb 2003). Equation (2) thus adds one more independent variable, the proportion of registered voters who are White Republican, into the model. The  $R^2$  increased from .008 to .201, indicating that the new model explains about 20 percent more variation in the dependent variable. This strongly suggests that contrary to political incorporation theory, Nagin's White support was from White Republicans. Moreover, the Black density variable also becomes statistically significant, which shows that with the level of White Republican voters held constant, White support for Nagin is increased if Black density is increased.

It is important to note that Table 4 uses King's EI precinct-level estimates of White crossover vote as the dependent variable. This second-stage regression approach in Table 4 may cause a problem of logical inconsistency (Herron and Shotts 2004). This is because King's EI first-stage operation, which computes precinct-level quantities of interest, must assume that there is no aggregation bias in the process of inferring individual-level phenomenon through using aggregate-level data (i.e.,  $X_i$  should not be correlated to  $\lambda_i^b$ ).<sup>16</sup> However, when one finds that  $\lambda_i^b$  as the dependent variable in the second-stage regression analysis is a linear function of an

independent variable, one introduces a logical inconsistency error if it is also true that this independent variable is correlated to  $X_i$ , because the no-aggregation-bias-rule is certainly violated.<sup>17</sup>

This is exactly what equation (2) of Table 4 reveals. The proportion of White Republican voters at the precinct level is included in the regression analysis, and it is statistically significant. The issue is whether this independent variable is correlated with the proportion of Black registered voters at the precinct level ( $X_i$ ). A correlation analysis shows that these two variables are indeed highly correlated ( $r = .936$ ,  $p < .001$ ). Thus there is a contextual effect because of the clear dependence between the observed variable ( $X_i$ ) and the quantities of interest ( $\lambda_i^b$ ), and using the point estimates of  $\lambda_i^b$  in equation (2) is logically inconsistent.

### **The King's EI Extended Model as a Method to Reduce Bias**

One solution to this problem is to use King's EI extended model (see Adolph et al. 2003; Herron and Shotts 2004). This solution requires that King's EI includes observed exogenous variables  $Z_i$  in the model from the start and to estimate all parameters jointly. King's EI extended model can incorporate covariate  $Z_i$  in the maximum likelihood estimation. This approach then releases the need to do regression analyses if one's goal is to infer the effect of  $Z_i$  on  $\lambda_i^b$  and therefore avoid logical inconsistency (for a critique of weighted least squares approach, see Herron and Shotts 2004:175). To do this for this study, we include only one covariate, the proportion of registered voters that are White Republicans in a precinct, in  $Z_i$ . More specific, we use the proportion of White Republican voters as  $Z_i^b$  in King's equation (9.2) to estimate  $\alpha^b$  (see King 1997:170; also see Herron and Shotts 2004:178 for the superpopulation approach).

The extended King's EI model estimated  $\alpha^b$  as .0259 with a standard error of .0568. This point estimate and the corresponding standard error lead to  $t$  statistics of approximately .456, which certainly does not satisfy the conventional statistically significant level. Thus we cannot draw a conclusion based on the extended King's EI that the White Republican proportion of registered voters is a significant factor that influenced the White crossover for Ray Nagin.<sup>18</sup> This finding is contrary to that of equation (2) of Table 4. With these contradictory findings, the remaining question is whether King's EI extended model certainly outperformed the King's EI basic model, despite the illogical consistency problem for King's EI basic model discussed previously.

Because we know for sure, based on the official postelection racial data, that White turnout in the 2002 New Orleans mayoral runoff was 47.79 percent, we can test the improvement of the extended King's EI over King's EI basic model. Using King's basic model (i.e., without any covariate), it is estimated that the White turnout at the city level was 51.75 percent, with a standard error of .48 percent (the vector  $\hat{\psi} = \{0.498, 0.408, 0.103, 0.097, 0.665\}$ ), almost 4 percent more than the actual White turnout. Using the percentage White Republican covariate in the extended King's EI model, however, it is estimated that the White turnout rate was 48.14 percent, which is very close to the truth (an error of less than .5 percent compared to the actual value, the vector  $\hat{\psi} = \{0.395, 0.430, 0.078, 0.105, 0.747\}$ ).<sup>19</sup> This estimate is also better than regression and neighborhood estimates reported in Table 3.

