STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
No. 21 CVS 015426
No. 21 CVS 500085

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC., *et al.*,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, IN HIS OFFICIAL CAPACITY AS SENIOR CHAIR OF THE HOUSE STANDING COMMITTEE ON REDISTRICTING, *et al.*,

Defendants.

REBECCA HARPER, *et al.*,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, IN HIS OFFICIAL CAPACITY AS SENIOR CHAIR OF THE HOUSE STANDING COMMITTEE ON REDISTRICTING, *et al.*,

Defendants.

COMMON CAUSE,

Plaintiff,

v.

REPRESENTATIVE DESTIN HALL, IN HIS OFFICIAL CAPACITY AS SENIOR CHAIR OF THE HOUSE STANDING COMMITTEE ON REDISTRICTING, *et al.*,

Defendants.

EXPERT REPORT OF DR. JOWEI CHEN

NCLCV v. Hall
21 CVS 15426
LDTX153

Exhibit #
CHEN 2
12/29/2021 - JNC

I, Dr. Jowei Chen, upon my oath, declare and say as follows:

1. I am over the age of eighteen (18) and competent to testify as to the matters set forth herein.

2. I am an Associate Professor in the Department of Political Science at the University of Michigan, Ann Arbor. I am also a Research Associate Professor at the Center for Political Studies of the Institute for Social Research at the University of Michigan and a Research Associate at the Spatial Social Science Laboratory at Stanford University. In 2007, I received a M.S. in Statistics from Stanford University, and in 2009, I received a Ph.D. in Political Science from Stanford University.

3. I have published academic papers on legislative districting and political geography in several political science journals, including The American Journal of Political Science and The American Political Science Review, and Election Law Journal. My academic areas of expertise include legislative elections, spatial statistics, geographic information systems (GIS) data, redistricting, racial politics, legislatures, and political geography. I have expertise in the use of computer simulations of legislative districting and in analyzing political geography, elections, and redistricting.

4. I have authored expert reports in the following redistricting court cases: *The League of Women Voters of Florida v. Detzner* (Fla. 2d Judicial Cir. Leon Cnty. 2012); *Romo v. Detzner* (Fla. 2d Judicial Cir. Leon Cnty. 2013); *Missouri National Association for the Advancement of Colored People v. Ferguson-Florissant School District & St. Louis County Board of Election Commissioners* (E.D. Mo. 2014); *Raleigh Wake Citizens Association v. Wake County Board of Elections* (E.D.N.C. 2015); *Brown v. Detzner* (N.D. Fla. 2015); *City of Greensboro v. Guilford County Board of Elections* (M.D.N.C. 2015); *Common Cause v. Rucho*

(M.D.N.C 2016); *The League of Women Voters of Pennsylvania v. Commonwealth of Pennsylvania* (No. 261 M.D. 2017); *Georgia State Conference of the NAACP v. The State of Georgia* (N.D. Ga. 2017); *The League of Women Voters of Michigan v. Johnson* (E.D. Mich. 2017); *Whitford v. Gill* (W.D. Wis. 2018); *Common Cause v. Lewis* (N.C. Super. 2018); *Harper v. Lewis* (N.C. Super. 2019); *Baroody v. City of Quincy, Florida* (N.D. Fla. 2020*); McConchie v. Illinois State Board of Elections* (N.D. Ill. 2021). I have testified either at deposition or at trial in the following cases: *Romo v. Detzner* (Fla. 2d Judicial Cir. Leon Cnty. 2013); *Missouri National Association for the Advancement of Colored People v. Ferguson-Florissant School District & St. Louis County Board of Election Commissioners* (E.D. Mo. 2014); *Raleigh Wake Citizens Association v. Wake County Board of Elections* (E.D.N.C. 2015); *City of Greensboro v. Guilford County Board of Elections* (M.D.N.C. 2015); *Common Cause v. Rucho* (M.D.N.C. 2016); *The League of Women Voters of Pennsylvania v. Commonwealth of Pennsylvania* (No. 261 M.D. 2017); *Georgia State Conference of the NAACP v. The State of Georgia* (N.D. Ga. 2017); *The League of Women Voters of Michigan v. Johnson* (E.D. Mich. 2017); *Whitford v. Gill* (W.D. Wis. 2018); *Common Cause v. Lewis* (N.C. Super. 2018); *Baroody v. City of Quincy, Florida* (N.D. Fla. 2020); *McConchie v. Illinois State Board of Elections* (N.D. Ill. 2021).

5. I have been retained by Plaintiffs in the above-captioned matter. I am being compensated $550 per hour for my work in this case.

6. Plaintiffs' counsel asked me to analyze the SB 740 districting plan for North Carolina's congressional districts (the "Enacted Plan"), as passed on November 4, 2021. Plaintiffs' counsel asked me to produce a set of computer-simulated plans for North Carolina's congressional districts by following the criteria adopted by the North Carolina General Assembly's Joint Redistricting Committee on August 12, 2021 (the "Adopted Criteria").

Plaintiffs' counsel asked me to compare the district-level partisan attributes of the Enacted Plan to those of the computer-simulated plans and to identify any districts in the Enacted Plan that are partisan outliers. Plaintiffs' counsel also asked me to compare the partisan composition of the individual Plaintiffs' congressional districts under the Enacted Plan to the partisan composition of Plaintiffs' districts under the computer-simulated plans and to identify any Plaintiffs whose Enacted Plan districts are partisan outliers.

7. The Use of Computer-Simulated Districting Plans: In conducting my academic research on legislative districting, partisan and racial gerrymandering, and electoral bias, I have developed various computer simulation programming techniques that allow me to produce a large number of nonpartisan districting plans that adhere to traditional districting criteria using US Census geographies as building blocks. This simulation process ignores all partisan and racial considerations when drawing districts. Instead, the computer simulations are programmed to draw districting plans following various traditional districting goals, such as equalizing population, avoiding county and Voting Tabulation District (VTD) splits, and pursuing geographic compactness. By randomly generating a large number of districting plans that closely adhere to these traditional districting criteria, I am able to assess an enacted plan drawn by a state legislature and determine whether partisan goals motivated the legislature to deviate from these traditional districting criteria. More specifically, by holding constant the application of nonpartisan, traditional districting criteria through the simulations, I am able to determine whether the enacted plan could have been the product of something other than partisan considerations. With respect to North Carolina's 2021 Congressional Enacted Plan, I determined that it could not.

8. I produced a set of 1,000 valid computer-simulated plans for North Carolina's congressional districts using a computer algorithm programmed to strictly follow the required districting criteria enumerated in the August 12, 2021 Adopted Criteria of the General Assembly's Joint Redistricting Committee. In following these Adopted Criteria, the computer algorithm uses the same general approach that I employed in creating the simulated state House and state Senate plans that I analyzed in *Common Cause v. Lewis* (2019) and the simulated congressional plans that I used in *Harper v. Lewis* (2019).

9. By randomly drawing districting plans with a process designed to strictly follow nonpartisan districting criteria, the computer simulation process gives us an indication of the range of districting plans that plausibly and likely emerge when map-drawers are not motivated primarily by partisan goals. By comparing the Enacted Plan against the distribution of simulated plans with respect to partisan measurements, I am able to determine the extent to which a map-drawer's subordination of nonpartisan districting criteria, such as geographic compactness and preserving precinct boundaries, was motivated by partisan goals.

10. These computer simulation methods are widely used by academic scholars to analyze districting maps. For over a decade, political scientists have used such computer-simulated districting techniques to analyze the racial and partisan intent of legislative map-drawers.[1] In recent years, several courts have also relied upon computer simulations to assess partisan bias in enacted districting plans.[2]

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[1] *E.g.*, Carmen Cirincione, Thomas A. Darling, Timothy G. O'Rourke. "Assessing South Carolina's 1990s Congressional Districting," Political Geography 19 (2000) 189–211; Jowei Chen, "The Impact of Political Geography on Wisconsin Redistricting: An Analysis of Wisconsin's Act 43 Assembly Districting Plan." Election Law Journal.

[2] *See, e.g.*, *League of Women Voters of Pa. v. Commonwealth*, 178 A. 3d 737, 818-21 (Pa. 2018); *Raleigh Wake Citizens Association v. Wake County Board of Elections*, 827 F.3d 333, 344-45 (4th Cir. 2016); *City of Greensboro v. Guilford County Board of Elections*, No. 1:15-CV-599, 2017 WL 1229736 (M.D.N.C. Apr 3, 2017); *Common Cause v. Rucho*, No. 1:16-CV-1164 (M.D.N.C. Jan 11, 2018); *The League of Women Voters of Michigan v. Johnson* (E.D. Mich. 2017); *Common Cause v. David Lewis* (N.C. Super. 2018).

11. Redistricting Criteria: I programmed the computer algorithm to create 1,000 independent simulated plans adhering to the following seven districting criteria, as specified in the Adopted Criteria[3]:

a) Population Equality[4]: Because North Carolina's 2020 Census population was 10,439,388, districts in every 14-member congressional plan have an ideal population of 745,670.6. Accordingly, the computer simulation algorithm populated each districting plan such that precisely six districts have a population of 745,670, while the remaining eight districts have a population of 745,671.

b) Contiguity[5]: The simulation algorithm required districts to be geographically contiguous. Water contiguity is permissible. I also programmed the simulation algorithm to avoid double-traversals within a single county. In other words, for every simulated district, the portion of that district within any given county will be geographically contiguous.

c) Minimizing County Splits[6]: The simulation algorithm avoided splitting any of North Carolina's 100 counties, except when doing so is necessary to avoid violating one of the aforementioned criteria. When a county is divided into two districts, the county is considered to have one split. A county divided into three districts is considered to have two splits. A county divided into four districts is considered to have

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[3] Since my November 30 report, I made the following changes to the computer simulation algorithm. First, I added additional code at the conclusion of the algorithm that checks for the occurrence of double traversals. The computer is instructed to automatically reject any simulated plan that contains a double traversal. Second, the algorithm now contains several steps that further increase the preservation of municipal boundaries, discussed further below.

[4] The Adopted Criteria state: "The number of persons in each congressional district shall be as nearly as equal as practicable, as determined under the most recent federal decennial census."

[5] The Adopted Criteria state: "No point contiguity shall be permitted in any 2021 Congressional, House, and Senate plan. Congressional, House, and Senate districts shall be compromised of contiguous territory. Contiguity by water is sufficient."

[6] The Adopted Criteria state: "Division of counties in the 2021 Congressional plan shall only be made for reasons of equalizing population and consideration of double bunking."

three splits, and so on. For the purpose of creating equally populated districts, each newly drawn congressional district requires only one county split. But the fourteenth and final district drawn in North Carolina does need not create an additional county split, since this final district should simply be the remaining area unassigned to the first thirteen districts. Therefore, an entire plan of 14 congressional districts requires only 13 county splits. Accordingly, I require that every simulated plan contain only 13 county splits. The 2021 Adopted Criteria do not prohibit splitting a county more than once, so I allow some of these 13 county splits to occur within the same county. As a result, the total number of counties containing one or more splits may be fewer than 13. The algorithm also follows the Adopted Criteria in that it draws a congressional district wholly within Mecklenburg and Wake counties, which each have sufficient population size to contain an entire congressional district within their boundaries.

d) Minimizing VTD Splits[7]: North Carolina is divided into 2,666 VTDs. The computer simulation algorithm attempted to keep these VTDs intact and not split them into multiple districts, except when doing so is necessary for creating equally populated districts. For the purpose of creating equally populated districts, each newly drawn congressional district requires one VTD split. But the fourteenth and final district drawn in North Carolina does need not create an additional VTD split, since this final district should simply be the remaining area unassigned to the first thirteen districts. Therefore, an entire plan of 14 congressional districts requires only 13 VTD splits. I therefore require that every simulated plan split only 13 VTDs in total.

[7] The Adopted Criteria state: “Voting districts (‘VTDs’) should be split only when necessary.”

e) Geographic Compactness[8]: The simulation algorithm prioritized the drawing of geographically compact districts whenever doing so does not violate any of the aforementioned criteria.

f) Avoiding Incumbent Pairings: North Carolina's current congressional delegation includes two incumbents, Representatives Ted Budd and David Price, who announced before the Enacted Plan was adopted that they will not run for reelection in 2022. For the remaining eleven congressional incumbents, the simulation algorithm intentionally avoids pairing multiple incumbents in the same district. Hence, in every computer-simulated plan, each district contains no more than one incumbent's residence.

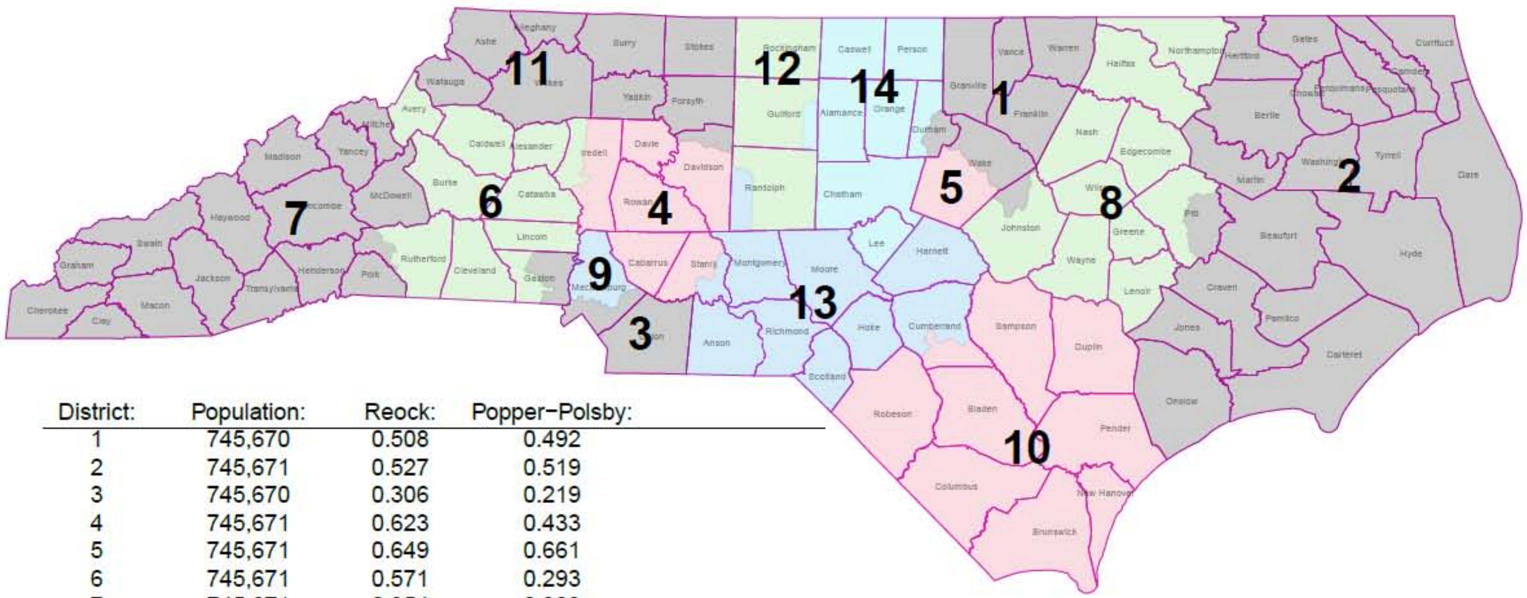
g) Municipal Boundaries[9]: The simulation algorithm generally favors not splitting municipalities. The algorithm contains several steps that favor the preservation of municipal boundaries, so long as other considerations required by the Adopted Criteria are not subordinated. To the extent that the algorithm avoids unnecessary splitting of counties, the municipalities within non-split counties are of course preserved. When the algorithm splits up a county by assigning the county's various VTDs to two different districts, the algorithm only allows one municipality to be split in this process of assigning the county's VTDs to different districts. Finally, as explained earlier, VTDs are only split when doing so is necessary for equalizing district populations. When a single VTD is split for this population equalization purpose, the algorithm attempts to split the VTD in such a way that minimizes the number of municipalities split within the VTD. In

[8] The Adopted Criteria state: "The Committees shall make reasonable efforts to draw legislative districts in the 2021 Congressional, House and Senate plans that are compact."
[9] The Adopted Criteria state: "The Committees may consider municipal boundaries when drawing districts in the 2021 Congressional, House, and Senate plans."

other words, the algorithm attempts to draw the district border within the VTD without crossing municipal boundaries.

12. On the following page of this report, Map 1 displays an example of one of the computer-simulated plans produced by the computer algorithm. The lower half of this Map also reports the population of each district, the compactness scores for each district, and the county splits and VTD splits created by the plan. As with every simulated plan, this plan contains exactly 13 VTD splits and 13 county splits, with 11 counties split into two or more districts.

Map 1:
Example of a Computer−Simulated Congressional Plan Protecting all 11 Incumbents



District:	Population:	Reock:	Popper−Polsby:
1	745,670	0.508	0.492
2	745,671	0.527	0.519
3	745,670	0.306	0.219
4	745,671	0.623	0.433
5	745,671	0.649	0.661
6	745,671	0.571	0.293
7	745,671	0.354	0.303
8	745,670	0.468	0.352
9	745,670	0.576	0.405
10	745,671	0.649	0.534
11	745,670	0.377	0.424
12	745,671	0.4	0.48
13	745,671	0.46	0.301
14	745,670	0.457	0.519
Plan Average:	745,670.6	0.495	0.424

13 Split Counties:
Alamance (Districts 12, 13)
Burke (Districts 10, 3)
Davie (Districts 2, 8)
Granville (Districts 1, 14)
Hoke (Districts 13, 6)
Mecklenburg (Districts 5, 9)
Nash (Districts 1, 11)
Orange (Districts 1, 13)
Pitt (Districts 11, 7)
Rockingham (Districts 12, 2)
Rowan (Districts 10, 8)
Rutherford (Districts 3, 9)
Wake (Districts 14, 4)

13 Split VTD's:
VTD 00008N in Alamance County (Districts 12 and 13)
VTD 000053 in Burke County (Districts 10 and 3)
VTD 000011 in Davie County (Districts 2 and 8)
VTD 00TYHO in Granville County (Districts 1 and 14)
VTD 000063 in Hoke County (Districts 13 and 6)
VTD 000018 in Mecklenburg County (Districts 5 and 9)
VTD 00P09A in Nash County (Districts 1 and 11)
VTD 0000CX in Orange County (Districts 1 and 13)
VTD 001301 in Pitt County (Districts 11 and 7)
VTD 0000LI in Rockingham County (Districts 12 and 2)
VTD 000033 in Rowan County (Districts 10 and 8)
VTD 000018 in Rutherford County (Districts 3 and 9)
VTD 008−03 in Wake County (Districts 14 and 4)

The Enacted Plan's Compliance with the Adopted Criteria

13. Although all seven of the criteria listed above are part of the General Assembly's Adopted Criteria, five of these criteria are ones that the Joint Redistricting Committee "shall" or "should" follow in the process of drawing its Congressional districting plan. These five mandated criteria are equal population, contiguity, minimizing county splits, minimizing VTD splits, and geographic compactness.[10]

14. I assessed whether the 2021 Enacted Plan complies with these five mandated criteria, and I describe my findings in this section. I found that the Enacted Plan does not violate the equal population requirement, nor do any of its districts violate contiguity.

15. However, by comparing the Enacted Plan to the 1,000 computer-simulated plans, I found that the Enacted Plan fails to minimize county splits, fails to minimize VTD splits, and is significantly less geographically compact than is reasonably possible. I describe these findings below in detail.

16. ***Minimizing County Splits:*** In comparing the total number of county splits in the Enacted Plan and in the computer-simulated plans, I counted the total number of times a county is split into more than one district. Specifically, a county fully contained within a single district counts as zero splits. A county split into two full or partial districts counts as one split. And a county split into three full or partial districts counts as two splits. And so on.

17. Using this standard method of accounting for total county splits, I found that the Enacted Plan contains 14 total county splits, which are detailed in Table 1. These 14 total county splits are spread across 11 counties. Eight of these 11 counties are split only once, but Guilford,

[10] In listing these five mandated criteria, I am not including the Adopted Criteria's prohibitions on the use of racial data, partisan considerations, and election results data. I did not assess whether the Enacted Plan complies with theprohibition on racial considerations.

Mecklenburg, and Wake Counties are each split into three districts, thus accounting for two splits each. Thus, the Enacted Plan has 14 total county splits, as listed in Table 1.

