STATE OF NORTH CAROLINA COUNTY OF WAKE


IN THE GENERAL COURT OF JUSTICE SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

## AFFIDAVIT OF PROFESSOR MOON DUCHIN

I, Dr. Moon, Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.
2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

## Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.
4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.
5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.
6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013-2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.
7. A current copy of my full CV is attached to this report.
8. I am compensated at the rate of $\$ 400$ per hour.

# Analysis of 2021 enacted redistricting plans in North Carolina 

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## 1 Introduction

On November 4, 2021, the North Carolina General Assembly enacted three districting plans: maps of 14 U.S. Congressional districts, 50 state Senate districts, and 120 state House districts. This affidavit contains a brief summary of my evaluation of the properties of these plans. My focus will be on the egregious partisan imbalance and racial vote dilution in the enacted plans, following a brief review of the traditional districting principles.

Because redistricting inevitably involves complex interactions of rules, which can create intricate tradeoffs, it will be useful to employ a direct comparison to an alternative set of plans. These demonstrative plans illustrate that it is possible to simultaneously maintain or improve metrics for all of the most important redistricting principles that are operative in North Carolina's constitution and state and federal law. Crucially, this shows that nothing about the state's political geography compels us to draw a plan with a massive and entrenched partisan skew or a significant dilutive effect on Black voters.

To this end, I will be comparing the following plans: the enacted plans SL-174, SL-173, and SL-175 and a corresponding set of alternative plans labeled NCLCV-Cong, NCLCV-Sen, and NCLCV-House (proposed by plaintiffs who include the North Carolina League of Conservation Voters). The accompanying block assignment files are Appendices A1, A2, A3 to this affidavit, and I understand that they will be provided to the court in native format.


Figure 1: The six plans under discussion in this affidavit.

## 2 Partisan gerrymandering

### 2.1 Abstract partisan fairness

There are many notions of partisan fairness that can be found in the scholarly literature and in redistricting practitioner guides and software. Most of them are numerical, in the sense that they address how a certain quantitative share of the vote should be translated to a quantitative share of the seats in a state legislature or Congressional delegation.

The numerical notions of partisan fairness all tend to agree on one central point: an electoral climate with a roughly 50-50 split in partisan preference should produce a roughly 50-50 representational split. I will call this the Close-Votes-Close-Seats principle. North Carolina voting has displayed a partisan split staying consistently close to even between the two major parties over the last ten years, but the plans released by the General Assembly after the 2010 census were very far from realizing the ideal of converting even voting to even representation. This time, with a 14th seat added to North Carolina's apportionment, an exactly even seat outcome is possible. But the new enacted plans, like the plans from ten years ago, are decidedly not conducive to even representation.

Importantly, Close-Votes-Close-Seats is not tantamount to a requirement for proportionality. Rather, it is closely related to the principle of Majority Rule: a party or group with more than half of the votes should be able to secure more than half of the seats. In fact, Close-Votes-Close-Seats is essentially a corollary (or byproduct) of Majority Rule. It is not practicable to design a map that always attains these properties, but by contrast a map that consistently thwarts them should be closely scrutinized and usually rejected.

Unlike proportionality, neither Close-Votes-Close-Seats nor Majority Rule has any bearing on the preferred representational outcome when one party has a significant voting advantage: these principles are silent about whether $70 \%$ vote share should secure $70 \%$ of the seats, as proportionality would dictate, or $90 \%$ of the seats, as supporters of the efficiency gap would prefer. The size of the "winner's bonus" is not at all prescribed by a Close-Votes-Close-Seats norm.

### 2.2 Geography and fairness

Some scholars have argued that all numerical ideals, including Close-Votes-Close-Seats, ignore the crucial political geography-this school of thought reminds us that the location of votes for each party, and not just the aggregate preferences, has a major impact on redistricting outcomes. In [5], my co-authors and I gave a vivid demonstration of the impacts of political geography in Massachusetts: we showed that for a ten-year span of observed voting patterns, even though Republicans tended to get over one-third of the statewide vote, it was impossible to draw a single Congressional district with a Republican majority. That is, the geography of Massachusetts Republicans locked them out of Congressional representation. It is therefore not reasonable to charge the Massachusetts legislature with gerrymandering for having produced maps which yielded all-Democratic delegations; they could not have done otherwise.

In North Carolina, this is not the case. The alternative plans demonstrate that it is possible to produce maps that give the two major parties a roughly equal opportunity to elect their candidates. These plans are just examples among many thousands of plausible maps that convert voter preferences to far more even representation by party. In Congressional redistricting, present-day North Carolina geography is easily conducive to a seat share squarely in line with the vote share. In Senate and House plans, even following the strict detail of the Whole County Provisions, there are likewise many alternatives converting nearly even voting patterns to nearly even representation, across a large set of recent elections.

The clear conclusion is that the political geography of North Carolina today does not obstruct the selection of a map that treats Democratic and Republican voters fairly and evenhandedly.

### 2.3 Overlaying elections and plans

The enacted plans behave as though they are built to resiliently safeguard electoral advantage for Republican candidates. We can examine this effect without invoking any predictions or assumptions about future voting behavior by using a standard technique in election analysis: pairing proposed plans with actual recent elections. This method works by overlaying (or superimposing) the districting plans on a series of observed voting patterns from the recent past; this lets us take advantage of the rich dataset of real electoral outcomes in North Carolina in the last ten years to avoid speculative or predictive modeling about voting trends in the future. ${ }^{1}$

The overlay method works best when there is a large set of statewide elections to apply, which is certainly true in North Carolina. Of the 52 statewide party-ID general elections from the last cycle, 29 are elections for Council of State (ten offices elected three times, with the Attorney General race uncontested in 2012), three are presidential races, three are for U.S. Senate, and 17 are judicial races since mid-decade, when those became partisan contests. See Table 1 for more detail on the election dataset.

### 2.4 Partisanship outcomes

North Carolina is a very "purple" state. In 38 out of the 52 contests in our dataset, the statewide partisan outcome is within a 6-point margin: 47-53 or closer.

To understand how the enacted plans create major shortfalls for Democratic representation, we will overlay the plans with voting patterns from individual elections in the past Census cycle. We can make a striking observation by laying our six plans over the vote patterns, shown in Table 1. This reveals that the enacted Congressional plan (SL-174) shows a remarkable lack of responsiveness, giving 10-4 partisan outcomes across a wide range of recent electoral conditions, meaning that 10 Republicans and only 4 Democrats would represent North Carolina in Congress. The alternative plan (NCLCV-Cong) is far more faithful to the vote share, far more responsive, and tends to award more seats to the party with more votes-usually upholding both basic small-d-democratic principles of Majority Rules and Close-Votes-Close-Seats, which are violated by the enacted plan.

The same patterns are visible at the Senate and House level. Overall, the three enacted plans combine with those 38 relatively even vote patterns to produce 114 outcomes. Every single pairing of an enacted plan with a close statewide contest-a complete sweep of 114 opportunities-gives an outright Republican majority of seats. All three enacted plans will lock in an extreme, resilient, and unnecessary advantage for one party.

By every measure considered above that corresponds to a clear legal or good-government redistricting goal or value, the alternative plans meet or exceed the performance of the enacted plans. This demonstrates that it is possible, without any cost to the redistricting principles in play, to select maps that are far fairer to the voters of North Carolina.

Below, the outcomes of overlaying the plans on the elections will be presented in a series of tables and figures. First, Table 1 overviews the overlays with numbers. ${ }^{2}$ Then, Figure 2 offers a visualization to depict the same big picture of entrenched partisan advantage in the enacted plans with the full 52-election dataset. The diagonals show various lines of responsiveness that pivot around the central point of fairness: half of the votes securing half of the seats.

Finally, we will restrict to a smaller set of the 14 "up-ballot" races and consider the comparison for one office at a time in Figures 3-5.