The precinct-level point estimates are also much better after the extended model is used. To statistically compare the EI extended model with other methods, it is necessary to introduce a measure that can not only assess the accuracy of point estimates (i.e., how biased is the estimator) but also assess the efficiency of estimators. Mean squared error (MSE) is such a measure that takes into consideration both the bias of an estimate and its variance. In other words, one should not only look for the smallest bias of the estimators but also the smallest variance of the estimators to find the best overall efficiency of estimation. To be more specific,

$$\text{MSE} = E(\lambda - \theta)^2 \quad (12)$$

where  $\lambda$  is the estimator and  $\theta$  is the true parameter.

Table 5 provides the MSE for all the estimators and their performance rankings based on the magnitudes of the MSE scores. King's EI extended model, with the smallest MSE scores, is clearly the best estimator for both White and Black turnout figures, whereas the traditional regression methods are the worst.

Figures 1 and 2 further examine the results from King's basic and extended models. The basic model (Figure 1) clearly shows that King's EI overestimated the White turnout rates for most precincts, because most  $y$  values in the scatterplot are larger than 0 (mean of errors = .089,  $SD = .151$ ).<sup>20</sup> The extended model shifted the precinct-level estimates to the right direction, and the  $y$  values in the scatterplot (Figure 2) are much closer to 0 (mean of errors = -.007,  $SD = .13$ ).

Figures 1 and 2 also show that King's EI errors tend to increase as the proportion of registered voters who are White decreases. In other words,

**Table 5**  
**Mean Squared Error (MSE) for Turnout Estimates at the Precinct Level**

	$\beta_i^b$ : Black MSE <sup>a</sup>	Ranking <sup>b</sup>	$\beta_i^w$ : White MSE	Ranking
Goodman	.02172	6	.03920	6
Goodman weighted	.02075	4	.04275	8
Double regression	.02167	5	.03920	6
Double-regression weighted	.02199	8	.03788	5
Neighborhood nonlinear	.01319	2	.02094	2
Neighborhood linear	.02191	7	.02760	3
King's basic	.01410	3	.03074	4
King's extended	.01074	1	.01691	1

a.  $MSE = E(\lambda - \theta)^2$  where  $\lambda$  is the estimator and  $\theta$  is the true parameter.

b. The performance ranking is based on the magnitudes of MSE scores, the smaller the MSE scores, the better the rankings are.

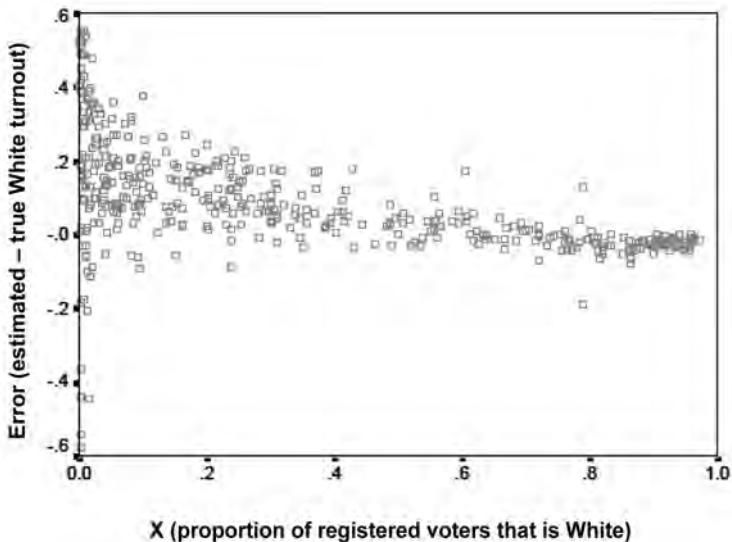
King's EI did a better job estimating White turnout rates for precincts with a high White density (HWD) than for precincts with a lower White density (LWD). This is because King's EI incorporates the method of bound approach, and the LWD precincts display larger bounds or a greater level of uncertainty for White point estimates. The reason why King's EI basic model overestimated White turnout rates is also related to its "borrowing strength" principle, which uses the informative precincts (precincts with small bounds, i.e., HWD precincts) to infer the uninformative precincts.