Table 1: Total Number of County Splits in the 2021 Enacted Plan

	County:	Congressional Districts:	Total County Splits:
1	Davidson	7 and 10	1
2	Guilford	7, 10, and 11	2
3	Harnett	4 and 7	1
4	Iredell	10 and 12	1
5	Mecklenburg	8, 9, and 13	2
6	Onslow	1 and 3	1
7	Pitt	1 and 2	1
8	Robeson	3 and 8	1
9	Wake	5, 6, and 7	2
10	Watauga	11 and 14	1
11	Wayne	2 and 4	1
Total County Splits:			**14**

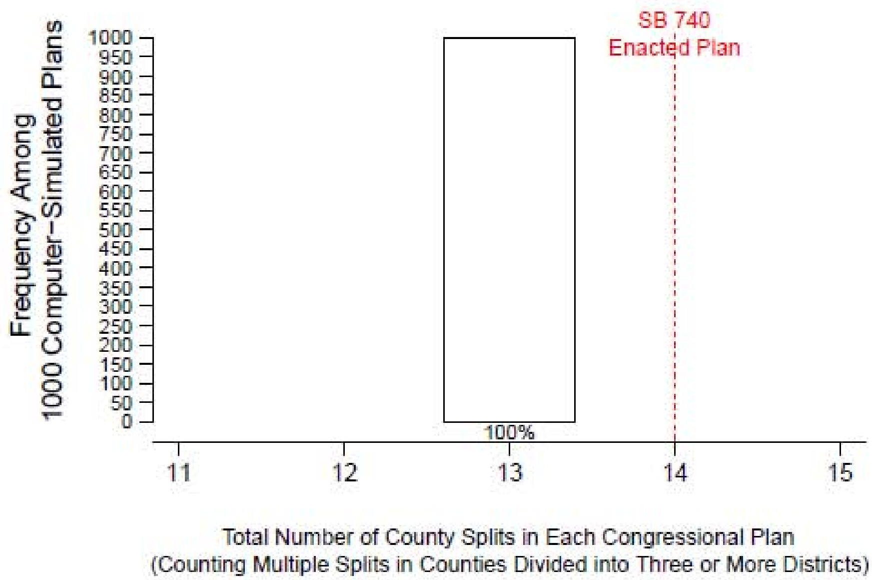
As explained in the previous section, a congressional plan in North Carolina needs to contain only 13 county splits if the map-drawer is attempting to minimize the splitting of counties. The Enacted Plan's 14 county splits is therefore one more split than is necessary. This "extra" split is specifically found at the border between District 7 and District 10. In general, the border between any two congressional districts in North Carolina needs to split only one county, at most. But in the Enacted Plan, the border between Districts 7 and 10 creates two county splits: One split of Davidson County and one split of Guilford County. Creating two county splits of Davidson and Guilford Counties was not necessary for equalizing district populations. Nor was it necessary for protecting incumbents, as no incumbents reside in the portions of Davidson and Guilford Counties within District 7 and District 10. Hence, the "extra" county split in Davidson and Guilford Counties does not appear to be consistent with the 2021 Adopted Criteria, which mandate that "Division of counties in the 2021 Congressional plan shall only be made for reasons of equalizing population and consideration of double bunking."

18. Indeed, I found that the computer simulation algorithm was always able to draw districts complying with the Adopted Criteria without using an "extra" 14th county split. As the upper half of Figure 1 illustrates, all 1,000 computer-simulated plans contain exactly 13 county splits. The Enacted Plan clearly contains more county splits than one would expect from a map-drawing process complying with the Adopted Criteria. Therefore, I conclude that the Enacted Plan does not comply with the Adopted Criteria's rule against unnecessary division of counties.

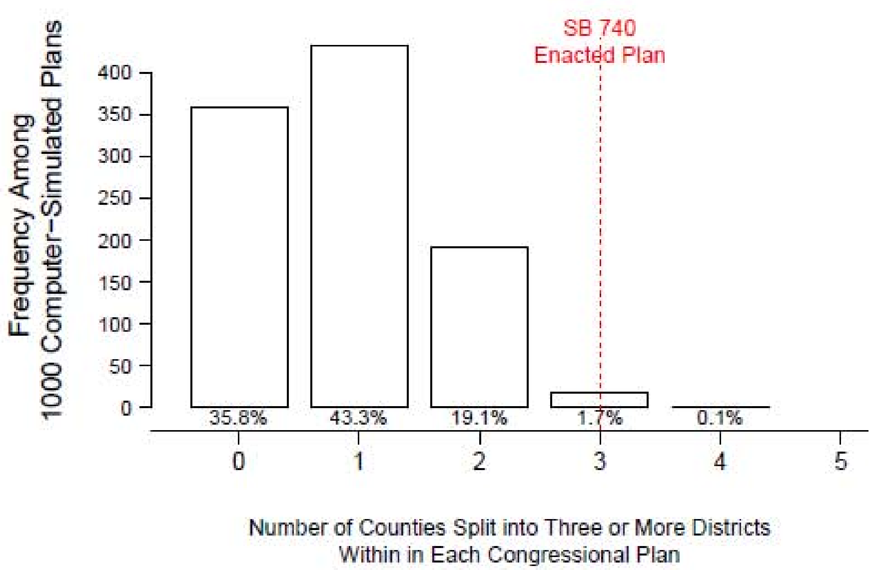
19. The Adopted Criteria do not explicitly limit the number of county splits within any single county. Nevertheless, it is notable that under the Enacted Plan, three different counties (Guilford, Mecklenburg, and Wake) are split multiple times. These three counties are each split into three districts under the Enacted Plan. This is an outcome that rarely occurs under the computer-simulated plans. As the lower half of Figure 1 illustrates, only 1.8% of the computer-simulated plans similarly split three or more counties multiple times. Thus, it is clear that the Enacted Plan's level of concentrating multiple county splits within a single county is an outcome that generally does not occur in a vast majority of the simulated plans drawn according to the Adopted Criteria. Additionally, not once in the small number of simulated plans that split at least three counties three ways are Guilford, Mecklenburg, and Wake Counties all split multiple times.

Figure 1:

Comparison of Total County Splits in Enacted SB 740 Plan and 1,000 Computer-Simulated Plans



Number of Counties Split Multiple Times in Enacted SB 740 Plan and 1,000 Computer-Simulated Plans



21. **Minimizing VTD Splits:** The Adopted Criteria mandates that "Voting districts ('VTDs') should be split only when necessary." As explained earlier in this report, each newly drawn congressional district needs to create only one VTD split for the purpose of equalizing the district's population. But the fourteenth and final district drawn in North Carolina does need not create an additional VTD split, since this final district should simply be the remaining area unassigned to the first 13 districts. Therefore, an entire plan of 14 congressional districts needs to create only 13 VTD splits.

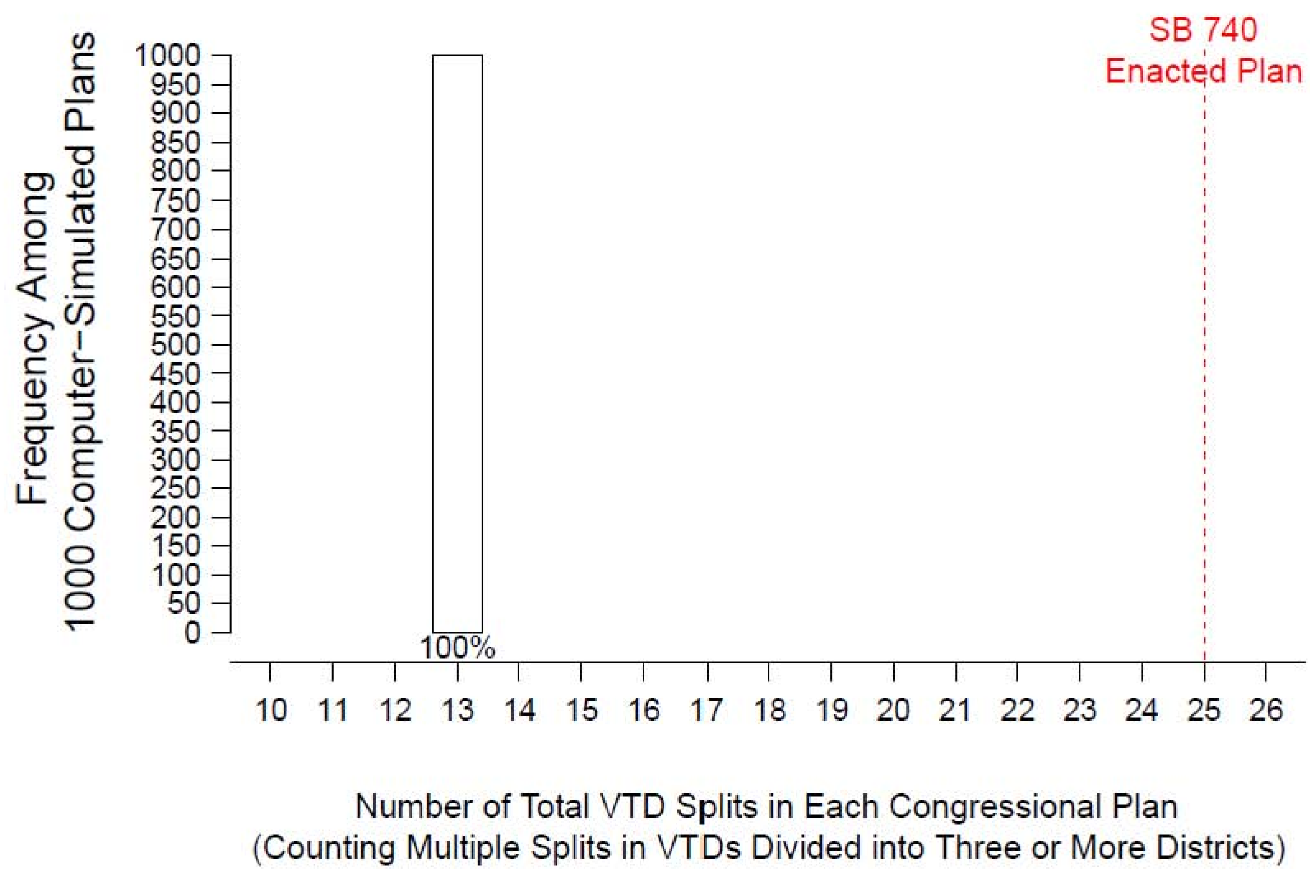
22. However, the Enacted Plan creates far more VTD splits than is necessary. As the General Assembly's "StatPack" Report[11] for the Enacted SB 740 Plan details, the Enacted Plan splits 24 VTDs into multiple districts. Among these 24 split VTDs, 23 VTDs are split into two districts, while one VTD (Wake County VTD 18-02) is split into three districts. Thus, using the same method of accounting for splits described earlier, the Enacted Plan contains 25 total VTD splits, and 24 VTDs are split into two or more districts.

23. The Enacted Plan's 25 total VTD splits is far more than is necessary to comply with the Adopted Criteria' equal population requirement. As explained earlier, only 13 VTD splits are necessary in order to produce an equally populated congressional plan in North Carolina. Thus, as Figure 2 illustrates, every one of the 1,000 computer-simulated plans contains exactly 13 VTD splits, and the Enacted Plan's 25 total VTD splits is clearly not consistent with the Adopted Criteria's requirement that "Voting districts ('VTDs') should be split only when necessary."

[11] Available at:
https://webservices.ncleg.gov/ViewBillDocument/2021/53447/0/SL%202021-174%20-%20StatPack%20Report.

Figure 2:

Comparison of Total VTD Splits in Enacted SB 740 Plan and 1,000 Computer−Simulated Plans



24. ***Measuring Geographic Compactness***: The August 12, 2021 Adopted Criteria mandates that the Joint Redistricting Committee "shall" attempt to draw geographically compact congressional districts. The Adopted Criteria also specify two commonly used measures of district compactness: the Reock score and the Polsby-Popper score.

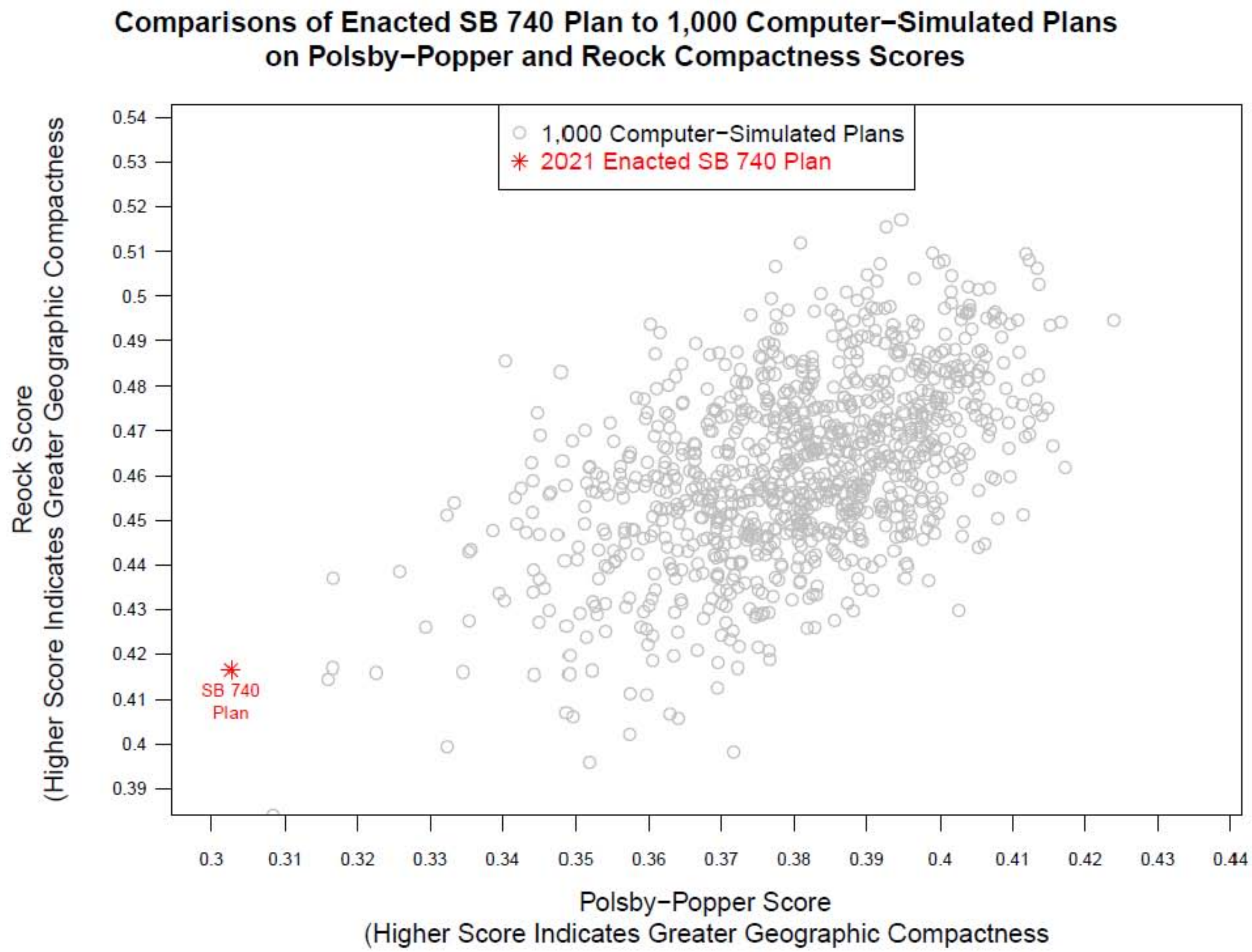
25. In evaluating whether the Enacted Plan follows the compactness requirement of the Adopted Criteria, it is useful to compare the compactness of the Enacted Plan and the 1,000 computer-simulated plans. The computer-simulated plans were produced by a computer algorithm adhering strictly to the traditional districting criteria mandated by the Adopted Criteria and ignoring any partisan or racial considerations. Thus, the compactness scores of these computer-simulated plans illustrate the statistical range of compactness scores that could be reasonably expected to emerge from a districting process that solely seeks to follow the Adopted Criteria while ignoring partisan and racial considerations. I therefore compare the compactness of the simulated plans and the Enacted Plan using the two measures of compactness specified by the 2021 Adopted Criteria.

26. First, I calculate the average Polsby-Popper score of each plan's districts. The Polsby-Popper score for each individual district is calculated as the ratio of the district's area to the area of a hypothetical circle whose circumference is identical to the length of the district's perimeter; thus, higher Polsby-Popper scores indicate greater district compactness. The 2021 Enacted Plan has an average Polsby-Popper score of 0.3026 across its 14 congressional districts. As illustrated in Figure 3, every single one of the 1,000 computer-simulated House plans in this report exhibits a higher Polsby-Popper score than the Enacted Plan. In fact, the middle 50% of these 1,000 computer-simulated plans have an average Polsby-Popper score ranging from 0.37 to 0.39, and the most compact computer-simulated plan has a Polsby-Popper score of 0.42. Hence,

it is clear that the Enacted Plan is significantly less compact, as measured by its Polsby-Popper score, than what could reasonably have been expected from a districting process adhering to the Adopted Criteria.

27. Second, I calculate the average Reock score of the districts within each plan. The Reock score for each individual district is calculated as the ratio of the district's area to the area of the smallest bounding circle that can be drawn to completely contain the district; thus, higher Reock scores indicate more geographically compact districts. The 2021 Enacted Plan has an average Reock score of 0.4165 across its 14 congressional districts. As illustrated in Figure 3, 98.2% of the 1,000 computer-simulated plans exhibit a higher Reock score than the Enacted Plan. In fact, the middle 50% of these 1,000 computer-simulated plans have an average Reock score ranging from 0.45 to 0.46, and the most compact computer-simulated plan has an average Reock score of 0.52. Hence, it is clear that the Enacted Plan is significantly less compact, as measured by its Reock score, than what could reasonably have been expected from a districting process adhering to the Adopted Criteria.

Figure 3:


Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
on Polsby−Popper and Reock Compactness Scores
1,000 Computer−Simulated Plans
2021 Enacted SB 740 Plan
SB 740
Plan
Reock Score
(Higher Score Indicates Greater Geographic Compactness
0.54
0.53
0.52
0.51
0.5
0.49
0.48
0.47
0.46
0.45
0.44
0.43
0.42
0.41
0.4
0.39
0.3
0.31
0.32
0.33
0.34
0.35
0.36
0.37
0.38
0.39
0.4
0.41
0.42
0.43
0.44
Polsby−Popper Score
(Higher Score Indicates Greater Geographic Compactness

Measuring the Partisanship of Districting Plans

28. In general, I use actual election results from recent, statewide election races in North Carolina to assess the partisan performance of the Enacted Plan and the computer-simulated plans analyzed in this report. Overlaying these past election results onto a districting plan enables me to calculate the Republican (or Democratic) share of the votes cast from within each district in the Enacted Plan and in each simulated plan. I am also able to count the total number of Republican and Democratic-leaning districts within each simulated plan and within the Enacted Plan. All of these calculations thus allow me to directly compare the partisanship of the Enacted Plan and the simulated plans. These partisan comparisons allow me to determine whether or not the partisanship of individual districts and the partisan distribution of seats in the Enacted Plan could reasonably have arisen from a districting process adhering to the Adopted Criteria and its explicit prohibition on partisan considerations. Past voting history in federal and statewide elections is a strong predictor of future voting history. Mapmakers thus can and do use past voting history to identify the class of voters, at a precinct-by-precinct level, who are likely to vote for Republican or Democratic congressional candidates.

29. In the 2011, 2016, and 2017 rounds of state legislative and congressional redistricting last decade, the North Carolina General Assembly publicly disclosed that it was relying solely on recent statewide elections in measuring the partisanship of the districting plans being created. I therefore follow the General Assembly's past practice from last decade by using results from a similar set of recent statewide elections in order to measure the partisanship of districts in the Enacted Plan and in the computer-simulated plans.

30. ***The 2016-2020 Statewide Election Composite:*** During the General Assembly's 2017 legislative redistricting process, Representative David Lewis announced at the Joint Redistricting Committee's August 10, 2017 meeting that the General Assembly would measure

the partisanship of legislative districts using the results from some of the most recent elections held in North Carolina for the following five offices: US President, US Senator, Governor, Lieutenant Governor, and Attorney General.

31. To measure the partisanship of all districts in the computer-simulated plans and the 2021 Enacted Plan, I used the two most-recent election contests held in North Carolina for these same five offices during 2016-2020. In other words, I used the results of the following ten elections: 2016 US President, 2016 US Senator, 2016 Governor, 2016 Lieutenant Governor, 2016 Attorney General, 2020 US President, 2020 US Senator, 2020 Governor, 2020 Lieutenant Governor, and 2020 Attorney General. I use these election results because these are the same state and federal offices whose election results were used by the General Assembly during its 2017 legislative redistricting process, and the 2017 redistricting process was the most recent one in which the leadership of the General Assembly's redistricting committees publicly announced how the General Assembly would evaluate the partisanship of its own districting plans.