[^0]Do close votes translate to close seats?
The table records the number of districts in each plan with a Democratic win. This shows that the enacted maps systematically violate the principles of Close-Votes-Close-Seats and Majority Rule.

|  | D Vote Share | SL-174 | NCLCV-Cong | SL-173 | NCLCV-Sen | SL-175 | NCLCV-House |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GOV12 | 0.4418 | 4 | 4 | 16 | 18 | 41 | 44 |
| AGC16 | 0.4444 | 4 | 4 | 17 | 17 | 40 | 42 |
| LAC16 | 0.4475 | 4 | 5 | 18 | 20 | 42 | 45 |
| JHU16 | 0.4563 | 4 | 5 | 18 | 19 | 42 | 49 |
| AGC20 | 0.4615 | 3 | 4 | 17 | 19 | 40 | 51 |
| JZA16 | 0.4619 | 4 | 5 | 19 | 21 | 43 | 50 |
| JDI16 | 0.4653 | 4 | 6 | 19 | 21 | 44 | 53 |
| LTG16 | 0.4665 | 4 | 6 | 19 | 21 | 44 | 54 |
| LAC12 | 0.4674 | 4 | 5 | 20 | 20 | 44 | 51 |
| AGC12 | 0.4678 | 4 | 5 | 18 | 18 | 43 | 50 |
| SEN16 | 0.4705 | 4 | 6 | 19 | 21 | 43 | 55 |
| TRS16 | 0.4730 | 4 | 6 | 19 | 21 | 45 | 53 |
| TRS20 | 0.4743 | 4 | 6 | 17 | 20 | 45 | 51 |
| JA620 | 0.4806 | 4 | 7 | 17 | 21 | 46 | 55 |
| PRS16 | 0.4809 | 4 | 7 | 19 | 22 | 48 | 56 |
| JA420 | 0.4822 | 4 | 7 | 17 | 22 | 47 | 56 |
| INC20 | 0.4823 | 4 | 7 | 18 | 23 | 47 | 56 |
| LTG20 | 0.4836 | 4 | 7 | 18 | 21 | 46 | 55 |
| JA720 | 0.4842 | 4 | 7 | 17 | 22 | 48 | 56 |
| SUP20 | 0.4862 | 4 | 7 | 19 | 23 | 49 | 56 |
| JA520 | 0.4874 | 4 | 7 | 18 | 22 | 49 | 57 |
| JA218 | 0.4876 | 4 | 7 | 18 | 22 | 45 | 55 |
| JS420 | 0.4879 | 4 | 7 | 19 | 24 | 49 | 56 |
| J1320 | 0.4885 | 4 | 7 | 19 | 23 | 49 | 56 |
| PRS12 | 0.4897 | 4 | 6 | 20 | 21 | 46 | 55 |
| SEN20 | 0.4910 | 4 | 7 | 20 | 24 | 48 | 56 |
| LAC20 | 0.4918 | 4 | 8 | 21 | 25 | 51 | 58 |
| SEN14 | 0.4919 | 4 | 6 | 20 | 22 | 46 | 52 |
| PRS20 | 0.4932 | 4 | 8 | 20 | 25 | 50 | 60 |
| JS220 | 0.4934 | 4 | 8 | 21 | 24 | 51 | 59 |
| SUP16 | 0.4941 | 4 | 6 | 22 | 23 | 49 | 57 |
| JS118 | 0.4955 | 4 | 7 | 20 | 25 | 50 | 58 |
| INC16 | 0.4960 | 4 | 6 | 22 | 22 | 50 | 57 |
| JST16 | 0.4976 | 4 | 7 | 21 | 23 | 50 | 58 |
| LTG12 | 0.4992 | 5 | 7 | 22 | 22 | 50 | 58 |
| JS120 | 0.5000 | 4 | 8 | 22 | 27 | 52 | 60 |
| AUD16 | 0.5007 | 5 | 8 | 22 | 23 | 51 | 56 |
| GOV16 | 0.5011 | 4 | 7 | 20 | 27 | 50 | 58 |
| ATG20 | 0.5013 | 4 | 8 | 21 | 25 | 51 | 58 |
| ATG16 | 0.5027 | 4 | 7 | 20 | 23 | 50 | 57 |
| JA118 | 0.5078 | 4 | 8 | 22 | 26 | 51 | 58 |
| AUD20 | 0.5088 | 4 | 8 | 24 | 28 | 54 | 61 |
| JA318 | 0.5091 | 4 | 8 | 21 | 26 | 52 | 59 |
| SOS20 | 0.5116 | 5 | 8 | 24 | 28 | 53 | 62 |
| JGE16 | 0.5131 | 5 | 8 | 22 | 25 | 52 | 59 |
| INC12 | 0.5186 | 5 | 8 | 22 | 22 | 55 | 61 |
| SOS16 | 0.5226 | 5 | 9 | 24 | 24 | 57 | 62 |
| GOV20 | 0.5229 | 4 | 8 | 23 | 27 | 58 | 63 |
| AUD12 | 0.5371 | 8 | 9 | 27 | 28 | 61 | 65 |
| SOS12 | 0.5379 | 7 | 9 | 26 | 26 | 59 | 63 |
| TRS12 | 0.5383 | 7 | 9 | 25 | 24 | 59 | 65 |
| SUP12 | 0.5424 | 8 | 9 | 28 | 28 | 61 | 66 |

AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; LTG = Lieutenant Governor; PRS = President; SEN = Senator; SOS = Secretary of State; SUP $=$ Superintendent of Public Instruction; TRS $=$ Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), JS* are elections to the state Supreme Court. All other $J^{*}$ prefixes refer to an election to replace a specific judge on the Court of Appeals. Where there was more than one judicial candidate from a given party on the ballot, they were combined for this analysis. The two-digit suffix designates the election year.

Table 1: 52 general elections, sorted from lowest to highest Democratic share.

## Seats vs. Votes

Majority Rule says that outcomes should tend to fall in the Northeast and Southwest quadrants, avoiding the Southeast and Northwest. Close-Votes-Close-Seats says that points should not miss the bulls-eye near the center by systematically deviating to the North or the South. These principles are clearly upheld by the alternative plans (green) and violated by the enacted plans (maroon).

Congress




Figure 2: On these seats-vs.-votes plots, we see the election results when overlaying the six maps on the 52 general election contests in the last decade; each colored dot is plotted as the coordinate pair (vote share, seat share).

### 2.5 Up-ballot races

The same patterns are apparent if we narrow our focus to the smaller set of better-known "up-ballot" races: in order, the first five to appear on the ballot are the contests for President, U.S. Senator, Governor, Lieutenant Governor, and Attorney General. Together these occurred 14 times in the last Census cycle.

|  | Up-ballot generals (14) |  | All generals (52) |  |
| :---: | :---: | :---: | :---: | :---: |
| SL-174 | 4883 | . 2908 | 4911 | . 3118 |
| NCLCV-Cong | . 4883 | . 4796 | . 4911 | . 4931 |
| SL-173 | 4883 | . 3957 | 4911 | . 4065 |
| NCLCV-Sen | . 4883 | . 4557 | 4911 | . 4592 |
| SL-175 |  | . 3994 |  | . 4080 |
| NCLCV-House | . 4883 | . 4649 | . 4911 | . 4684 |

Table 2: Comparing overall fidelity of representation to the voting preferences of the electorate. Vote shares are computed with respect to the major-party vote total.

Figure 3 shows the performance of the Congressional maps in the three Presidential contests in the last Census cycle, where the Democratic vote share (pink box) was between $48 \%$ and $50 \%$ of the major-party total each time. For a contest that is so evenly divided, we would expect a fair map to have 6,7 , or 8 out of 14 districts favoring each party. The alternative Congressional map NCLCV-Cong does just that, while the enacted plan SL-174 has just 4 out of 14 Democratic-majority districts each time (green and maroon circles). The alternative plan is far more successful at reflecting the even split of voter preferences.

Congressional plan comparison in Presidential elections
Do close votes translate to close seats?


Figure 3: When Presidential voting is overlaid on the plans, we can compare the Democratic seat share in the enacted Congressional plan SL-174 (maroon) and the alternative Congressional plan NCLCV-Cong (green) to the vote share ( $\mathrm{p} / \mathrm{m}$ ) for Democratic candidates. The $50 \%$ line is marked.

Next, simplified versions of the same type of graphic are presented for all five up-ballot offices. Figure 4 compares Congressional maps, and Figure 5 compares legislative maps in the same fashion.

In these figures, we can view whether the plans display a tendency to uphold the Close-Votes-Close-Seats norm, for one office at a time. The pink squares are the vote share. If they are close to the 50-50 mark, then a fair map would also produce seat shares that are close to that mark. This is consistently true for the alternative plans and consistently false for the enacted plans.

## Congressional plan comparison across up-ballot races



Figure 4: For up-ballot general election contests across the previous Census cycle, we can compare the seat share under the enacted Congressional plan SL-174 (maroon) and the seat share under the alternative Congressional plan NCLCV-Cong (green) to the vote share (pink) for Democratic candidates. The presidential comparison from the previous figure is repeated here, alongside the other four up-ballot offices. The $50 \%$ line is marked each time.