A check on the real turnout rates at the precinct level shows that the level of White turnout is greater in HWD precincts than in LWD precincts (a sign of aggregation bias and the correlation coefficient between the true White turnout and  $X_i$  is .478,  $p < .01$ ). Moreover, the variance of White turnout in the LWD precincts is much larger than that for HWD precincts: a problem of heteroskedasticity (King 1997:62-66). Adolph et al. (2003) suggest using weighted least squares (WLS) with weights given by the EI standard errors rather than EI with least squares, which seems to have much worse properties in Monte Carlo tests. This type of WLS, however, does not take consideration of heteroskedasticity problem revealed in Figure 1.

Although we find that the King's EI extended model does improve the White turnout estimate to an impressive level of accuracy, how to find useful covariates is still a question to be answered. In this vein, using the estimated  $\alpha^b$  and its standard error to calculate  $t$  statistics is not always reliable (see Adolph et al. 2003:86-94).<sup>21</sup> Using King's diagnostics to

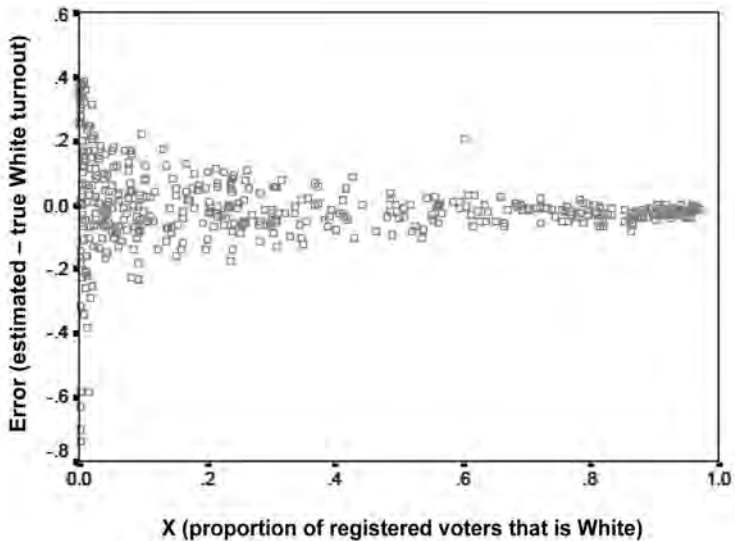


**Figure 1**  
**King's EI Basic Estimates**



compare results, on the other hand, is useful. In our case, the diagnostic tests did show that the extended model improves the accuracy of estimates. First, compared to that for the basic model, the tomography plot for the extended model exhibits smaller/narrower contours, which more precisely identify “the proportions of the lines with the highest probability of containing the true coordinate” (i.e., the true quantities of interests; see King 1997:204). The contours also capture some outliers that were visible in the basic model (the graphs are not shown in the article because of space limit). A much more useful way to assess the effectiveness of EI extended model is to check the relationship between  $\beta_i^b$  and  $X_i$ . Indeed, the plot of  $\beta_i^b$  by  $X_i$  indicates that the extended model, unlike the basic model, does pick up the pattern of aggregation bias exhibited in the boundx graph ( $r = .65$ ,  $p < .01$ , in the extended model, compared to  $r = .004$ ,  $p = .929$  in the basic model). Thus, they are correlated in the extended model, not in the basic model).<sup>22</sup>

**Figure 2**  
**King's EI Extended Estimates**



## Conclusion and Suggestions

The fear of ecological fallacy has led to methodological individualism in the political science discipline. Although survey method has been proved to be reliable in empirical research of many subfields, aggregate data have not been used to their full potential because of the faith in survey data only. Despite the danger of ecological fallacy, Goodman and double-regression approaches had been used to infer individual-level behavior in academic studies and voting rights cases when survey data are not easily available. More recently, King's EI procedure has provided a new round of interests in ecological inferences. These methods, however, have all faced constant skepticism and critiques in the discipline.

This article takes advantage of a unique opportunity of comparing all major methods, including survey results, by using aggregate as well as individual-level data concerning the 2002 New Orleans mayoral runoff election. One more advantage of this research is that the true quantities of

interest are also available so that the conclusion drawn here based on the comparisons of the methods is guaranteed to be accurate. Therefore, this verification study offered a much better and reliable test of the ecological inference methods than did previous empirical analyses in which simulated data were used and/or true quantities of interests were unknown.

Our empirical analysis shows that survey method should not always be regarded as the panacea to cure the disease of ecological fallacy. When the nature of a research problem is racially sensitive, survey respondents may choose not to tell the whole truth, and survey results can be much more misleading than what aggregate data would reveal. In fact, survey analysis should be checked with aggregate data to avoid the pitfall of methodological individualism.