32. I obtained precinct-level results for these ten elections, and I disaggregated these election results down to the census block level. I then aggregated these block-level election results to the district level within each computer-simulated plan and the Enacted Plan, and I calculated the number of districts within each plan that cast more votes for Republican than Democratic candidates. I use these calculations to measure the partisan performance of each simulated plan analyzed in this report and of the Enacted Plan. In other words, I look at the census blocks that would comprise a particular district in a given simulation and, using the actual election results from those census blocks, I calculate whether voters in that simulated district collectively cast more votes for Republican or Democratic candidates in the 2016-2020 statewide election contests. I performed such calculations for each district under each simulated plan to

measure the number of districts Democrats or Republicans would win under that particular simulated districting map.

33. I refer to the aggregated election results from these ten statewide elections as the "2016-2020 Statewide Election Composite." For the Enacted Plan districts and for all districts in each of the 1,000 computer-simulated plans, I calculate the percentage of total two-party votes across these ten elections that were cast in favor of Republican candidates in order to measure the average Republican vote share of the district. In the following section, I present district-level comparisons of the Enacted Plan and simulated plan districts in order to identify whether any individual districts in the Enacted Plan are partisan outliers. I also present plan-wide comparisons of the Enacted Plan and the simulated plans in order to identify the extent to which the Enacted Plan is a statistical outlier in terms of common measures of districting plan partisanship.

District-Level and Plan-Wide Partisan Comparisons of the Enacted Plan and Simulated Plans

34. In this section, I present partisan comparisons of the Enacted Plan to the computer-simulated plans at both a district-by-district level as well as a plan-wide level using several common measures of districting plan partisanship. First, I compare the district-level Republican vote share of the Enacted Plan's districts and the districts in the computer-simulatedplans. Next, I compare the number of Republican-favoring districts in the Enacted Plan and in the computer-simulated plans. Finally, I use several common measures of partisan bias to compare the Enacted Plan to the computer-simulated plans. Overall, I find that the several individual districts in the Enacted Plan are statistical outliers, exhibiting extreme partisan characteristics that are rarely or never observed in the computer-simulated plan districts drawn with strict adherence to the Adopted Criteria. Moreover, I find that at the plan-wide level, the

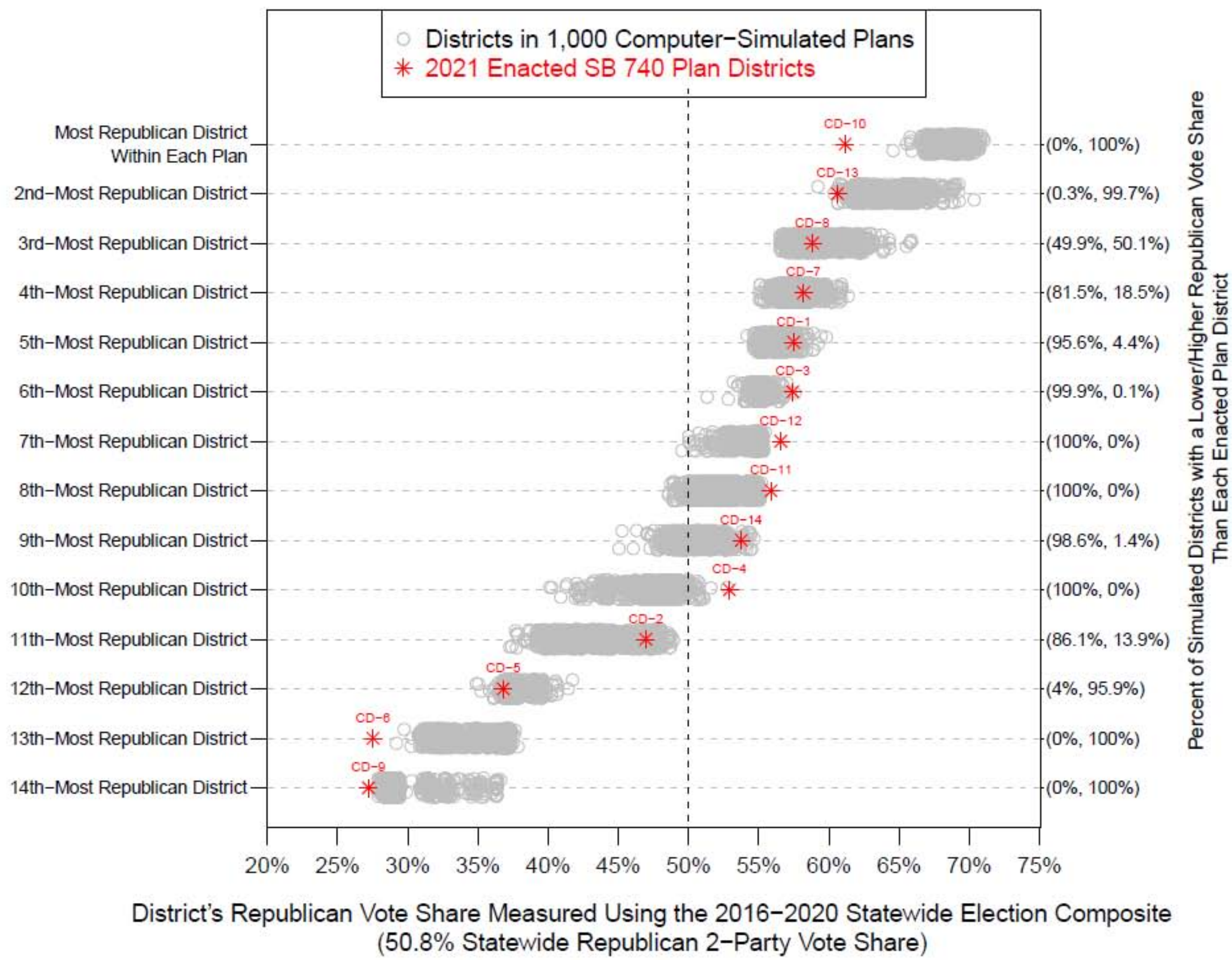
Enacted Plan creates a degree of partisan bias favoring Republicans that is more extreme than the vast majority of the computer-simulated plans. I describe these findings in detail below:

35. ***Partisan Outlier Districts in the Enacted Plan***: In Figure 4, I directly compare the partisan distribution of districts in the Enacted Plan to the partisan distribution of districts in the 1,000 computer-simulated plans. I first order the Enacted Plan's districts from the most to theleast-Republican district, as measured by Republican vote share using the 2016-2020 Statewide Election Composite. The most-Republican district appears on the top row, and the least- Republican district appears on the bottom row of Figure 4. Next, I analyze each of the 1,000 computer-simulated plans and similarly order each simulated plan's districts from the most- to the least-Republican district. I then directly compare the most-Republican Enacted Plan district (CD-10) to the most-Republican simulated district from each of the 1,000 computer-simulated plans. In other words, I compare one district from the Enacted Plan to 1,000 computer-simulated

districts, and I compare these districts based on their Republican vote share. I then directly compare the second-most-Republican district in the Enacted Plan to the second-most-Republican district from each of the 1,000 simulated plans. I conduct the same comparison for each district in the Enacted Plan, comparing the Enacted Plan district to its computer-simulated counterparts from each of the 1,000 simulated plans.

Figure 4:

Comparisons of Enacted SB 740 Plan Districts to 1,000 Computer–Simulated Plans' Districts



36. Thus, the top row of Figure 4 directly compares the partisanship of the most-Republican Enacted Plan district (CD-10) to the partisanship of the most-Republican district from each of the 1,000 simulated plans. The two percentages (in parentheses) in the right margin of this Figure report the percentage of these 1,000 simulated districts that are less Republican than, and more Republican than, the Enacted Plan district. Similarly, the second row of this Figure compares the second-most-Republican district from each plan, the third row compares the third-most-Republican district from each plan, and so on. In each row of this Figure, the Enacted Plan's district is depicted with a red star and labeled in red with its district number; meanwhile, the 1,000 computer-simulated districts are depicted with 1,000 gray circles on each row.

37. As the bottom row of Figure 4 illustrates, the most-Democratic district in the Enacted Plan (CD-9) is more heavily Democratic than 100% of the most-Democratic districts in each of the 1,000 computer-simulated plans. This calculation is numerically reported in the right margin of the Figure. Every single one of the computer-simulated counterpart districts would have been more politically moderate than CD-9 in terms of partisanship: CD-9 exhibits a Republican vote share of 27.2%, while all 1,000 of the most-Democratic districts in the computer-simulated plans would have exhibited a higher Republican vote share and would therefore have been more politically moderate. It is thus clear that CD-9 packs together Democratic voters to a more extreme extent than the most-Democratic district in 100% of the computer-simulated plans. I therefore identify CD-9 as an extreme partisan outlier when compared to its 1,000 computer-simulated counterparts, using a standard threshold test of 95% for statistical significance.

38. The next-to-bottom row of Figure 4 reveals a similar finding regarding CD-6 in the Enacted Plan. This row illustrates that the second-most-Democratic district in the Enacted

Plan (CD-6) is more heavily Democratic than 100% of the second-most-Democratic districts in each of the 1,000 computer-simulated plans. Every single one of its computer-simulated counterpart districts would have been more politically moderate than CD-6 in terms of partisanship: CD-6 exhibits a Republican vote share of 27.5%, while 100% of the second-most-Democratic districts in the computer-simulated plans would have exhibited a higher Republican vote share and would therefore have been more politically moderate. In other words, CD-6 packs together Democratic voters to a more extreme extent than the second-most-Democratic district in 100% of the computer-simulated plans. I therefore identify CD-6 as an extreme partisan outlier when compared to its 1,000 computer-simulated counterparts, using a standard threshold test of 95% for statistical significance.

39. Meanwhile, the top two rows of Figure 4 reveal a similar finding: As the top row illustrates, the most-Republican district in the Enacted Plan (CD-10) is less heavily Republican than 100% of the most-Republican districts in each of the 1,000 computer-simulated plans. A similar pattern appears in the second-to-top row of Figure 4, which illustrates that the second-most-Republican district in the Enacted Plan (CD-13) is less heavily Republican than 99.7% of the second-most-Republican districts in each of the 1,000 computer-simulated plans.

40. It is especially notable that these four aforementioned Enacted Plan districts – the two most Republican districts (CD-10 and CD-13) and the two most Democratic districts (CD-9 and CD-6) in the Enacted Plan – were drawn to include more Democratic voters than virtually allof their counterpart districts in the 1,000 computer-simulated plans. These "extra" Democratic voters in the four most partisan-extreme districts in the Enacted Plan had to come from the remaining ten more moderate districts in the Enacted Plan. Having fewer Democratic voters in these more moderate districts enhances Republican candidate performance in these districts.

41. Indeed, the middle six rows in Figure 4 (i.e., rows 5 through 10) confirm this precise effect. The middle six rows in Figure 4 compare the partisanship of districts in the fifth, sixth, seventh, eighth, ninth, and tenth-most Republican districts within the Enacted Plan and the 1,000 computer-simulated plans. In all six of these rows, the Enacted Plan district is a partisan outlier. In each of these six rows, the Enacted Plan's district is more heavily Republican than over 95% of its counterpart districts in the 1,000 computer-simulated plans. Three of these six rows illustrate Enacted Plan districts that are more heavily Republican than 100% of their counterpart districts in the computer-simulated plans. The six Enacted Plan districts in these six middle rows (CD-1, 3, 4, 11, 12, and 14) are more heavily Republican than nearly all of their counterpart computer-simulated plan districts because the four most partisan-extreme districts inthe Enacted Plan (CD-6, 9, 10, and 13) are more heavily Democratic than nearly all of their counterpart districts in the computer-simulated plans.

42. I therefore identify the six Enacted Plan districts in the six middle rows (CD-1, 3,4, 11, 12, and 14) of Figure 4 as partisan statistical outliers. Each of these six districts has a Republican vote share that is higher than over 95% of the computer-simulated districts in its respective row in Figure 4. I also identify the four Enacted Plan districts in the top rows and the bottom two rows (CD-6, 9, 10, and 13) of Figure 4 as partisan statistical outliers. Each of these four districts has a Republican vote share that is lower than at least 99.7% of the computer-simulated districts in its respective row in Figure 4.

43. In summary, Figure 4 illustrates that 10 of the 14 districts in the Enacted Plan are partisan outliers: Six districts (CD-1, 3, 4, 11, 12, and 14) in the Enacted Plan are more heavily Republican than over 95% of their counterpart computer-simulated plan districts, while four

districts (CD-6, 9, 10, and 13) are more heavily Democratic than at least 99.7% of their counterpart districts in the computer-simulated plans.

44. The Appendix of this report contains ten additional Figures (Figures A1 through A10) that each contain a similar analysis of the Enacted Plan districts and the computer- simulated plan districts. Each of these ten Figures in the Appendix measures the partisanship ofdistricts using one of the individual ten elections included in the 2016-2020 Statewide ElectionComposite. These ten Figures generally demonstrate that the same extreme partisan outlier patterns observed in Figure 4 are also present when district partisanship is measured using any one of the ten statewide elections held in North Carolina during 2016-2020.

45. ***"Mid-Range" Republican Districts:*** Collectively, the upper ten rows in Figure 4 illustrate that the Enacted Plan's ten most-Republican districts exhibit a significantly narrower range of partisanship than is exhibited by the ten most-Republican districts in each of the computer-simulated plans. Specifically, the Enacted Plan's ten most-Republican districts all have Republican vote shares within the narrow range of 52.9% to 61.2%. As explained earlier, this narrow range is the product of two distinct dynamics: In the top two rows of Figure 4, the Enacted Plan's districts are significantly less Republican than nearly all of the simulated plans' districts in these rows. But in the fifth to tenth rows of Figure 4, the Enacted Plan's districts are more safely Republican-leaning than over 95% of the computer-simulated districts within each of these six rows. The overall result of these two distinct dynamics is that the Enacted Plan contains ten districts that all have Republican vote shares within the narrow range of 52.9% to 61.2%. I label any districts within this narrow range of partisanship as "mid-range" Republican-leaning districts, reflecting the fact that these districts have generally favored Republican candidates, but not by overwhelmingly large margins.

46. Is the Enacted Plan's creation of ten such "mid-range" Republican-leaning districts an outcome that ever occurs in the 1,000 computer-simulated plans? I analyzed the simulated plans and counted the number of districts within each plan that are similarly "mid- range" with a Republican vote share between 52.9% and 61.2%. As Figure 5 illustrates, the Enacted Plan's creation of ten "mid-range" Republican districts is an extreme statistical outlier. None of the 1,000 simulated plans comes close to creating ten such districts. Virtually all of the simulated plans contain from two to six "mid-range" Republican districts, and the most commonoutcome among the simulations is four such districts. Hence, the Enacted Plan is clearly an extreme partisan outlier in terms of its peculiar focus on maximizing the number of "mid-range"Republican districts, and the Enacted Plan did so to an extreme degree far beyond any of the 1,000 simulated plans created using a partisan-blind computer algorithm that follows the Adopted Criteria.

47. ***Competitive Districts:*** The Enacted Plan's maximization of "mid-range" Republican districts necessarily comes at the expense of creating more competitive districts. As Figure 4 illustrates, the Enacted Plan contains zero districts whose Republican vote share is higher than 47.0% and lower than 52.9%, as measured using the 2016-2020 Statewide ElectionComposite. In other words, there are zero districts in which the Republican vote share is within5% of the Democratic vote share.

48. I label districts with a Republican vote share from 47.5% to 52.5% as "competitive" districts to reflect the fact that such districts have a nearly even share of Republican and Democratic voters, and election outcomes in the district could therefore swing in favor of either party. The Enacted Plan contains zero "competitive" districts, as measured using the 2016-2020 Statewide Election Composite.

Figure 5:

Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans On Number of Mid−Range Republican Districts

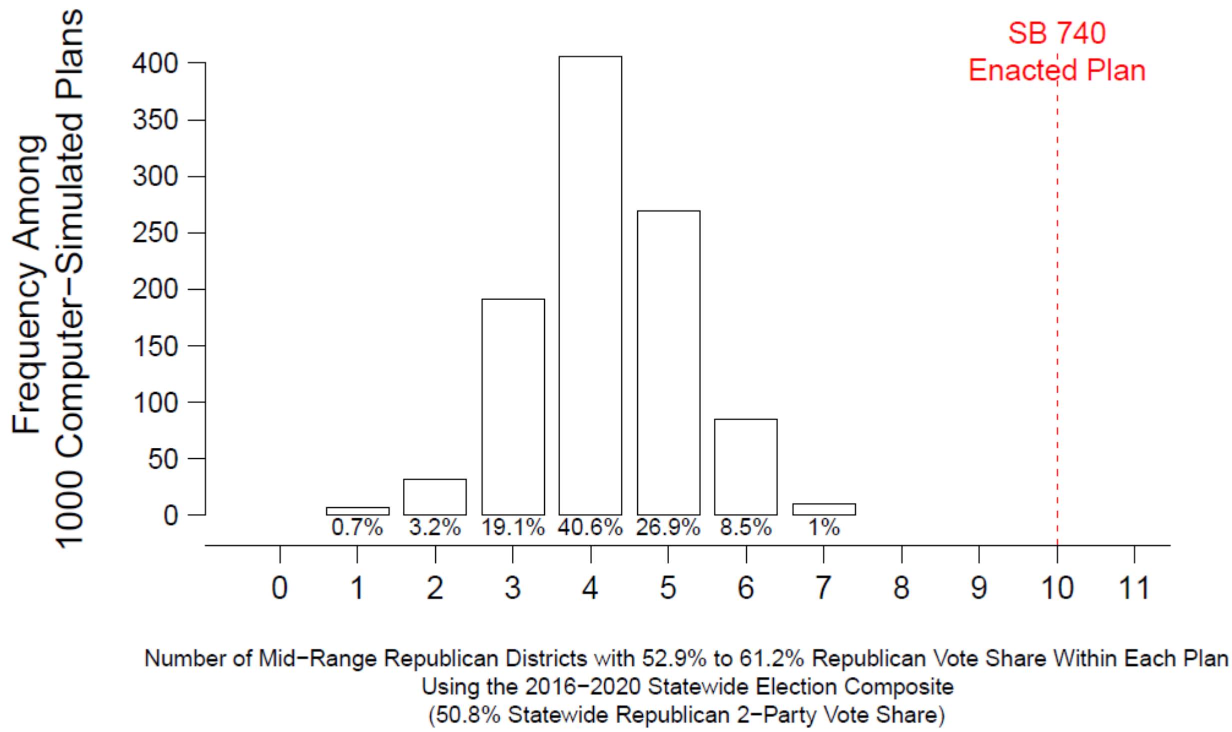
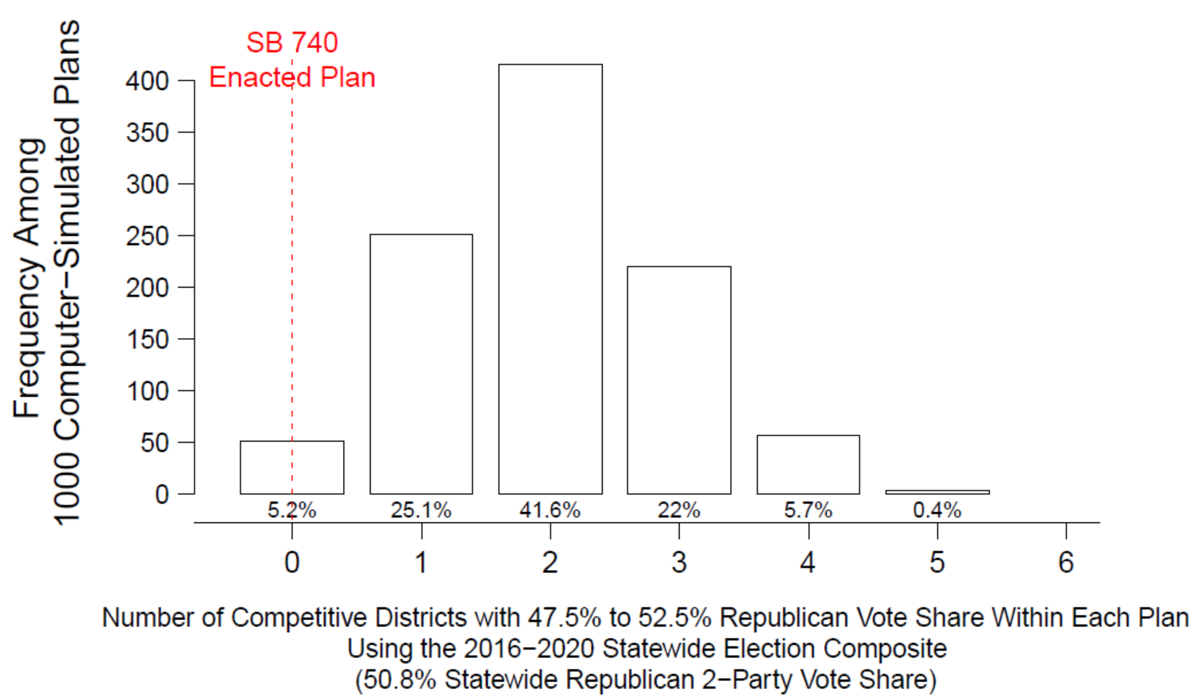