## State Senate plan comparison across up-ballot races



Figure 5: Legislative plans overlaid with voting patterns from up-ballot elections. The enacted plans SL-173 and SL-175 are shown in maroon. The alternative plans NCLCV-Sen and NCLCVHouse, in green, have seat shares tracking much closer to the nearly even voting preferences.

## 3 Racial vote dilution

North Carolina has a large minority of Black-identified residents. Over two million North Carolinians-2,107,526 out of 10,439,388 to be precise, or about $20.2 \%$-were identified as non-Hispanic Black-alone on the Census. Within the voting-age population, the numbers shift to $1,620,569$ out of $8,155,099$, or about $19.9 \%$. Increasing numbers of Americans identify as Black in combination with other races and/or Hispanic ethnicity. Passing to this more expansive definition of Black voting age population raises the numbers to $1,743,052$ out of $8,155,099$, or 21.4\%.

Minority groups' opportunity to elect candidates of choice is protected by both state and federal law. A detailed assessment of opportunity must not primarily hinge on the demographics of the districts, but must also rely on electoral history and an assessment of polarization patterns. ${ }^{3}$

I have used industry-leading techniques to study the racial polarization patterns in North Carolina general and primary elections from the last decade. They indicate a consistent pattern of polarization in statewide general elections, such that White voters are estimated to support the Republican candidate at a rate of over $61 \%$ in every general election, and Black voters are estimated to support the Democratic candidate at a rate of over $94 \%$ each time. PoIarization is present in many Democratic primary elections as well, particularly in elections in which there is a Black Democratic candidate. I have designated a selection of eight electionsfour generals and four primaries-chosen to be particularly informative in determining whether Black voters have an opportunity to elect their candidates of choice.

## Democratic Primaries

- Sutton preferred over Mangrum in the 2020 Superintendent primary;
- Smith preferred over Wadsworth in the 2020 Ag. Commissioner primary;
- Williams preferred over Stein in the 2016 Attorney General primary;
- Coleman preferred over the field in the 2016 Lieutenant Governor primary.


## General Elections

- Holley preferred over Robinson in the 2020 Lieutenant Governor election;
- Cunningham preferred over Tillis in the 2020 U.S. Senate election;
- Coleman preferred over Forest in the 2016 Lieutenant Governor election;
- Blue preferred over Folwell in the 2016 Treasurer election.

These eight contests were chosen by a combination of factors that combine to make an election particularly informative with respect to the preferences of Black voters. Namely: I prioritized elections that are more recent, that have a Black candidate on the ballot, that are clearly polarized, and that are close enough to produce variation at the district level. ${ }^{4}$

The electoral alignment score derived from these elections is a value from 0 to 8 . I consider a district in which the Black candidate of choice prevails in at least 6 of these 8 contests to be aligned with Black voting preferences in the state. ${ }^{5}$ If, in addition, at least $25 \%$ of the voting age population is Black, then I label the district to be effective for Black voters.

I note that the use of electoral history is not just cosmetic: there are House-sized districts with $35-39 \%$ BVAP that are nonetheless not labeled effective in these lists because they fall short of the standard of inclining to the Black candidate of choice in at least six out of the eight chosen elections.

[^1]At all three levels, the NCLCV alternative maps provide more effective opportunity-to-elect districts for Black voters than the corresponding enacted plans.

## Effective districts for Black voters

Out of 14 Congressional districts, SL-174 has 2 effective districts, while NCLCV-Cong has 4.
Out of 50 Senate districts, SL-173 has 8 effective districts, while NCLCV-Sen has 12.
Out of 120 House districts, SL-175 has 24 effective districts, while NCLCV-House has 36.

## effective districts in state plan effective districts in alternative plan

CD2, 9
SD5, 11, 14, 19, 28, 38, 39, 40
HD8, 23, 24, 25, 27, 32, 38, 39, 42, 44, 48, $57,58,60,66,71,92,99,100,101,102$, 106, 107, 112

CD2, 4, 9, 11
SD1, 5, 11, 14, 18, 19, 26, 27, 32, 38, 39, 40
HD2, 8, 9, 10, 23, 24, 25, 27, 31, 32, 33, 38, $39,40,42,43,44,45,48,57,58,59,60,61$, $63,66,71,88,92,99,100,101,102,106$, 107, 112

## 4 Detailed plan comparison

Detailed maps showing how the district lines cut through the patterns of Democratic and Republican support, and how they cut through the demographic location of Black voting age population, can be found in Appendix B.

### 4.1 Traditional districting principles

Principles that are relevant to North Carolina redistricting include the following.

- Population balance. The standard interpretation of One Person, One Vote for Congressional districts is that districts should be fine-tuned so that their total Census population deviates by no more than one person from any district to any other.
There is more latitude with legislative districts; they typically vary top-to-bottom by no more than $10 \%$ of ideal district size. In North Carolina, the Whole County Provisions make it very explicit that $5 \%$ deviation must be tolerated if it means preserving more counties intact.
All six plans have acceptable population balance.
Population deviation

|  | Max Positive Deviation | District | Max Negative Deviation | District |
| :---: | :---: | :---: | :---: | :---: |
| SL-174 | 0 | (eight districts) | -1 | (six districts) |
| NCLCV-Cong | 0 | (eight districts) | -1 | (six districts) |
| SL-173 | $10,355(4.960 \%)$ | 5 | $-10,434(4.997 \%)$ | 13,18 |
| NCLCV-Sen | $10,355(4.960 \%)$ | 5 | $-10,427(4.994 \%)$ | 15 |
| SL-175 | $4250(4.885 \%)$ | 18 | $-4189(4.815 \%)$ | 112 |
| NCLCV-House | $4341(4.990 \%)$ | 82 | $-4323(4.969 \%)$ | 87 |

Table 3: Deviations are calculated with respect to the rounded ideal district populations of 745,671 for Congress, 208,788 for Senate, and 86,995 for House.

- Contiguity. All six plans are contiguous; for each district, it is possible to transit from any part of the district to any other part through a sequence of census blocks that share boundary segments of positive length. As is traditional in North Carolina, contiguity through water is accepted.
- Compactness. The two compactness metrics most commonly appearing in litigation are the Polsby-Popper score and the Reock score. Polsby-Popper is the name given in redistricting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula $4 \pi A / P^{2}$. Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1 . Reock is a different measurement of how much a shape differs from a circle: it is computed as the ratio of a region's area to that of its circumcircle, defined as the smallest circle in which the region can be circumscribed. From this definition, it is clear that it too is optimized at a value of 1 , which is achieved only by circles.
These scores depend on the contours of a district and have been criticized as being too dependent on map projections or on cartographic resolution [1, 2]. Recently, some mathematicians have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built. The most commonly cited discrete score for districts is the cut edges score, which counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations. This score improves on the contour-based scores by better controlling for factors like coastline and other natural boundaries, and by focusing on the units actually available to redistricters rather than treating districts like free-form Rorschach blots.
The alternative plans are significantly more compact than the enacted plans in all three compactness metrics.


## Compactness

|  | block cut edges <br> (lower is better) | average Polsby-Popper <br> (higher is better) | average Reock <br> (higher is better) |
| :---: | :---: | :---: | :---: |
| SL-174 | 5194 | 0.303 | 0.417 |
| NCLCV-Cong | 4124 | 0.383 | 0.470 |
| SL-173 | 9702 | 0.342 | 0.416 |
| NCLCV-Sen | 9249 | 0.369 | 0.428 |
| SL-175 | 16,182 | 0.351 | 0.437 |
| NCLCV-House | 13,963 | 0.414 | 0.465 |

Table 4: Comparing compactness scores via one discrete and two contour-based metrics. These scores were computed using dissolved districts based on the census blocks that were assigned in the plans under discussion.