King's EI method indeed offers unique advantages as shown in Table 1. However, based on our empirical analysis, the basic King's EI model produced a better estimate than did the traditional Goodman and double-regression approaches only for the Black turnout rate in the 2002 New Orleans runoff election. King's EI basic model in fact overestimated White turnout rate by almost 4 percentage points, worse than Goodman as well as double-regression estimates.

There is a way to improve King's EI basic model, however. Through using an appropriate covariate, King's EI extended model can drastically improve the accuracy of King's EI precinct estimates, and therefore the city level estimate too. Our MSE scores show that after a covariate (the proportion of registered voters who are White Republican in the precinct) is used in the King's EI extended model outperformed all previous methods, and the White turnout estimate is improved to the extent that the error is greatly reduced to less than .5 percent. Thus, to conquer the fear of ecological fallacy, the King's EI extended model, with its great power and flexibility, may indeed represent the future of ecological inferences.<sup>23</sup>

Based on the empirical findings, this article also offers the following advice on the proper use of King's EI extended models. First a solid understanding of King's EI model assumptions is necessary. In fact there is a range of King's EI model specifications from which to choose. The assumptions for any specification will apply in some data sets and not in others. Thus "checking your data" is always a good beginning (see King 1997, especially pp. 283-289, for diagnostics). Second, if one sees a clear sign of aggregation bias, the extended model may be called for (e.g., by looking at the XT fit, tomography plot, and especially boundx; see King 1997:282-3). Furthermore it is also important to check the possibility of logical inconsistency in a second-stage regression analysis (see above

example), which is clearly a situation where the King's EI basic model should be avoided, and an extended model is needed.

Which covariate is appropriate for improving the accuracy of estimation, if an extended model is needed? There are two ways, based on the above empirical analysis, to choose and justify a covariate in a King's EI extended model. First one can examine whether there is a good theoretic reason to include a certain covariate. For example, does the literature in the field strongly suggest that ideology is a factor affecting White voters' choice for minority candidates? If so, one can try to include ideology in the extended model. Second, one can also examine whether the diagnostic tests reveal a better fit for the data with the covariate included in the model. In particular, is the pattern of aggregation bias picked up by the new extended model? In this regard, comparing the tomography contour of the basic model with that of the extended model may be especially useful in evaluating the effectiveness of King's EI extended model (i.e., whether King's EI extended model is more precise with narrower and reasonable contours). Running the simple plot of  $\beta_i^b$  by  $X_i$  and computing their bivariate correlation coefficient ( $r$ ) for both the basic and extended models are also very important to evaluate whether the extended model outperforms the basic model in identifying and correcting the pattern of aggregation bias. A remaining question for more future research, as far as the application of King's EI extended models is concerned, is how to test statistically the improvement of King's EI extended model over the basic model when the true quantities of interests are unknown. This study, nevertheless, does convincingly show that King's EI extended model, when used properly, can offer a great tool for scholars and practitioners to correct the problem of aggregation bias and attack ecological fallacy.

## Notes

1. Ecological fallacy can be defined as the assumption that something learned about an ecological unit reveals something about the individuals making up the unit, which certainly may be false in reality.

2. Daron Shaw (1997) focused on "the potential pitfalls of using aggregate-level data to infer intergroup voting differences" (p. 49). By comparing double-regression estimates of group voting in each of the states with estimates from national tracking polls for the 1992 presidential election, Shaw shows that double regression overstated the degree of racial polarization.

3. King (1997) did have several empirical tests based on voter turnout data where the truth is known. But this article is the first verification study that uses King's Ecological Inference (EI) extended model and all major ecological inference methods as well as survey research and covariates to infer quantities of interests.

4. Because the ecological inference problem has more unknowns than knowns, there is in fact no single solution, which means that one can only turn to simulations or real data set comparisons to evaluate which models work better than others.

5. One can easily change the dependent variable of this equation into the proportion of registered voters who voted ( $T_i$ ) to calculate  $\beta^b$  and  $\beta^w$ .

6. Tomz and Van Houweling (2003) recently used the quadratic ecological regression model of Achen and Shively (1995), an extension of Goodman regression, to estimate the Black and White discrepancy in uncounted ballots.