Figure 6:

Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans On Number of Competitive Districts



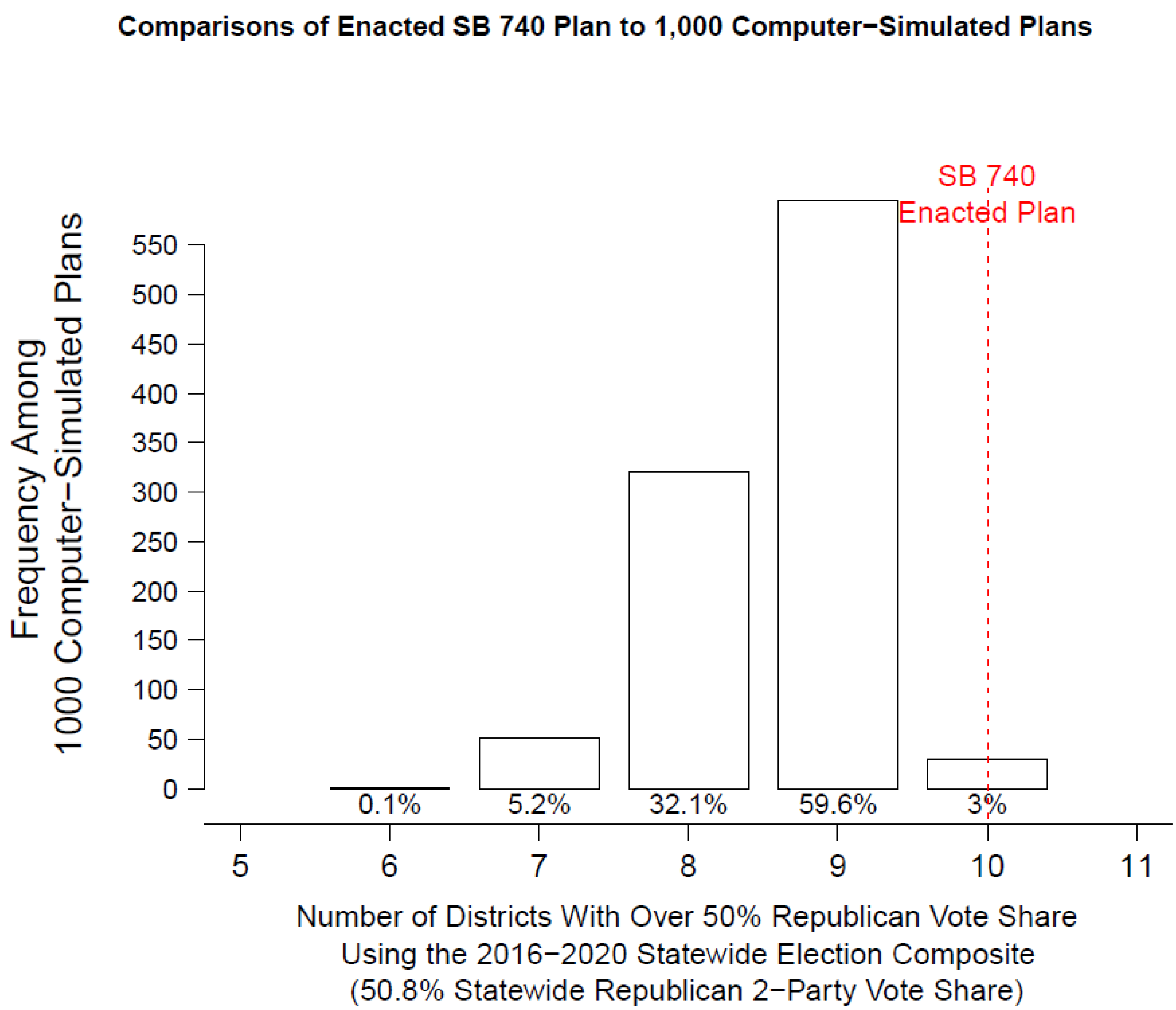
49. Is the Enacted Plan's failure to create any "competitive" districts an outcome that ever occurs in the 1,000 computer-simulated plans? I analyzed the simulated plans and counted the number of districts within each plan that are "competitive" districts with a Republican vote share between 47.5% and 52.5%. As Figure 6 illustrates, the Enacted Plan's creation of zero "competitive" districts is almost a statistical outlier: Only 5.2% of the 1,000 simulated plans similarly fail to have a single "competitive" district. The vast majority of the computer-simulated plans contain two or more "competitive" districts. Almost 95% of the computer-simulated plans create more "competitive" districts than the Enacted Plan does.

50. ***Number of Democratic and Republican Districts:*** Figure 7 compares the partisan breakdown of the computer-simulated plans to the partisanship of the Enacted Plan. Specifically, Figure 7 uses the 2016-2020 Statewide Election Composite to measure the number of Republican-favoring districts created in each of the 1,000 simulated plans. Across the entire state, Republican candidates collectively won a 50.8% share of the votes in the ten elections in the 2016-2020 Statewide Election Composite. But within the 14 districts in the Enacted Plan, Republicans have over a 50% vote share in 10 out of 14 districts. In other words, the Enacted Plan created 10 Republican-favoring districts, as measured using the 2016-2020 Statewide Election Composite. By contrast, only 3% of the computer-simulated plans create 10 Republican-favoring districts, and no computer-simulated plan ever creates more than 10 Republican districts.

51. Hence, in terms of the total number of Republican-favoring districts created by the plan, the 2021 Enacted Plan is a statistical outlier when compared to the 1,000 computer- simulated plans. The Enacted Plan creates the maximum number of Republican districts that everoccurs in any computer-simulated plan, and the Enacted Plan creates more Republican districts

than 97% of the computer-simulated plans, which were drawn using a non-partisan districting process adhering to the General Assembly's 2021 Adopted Criteria. I characterize the Enacted Plan's creation of 10 Republican districts as a statistical outlier among the computer-simulated plans because the Enacted Plan exhibits an outcome that is more favorable to Republicans than over 95% of the simulated plans.

Figure 7:



52. Notably, the ten elections included in the Statewide Election Composite all occurred in two election years and in electoral environments that were relatively favorable to Republicans across the country (November 2016 and November 2020). North Carolina did not hold any statewide elections for non-judicial offices in November 2018, which was an electoral environment more favorable to Democrats across the country.

53. Hence, the projected number of Republican seats would be even lower in the computer-simulated plans if one measured district partisanship using a statewide election whose outcome was more partisan-balanced or even favorable to Democrats. In the Appendix, I presentten histograms (labeled as Figures B1 to B10), each presenting the projected number of Republican seats across all of the simulated plans and the Enacted Plan using only one of the tenelections in the Statewide Election Composite.

54. The ten histograms in Figures B1 to B10 illustrate how the partisanship of the Enacted Plan compares to the partisanship of the 1,000 computer-simulated plans under a range of different electoral environments, as reflected by the ten elections in the Statewide Election Composite. Most notably, under all ten of these elections, the Enacted Plan always contains exactly 10 Republican-favoring districts and 4 Democrat-favoring districts. Hence, it is clear thatthe Enacted Plan creates a 10-to-4 distribution of seats in favor of Republican candidates that is durable across a range of different electoral conditions.

55. Moreover, the histograms in Figures B1 to B10 demonstrate that the Enacted Plan becomes a more extreme partisan outlier relative to the computer-simulated plans under electoral conditions that are slightly to moderately favorable to the Democratic candidate. For example, Figure B1 compares the Enacted Plan to the computer-simulated plan using the results of the 2016 Attorney General election, which was a near-tied statewide contest in which Democrat Josh

Stein defeated Republican Buck Newton by a very slim margin. Using the 2016 Attorney General election to measure district partisanship, the 2021 Enacted Plan contains 10 Republican-favoring districts out of 14. The Enacted Plan's creation of 10 districts favoring Republican BuckNewton over Democrat Josh Stein is an outcome that never occurs in the 1,000 computer-simulated plans, indicating that the Enacted Plan is a partisan statistical outlier under electoral conditions that are more favorable for Democrats (and thus relatively more unfavorable for Republicans) than is normal in North Carolina.

56. An even more favorable election for the Democratic candidate was the 2020 gubernatorial contest, in which Democrat Roy Cooper defeated Republican Dan Forest by a 4.5% margin. Figure B7 compares the Enacted Plan to the computer-simulated plans using the resultsof this 2020 gubernatorial election. Using the results from this election, the 2021 Enacted Plan contains 10 Republican-favoring districts out of 14. None of the 1,000 simulated plans ever contain 10 districts favoring the Republican candidate. The Enacted Plan's creation of 10 Republican-favoring districts is therefore an extreme partisan outlier that is durable even in Democratic-favorable electoral conditions. In fact, the 10-to-4 Republican partisan advantage under the Enacted Plan appears to become even more of an extreme partisan outlier under Democratic-favorable elections.

57. ***The Mean-Median Difference:*** I also calculate each districting plan's mean-median difference, which is another accepted method that redistricting scholars commonly use to compare the relative partisan bias of different districting plans. The mean-median difference for any given plan is calculated as the mean district-level Republican vote share, minus the median district-level Republican vote share. For any congressional districting plan, the mean is calculated as the average of the Republican vote shares in each of the 14 districts. The median, in

turn, is the Republican vote share in the district where Republican performed the middle-best, which is the district that Republican would need to win to secure a majority of the congressional delegation. For a congressional plan containing 14 districts, the median district is calculated as the average of the Republican vote share in the districts where Republican performed the 7th and8th-best across the state.

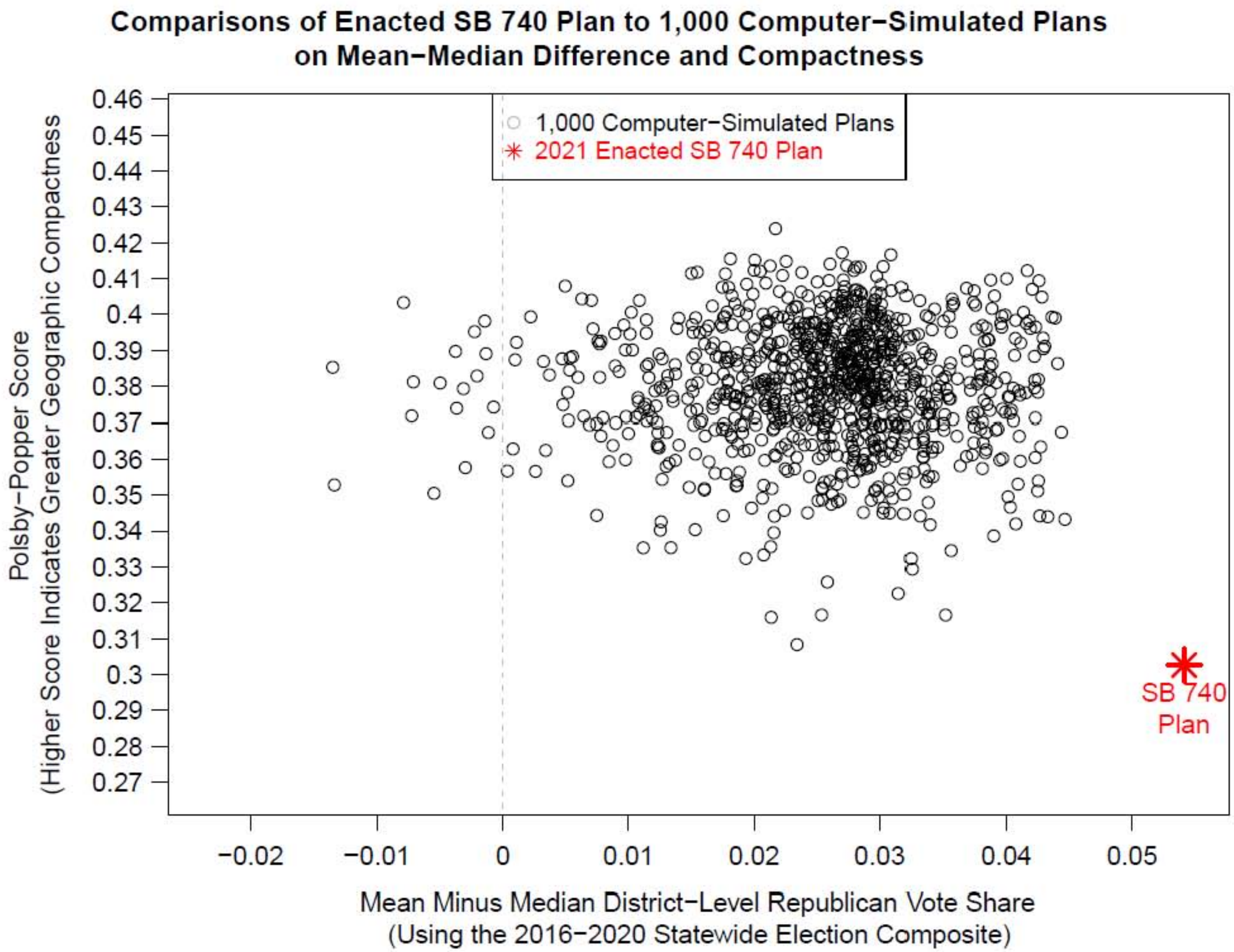
58. Using the 2016-2020 Statewide Election Composite to measure partisanship, the districts in the 2021 Enacted Plan have a mean Republican vote share of 50.8%, while the median district has a Republican vote share of 56.2%. Thus, the Enacted Plan has a mean-mediandifference of +5.4%, indicating that the median district is skewed significantly more Republican than the plan's average district. The mean-median difference thus indicates that the Enacted Plandistributes voters across districts in such a way that most districts are significantly more Republican-leaning than the average North Carolina congressional district, while Democratic voters are more heavily concentrated in a minority of the Enacted Plan's districts.

59. I perform this same mean-median difference calculation on all computer-simulated plans in order to determine whether this partisan skew in the median congressional districts could have resulted naturally from North Carolina's political geography and the application of the Adopted Criteria. Figure 8 compares the mean-median difference of the Enacted Plan to the mean-median difference for each the 1,000 computer-simulated plans.

60. Figure 8 contains 1,000 gray circles, representing the 1,000 computer-simulated plans, as well as a red star, representing the 2021 Enacted Plan. The horizontal axis in this Figure measures the mean-median difference of the 2021 Enacted Plan and each simulated plan using the 2016-2020 Statewide Election Composite, while the vertical axis measures the average Polsby-Popper compactness score of the districts within each plan, with higher Polsby-Popper

scores indicating more compact districts. Figure 8 illustrates that the Enacted Plan's mean-median difference is +5.4%, indicating that the median district is skewed significantly more Republican than the plan's average district. Figure 8 further indicates that this difference is an extreme statistical outlier compared to the 1,000 computer-simulated plans. Indeed, the Enacted Plan's +5.4% mean-median difference is an outcome never observed across these 1,000 simulated plans. The 1,000 simulated plans all exhibit mean-median differences that range from -0.1% to +4.6%. In fact, the middle 50% of these computer-simulated plans have mean-median differences ranging from +2.1% to +3.1%, indicating a much smaller degree of skew in the median district than occurs under the 2021 Enacted Plan. These results confirm that the Enacted Plan creates an extreme partisan outcome that cannot be explained by North Carolina's voter geography or by strict adherence to the required districting criteria set forth in the General Assembly's Adopted Criteria.

Figure 8:


Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
on Mean−Median Difference and Compactness
1,000 Computer−Simulated Plans
2021 Enacted SB 740 Plan
SB 740
Plan
Polsby−Popper Score
(Higher Score Indicates Greater Geographic Compactness
0.46
0.45
0.44
0.43
0.42
0.41
0.4
0.39
0.38
0.37
0.36
0.35
0.34
0.33
0.32
0.31
0.3
0.29
0.28
0.27
−0.02
−0.01
0
0.01
0.02
0.03
0.04
0.05
Mean Minus Median District−Level Republican Vote Share
(Using the 2016−2020 Statewide Election Composite)

61. Figure 8 illustrates that the Enacted Plan is less geographically compact than every single one of the computer-simulated plans, as measured by each plan's average Polsby-Popper score. The simulated plans have Polsby-Popper scores ranging from 0.31 to 0.42. In fact, the middle 50% of these computer-simulated plans have Polsby-Popper scores ranging from 0.37 to 0.39. Meanwhile, the Enacted Plan exhibits a Polsby-Popper score of only 0.30, which is lower than all 1,000 of the computer-simulated plans. Hence, it is clear that the Enacted Plan did not seek to draw districts that were as geographically compact as reasonably possible. Instead, the Enacted Plan subordinated geographic compactness, which enabled the Enacted Plan to create a partisan skew in North Carolina's congressional districts favoring Republican candidates.

62. ***The Efficiency Gap:*** Another commonly used measure of a districting plan's partisan bias is the efficiency gap.[12] To calculate the efficiency gap of the Enacted Plan and every computer-simulated plan, I first measure the number of Republican and Democratic votes within each Enacted Plan district and each computer-simulated district, as measured using the 2016- 2020 Statewide Election Composite. Using this measure of district-level partisanship, I then calculate each districting plan's efficiency gap using the method outlined in *Partisan Gerrymandering and the Efficiency Gap.*[13] Districts are classified as Democratic victories if, using the 2016-2020 Statewide Election Composite, the sum total of Democratic votes in the district during these elections exceeds the sum total of Republican votes; otherwise, the district is classified as Republican. For each party, I then calculate the total sum of surplus votes in districts the party won and lost votes in districts where the party lost. Specifically, in a district lost by a

[12] Eric McGhee, "Measuring Partisan Bias in Single-Member District Electoral Systems." Legislative Studies Quarterly Vol. 39, No. 1: 55–85 (2014).
[13] Nicholas O. Stephanopoulos & Eric M. McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 University of Chicago Law Review 831 (2015).

given party, all of the party's votes are considered lost votes; in a district won by a party, only the party's votes exceeding the 50% threshold necessary for victory are considered surplus votes. A party's total wasted votes for an entire districting plan is the sum of its surplus votes in districts won by the party and its lost votes in districts lost by the party. The efficiency gap is then calculated as total wasted Democratic votes minus total wasted Republican votes, divided by the total number of two-party votes cast statewide across all seven elections.

63. Thus, the theoretical importance of the efficiency gap is that it tells us the degree to which more Democratic or Republican votes are wasted across an entire districting plan. A significantly positive efficiency gap indicates far more Democratic wasted votes, while a significantly negative efficiency gap indicates far more Republican wasted votes.