District-by-district compactness scores for the contour-based metrics are shown in Tables 5-7.

|  | Reock |  | Polsby-Popper |  |
| :---: | :---: | :---: | :---: | :---: |
| CD | SL-174 | NCLCV-Cong | SL-174 | NCLCV-Cong |
| 1 | 0.517 | 0.534 | 0.324 | 0.403 |
| 2 | 0.303 | 0.47 | 0.278 | 0.323 |
| 3 | 0.484 | 0.212 | 0.331 | 0.228 |
| 4 | 0.487 | 0.412 | 0.39 | 0.304 |
| 5 | 0.468 | 0.582 | 0.347 | 0.514 |
| 6 | 0.418 | 0.472 | 0.231 | 0.483 |
| 7 | 0.424 | 0.664 | 0.199 | 0.434 |
| 8 | 0.472 | 0.523 | 0.532 | 0.398 |
| 9 | 0.678 | 0.579 | 0.469 | 0.43 |
| 10 | 0.41 | 0.285 | 0.197 | 0.254 |
| 11 | 0.282 | 0.533 | 0.207 | 0.532 |
| 12 | 0.247 | 0.388 | 0.243 | 0.368 |
| 13 | 0.41 | 0.558 | 0.266 | 0.379 |
| 14 | 0.232 | 0.354 | 0.221 | 0.313 |

Table 5: Compactness scores by district for the Congressional plans.

|  | Reock |  |  | Polsby-Popper |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SD | SL-173 | NCLCV-Sen | SL-173 | NCLCV-Sen |  |
| 1 | 0.263 | 0.297 | 0.213 | 0.174 |  |
| 2 | 0.231 | 0.397 | 0.105 | 0.178 |  |
| 3 | 0.409 | 0.409 | 0.179 | 0.179 |  |
| 4 | 0.564 | 0.564 | 0.406 | 0.406 |  |
| 5 | 0.403 | 0.403 | 0.335 | 0.335 |  |
| 6 | 0.616 | 0.616 | 0.595 | 0.595 |  |
| 7 | 0.213 | 0.553 | 0.219 | 0.411 |  |
| 8 | 0.446 | 0.457 | 0.439 | 0.478 |  |
| 9 | 0.443 | 0.441 | 0.217 | 0.226 |  |
| 10 | 0.618 | 0.618 | 0.614 | 0.614 |  |
| 11 | 0.464 | 0.464 | 0.376 | 0.376 |  |
| 12 | 0.42 | 0.388 | 0.395 | 0.404 |  |
| 13 | 0.284 | 0.357 | 0.257 | 0.4 |  |
| 14 | 0.399 | 0.523 | 0.247 | 0.45 |  |
| 15 | 0.397 | 0.52 | 0.231 | 0.398 |  |
| 16 | 0.619 | 0.51 | 0.473 | 0.388 |  |
| 17 | 0.488 | 0.54 | 0.361 | 0.505 |  |
| 18 | 0.376 | 0.644 | 0.309 | 0.514 |  |
| 19 | 0.53 | 0.53 | 0.34 | 0.34 |  |
| 20 | 0.384 | 0.387 | 0.363 | 0.344 |  |
| 21 | 0.218 | 0.218 | 0.137 | 0.137 |  |
| 22 | 0.473 | 0.459 | 0.471 | 0.517 |  |
| 23 | 0.498 | 0.498 | 0.529 | 0.529 |  |
| 24 | 0.52 | 0.52 | 0.452 | 0.452 |  |
| 25 | 0.283 | 0.325 | 0.271 | 0.276 |  |
| 26 | 0.451 | 0.397 | 0.301 | 0.331 |  |
| 27 | 0.541 | 0.364 | 0.437 | 0.321 |  |
| 28 | 0.444 | 0.544 | 0.248 | 0.457 |  |
| 29 | 0.317 | 0.378 | 0.202 | 0.252 |  |
| 30 | 0.4 | 0.4 | 0.456 | 0.456 |  |
| 31 | 0.482 | 0.429 | 0.344 | 0.355 |  |
| 32 | 0.62 | 0.455 | 0.422 | 0.354 |  |
| 33 | 0.322 | 0.322 | 0.294 | 0.294 |  |
| 34 | 0.49 | 0.477 | 0.523 | 0.489 |  |
| 35 | 0.375 | 0.342 | 0.225 | 0.348 |  |
| 36 | 0.463 | 0.314 | 0.411 | 0.294 |  |
| 37 | 0.401 | 0.397 | 0.421 | 0.437 |  |
| 38 | 0.523 | 0.566 | 0.334 | 0.444 |  |
| 39 | 0.356 | 0.391 | 0.295 | 0.368 |  |
| 40 | 0.381 | 0.453 | 0.382 | 0.538 |  |
| 41 | 0.287 | 0.519 | 0.294 | 0.531 |  |
| 42 | 0.429 | 0.397 | 0.273 | 0.469 |  |
| 43 | 0.533 | 0.341 | 0.522 | 0.274 |  |
| 44 | 0.386 | 0.425 | 0.46 | 0.357 |  |
| 45 | 0.343 | 0.391 | 0.25 | 0.3 |  |
| 46 | 0.229 | 0.249 | 0.184 | 0.213 |  |
| 47 | 0.186 | 0.116 | 0.127 | 0.113 |  |
| 48 | 0.404 | 0.373 | 0.38 | 0.264 |  |
| 49 | 0.479 | 0.424 | 0.358 | 0.22 |  |
| 50 | 0.422 | 0.312 | 0.441 | 0.335 |  |
|  |  |  |  |  |  |

Table 6: Compactness scores by district for the Senate plans.