7. King (1997) provides the detailed description of his theory and application of his EI method.

8. The three assumptions of basic King's EI include no aggregation bias and spatial autocorrelation and a truncated bivariate normal distribution of model parameters. It should also be noted that King's method not only combines the strength of Goodman regression and the method of bounds but also contains a maximum likelihood estimation framework assisted by Monte Carlo simulation. Furthermore the assumption of a truncated bivariate normal distribution (TBVN) of parameters may be relaxed in more recently developed Bayesian approaches. Jonathan Wakefield (2004), in particular, suggested a three-stage Bayesian hierarchical model to incorporate substantive prior knowledge and additional data in parameter estimation. This method, however, as Wakefield himself noted, cannot estimate the contextual effect without "an informative prior distribution, or surveys from within a sample of areas" (p. 46).

9. It should be noted that the fact that ecological inference is ill-posed inverse problem doesn't invalid any method (see King 1997:129).

10. The citywide racial turnout rates, again based on the government's data set required by the voting rights law, are also the true turnout rates rather than a weighted version of the district-level truths.

11. Although one should not conclude that all survey methods are inaccurate just based on one study, the findings presented here, however, indicated that survey responses based on elections can suffer from many problems that actually do not affect real voting returns, such as respondents lie, forget, or refuse to respond.

12. This finding confirms Voss (2004) about the tendency of King's EI to underestimate standard errors. In the same time, the reason why standard errors can be reported only for King's EI and not for Goodman or double regression is that proper methods of computing the standard errors for those methods haven't been developed, unlike King's EI.

13. Zax (2002) did correctly identify a mathematical error in Grofman and Migalski (1988) concerning using double regression for multimember district elections.

14. The diagnostic charts are not shown in this article because of space limit.

15. One may certainly argue that all major statistical models including OLS make some assumptions that in reality are difficult to reach. One thing that Herron and Shotts (2004) did fail to notice is that in King's EI, the bounds provide information on the true first-stage quantities of interests and thus implicitly contain information on any relationship between the beta coefficients and the covariates. Especially, as the bounds get narrower, the basic EI does a greater job in estimation, regardless of the omission of covariates from the probability model. This is because EI is not just the probability model, it's the probability model conditioned on the bounds as well.

16. For a discussion of contextual effect, aggregation effect, and distribution effect that may influence ecological inference, see Imai and Lu (2004).

17. Note that one of King's EI model specifications may be inappropriate for a given data set, but the model itself has never been proved to be logically inconsistent. In other words, one should always pay attention to the nature of the data to see whether a model fits them.

18. Using both  $X_i$  and White Republican proportion as two covariates in the King's EI extended model, or different prior/no prior specifications, led to the same conclusion, that is, neither White Republican proportion of registered voters nor  $X_i$  revealed a significant effect on White support for Nagin.

19. Note that this vector is critical to computing the final parameters, the vector  $\hat{\psi}$  itself, however, is not the final quantities of interests (see King 1997:138).

20. For Figures 1 and 2,  $y$  value is measured by the error of estimate for each precinct (i.e., estimated White turnout in the precinct minus the true White turnout rate).  $X$  value is the proportion of registered voters that are White in the precinct. Thus, if King's EI provides perfect estimates for all precincts, it should be reflected by a perfect horizontal line with  $y$  values fixed as exactly zero across all precincts. The data points above the  $y$  value of zero indicate overestimated values.

21. We did find a  $t$  value of 7.09 for the White turnout estimate by using the proportion of registered voters that are White Republican in a precinct as the covariate explained above, which indicates the effectiveness of this covariate. But as we included other covariates, the  $t$  test method did not work. For instance, when we included two covariates (proportion of registered voters who are Black and proportion of registered voters who are White Republican) together in the King's EI extended model, the  $t$  values are 5.64 and 9.25, respectively. This finding would suggest that these two covariates are both effective in the extended model and should improve the turnout estimate overall. The White turnout estimate based on this extended model, however, is 52.18, which is even worse than the King's EI basic model estimation.

22. King's EI does allow users to adjust priors to produce the best fit to the data. The extended model used in this model, however, adopted the default settings, which provides a reasonably good fit to the data.

23. The first stage to find turnout estimates is the same for Goodman and double regressions. A key reason why the extended model is better in our case is because we have an additional covariate. Certainly, adding useful information can lead to a better fit.

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