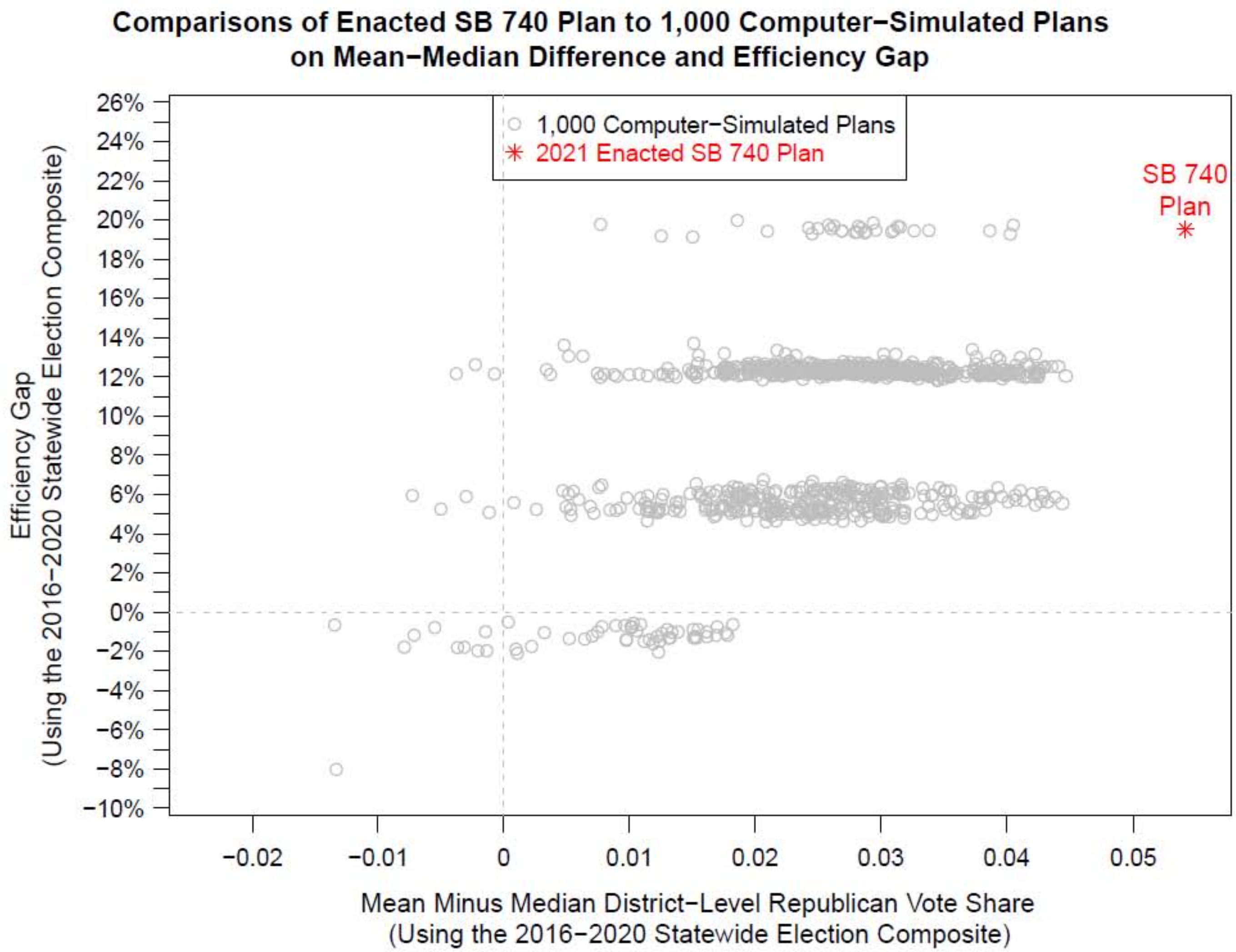
64. I analyze whether the Enacted Plan's efficiency gap arises naturally from a map-drawing process strictly adhering to the mandated criteria in the General Assembly's Adopted Criteria, or rather, whether the skew in the Enacted Plan's efficiency gap is explainable only as the product of a map-drawing process that intentionally favored one party over the other. By comparing the efficiency gap of the Enacted Plan to that of the computer-simulated plans, I am able to evaluate whether or not such the Enacted Plan's efficiency gap could have realistically resulted from adherence to the Adopted Criteria.

65. Figure 9 compares the efficiency gaps of the Enacted Plan and of the 1,000 computer-simulated plans. As before, the 1,000 circles in this Figure represent the 1,000 computer-simulated plans, while the red star in the upper right corner represents the Enacted Plan. Each plan is plotted along the vertical axis according to its efficiency gap, while each plan is plotted along the horizontal axis according to its mean-median difference.

66. The results in Figure 9 illustrate that the Enacted Plan exhibits an efficiency gap

of +19.5%, indicating that the plan results in far more wasted Democratic votes than wasted Republican votes. Specifically, the difference between the total number of wasted Democratic votes and wasted Republican votes amounts to 19.5% of the total number of votes statewide. The Enacted Plan's efficiency gap is larger than the efficiency gaps exhibited by 98.7% of the computer-simulated plans. This comparison reveals that the significant level of Republican bias exhibited by the Enacted Plan cannot be explained by North Carolina's political geography or the Adopted Criteria alone.

Figure 9:


Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
on Mean−Median Difference and Efficiency Gap
1,000 Computer−Simulated Plans
2021 Enacted SB 740 Plan
SB 740
Plan
Efficiency Gap
(Using the 2016−2020 Statewide Election Composite)
26%
24%
22%
20%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
−2%
−4%
−6%
−8%
−10%
−0.02
−0.01
0
0.01
0.02
0.03
0.04
0.05
Mean Minus Median District−Level Republican Vote Share
(Using the 2016−2020 Statewide Election Composite)

67. ***The Lopsided Margins Measure:*** Another measure of partisan bias in districting plans is the "lopsided margins" test. The basic premise captured by this measure is that a partisan-motivated map-drawer may attempt to pack the opposing party's voters into a small number of extreme districts that are won by a lopsided margin. Thus, for example, a map-drawer attempting to favor Party A may pack Party B's voters into a small number of districts that very heavily favor Party B. This packing would then allow Party A to win all the remaining districts with relatively smaller margins. This sort of partisan manipulation in districting would result in Party B winning its districts by extremely large margins, while Party A would win its districts by relatively small margins.

68. Hence, the lopsided margins test is performed by calculating the difference between the average margin of victory in Republican-favoring districts and the average margin of victory in Democratic-favoring districts. The 2021 Enacted Plan contains four Democratic-favoring districts (CD-2, 5, 6, and 9), and these four districts have an average Democratic vote share of 65.4%, as measured using the 2016-2020 Statewide Election Composite. By contrast, the Enacted Plan contains ten Republican-favoring districts (CD-1, 3, 4, 7, 8, 10, 11, 12, 13, and 14), and these ten districts have an average Republican vote share of 57.3%. Hence, the difference between the average Democratic margin of victory in Democratic-favoring districts and the average Republican margin of victory in Republican-favoring districts is +8.1%, which is calculated as 65.4% - 57.3%. I refer to this calculation of +8.1% as the Enacted Plan's lopsided margins measure.

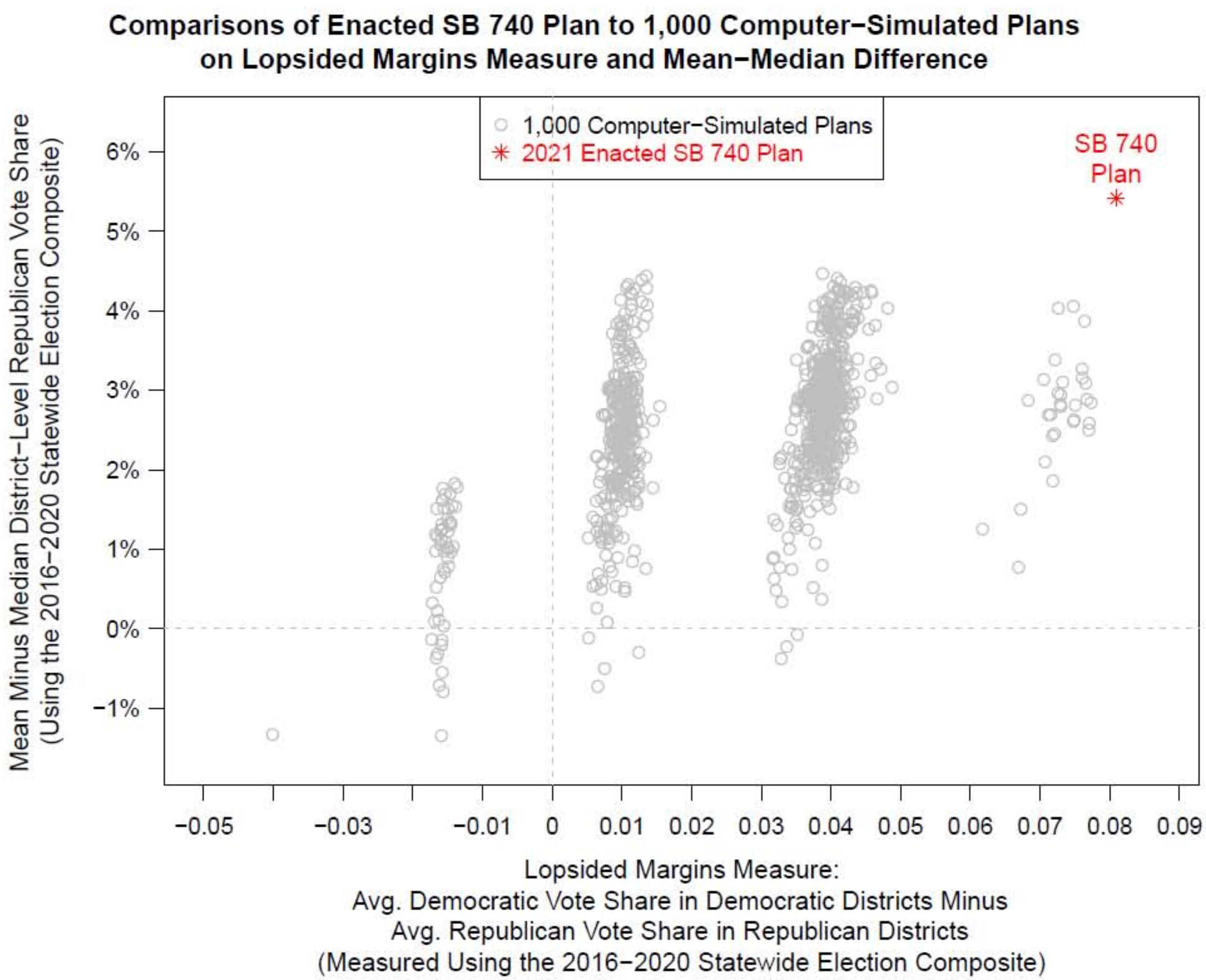
69. How does the 8.1% lopsided margins measure of the Enacted Plan compare to the same calculation for the 1,000 computer-simulated plans? Figure 10 reports the lopsided margins calculations for the Enacted Plan and for the simulated plans. In Figure 10, each plan is plotted

along the horizontal axis according to its lopsided margins measure and along the vertical axis according to its mean-median difference.

70. Figure 10 reveals that the Enacted Plan's +8.1% lopsided margins measure is an extreme outlier compared to the lopsided margins measures of the 1,000 computer-simulated plans. All 1,000 of the simulated plans have a smaller lopsided margins measure than the Enacted Plan. In fact, a significant minority (37.3%) of the 1,000 simulated plans have a lopsided margins measure of between -2% to +2%, indicating a plan in which Democrats and Republicans win their respective districts by similar average margins.

71. By contrast, the Enacted Plan's lopsided margins measure of +8.1% indicates that the Enacted Plan creates districts in which Democrats are extremely packed into their districts, while the margin of victory in Republican districts is significantly smaller. The "lopsidedness" of the two parties' average margin of victory is extreme when compared to the computer-simulated plans. The finding that all 1,000 simulated plans have a smaller lopsided margins measure indicates that the Enacted Plan's extreme packing of Democrats into Democratic-favoring districts was not simply the result of North Carolina's political geography, combined with adherence to the Adopted Criteria.

Figure 10:


Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
on Lopsided Margins Measure and Mean−Median Difference
1,000 Computer−Simulated Plans
2021 Enacted SB 740 Plan
SB 740
Plan
Mean Minus Median District−Level Republican Vote Share
(Using the 2016−2020 Statewide Election Composite)
6%
5%
4%
3%
2%
1%
0%
−1%
−0.05
−0.03
−0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
Lopsided Margins Measure:
Avg. Democratic Vote Share in Democratic Districts Minus
Avg. Republican Vote Share in Republican Districts
(Measured Using the 2016−2020 Statewide Election Composite)

72. ***Partisan Symmetry Based on Uniform Swing:*** Another common measure of partisan bias is based on the concept of partisan symmetry and asks the following question: Under a given districting plan and given a particular election-based measure of district partisanship, what share of seats would each party win in a hypothetical tied election (i.e., 50% vote share for each of two parties). To approximate the district-level outcomes in a hypothetical tied election, one normally uses a uniform swing in order to simulate a tied statewide election. We then calculate whether each party would receive more than or less than 50% of the seats under this hypothetical tied election in a given districting plan. This particular measure is often referred to in the academic literature as "partisan bias." In order to avoid confusion with other measures of partisan bias described in this report, I will refer to this measure as "Partisan Symmetry Based on Uniform Swing."

73. Specifically, I use the 2016-2020 Statewide Election Composite to calculate the Partisan Symmetry measure for both the Enacted Plan and for the computer-simulated plans. The 2016-2020 Statewide Election Composite produces a statewide Republican vote share of 50.8%. Therefore, I use a uniform swing of -0.8% in order to estimate the partisanship of districts under a hypothetical tied election in which each party wins exactly 50% of the statewide vote. In other words, this uniform swing subtracts 0.8% from the Republican vote share in every district, both in the Enacted Plan and in all simulated plans.

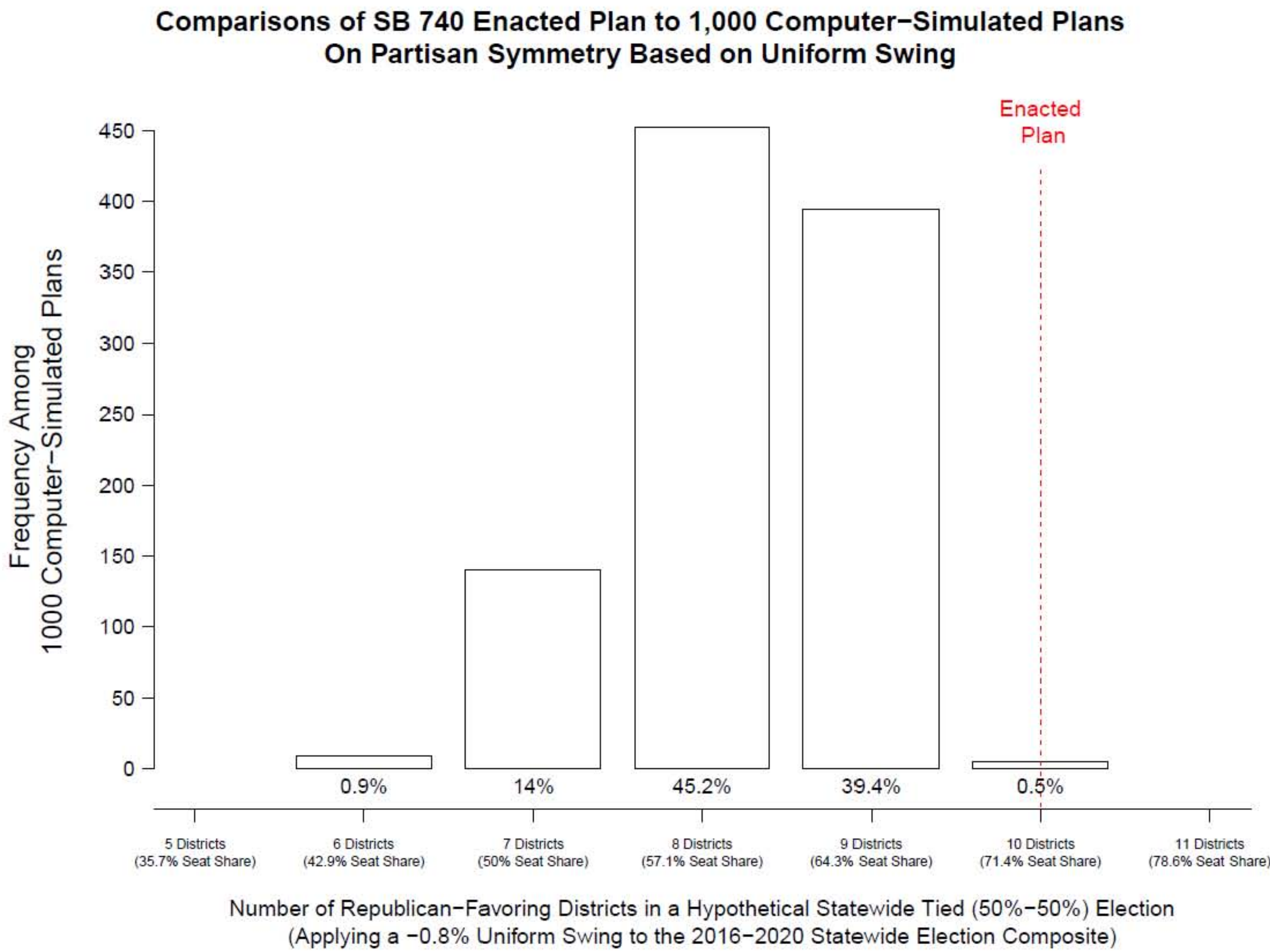
74. After applying this -0.8% uniform swing, I compare the number of Republican-favoring districts in the Enacted Plan and the simulated plans. In the Enacted Plan, 71.4% of the districts (10 out of 14) are Republican-favoring after applying the uniform swing. I then report the Republicans' seat share (71.4%) under this hypothetical tied election in Figure 11 as the "Partisan

Symmetry Based on Uniform Swing" measure for the Enacted Plan. Figure 11 also reports the calculations for all 1,000 simulated plans using this identical method.

75. Figure 11 reveals 99.5% of the 1,000 simulated plans have a "Partisan Symmetry Based on Uniform Swing" measure that is closer to 50% than the Enacted Plan's measure. In fact, 14% of the simulated plans have a measure that is exactly 50% (7 out of 14 districts), while over 60% of the simulated plans are between 40% and 60%.

76. By contrast, the Enacted Plan's measure of 71.4% in Figure 11 would be a statistical outlier and is more favorable to Republicans than in 99.5% of the simulated plans. Substantively, this 71.4% measure reflects the Enacted Plan's creation of a durable Republican majority for North Carolina's congressional delegation, such that even when Democrats win 50% of the statewide vote, Republicans will still be favored in 10 out of 14 (71.4%) of the congressional districts, while Democrats will only be favored in only 4 out of the 14 (28.6%) districts.

Figure 11:


Comparisons of SB 740 Enacted Plan to 1,000 Computer−Simulated Plans
On Partisan Symmetry Based on Uniform Swing
Enacted
Plan
Frequency Among
1000 Computer−Simulated Plans
450
400
350
300
250
200
150
100
50
0
0.9%
14%
45.2%
39.4%
0.5%
5 Districts
(35.7% Seat Share)
6 Districts
(42.9% Seat Share)
7 Districts
(50% Seat Share)
8 Districts
(57.1% Seat Share)
9 Districts
(64.3% Seat Share)
10 Districts
(71.4% Seat Share)
11 Districts
(78.6% Seat Share)
Number of Republican−Favoring Districts in a Hypothetical Statewide Tied (50%−50%) Election
(Applying a −0.8% Uniform Swing to the 2016−2020 Statewide Election Composite)

Conclusions Regarding Partisanship and Traditional Districting Criteria

77. The analysis described thus far in this report lead me to reach two main findings: First, among the five traditional districting criteria mandated by the General Assembly's 2021 Adopted Criteria, the Enacted Plan fails to minimize county splits, fails to minimize VTD splits, and is significantly less geographically compact than is reasonably possible under a districting process that follows the Adopted Criteria. Second, I found that the Enacted Plan is an extreme partisan outlier when compared to computer-simulated plans produced by a process following the Adopted Criteria. The Enacted Plan contains 10 districts that are partisan outliers when compared to the simulated plans' districts, and using several different common measures of partisan bias, the Enacted Plan creates a level of pro-Republican bias more extreme than in over 95% of the computer-simulated plans. In particular, the Enacted Plan creates more "mid-range" Republican districts than is created in 100% of the computer-simulated plans (Paragraphs 45-46).

78. Based on these two main findings, I conclude that partisanship predominated in the drawing of the 2021 Enacted Plan and subordinated the traditional districting principles of avoiding county splits, avoiding VTD splits, and geographic compactness. Because the Enacted Plan fails to follow three of the Adopted Criteria's mandated districting principles while simultaneously creating an extreme level of partisan bias, I therefore conclude that the partisan bias of the Enacted Plan did not naturally arise by chance from a districting process adhering to the Adopted Criteria. Instead, I conclude that partisan goals predominated in the drawing of the Enacted Plan. By subordinating traditional districting criteria, the General Assembly's Enacted Plan was able to achieve partisan goals that could not otherwise have been achieved under a partisan-neutral districting process that follows the Adopted Criteria.

Regional Comparisons of Enacted Plan and Simulated Plan Districts

79. I have thus far compared the Enacted Plan to the simulated plans at a statewide level using several common measures of partisan bias and by identifying individual districts that are partisan outliers. However, I also analyzed the extent to which partisan bias affected the map-drawing process within specific cities and geographic regions of North Carolina. I found that the Enacted Plan's individual districts in certain regions exhibit extreme political bias when compared to the computer-simulated districts in the same regions. Below, I describe my findings regarding the partisan bias caused by the Enacted Plan's district boundaries in the Piedmont Triad area, in the Research Triangle, and in Mecklenburg County.

80. ***The Piedmont Triad Area:*** The Enacted Plan splits Guilford County into three different districts: CD-7, 10, and 11. These three fragments of Guilford County, which has voted solidly Democratic in recent statewide elections, are each combined with more Republican areas in surrounding counties across the Piedmont Triad area. This three-way splitting of Guilford County results in CD-7, 10, and 11 being safely Republican, each with a Republican vote share between 55.9% and 61.2%, as measured using the 2016-2020 Statewide Election Composite.