|  | Reock |  | Polsby-Popper |  |  | Reock |  | Polsby-Popper |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HD | SL-175 | NCLCV-House | SL-175 | NCLCV-House | HD | SL-175 | NCLCV-House | SL-175 | NCLCV-House |
| 1 | 0.413 | 0.393 | 0.213 | 0.168 | 61 | 0.388 | 0.356 | 0.294 | 0.346 |
| 2 | 0.316 | 0.404 | 0.326 | 0.468 | 62 | 0.318 | 0.651 | 0.312 | 0.589 |
| 3 | 0.377 | 0.448 | 0.298 | 0.329 | 63 | 0.56 | 0.596 | 0.353 | 0.533 |
| 4 | 0.482 | 0.337 | 0.448 | 0.237 | 64 | 0.329 | 0.48 | 0.257 | 0.459 |
| 5 | 0.28 | 0.28 | 0.3 | 0.3 | 65 | 0.594 | 0.594 | 0.764 | 0.764 |
| 6 | 0.389 | 0.539 | 0.479 | 0.549 | 66 | 0.457 | 0.46 | 0.264 | 0.293 |
| 7 | 0.476 | 0.442 | 0.44 | 0.403 | 67 | 0.444 | 0.444 | 0.486 | 0.486 |
| 8 | 0.394 | 0.437 | 0.327 | 0.314 | 68 | 0.45 | 0.577 | 0.305 | 0.502 |
| 9 | 0.587 | 0.698 | 0.411 | 0.425 | 69 | 0.539 | 0.49 | 0.346 | 0.364 |
| 10 | 0.589 | 0.606 | 0.567 | 0.398 | 70 | 0.542 | 0.638 | 0.535 | 0.65 |
| 11 | 0.359 | 0.654 | 0.246 | 0.473 | 71 | 0.267 | 0.488 | 0.275 | 0.509 |
| 12 | 0.312 | 0.312 | 0.291 | 0.291 | 72 | 0.521 | 0.495 | 0.27 | 0.398 |
| 13 | 0.379 | 0.367 | 0.425 | 0.488 | 73 | 0.487 | 0.46 | 0.421 | 0.612 |
| 14 | 0.384 | 0.305 | 0.291 | 0.204 | 74 | 0.367 | 0.548 | 0.299 | 0.425 |
| 15 | 0.546 | 0.468 | 0.371 | 0.395 | 75 | 0.388 | 0.468 | 0.266 | 0.53 |
| 16 | 0.404 | 0.483 | 0.242 | 0.388 | 76 | 0.43 | 0.43 | 0.497 | 0.497 |
| 17 | 0.416 | 0.668 | 0.227 | 0.473 | 77 | 0.408 | 0.408 | 0.297 | 0.297 |
| 18 | 0.589 | 0.336 | 0.37 | 0.374 | 78 | 0.341 | 0.479 | 0.204 | 0.447 |
| 19 | 0.462 | 0.482 | 0.285 | 0.359 | 79 | 0.523 | 0.353 | 0.36 | 0.2 |
| 20 | 0.463 | 0.172 | 0.557 | 0.173 | 80 | 0.285 | 0.413 | 0.319 | 0.359 |
| 21 | 0.45 | 0.591 | 0.206 | 0.469 | 81 | 0.481 | 0.434 | 0.312 | 0.359 |
| 22 | 0.528 | 0.528 | 0.361 | 0.361 | 82 | 0.311 | 0.444 | 0.32 | 0.477 |
| 23 | 0.453 | 0.453 | 0.359 | 0.359 | 83 | 0.474 | 0.473 | 0.328 | 0.342 |
| 24 | 0.463 | 0.554 | 0.538 | 0.638 | 84 | 0.498 | 0.57 | 0.515 | 0.645 |
| 25 | 0.463 | 0.402 | 0.511 | 0.455 | 85 | 0.501 | 0.493 | 0.315 | 0.299 |
| 26 | 0.45 | 0.474 | 0.4 | 0.412 | 86 | 0.49 | 0.49 | 0.437 | 0.437 |
| 27 | 0.433 | 0.433 | 0.353 | 0.353 | 87 | 0.538 | 0.512 | 0.437 | 0.526 |
| 28 | 0.573 | 0.411 | 0.498 | 0.43 | 88 | 0.233 | 0.367 | 0.211 | 0.364 |
| 29 | 0.36 | 0.519 | 0.333 | 0.645 | 89 | 0.304 | 0.462 | 0.291 | 0.338 |
| 30 | 0.381 | 0.306 | 0.356 | 0.389 | 90 | 0.508 | 0.431 | 0.349 | 0.381 |
| 31 | 0.415 | 0.476 | 0.323 | 0.533 | 91 | 0.541 | 0.563 | 0.522 | 0.583 |
| 32 | 0.534 | 0.528 | 0.587 | 0.543 | 92 | 0.28 | 0.399 | 0.244 | 0.455 |
| 33 | 0.491 | 0.254 | 0.289 | 0.252 | 93 | 0.317 | 0.33 | 0.288 | 0.319 |
| 34 | 0.414 | 0.383 | 0.289 | 0.349 | 94 | 0.507 | 0.496 | 0.348 | 0.371 |
| 35 | 0.28 | 0.528 | 0.292 | 0.464 | 95 | 0.616 | 0.49 | 0.596 | 0.516 |
| 36 | 0.586 | 0.396 | 0.532 | 0.443 | 96 | 0.358 | 0.316 | 0.351 | 0.33 |
| 37 | 0.417 | 0.372 | 0.369 | 0.379 | 97 | 0.321 | 0.321 | 0.515 | 0.515 |
| 38 | 0.377 | 0.522 | 0.247 | 0.383 | 98 | 0.593 | 0.574 | 0.576 | 0.589 |
| 39 | 0.649 | 0.399 | 0.519 | 0.245 | 99 | 0.469 | 0.471 | 0.322 | 0.443 |
| 40 | 0.413 | 0.342 | 0.336 | 0.242 | 100 | 0.537 | 0.359 | 0.333 | 0.312 |
| 41 | 0.521 | 0.581 | 0.423 | 0.498 | 101 | 0.488 | 0.518 | 0.31 | 0.515 |
| 42 | 0.537 | 0.402 | 0.395 | 0.258 | 102 | 0.392 | 0.621 | 0.23 | 0.36 |
| 43 | 0.52 | 0.415 | 0.281 | 0.372 | 103 | 0.278 | 0.546 | 0.349 | 0.479 |
| 44 | 0.587 | 0.564 | 0.419 | 0.564 | 104 | 0.573 | 0.432 | 0.32 | 0.313 |
| 45 | 0.248 | 0.555 | 0.274 | 0.495 | 105 | 0.395 | 0.437 | 0.419 | 0.391 |
| 46 | 0.316 | 0.432 | 0.239 | 0.275 | 106 | 0.599 | 0.485 | 0.419 | 0.503 |
| 47 | 0.604 | 0.535 | 0.498 | 0.453 | 107 | 0.304 | 0.529 | 0.183 | 0.556 |
| 48 | 0.479 | 0.479 | 0.442 | 0.442 | 108 | 0.374 | 0.402 | 0.24 | 0.288 |
| 49 | 0.447 | 0.555 | 0.358 | 0.604 | 109 | 0.466 | 0.485 | 0.421 | 0.522 |
| 50 | 0.375 | 0.384 | 0.343 | 0.388 | 110 | 0.355 | 0.514 | 0.277 | 0.39 |
| 51 | 0.48 | 0.427 | 0.283 | 0.262 | 111 | 0.348 | 0.641 | 0.24 | 0.436 |
| 52 | 0.352 | 0.468 | 0.214 | 0.28 | 112 | 0.58 | 0.266 | 0.397 | 0.229 |
| 53 | 0.322 | 0.597 | 0.256 | 0.449 | 113 | 0.392 | 0.368 | 0.224 | 0.186 |
| 54 | 0.459 | 0.486 | 0.376 | 0.442 | 114 | 0.307 | 0.549 | 0.182 | 0.46 |
| 55 | 0.458 | 0.534 | 0.312 | 0.399 | 115 | 0.559 | 0.308 | 0.349 | 0.289 |
| 56 | 0.502 | 0.652 | 0.37 | 0.691 | 116 | 0.401 | 0.532 | 0.159 | 0.332 |
| 57 | 0.436 | 0.589 | 0.368 | 0.475 | 117 | 0.422 | 0.581 | 0.271 | 0.393 |
| 58 | 0.397 | 0.521 | 0.257 | 0.432 | 118 | 0.412 | 0.412 | 0.247 | 0.247 |
| 59 | 0.455 | 0.463 | 0.334 | 0.56 | 119 | 0.276 | 0.276 | 0.22 | 0.22 |
| 60 | 0.383 | 0.361 | 0.261 | 0.407 | 120 | 0.4 | 0.4 | 0.367 | 0.367 |

Table 7: Compactness scores by district for the House plans.

- Respect for political subdivisions. For legislative redistricting, North Carolina has one of the strongest requirements for county consideration of any state in the nation. In my understanding, courts have interpreted the Whole County Provisions as follows. ${ }^{6}$
- First, if any county is divisible into a whole number of districts that will be within $\pm 5 \%$ of ideal population, then it must be subdivided accordingly without districts crossing into other counties.
- Next, seek any contiguous grouping of two counties that is similarly divisible into a whole number of districts.
- Repeat for groupings of three, and so on, until all counties are accounted for.

Once clusters have been formed, there are more rules about respecting county lines within clusters. The legal language is again explicit: "[T]he resulting interior county lines created by any such groupings may be crossed or traversed in the creation of districts within said multi-county grouping but only to the extent necessary" to meet the $\pm 5 \%$ population standard for districts. To address this, I have counted the country traversals in each plan, i.e., the number of times a district crosses between adjacent counties within a grouping.
Table 8 reflects the county integrity metric that is most relevant at each level: the enacted congressional plan splits 11 counties into 25 pieces while the alternative plan splits 13 , but splits no county three ways. (The enacted plans unnecessarily split three counties into three pieces.) In the legislative plans, the law specifies traversals as the fundamental integrity statistic.

County and municipality preservation

| \# county pieces |  | \# traversals |  |
| :---: | :---: | :---: | :---: |
| SL-174 | 25 | SL-173 | 97 |
| NCLCV-Cong | 26 | NCLCV-Sen | 89 |
|  |  | SL-175 | 69 |
|  |  | NCLCV-House | 66 |
|  | \# municipal pieces (considering all blocks) | \# municipal pieces(considering populated blocks) |  |
| SL-174 | 90 |  |  |
| NCLCV-Cong | 58 |  |  |
| SL-173 | 152 |  |  |
| NCLCV-Sen | 125 |  |  |
| SL-175 | 292 |  |  |
| NCLCV-House | 201 |  |  |

Table 8: Comparing the plans' conformance to political boundaries.

[^2]The alternative plans are comparable to the enacted plans, and often superior, in each of these key metrics regarding preservation of political boundaries. This remains true whether splits of municipalities are counted by the division of any of their census blocks, or only by the division of populated census blocks.

I will briefly mention several additional redistricting principles.

- Communities of interest. In North Carolina, there was no sustained effort by the state or by community groups to formally collect community of interest (COI) maps, to my knowledge. Without this, it is difficult to produce a suitable metric.
- Cores of prior districts. In some states, there is statutory guidance to seek districting plans that preserve the cores of prior districts. In North Carolina, this is not a factor in the constitution, in statute, or in case law. In addition, attention to core preservation would be prohibitively difficult in the Senate and House because of the primacy of the Whole County Provisions, which forces major changes to the districts simply as a consequence of fresh population numbers.
- Incumbent pairing. In 2017, the North Carolina legislative redistricting committee listed "incumbency protection" as a goal in their itemization of principles. In 2021, this was softened to the statement that "Member residence may be considered" in the drawing of districts. I have counted the districts in each plan that contain more than one incumbent address; these are sometimes colorfully called "double-bunked" districts. For this statistic, it is not entirely clear whether a high or low number is preferable. When a plan remediates a gerrymandered predecessor, we should not be surprised if it ends up pairing numerous incumbents.