81. Is this three-way splitting of Guilford County, and the resulting creation of three safe Republican districts, a districting outcome that could have resulted naturally from the region's political geography, combined with the districting principles required by the Adopted Criteria? A comparison of the Enacted Plan's districts to the simulated districts in the Piedmont Triad area reveals that the Enacted Plan managed to crack Democratic voters in the region to a more extreme extent than in virtually all of the computer-simulated plans. Moreover, the Enacted Plan achieved this extreme cracking of Democrats by creating districts that are significantly less compact than virtually all of the Guilford County districts in the computer-simulated plans.

82. Figure 12 directly compares the partisanship of the Enacted Plan's districts to the simulated plans' districts in the Piedmont Triad area at a local level. Specifically, the top row of Figure 12 describes the district within each plan that contains the most amount of Greensboro's population. In the Enacted Plan, this district is CD-11, and Figure 12 directly compares the Republican vote share of CD-11 to the Republican vote shares of all simulated districts that contain the largest portion of Greensboro residents among all districts in their respective simulated plans. The Figure reveals that the Enacted Plan's CD-11 is more safely Republican than 99.6% of the computer-simulated Greensboro districts. In fact, although CD-11 exhibits a 55.9% Republican vote share, 96.1% of the simulated districts containing Greensboro are Democratic-favoring districts. Hence, it is clear that the Enacted Plan created a safe Republican district for Greensboro, even though a partisan-neutral districting process following the Adopted Criteria would almost always have placed Greensboro in a Democratic-favoring district.

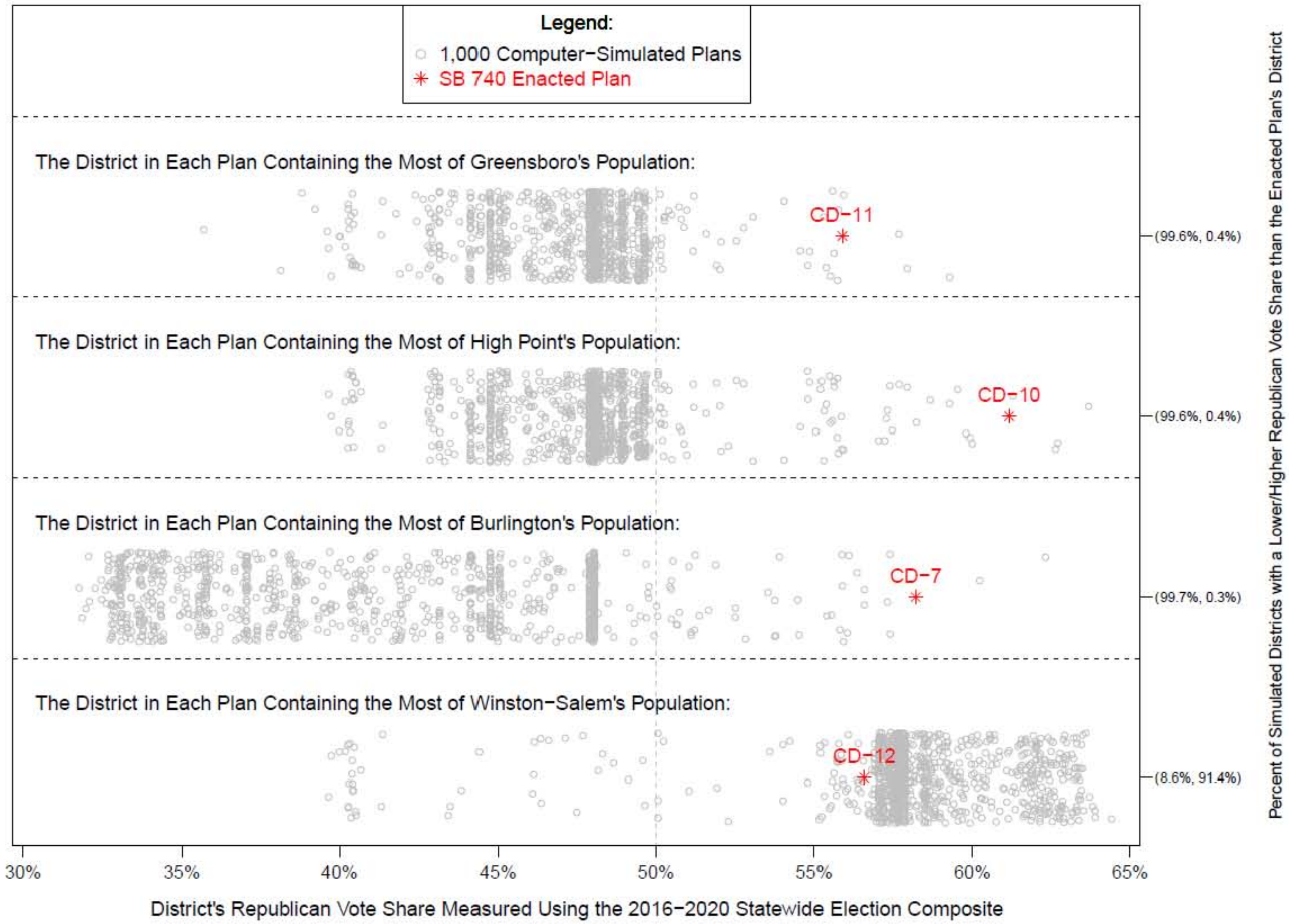
83. The second row of Figure 12 illustrates a similar finding regarding the city of High Point in Guilford County. The Enacted Plan places High Point into CD-10, which has a Republican vote share of 61.2%. CD-10 is more heavily Republican than 99.6% of the High Point-based district in the 1,000 computer-simulated plans. Once again, nearly all of the simulated plans place High Point into a Democratic-favoring district, but the Enacted Plan managed to place High Point into an anomalously Republican district.

84. The third row of Figure 12 reveals a similar finding regarding CD-7, the third district containing a fragment of Guilford County. The city of Burlington (Alamance and Guilford Counties) is assigned to the Enacted Plan's CD-7, which exhibits a 58.2% Republican vote share. CD-7 is more heavily Republican than 99.7% of the Burlington-based districts in the 1,000 computer-simulated plans. In fact, 95.5% of the Burlington districts in the simulated plans

favor the Democrats, often by an extremely wide margin. Thus, it is clear that the Enacted Plan created a far more Republican-favorable district for Burlington than could be reasonably expected from a partisan-blind districting process.

85. Of course, the creation of three safe Republican districts (CD-7, 10, and 11) in the Guilford County area required bringing in Republican voters from other, surrounding districts. One such district was CD-12, a safely Republican district covering areas in the Piedmont Triad region to the west of Guilford County. The fourth row of Figure 12 compares the partisanship of the Enacted Plan's district containing Winston-Salem (CD-12) to the simulated plans' districts containing Winston-Salem. The simulated plan results on this row illustrate that under a partisan-blind districting process, Winston-Salem would normally be placed into an even more heavily Republican district than the Enacted Plan's CD-12. The Enacted Plan's CD-12 is a safe Republican seat with a Republican vote share of 56.6%, but it is less heavily Republican than 91.4% of the computer-simulated districts containing the most of Winston-Salem's population. This finding suggests that CD-12 was drawn to be less extremely Republican than should be expected, given the political geography of the Piedmont Triad area. As a result, more Republican voters could be placed in the surrounding districts, particularly CD-10 and CD-11, that split up Guilford County.

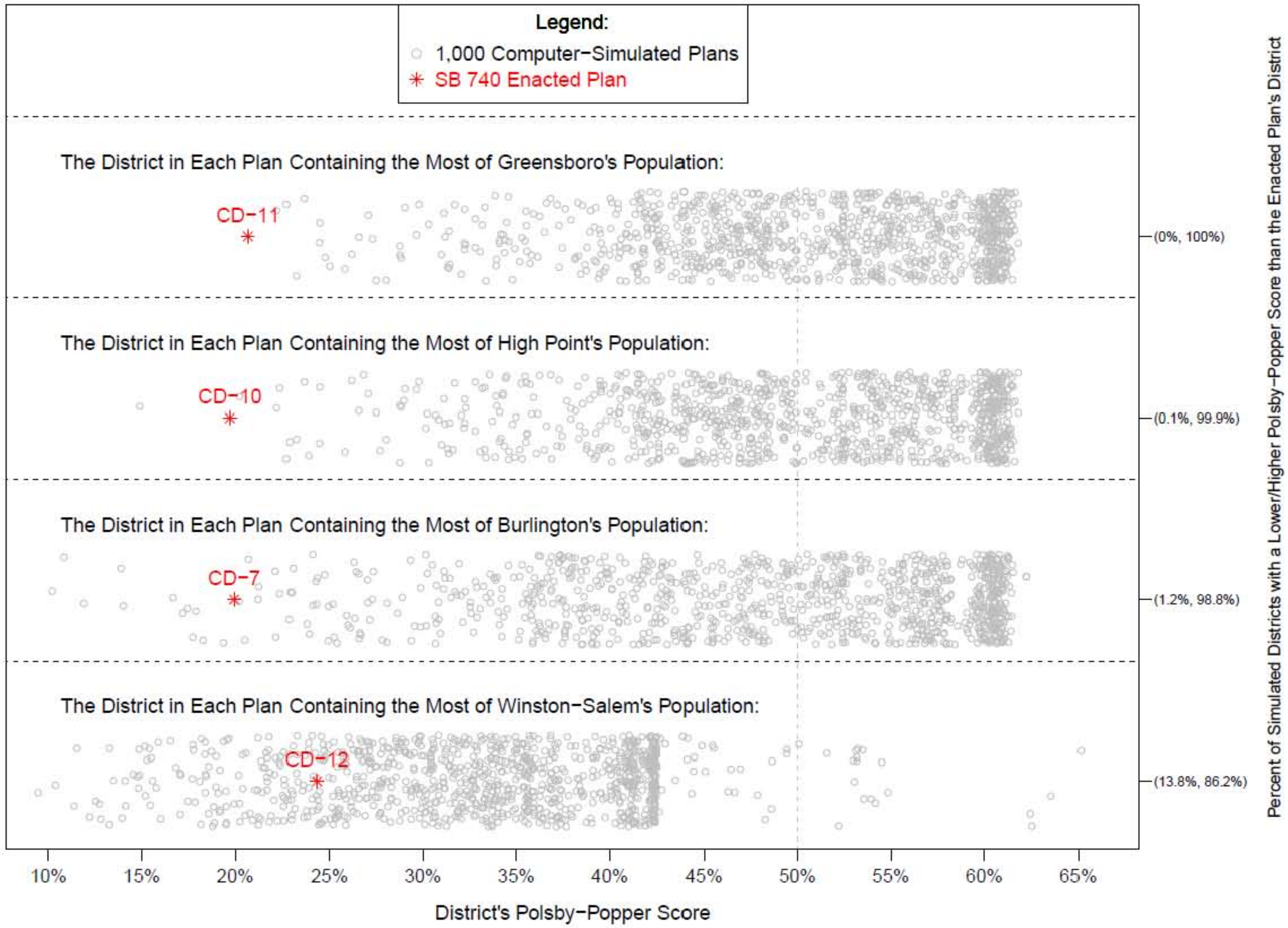
Figure 12: Piedmont Triad Area:
Comparison of Individual Districts' Republican Vote Shares
in the SB 740 Plan and in 1,000 Computer−Simulated Plans



86. Could the Enacted Plan's cracking of Guilford County Democrats into three districts (CD-7, 10, and 11) have resulted from a mapdrawing process attempting to follow the Adopted Criteria? The geographic characteristics of these three districts illustrate the opposite conclusion: The General Assembly managed to split Guilford County into three safe Republican districts by subordinating the districting principles required by the Adopted Criteria. Although the Adopted Criteria do not explicitly prohibit dividing Guilford County into three districts, doing so was not necessary to comply with the Adopted Criteria. Guilford County's population is well under that of an equally populated congressional district. In fact, the vast majority (75.6%) of the computer-simulated plans do not split Guilford County a single time. When Guilford County is split, the simulated plans usually split it only once.

87. Moreover, the compactness scores of the Enacted Plan's CD-7, 10, and 11 reveal that the General Assembly subordinated geographic compactness considerations in the process of cracking Democrats in Guilford County. The first row of Figure 13 illustrates that the Enacted Plan's CD-11 has a lower Polsby-Popper score than all 1,000 of the Greensboro-based districts in the computer-simulated plans. The second and third rows of Figure 13 reveal a nearly identical conclusion regarding the other two districts covering Guilford County (CD-7 and CD-10). In fact, there is a vast disparity between the compactness of the Enacted Plan's Guilford County districts and the simulated plans' districts in Guilford County. CD-7, 10, and 11 have Polsby-Popper scores of 0.197, 0.199, and 0.207. Meanwhile, over half of the simulated districts displayed in these upper three rows of Figure 13 have a Polsby-Popper score over 0.5. It is therefore clear that the Enacted Plan subordinated geographic compactness in the pursuit of Republican partisan advantage in the drawing of district boundaries in the Piedmont Triad area.

Figure 13: Piedmont Triad Area:
Comparison of Individual Districts' Compactness Scores
in the SB 740 Plan and in 1,000 Computer−Simulated Plans



88. ***The Research Triangle:*** Figures 14 and 15 present a similar analysis of the districts in the Research Triangle. The top row of Figure 14 compares the Republican vote shares of the Enacted Plan's and each computer-simulated plan's district containing the most of Raleigh's population. The second row of Figure 14 is a similar comparison of the Enacted Plan's and each simulated plan's district containing the most of Durham's population. Overall, these two rows illustrate that the Enacted Plan's Raleigh-based district (CD-5) and Durham-based district (CD-6) are more heavily packed with Democrats than almost 100% of the computer-simulated districts containing Raleigh and Durham.

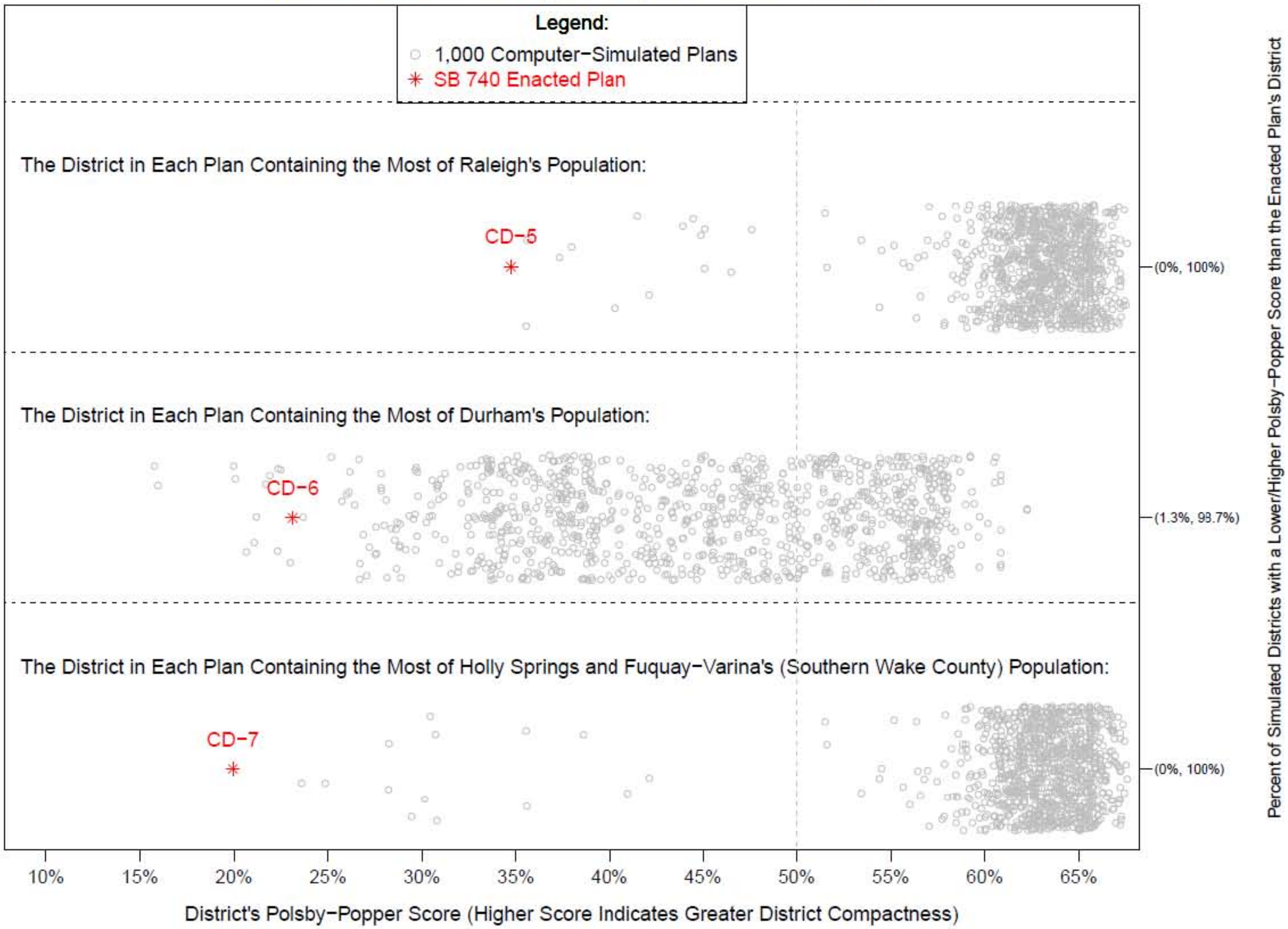
89. The top two rows of Figure 15 illustrate that extreme degree of Democratic voter packing in CD-5 and CD-6 is not the result of the Research Triangle's political geography or the Adopted Criteria. Instead, Figure 15 reveals that CD-5 and CD-6 are less geographically compact than nearly 100% of the computer-simulated districts containing Raleigh and Durham. Thus, the General Assembly managed to unnaturally pack Democrats in its Raleigh-based and Durham-based districts by subordinating geographic compactness in the drawing of these districts.

90. As a result of this packing of Democratic voters in CD-5 and CD-6, the surrounding districts in the Enacted Plan are more safely Republican than they would have been in the absence of such packing of Democrats. One example of these surrounding Republican districts in the Enacted Plan is CD-7, which combines Southern Wake County with various counties west of the Research Triangle. Southern Wake County is more politically moderate than the heavily Democratic cores of Raleigh and Durham. The third row of Figure 14 compares the partisanship of the Enacted Plan's district and each simulated plan's district containing the most of Holly Springs's and Fuquay-Varina's populations in Southern Wake County. The results on

this row illustrate that in the computer-simulated plans drawn according to the Adopted Criteria, Southern Wake County is generally placed into a heavily-Democratic district because it is generally placed into the same district with part of Raleigh. But the Enacted Plan packed Democrats into CD-5 (Raleigh) and CD-6 (Durham), so the General Assembly was able to create a safe Republican district by combining Southern Wake County with other Republican-favoring counties to the west of the Research Triangle. As the third row of Figure 14 illustrates, this outcome is an extreme statistical outlier compared to the computer-simulated districts in Southern Wake County. 99.2% of the simulated plans place Southern Wake County into a Democratic-favoring district, and 100% of the simulated districts containing Southern Wake County are less extremely Republican than CD-7. Hence, it is clear that CD-7 is a partisan outlier that was enabled by the packing of Democratic voters in CD-5 (Raleigh) and CD-6 (Durham).


Figure 14: Research Triangle Area:
Comparison of Individual Districts' Republican Vote Shares
in the SB 740 Plan and in 1,000 Computer−Simulated Plans
Legend:
1,000 Computer−Simulated Plans
SB 740 Enacted Plan
The District in Each Plan Containing the Most of Raleigh's Population:
CD−5
(0.4%, 99.6%)
The District in Each Plan Containing the Most of Durham's Population:
CD−6
(0%, 100%)
The District in Each Plan Containing the Most of Holly Springs and Fuquay−Varina's (Southern Wake County) Population:
CD−7
(100%, 0%)
25%
30%
35%
40%
45%
50%
55%
60%
65%
District's Republican Vote Share Measured Using the 2016−2020 Statewide Election Composite
Percent of Simulated Districts with a Lower/Higher Republican Vote Share than the Enacted Plan's District

Figure 15: Research Triangle Area: Comparison of Individual Districts' Compactness Scores in the SB 740 Plan and in 1,000 Computer−Simulated Plans



91. ***Mecklenburg County Districts:*** Figure 16 illustrates a similar finding regarding Mecklenburg County. The top row of Figure 16 compares the partisanship of the Enacted Plan's district and each simulated plan's district containing the most of Charlotte's population. The results in this row illustrate that the Enacted Plan's CD-9 is more heavily Democratic than 100% of the simulated plans' primary Charlotte districts.

92. As a result, the second and third rows of Figure 16 reveal that the surrounding suburban districts in the Enacted Plan are more safely Republican than their geographic counterparts in all of the computer-simulated plans. Specifically, the second row of Figure 16 compares the partisanship of the Enacted Plan's district and each simulated plan's district containing the most of Huntersville's (Northern Mecklenburg County) population. In the simulated plans, Huntersville is either placed into the same district as most of Charlotte, resulting in a heavily Democratic district, or it is grouped with other counties outside of Mecklenburg, thus forming a politically competitive district with a Republican vote share close to 50%. But the Enacted Plan places Huntersville into a district (CD-13) that is much more strongly Republican than all 100% of the simulated districts containing Huntersville.

93. The third row of Figure 16 reveals a similar finding regarding Eastern Mecklenburg County. Specifically, this row compares the partisanship of the Enacted Plan's district and each simulated plan's district containing the most of Mint Hill's and Matthews' (Eastern Mecklenburg County) population. Once again, the results reveal that the Enacted Plan places Eastern Mecklenburg County into a district (CD-8) that is more strongly Republican than all 100% of the computer-simulated districts containing Mint Hill and Matthews.

94. Thus, it is clear that the Enacted Plan packed Democrats in Mecklenburg County to an extent greater than what naturally occurs as a result of the area's political geography.

Democratic voters are residentially concentrated in Charlotte, and this political geography tends to cause a clustering of Democratic voters in Mecklenburg County districts, as reflected in the simulation results in Figure 16. But the Enacted Plan's packing of Democratic voters in Mecklenburg goes beyond what is caused by political geography, resulting in a Charlotte district that is even more heavily Democratic than what could be expected from a partisan-blind map-drawing process.

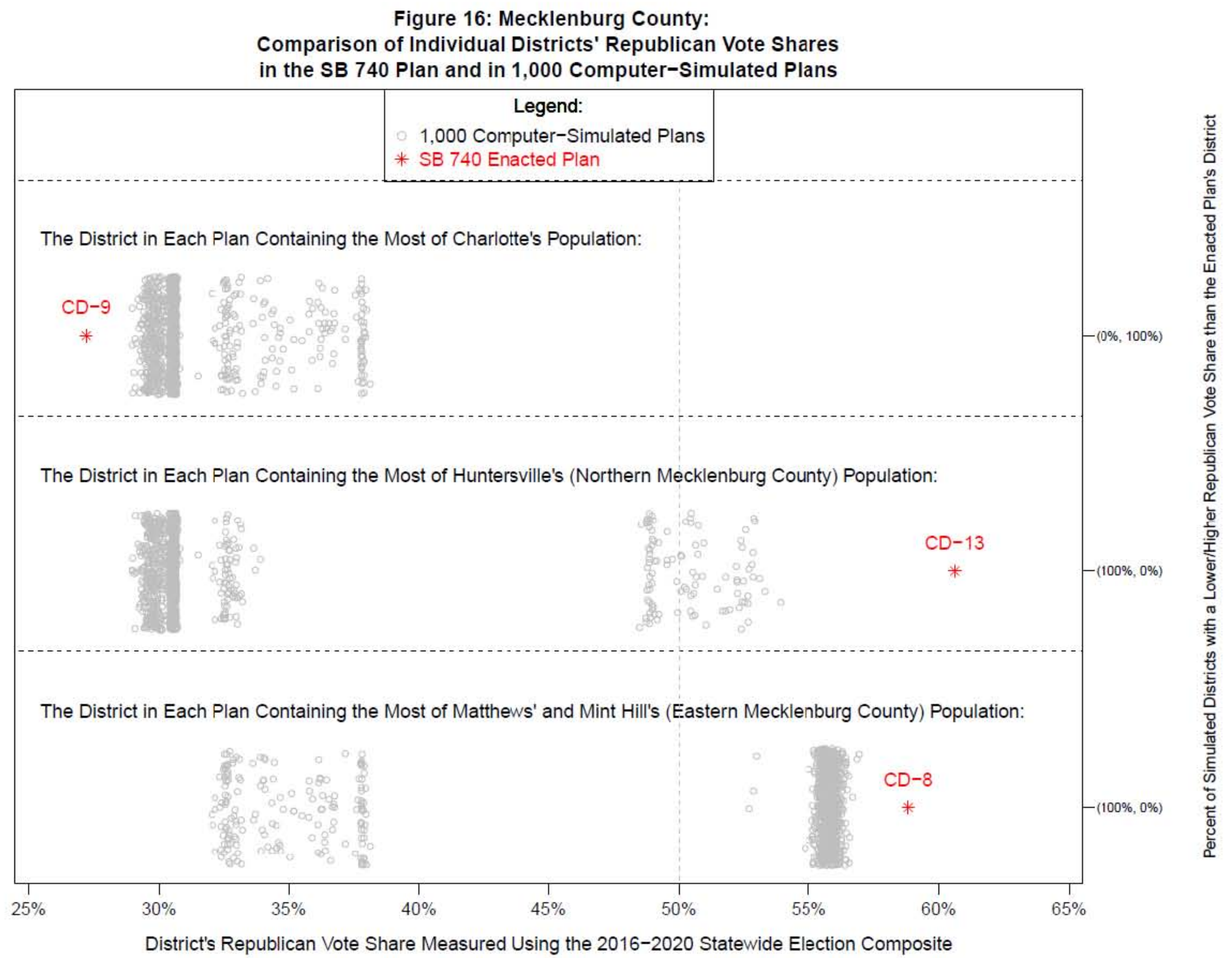


Figure 16: Mecklenburg County:
Comparison of Individual Districts' Republican Vote Shares
in the SB 740 Plan and in 1,000 Computer−Simulated Plans
Legend:
1,000 Computer−Simulated Plans
SB 740 Enacted Plan
The District in Each Plan Containing the Most of Charlotte's Population:
CD−9
(0%, 100%)
The District in Each Plan Containing the Most of Huntersville's (Northern Mecklenburg County) Population:
CD−13
(100%, 0%)
The District in Each Plan Containing the Most of Matthews' and Mint Hill's (Eastern Mecklenburg County) Population:
CD−8
(100%, 0%)
25%
30%
35%
40%
45%
50%
55%
60%
65%
District's Republican Vote Share Measured Using the 2016−2020 Statewide Election Composite
Percent of Simulated Districts with a Lower/Higher Republican Vote Share than the Enacted Plan's District

North Carolina's Political Geography Did Not Cause the Enacted Plan's Extreme Partisan Bias

95. How does North Carolina's political geography affect the partisan characteristics of the 2021 Enacted Plan? Democratic voters tend to be geographically concentrated in the urban cores of several of the state's largest cities, including Charlotte, Raleigh, and Greensboro. As I have explained in my prior academic research,[14] these large urban clusters of Democratic voters, combined with the common districting principle of drawing geographically compact districts, can sometimes result in urban districts that "naturally" pack together Democratic voters, thus boosting the Republican vote share of other surrounding suburban and rural districts.

96. More importantly, my prior academic research explained how I can estimate the precise level of electoral bias in districting caused by a state's unique political geography: I programmed a computer algorithm that draws districting plans using North Carolina's unique political geography, including the state's census population data and political subdivision boundaries. In this report, I have also programmed the algorithm to follow North Carolina's Adopted Criteria. I then analyzed the partisan characteristics of the simulated districting plans using North Carolina's precinct-level voting data from past elections (past elections that were themselves skewed towards Republicans). Hence, the entire premise of conducting districting simulations is to fully account for North Carolina's unique political geography, its political subdivision boundaries, and its districting criteria, as mandated by the General Assembly's Adopted Criteria.

97. This districting simulation analysis allowed me to identify how much of the

[14] Jowei Chen and Jonathan Rodden, 2013. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures" Quarterly Journal of Political Science, 8(3): 239-269; Jowei Chen and David Cottrell, 2016. "Evaluating Partisan Gains from Congressional Gerrymandering: Using Computer Simulations to Estimate the Effect of Gerrymandering in the U.S. House." Electoral Studies, Vol. 44, No. 4: 329-430.

electoral bias in the 2021 Enacted Plan is caused by North Carolina's political geography and how much is caused by the map-drawer's intentional efforts to favor one political party over the other. North Carolina's natural political geography, combined with the Adopted Criteria, almost never resulted in simulated congressional plans containing 10 Republican-favoring districts out of 14 total districts.

98. The 2021 Enacted Plan's creation of 10 electorally safe Republican districts, which persists across a range of electoral outcomes, goes beyond any "natural" level of electoral bias caused by North Carolina's political geography or the political composition of the state's voters. The Enacted Plan is a statistical outlier in terms of its partisan characteristics when compared to the 1,000 computer-simulated plans and cannot be explained by North Carolina's natural political geography.

99. The two most Republican districts (CD-10 and CD-13) and the two most Democratic districts (CD-9 and CD-6) in the Enacted Plan were drawn to include more Democratic voters than virtually all of their counterpart districts in the 1,000 computer-simulated plans. Six other districts (CD-1, 3, 4, 11, 12, and 14) were drawn to be more heavily Republican than over 95% of their counterpart computer-simulated plan districts. Ten districts were drawn precisely to have Republican vote shares within the narrow range of 52.9% to 61.2%—an outcome that never arises in the computer-simulated plans.

100. This extreme, additional level of partisan bias in the 2021 Enacted Plan can be directly attributed to the map-drawer's clear efforts to favor the Republican Party. This level of partisan bias was not caused by North Carolina's political geography.

The Effect of the Enacted Plan Districts on Plaintiffs

101. I evaluated the congressional districts in which each Plaintiff would reside under the 1,000 computer-simulated maps using a list of geocoded residential addresses for the Plaintiffs that counsel for the Plaintiffs provided me. I used these geocoded addresses to identify the specific district in which each Plaintiff would be located under each computer-simulated plan, as well as under the Enacted Plan. I then compared the partisanship of each individual Plaintiff's Enacted Plan district to the partisanship of the Plaintiff's 1,000 districts from the 1,000 computer-simulated plans. Using this approach, I identify whether each Plaintiff's district is a partisan outlier when compared to the Plaintiff's 1,000 computer-simulated districts.

102. Figures 17a and 17b present the results of this analysis. These Figures list the individual Plaintiffs and describes the partisanship of each Plaintiff's district of residence in the Enacted Plan, as well as the partisanship of the district the Plaintiff would have resided in under each of the 1,000 simulated congressional plans. The first half of the plaintiffs are analyzed in Figure 17a, while the second half of the plaintiffs appear in Figure 17b.

103. To explain these analyses with an example, each row in Figure 17a corresponds to a particular individual Plaintiff. In the first row, describing Plaintiff Bobby Jones, the red star depicts the partisanship of the Plaintiff's Enacted Plan district (CD-2), as measured by its Republican vote share using the 2016-2020 Statewide Election Composite. The 1,000 gray circles on this row depict the Republican vote share of each of the 1,000 simulated districts in which the Plaintiff would reside in each of the 1,000 computer-simulated plans, based on that Plaintiff's residential address. In the margin to the right of each row, I list in parentheses how many of the 1,000 simulated plans would place the plaintiff in a more Democratic-leaning district (on the left) and how many of the 1,000 simulations would place the plaintiff in a more

Republican-leaning district (on the right) than the Plaintiff's Enacted Plan district. Thus, for example, the first row of Figure 17a reports that 99% of the 1,000 computer-simulated plans would place Plaintiff Bobby Jones in a more Republican-leaning district than his actual Enacted Plan district (CD-2). Therefore, I can conclude that Plaintiff Bobby Jones' Enacted Plan district is a partisan statistical outlier when compared to his district under the 1,000 simulated plans.

Figure 17a:
Plaintiffs' Districts in the SB 740 Plan and in 1,000 Computer−Simulated Plans

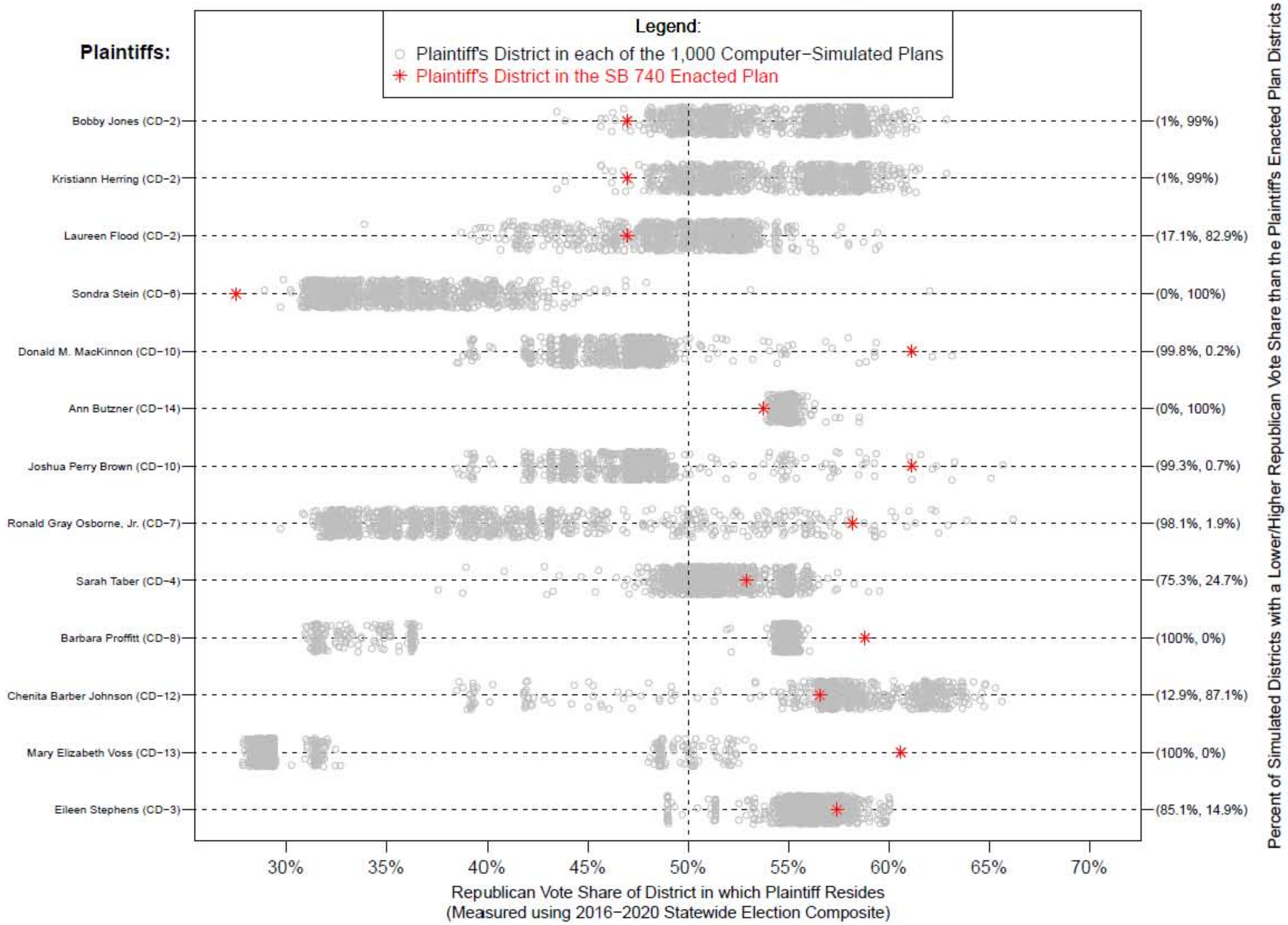
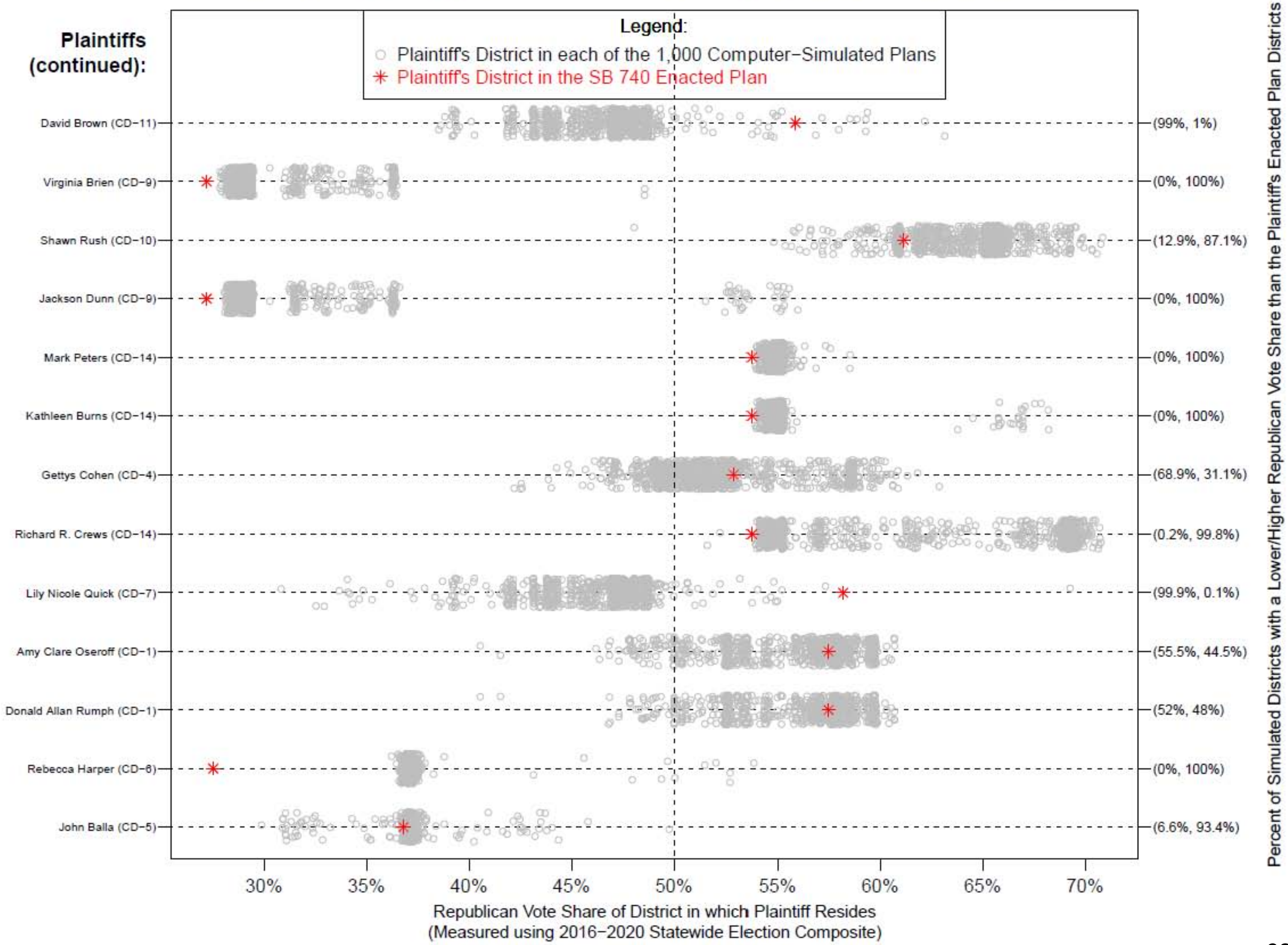


Figure 17b:
Plaintiffs' Districts in the SB 740 Plan and in 1,000 Computer−Simulated Plans



104. Figures 17a and 17b show that seven Plaintiffs residing in Republican-leaning districts under the Enacted Plan would be placed in a more Democratic-leaning district in over 95% of the computer-simulated plans: Donald M. MacKinnon (CD-10), Joshua Perry Brown (CD-10), Ronald Gray Osborne, Jr. (CD-7), Barbara Proffitt (CD-8), Mary Elizabeth Voss (CD-13); David Brown (CD-11) and Lily Nicole Quick (CD-7). Additionally, six Plaintiffs residing in Democratic-leaning districts under the Enacted Plan would be placed in a more Republican-leaning district in over 95% of the computer-simulated plans: Bobby Jones (CD-2), Kristiann Herring (CD-2), Sondra Stein (CD-6), Virginia Brien (CD-9), Jackson Dunn (CD-9), and Rebecca Harper (CD-6). Additionally, six Plaintiffs would be placed in a more Republican district in 99.9% or more of the simulated plans relative to their districts under the Enacted Plan: Ann Butzner (CD-14), Virginia Brien (CD-9), Jackson Dunn (CD-9), Mark Peters (CD-14), Kathleen Barnes (CD-14), Richard R. Crews (CD-14), and Rebecca Harper (CD-6).

I declare under penalty of perjury that the foregoing is true and correct to the best of my knowledge.

This 23rd day of December, 2021.

Dr. Jowei Chen

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Curriculum Vitae

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Research Associate Professor (2016-present), Faculty Associate (2009-2015), Center for Political Studies, University of Michigan.
W. Glenn Campbell and Rita Ricardo-Campbell National Fellow, Hoover Institution, Stanford University, 2013.
Principal Investigator and Senior Research Fellow, Center for Governance and Public Policy Research, Willamette University, 2013 – Present.