## Double-bunking

\# districts pairing incumbents

| SL-174 | 3 |
| :---: | :---: |
| NCLCV-Cong | 1 |
| SL-173 | 5 |
| NCLCV-Sen | 9 |
| SL-175 | 6 |
| NCLCV-House | 16 |

Table 9: For Congress and Senate, the enacted and alternative plans are comparable; at the House level, the alternative plan has more double-bunking. Note: These numbers were calculated using incumbent addresses that I understand were provided by the Legislative Defendants.

### 4.2 Swing districts and competitive contests

Another way to understand the electoral properties of districting plans is to investigate how many districts always give the same partisan result over a suite of observed electoral conditions, and how many districts can "swing" between the parties. Figure 6 compares the six plans across the up-ballot elections. The enacted plans lock in large numbers of always-Republican seats. In the Senate and House, nearly half the seats are locked down for Republicans. In the Congressional plan, it's well over half. This provides another view from which the NCLCV plans provide attractive alternatives.


Figure 6: These visuals show the breakdown of seats that always have a Republican winner, always have a Democratic winner, or are sometimes led by each party across the 14 up-ballot elections over the previous Census cycle. The 50-50 split is marked.

In interpreting this visualization, note that this is consistent with the discussion elsewhere of entrenched Republican majorities in the enacted maps. These Always-Republican districts provide a floor for Republican performance from the viewpoint of these up-ballot contests.

One more measure of partisan fairness, frequently referenced in the public discourse, is the tendency of a districting plan to promote close or competitive contests. We close with a comparison of the enacted and alternative plans that displays the number of times across the full dataset of 52 elections that a contest had a partisan margin of closer than 10 points, 6 points, or 2 points, respectively. This can occur up to $14 \cdot 52=728$ times in Congressional maps, $50 \cdot 52=2600$ times in state Senate maps, and $120 \cdot 52=6240$ times in state House maps. The figures below show horizontal rules at every $10 \%$ interval of the total number of possible competitive contests; we can see, for instance, that the alternative Congressional plan has contests within a 10 -point margin more than $40 \%$ of the time.

Competitive contests in the Congressional plans


Senate plans


House plans


Figure 7: These bar graphs show the number of competitive contests for the enacted plans (maroon) and the alternative plans (green). In each plot, we consider increasingly restrictive definitions of "competitive" from left to right, counting districts in which the major-party vote split is closer than 45-55, 47-53, and 49-51, respectively.

## 5 Location-specific comparison of electoral opportunity

I received information reflecting the residential locations of 147 individuals, who come from either of two groups:

- plaintiffs in the NCLCV v. Hall case; or
- registered voters belonging to the NCLCV membership who are Black and/or are registered as Democrats.

In Table 10 below, I summarize the impact on the identified individuals in terms of electoral opportunity if the enacted maps are compared to the alternative maps.

Subsequently, Figures 8 and 9 provide a visualization that pinpoints the geographical sites where the alternative plans improve electoral opportunities for plaintiffs and NCLCV membersthat is, places where the identified individuals (as Democrats and/or Black voters) have measurably greater ability to elect their candidates of choice under the alternative plans than under the existing plans.

This is backed up by the data in Tables 11-13 below, which identify the district numbers in the six enacted and alternative plans for each of these identified individuals. The district numbers were computed using census block information to specify the locations, but the table reports the locations by larger units (VTDs) in order to protect privacy.

## Lost opportunity for Democratic and Black voters

|  | greater Democratic opportunity <br> in alternative plan than enacted plan |
| :---: | :---: |
| Congress | 51 individuals |
| Senate | 37 individuals |
| House | 39 individuals |

resides in effective district in alternative plan but not enacted plan

| Congress | 28 Black voters |
| :--- | :--- |
| Senate | 21 Black voters |
| House | 21 Black voters |

Table 10: Of the 147 identified individuals, how many saw a change in their opportunity for Democratic representation? How many Black voters saw a change in their opportunity to elect Black candidates of choice?


Figure 8: Locations where identified individuals have less opportunity to be represented by a Democrat in Congress, state Senate, and state House under the enacted plans. The shading indicates the drop in Democratic wins across the 14 up-ballot races in the enacted map relative to the alternative map. There are 51 such individuals in the Congressional maps, 37 in the Senate maps, and 31 in the House maps.


Figure 9: Locations where Black voters from the identified individuals list would be in a district that provides effective electoral opportunity under the alternative plan, but not under the enacted plan. There are 28 such voters at the Congressional level and 21 at each of the Senate and House level.

| VTD Census ID | VTD/Precinct Name | SL-174 | NCLCV-Cong | SL-173 | NCLCV-Sen | SL-175 | NCLCV-House |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 37025001-07 | 01-07 | 10 | 10 | 34 | 34 | 73 | 73 |
| 37025012-03 | 12-03 | 10 | 10 | 34 | 34 | 82 | 82 |
| 37025002-07 | 02-07 | 10 | 10 | 34 | 34 | 83 | 73 |
| 37009000002 | CLIFTON | 11 | 12 | 47 | 47 | 93 | 93 |
| 37063000029 | GLENN ELEMENTARY | 6 | 2 | 22 | 22 | 2 | 2 |
| 37063000043 | FOREST VIEW ELEMENTARY | 6 | 6 | 22 | 20 | 30 | 30 |
| 37063000052 | EVANGEL ASSEMBLY OF GOD | 6 | 2 | 22 | 22 | 31 | 31 |
| 37063055-11 | 055-11 | 6 | 6 | 20 | 22 | 29 | 29 |
| 37071000012 | FLINT GROVES | 13 | 13 | 43 | 43 | 108 | 108 |
| 37071000004 | FOREST HEIGHTS | 13 | 13 | 43 | 43 | 109 | 109 |
| 37057000076 | THOMASVILLE 1076 | 7 | 8 | 30 | 30 | 80 | 80 |
| 371350000 FF | EFLAND | 6 | 6 | 23 | 23 | 50 | 50 |
| 371050000A2 | A2 | 7 | 7 | 12 | 12 | 51 | 54 |
| 37131NEWTOW | NEWTOWN | 2 | 2 | 1 | 1 | 27 | 27 |
| 371350000CF | CEDAR FALLS | 6 | 6 | 23 | 23 | 56 | 56 |
| 37081000 H 25 | H25 | 10 | 11 | 27 | 27 | 62 | 60 |
| 37093000061 | RAEFORD 1 | 8 | 4 | 24 | 24 | 48 | 48 |
| 37081000RC2 | RC2 | 7 | 11 | 26 | 26 | 59 | 59 |
| 3712700P15A | OAK LEVEL | 2 | 2 | 11 | 11 | 25 | 25 |
| 3707700 TYHO | OOTYHO | 2 | 2 | 13 | 13 | 32 | 32 |
| 370910000CO | COFIELD | 2 | 1 | 1 | 1 | 5 | 5 |
| 37057000038 | EASTSIDE 38 | 7 | 8 | 30 | 30 | 81 | 81 |
| 370210021.1 | HAW CREEK ELEMENTARY SCHOOL | 14 | 14 | 49 | 49 | 115 | 114 |
| 37019000015 | GRISSETTOWN | 3 | 3 | 8 | 8 | 17 | 19 |
| 37047000P15 | TATUM | 3 | 3 | 8 | 8 | 46 | 46 |
| 37019000002 | LELAND | 3 | 3 | 8 | 8 | 17 | 17 |
| 370450CASAR | CASAR | 13 | 13 | 44 | 44 | 110 | 111 |
| 370210007.1 | KENILWORTH PRESBYTE- RIAN CHURCH | 14 | 14 | 49 | 49 | 114 | 115 |
| 370210053.1 | LEICESTER 2 - COMMUNITY CENTER | 14 | 14 | 46 | 49 | 116 | 116 |
| 370210054.2 | LUTHERAN CHURCH OF THE NATIVITY | 14 | 14 | 49 | 49 | 116 | 115 |
| 37193000108 | FAIRPLAINS | 11 | 12 | 36 | 36 | 94 | 94 |
| 37173000 BC 2 | BC2 | 14 | 14 | 50 | 47 | 119 | 119 |
| 37119000054 | 54 | 9 | 9 | 40 | 42 | 102 | 112 |
| 37119000108 | 108 | 9 | 9 | 40 | 40 | 100 | 100 |
| 37119000208 | 208 | 13 | 10 | 37 | 38 | 98 | 98 |
| 371190204.1 | 204.1 | 9 | 10 | 40 | 40 | 99 | 106 |
| 37119000097 | 97 | 9 | 9 | 42 | 39 | 112 | 105 |
| 37119000222 | 222 | 9 | 9 | 38 | 39 | 101 | 101 |
| 37097000516 | STATESVILLE 6 | 12 | 10 | 37 | 37 | 84 | 84 |
| 370970 DV1-B | DAVIDSON 1-B | 10 | 10 | 37 | 37 | 95 | 95 |
| 37119000048 | 48 | 9 | 9 | 42 | 42 | 88 | 104 |
| 37119000216 | 216 | 8 | 9 | 41 | 41 | 103 | 99 |
| 37081000G27 | G27 | 11 | 11 | 28 | 28 | 57 | 57 |
| 37081000G43 | G43 | 11 | 11 | 27 | 28 | 58 | 62 |
| 37153000006 | WOLF PIT 3 | 8 | 4 | 29 | 29 | 52 | 52 |
| 371570000 MS | MOSS STREET | 11 | 6 | 26 | 26 | 65 | 65 |
| 3716300ROWA | ROWAN | 4 | 4 | 9 | 9 | 22 | 22 |
| 3719500PRWI | WILSON I | 2 | 2 | 4 | 4 | 24 | 24 |
| 37119000206 | 206 | 13 | 10 | 37 | 37 | 98 | 98 |
| 37119000236 | 236 | 8 | 10 | 41 | 40 | 103 | 99 |

Table 11: Locations of identified individuals, Part 1 of 3 . For each location, the district numbers are given for the six plans discussed here. VTDs are listed rather than the more precise census block in order to protect privacy. Rows highlighted blue indicate individuals who lose Democratic opportunity in at least one of the enacted plans, relative to the alternative plans. Rows highlighted orange indicate Black voters who lose the opportunity to be in an effective district for Black candidates of choice in at least one level. (As it turns out, every instance of lost opportunity for Black voters is also an instance of lost Democratic opportunity.)

| VTD Census ID | VTD/Precinct Name | SL-174 | NCLCV-Cong | SL-173 | NCLCV-Sen | SL-175 | NCLCV-House |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 37119000142 | 142 | 13 | 10 | 38 | 38 | 98 | 112 |
| 37081000G65 | G65 | 11 | 11 | 27 | 27 | 58 | 58 |
| 37081000G70 | G70 | 11 | 11 | 28 | 26 | 61 | 61 |
| 3708100 H 19 A | H19A | 10 | 11 | 27 | 27 | 60 | 60 |
| 3708100MON3 | MON3 | 11 | 11 | 26 | 28 | 59 | 57 |
| 37183015-01 | 15-01 | 5 | 7 | 17 | 14 | 37 | 38 |
| 37183019-17 | 19-17 | 5 | 5 | 18 | 18 | 39 | 66 |
| 37183001-31 | 01-31 | 5 | 5 | 15 | 15 | 11 | 33 |
| 37183012-02 | 12.02 | 7 | 7 | 17 | 17 | 37 | 37 |
| 37119000087 | 87 | 8 | 9 | 41 | 41 | 105 | 105 |
| 37119000068 | 68 | 9 | 9 | 42 | 41 | 104 | 100 |
| 371190223.1 | 223.1 | 13 | 9 | 39 | 39 | 101 | 101 |
| 37119000081 | 81 | 9 | 9 | 39 | 39 | 92 | 101 |
| 37119000237 | 237 | 9 | 10 | 38 | 40 | 106 | 106 |
| 37119000127 | 127 | 13 | 10 | 37 | 37 | 98 | 98 |
| 37191000014 | 14 | 2 | 1 | 4 | 4 | 4 | 10 |
| 37183005-01 | 05-01 | 6 | 7 | 16 | 16 | 41 | 41 |
| 37183020-09 | 20-09 | 6 | 7 | 16 | 17 | 36 | 36 |
| 37183004-18 | 04-18 | 6 | 7 | 16 | 16 | 49 | 11 |
| 37191000010 | 10 | 2 | 1 | 4 | 4 | 10 | 10 |
| 37183019-21 | 19-21 | 5 | 5 | 13 | 18 | 35 | 66 |
| 37183001-46 | 01-46 | 5 | 5 | 18 | 18 | 34 | 40 |
| 37183001-50 | 01-50 | 5 | 5 | 14 | 14 | 33 | 38 |
| 37183016-05 | 16-05 | 5 | 5 | 14 | 14 | 21 | 38 |
| 37119000145 | 145 | 9 | 10 | 38 | 38 | 107 | 107 |
| 37183008-03 | 08-03 | 5 | 5 | 15 | 15 | 40 | 49 |
| 37183017-05 | 17-05 | 5 | 5 | 14 | 18 | 38 | 40 |
| 37183013-09 | 13-09 | 5 | 5 | 18 | 18 | 66 | 66 |
| 370490000 N2 | FORT TOTTEN | 1 | 1 | 3 | 3 | 3 | 3 |
| 37049000002 | HAVELOCK | 1 | 1 | 3 | 3 | 13 | 13 |
| 37001000004 | MORTON | 7 | 6 | 25 | 25 | 64 | 63 |
| 37001000126 | BURLINGTON 6 | 7 | 6 | 25 | 25 | 63 | 64 |
| 3700100003 N | NORTH BOONE | 7 | 6 | 25 | 25 | 64 | 64 |
| 37001000124 | BURLINGTON 4 | 7 | 6 | 25 | 25 | 63 | 63 |
| 37165001-16 | 01-16/01 | 8 | 4 | 24 | 24 | 48 | 48 |
| 37067000063 | CASH ELEMENTARY SCHOOL | 12 | 12 | 31 | 32 | 75 | 75 |
| 37067000074 | MEADOWLARK MIDDLE SCHOOL | 12 | 12 | 31 | 31 | 74 | 74 |
| 37067000709 | WARD ELEMENTARY SCHOOL | 12 | 12 | 32 | 31 | 74 | 71 |
| 37067000065 | KERNERSVILLE 7TH DAY ADVENTIST CHURCH | 12 | 12 | 31 | 32 | 75 | 75 |
| 37067000507 | SEDGE GARDEN REC CTR | 12 | 11 | 32 | 32 | 71 | 75 |
| 371510000 AE | ASHEBORO EAST | 7 | 11 | 29 | 29 | 70 | 70 |
| 37067000905 | BETHABARA MORAVIAN CH | 12 | 12 | 32 | 31 | 91 | 72 |
| 37067000402 | FOURTEENTH STREET REC | 12 | 11 | 32 | 32 | 72 | 72 |
| 370890000FR | FLAT ROCK | 14 | 14 | 48 | 48 | 113 | 117 |
| $3708900 \mathrm{HV}-1$ | HENDERSONVILLE-1 | 14 | 14 | 48 | 48 | 117 | 117 |
| 37023000039 | MORGANTON 09 | 13 | 13 | 46 | 46 | 86 | 86 |
| 3710900 LB34 | LABORATORY | 12 | 13 | 44 | 46 | 97 | 97 |
| 3706100WARS | WARSAW | 3 | 4 | 9 | 9 | 4 | 4 |
| 3712900CF01 | CF01 | 3 | 3 | 8 | 7 | 18 | 17 |
| 370130BELHV | BELHAVEN | 1 | 1 | 3 | 3 | 79 | 1 |

Table 12: Locations of identified individuals, Part 2 of 3 . For each location, the district numbers are given for the six plans discussed here. VTDs are listed rather than the more precise census block in order to protect privacy. Rows highlighted blue indicate individuals who lose Democratic opportunity in at least one of the enacted plans, relative to the alternative plans. Rows highlighted orange indicate Black voters who lose the opportunity to be in an effective district for Black candidates of choice in at least one level. (As it turns out, every instance of lost opportunity for Black voters is also an instance of lost Democratic opportunity.)

| VTD Census ID | VTD/Precinct Name | SL-174 | NCLCV-Cong | SL-173 | NCLCV-Sen | SL-175 | NCLCV-House |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 37037NWM117 | NORTH WILLIAMS | 7 | 7 | 20 | 20 | 54 | 54 |
| 3714100CL05 | COLUMBIA | 3 | 3 | 9 | 9 | 16 | 16 |
| $3713300 \mathrm{BM08}$ | BRYNN MARR | 1 | 3 | 6 | 6 | 14 | 15 |
| 3713300NR02 | NEW RIVER | 1 | 3 | 6 | 6 | 15 | 15 |
| 37051SL78-3 | Spring Lake 3 | 4 | 4 | 21 | 21 | 42 | 44 |
| 3705100G10A | STONEY POINT 2-G10 | 4 | 4 | 19 | 19 | 45 | 45 |
| 37051000G1A | CROSS CREEK 02-G1 | 4 | 4 | 19 | 19 | 43 | 42 |
| 37035000035 | SWEETWATER | 12 | 13 | 45 | 45 | 96 | 96 |
| 37035000032 | SOUTH NEWTON | 12 | 13 | 45 | 45 | 89 | 89 |
| 3705100 CC 32 | CROSS CREEK 32 | 4 | 4 | 19 | 19 | 44 | 44 |
| 37059000007 | JERUSALEM | 10 | 8 | 30 | 30 | 77 | 77 |
| 3708500PR01 | ANDERSON CREEK | 4 | 7 | 12 | 12 | 6 | 6 |
| $3708500 P R 07$ | BARBECUE | 4 | 7 | 12 | 12 | 6 | 6 |
| $371070000 \mathrm{K8}$ | KINSTON-8 | 1 | 1 | 3 | 3 | 12 | 12 |
| 37189000009 | ELK | 14 | 12 | 47 | 47 | 87 | 93 |
| 371170000 BG | BEAR GRASS | 2 | 1 | 2 | 1 | 23 | 23 |
| 371010 PR 12 B | NORTH CLEVELAND 2 | 4 | 2 | 10 | 10 | 26 | 26 |
| 371010PR31B | SOUTHWEST CLEVELAND | 4 | 2 | 10 | 10 | 53 | 53 |
| 3710100PR24 | EAST SELMA | 4 | 2 | 10 | 10 | 28 | 28 |
| 3714701102 A | SIMPSON A | 1 | 1 | 5 | 5 | 9 | 8 |
| 37167000003 | ALBEMARLE NUMBER 3 | 8 | 8 | 33 | 33 | 67 | 67 |
| 3700700 LILE | LILESVILLE | 8 | 8 | 29 | 29 | 55 | 55 |
| $3704500 \mathrm{KM}-\mathrm{N}$ | KM N | 13 | 13 | 44 | 44 | 111 | 110 |
| 37143BETHEL | BETHEL | 1 | 1 | 1 | 2 | 1 | 1 |
| 37147000601 | CHICOD | 1 | 1 | 5 | 5 | 9 | 9 |
| 37147001201 | PACTOLUS | 1 | 1 | 5 | 5 | 8 | 8 |
| 37159000040 | NORTH WARD | 10 | 8 | 33 | 33 | 76 | 76 |
| 3712900 FP04 | FP04 | 3 | 3 | 7 | 8 | 19 | 20 |
| 37129000 W 16 | W16 | 3 | 3 | 7 | 7 | 20 | 18 |
| 37129000 H 11 | H11 | 3 | 3 | 7 | 7 | 18 | 20 |
| $37129000 \mathrm{HO2}$ | H02 | 3 | 3 | 7 | 7 | 20 | 20 |
| 37159000036 | SOUTH WARD | 10 | 8 | 33 | 33 | 76 | 76 |
| 37125000DHR | DEEP FALLS/RITTER RIVER/HIGH | 8 | 7 | 21 | 21 | 78 | 51 |
| 37069000015 | EAST FRANKLINTON | 2 | 2 | 11 | 11 | 7 | 7 |
| 3719908 -CRA | CRABTREE | 14 | 14 | 47 | 47 | 85 | 85 |
| $3719700 E B N D$ | EAST BEND | 12 | 12 | 36 | 31 | 77 | 77 |
| 37171000018 | MT AIRY 8 | 11 | 12 | 36 | 36 | 90 | 90 |
| $3708700 \mathrm{WS}-2$ | WAYNESVILLE SOUTH 2 | 14 | 14 | 50 | 50 | 118 | 118 |
| 3715500005 A | FAIRMONT | 3 | 4 | 24 | 24 | 46 | 47 |
| 37155000028 | RENNERT | 3 | 4 | 24 | 24 | 47 | 47 |
| 37113000011 | SMITHBRIDGE | 14 | 14 | 50 | 50 | 120 | 120 |
| 3714500WDSD | WOODSDALE | 2 | 6 | 23 | 23 | 2 | 2 |
| 3717900029A | SHILOH ELEMENTARY SCHOOL | 8 | 8 | 35 | 35 | 68 | 69 |
| 3717900037 A | NEXT LEVEL CHURCH | 8 | 8 | 35 | 35 | 69 | 69 |
| 37169000017 | WEST WALNUT COVE | 11 | 12 | 31 | 36 | 91 | 91 |
| 37185000007 | SHOCCO | 2 | 2 | 2 | 1 | 27 | 27 |
| 37185000013 | NORLINA | 2 | 2 | 2 | 1 | 27 | 27 |

Table 13: Locations of identified individuals, Part 3 of 3 . For each location, the district numbers are given for the six plans discussed here. VTDs are listed rather than the more precise census block in order to protect privacy. Rows highlighted blue indicate individuals who lose Democratic opportunity in at least one of the enacted plans, relative to the alternative plans. Rows highlighted orange indicate Black voters who lose the opportunity to be in an effective district for Black candidates of choice in at least one level. (As it turns out, every instance of lost opportunity for Black voters is also an instance of lost Democratic opportunity.)

## References

[1] Assaf Bar-Natan, Lorenzo Najt, and Zachary Schutzmann, The gerrymandering jumble: map projections permute districts' compactness scores. Cartography and Geographic Information Science, Volume 47, Issue 4, 2020, 321-335.
[2] Richard Barnes and Justin Solomon, Gerrymandering and Compactness: Implementation Flexibility and Abuse. Political Analysis, Volume 29, Issue 4, October 2021, 448-466.
[3] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, Computational redistricting and the Voting Rights Act. Election Law Journal.
Available at https://www.liebertpub.com/doi/epdf/10.1089/elj.2020.0704
[4] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, NC General Assembly County Clusterings from the 2020 Census. https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf
[5] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, Locating the Representational Baseline: Republicans in Massachusetts. Election Law Journal, Volume 18, Number 4, 2019, 388-401.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 23 day of December, 2021.


Professor Moon Duchin

Sworn and sybscribed before me this the 23 of December, 2021


Name: Shannan C Peterson
My commission expires: $05 / 10 / 2025$


[^0]:    ${ }^{1}$ Many authors have used this technique of overlaying "exogenous" statewide elections rather than using statistical regressions and other modeling to manipulate "endogenous" districted elections. For instance this can be found in peer-reviewed work and expert reports of scholar-practitioners such as Bernard Grofman and Steven Ansolabehere.
    ${ }^{2}$ The backup data supporting Table 1 is attached to this report as Appendix $C$ and I understand that it will be provided to the court in native format.

[^1]:    ${ }^{3}$ A detailed discussion of the inadequacy of using demographics alone as a proxy can be found in [3].
    ${ }^{4}$ Of the candidates above, Sutton, Williams, Coleman, Colley, and Blue are themselves Black-identified.
    ${ }^{5}$ I have used statewide ecological inference ("EI") runs to determine the candidate of choice for Black voters. I note that it is also possible to run El on smaller geographies (such as counties or county clusters) to detect regional candidates of choice rather than statewide candidates of choice; in most cases, these will be the same, but in some cases, regional effects may be meaningful and could affect these results at the margin.

[^2]:    ${ }^{6}$ A complete set of solutions is described in detail in the white paper of Mattingly et al.-though with the important caveat that the work "does not reflect... compliance with the Voting Rights Act" [4]. Absent a VRA conflict, the 2020 Decennial Census population data dictates that the North Carolina Senate plan must be decomposed into ten singledistrict fixed clusters and seven multi-district fixed clusters (comprising 2, 2, 3, 3, 4, 6, and 6 districts, respectively). It has four more areas in which there is a choice of groupings. In all, there are sixteen different possible clusterings for Senate, each comprising 26 county clusters. The House likewise has 11 single-district fixed clusters and 22 multidistrict fixed clusters (with two to thirteen districts per cluster), together with three more areas with a choice of groupings. In all, the House has only eight acceptable clusterings, each comprising 40 county clusters. Again, it is important to note that VRA compliance may present a compelling reason to select some clusterings and reject others.