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Publications:

Chen, Jowei and Neil Malhotra. 2007. "The Law of k/n: The Effect of Chamber Size on Government Spending in Bicameral Legislatures."
***American Political Science Review*. 101(4): 657-676.**

Chen, Jowei, 2010. "The Effect of Electoral Geography on Pork Barreling in Bicameral Legislatures."
***American Journal of Political Science.* 54(2): 301-322.**

Chen, Jowei, 2013. "Voter Partisanship and the Effect of Distributive Spending on Political Participation."
***American Journal of Political Science*. 57(1): 200-217.**

Chen, Jowei and Jonathan Rodden, 2013. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures"
***Quarterly Journal of Political Science*, 8(3): 239-269.**

Bradley, Katharine and Jowei Chen, 2014. "Participation Without Representation? Senior Opinion, Legislative Behavior, and Federal Health Reform."
***Journal of Health Politics, Policy and Law*. 39(2), 263-293.**

Chen, Jowei and Tim Johnson, 2015. "Federal Employee Unionization and Presidential Control of the Bureaucracy: Estimating and Explaining Ideological Change in Executive Agencies."
***Journal of Theoretical Politics*, Volume 27, No. 1: 151-174.**

Bonica, Adam, Jowei Chen, and Tim Johnson, 2015. "Senate Gate-Keeping, Presidential Staffing of 'Inferior Offices' and the Ideological Composition of Appointments to the Public Bureaucracy."
***Quarterly Journal of Political Science*. Volume 10, No. 1: 5-40.**

Chen, Jowei and Jonathan Rodden, 2015. "Cutting Through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders."
***Election Law Journal*. Volume 14, Number 4: 331-345.**

Chen, Jowei and David Cottrell, 2016. "Evaluating Partisan Gains from Congressional Gerrymandering: Using Computer Simulations to Estimate the Effect of Gerrymandering in the U.S. House."
***Electoral Studies*. Volume 44 (December 2016): 329-340.**

Chen, Jowei, 2017. "Analysis of Computer-Simulated Districting Maps for the Wisconsin State Assembly."
***Election Law Journal*. Volume 16, Number 4 (December 2017): 417-442.**

Chen, Jowei and Nicholas Stephanopoulos, 2020. "The Race-Blind Future of Voting Rights."
Yale Law Journal, Forthcoming. Volume 130, Number 4: 778-1049.

Kim, Yunsieg and Jowei Chen, 2021. "Gerrymandered by Definition: The Distortion of 'Traditional' Districting Principles and a Proposal for an Empirical Redefinition."
Wisconsin Law Review, Forthcoming, Volume 2021, Number 1.

Chen, Jowei and Nicholas Stephanopoulos, 2021. "Democracy's Denominator."
California Law Review, Accepted for Publication, Volume 109.

Non-Peer-Reviewed Publication:

Chen, Jowei and Tim Johnson. 2017. "Political Ideology in the Bureaucracy."
***Global Encyclopedia of Public Administration, Public Policy, and Governance*.**

Research Grants:

"How Citizenship-Based Redistricting Systemically Disadvantages Voters of Color". 2020 ($18,225). Combating and Confronting Racism Grant. University of Michigan Center for Social Solutions and Poverty Solutions.

Principal Investigator. National Science Foundation Grant SES-1459459, September 2015 – August 2018 ($165,008). "The Political Control of U.S. Federal Agencies and Bureaucratic Political Behavior."

"Economic Disparity and Federal Investments in Detroit," (with Brian Min) 2011. Graham Institute, University of Michigan ($30,000).

"The Partisan Effect of OSHA Enforcement on Workplace Injuries," (with Connor Raso) 2009. John M. Olin Law and Economics Research Grant ($4,410).

Invited Talks:

September, 2011. University of Virginia, American Politics Workshop.
October 2011. Massachusetts Institute of Technology, American Politics Conference.
January 2012. University of Chicago, Political Economy/American Politics Seminar.
February 2012. Harvard University, Positive Political Economy Seminar.
September 2012. Emory University, Political Institutions and Methodology Colloquium.
November 2012. University of Wisconsin, Madison, American Politics Workshop.
September 2013. Stanford University, Graduate School of Business, Political Economy Workshop.
February 2014. Princeton University, Center for the Study of Democratic Politics Workshop.
November 2014. Yale University, American Politics and Public Policy Workshop.
December 2014. American Constitution Society for Law & Policy Conference: Building the Evidence to Win Voting Rights Cases.
February 2015. University of Rochester, American Politics Working Group.
March 2015. Harvard University, Voting Rights Act Workshop.
May 2015. Harvard University, Conference on Political Geography.
Octoer 2015. George Washington University School of Law, Conference on Redistricting Reform.
September 2016. Harvard University Center for Governmental and International Studies, Voting Rights Institute Conference.
March 2017. Duke University, Sanford School of Public Policy, Redistricting Reform Conference.
October 2017. Willamette University, Center for Governance and Public Policy Research
October 2017, University of Wisconsin, Madison. Geometry of Redistricting Conference.
February 2018: University of Georgia Law School
September 2018. Willamette University.
November 2018. Yale University, Redistricting Workshop.

November 2018. University of Washington, Severyns Ravenholt Seminar in Comparative Politics.
January 2019. Duke University, Reason, Reform & Redistricting Conference.
February 2019. Ohio State University, Department of Political Science. Departmental speaker series.
March 2019. Wayne State University Law School, Gerrymandering Symposium.
November 2019. Big Data Ignite Conference.
November 2019. Calvin College, Department of Mathematics and Statistics.
September 2020 (Virtual). Yale University, Yale Law Journal Scholarship Workshop

Conference Service:

Section Chair, 2017 APSA (San Francisco, CA), Political Methodology Section
Discussant, 2014 Political Methodology Conference (University of Georgia)
Section Chair, 2012 MPSA (Chicago, IL), Political Geography Section.
Discussant, 2011 MPSA (Chicago, IL) "Presidential-Congressional Interaction."
Discussant, 2008 APSA (Boston, MA) "Congressional Appropriations."
Chair and Discussant, 2008 MPSA (Chicago, IL) "Distributive Politics: Parties and Pork."

Conference Presentations and Working Papers:

"Ideological Representation of Geographic Constituencies in the U.S. Bureaucracy," (with Tim Johnson). 2017 APSA.

"Incentives for Political versus Technical Expertise in the Public Bureaucracy," (with Tim Johnson). 2016 APSA.

"Black Electoral Geography and Congressional Districting: The Effect of Racial Redistricting on Partisan Gerrymandering". 2016 Annual Meeting of the Society for Political Methodology (Rice University)

"Racial Gerrymandering and Electoral Geography." Working Paper, 2016.

"Does Deserved Spending Win More Votes? Evidence from Individual-Level Disaster Assistance," (with Andrew Healy). 2014 APSA.

"The Geographic Link Between Votes and Seats: How the Geographic Distribution of Partisans Determines the Electoral Responsiveness and Bias of Legislative Elections," (with David Cottrell). 2014 APSA.

"Gerrymandering for Money: Drawing districts with respect to donors rather than voters." 2014 MPSA.

"Constituent Age and Legislator Responsiveness: The Effect of Constituent Opinion on the Vote for Federal Health Reform." (with Katharine Bradley) 2012 MPSA.

"Voter Partisanship and the Mobilizing Effect of Presidential Advertising." (with Kyle Dropp) 2012 MPSA.

"Recency Bias in Retrospective Voting: The Effect of Distributive Benefits on Voting Behavior." (with Andrew Feher) 2012 MPSA.

"Estimating the Political Ideologies of Appointed Public Bureaucrats," (with Adam Bonica and Tim Johnson) 2012 Annual Meeting of the Society for Political Methodology (University of North Carolina)

"Tobler's Law, Urbanization, and Electoral Bias in Florida." (with Jonathan Rodden) 2010 Annual Meeting of the Society for Political Methodology (University of Iowa)

"Unionization and Presidential Control of the Bureaucracy" (with Tim Johnson) 2011 MPSA.

"Estimating Bureaucratic Ideal Points with Federal Campaign Contributions" 2010 APSA. (Washington, DC).

"The Effect of Electoral Geography on Pork Spending in Bicameral Legislatures," Vanderbilt University Conference on Bicameralism, 2009.

"When Do Government Benefits Influence Voters' Behavior? The Effect of FEMA Disaster Awards on US Presidential Votes," 2009 APSA (Toronto, Canada).

"Are Poor Voters Easier to Buy Off?" 2009 APSA (Toronto, Canada).

"Credit Sharing Among Legislators: Electoral Geography's Effect on Pork Barreling in Legislatures," 2008 APSA (Boston, MA).

"Buying Votes with Public Funds in the US Presidential Election," Poster Presentation at the 2008 Annual Meeting of the Society for Political Methodology (University of Michigan).

"The Effect of Electoral Geography on Pork Spending in Bicameral Legislatures," 2008 MPSA.

"Legislative Free-Riding and Spending on Pure Public Goods," 2007 MPSA (Chicago, IL).

"Free Riding in Multi-Member Legislatures," (with Neil Malhotra) 2007 MPSA (Chicago, IL).

"The Effect of Legislature Size, Bicameralism, and Geography on Government Spending: Evidence from the American States," (with Neil Malhotra) 2006 APSA (Philadelphia, PA).

Figure A1: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2016 Attorney General Election Results

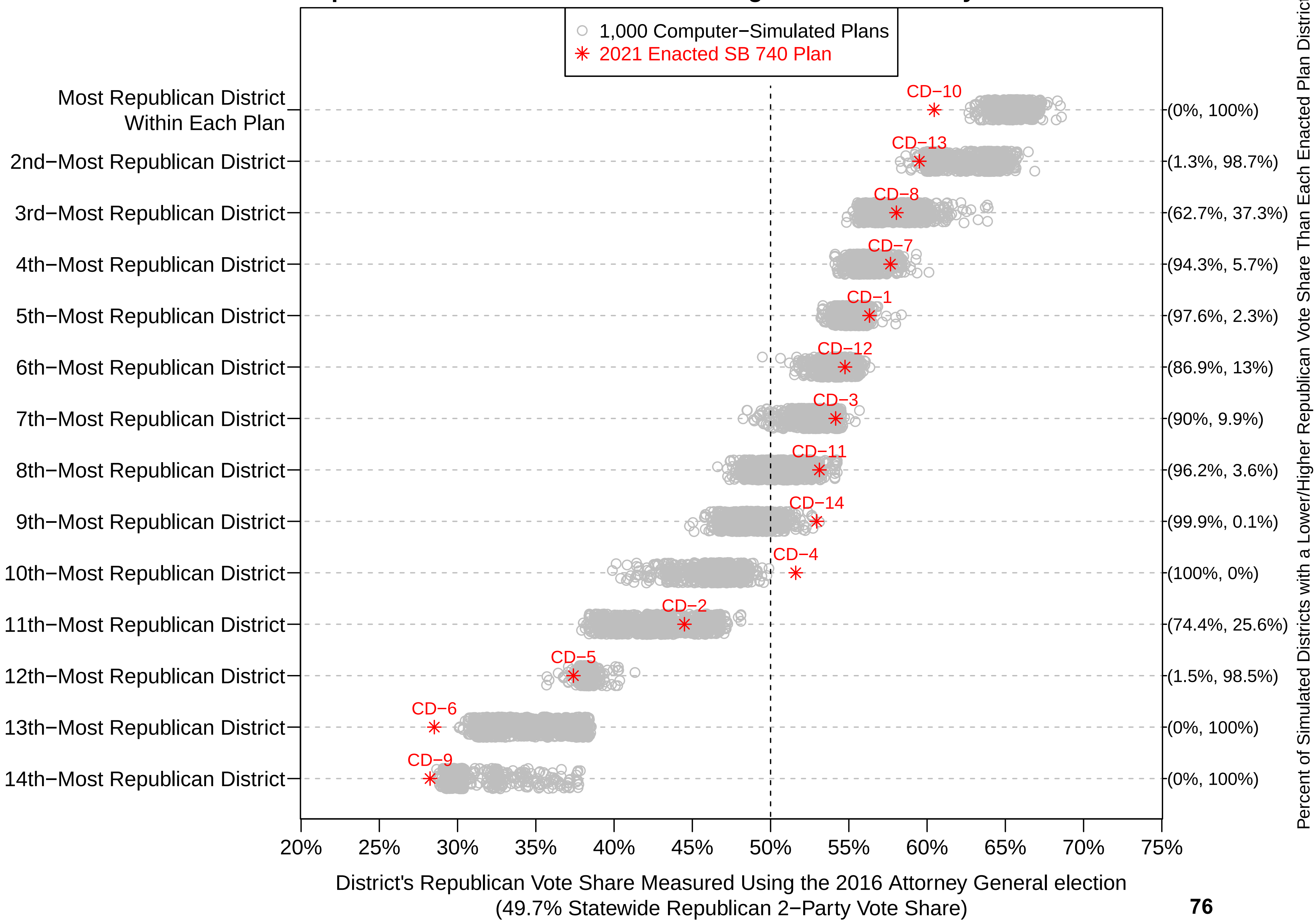


Figure A2: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2016 Governor Election Results

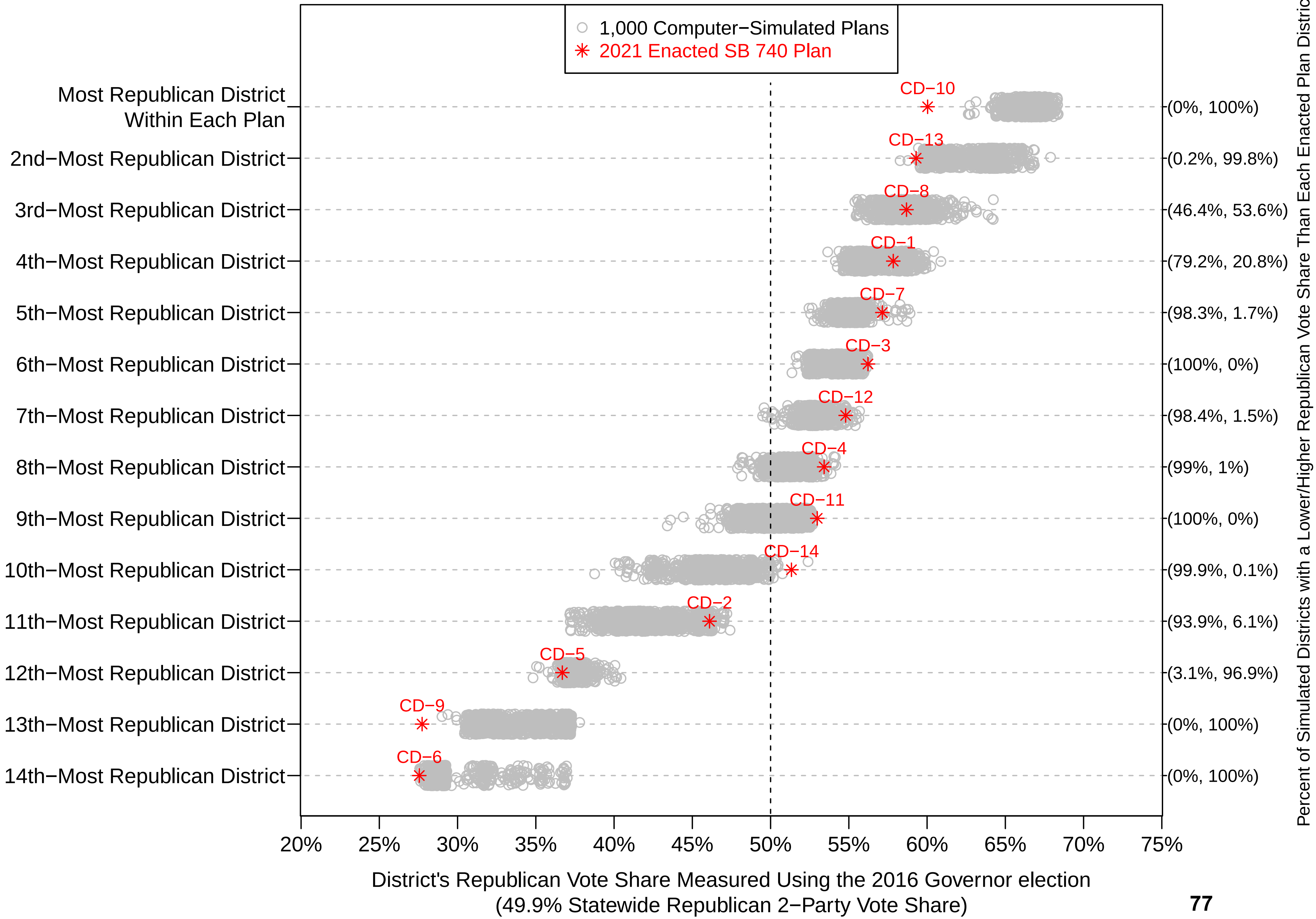


Figure A3: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2016 Lieutenant Governor Election Results

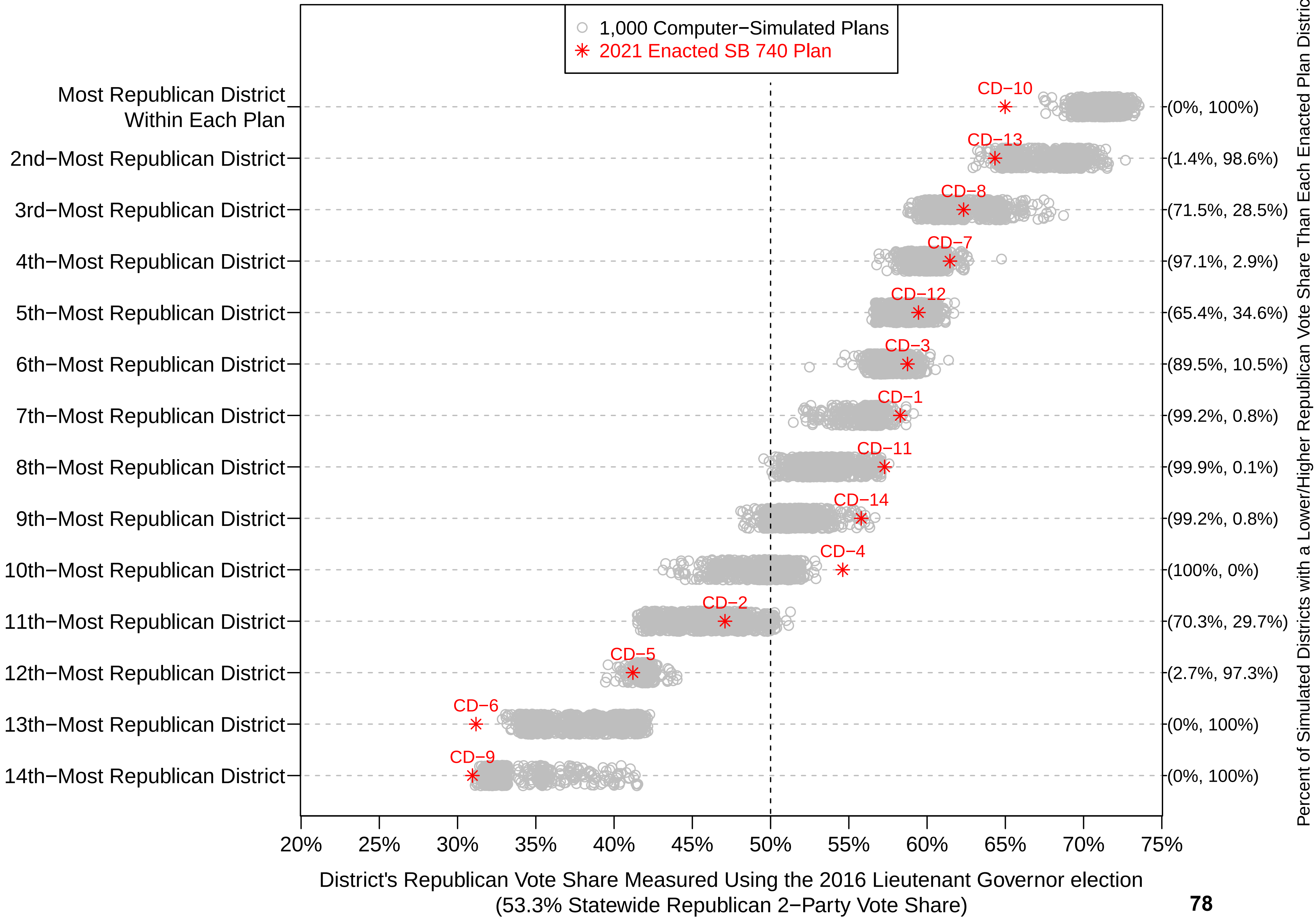


Figure A4: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2016 US President Election Results

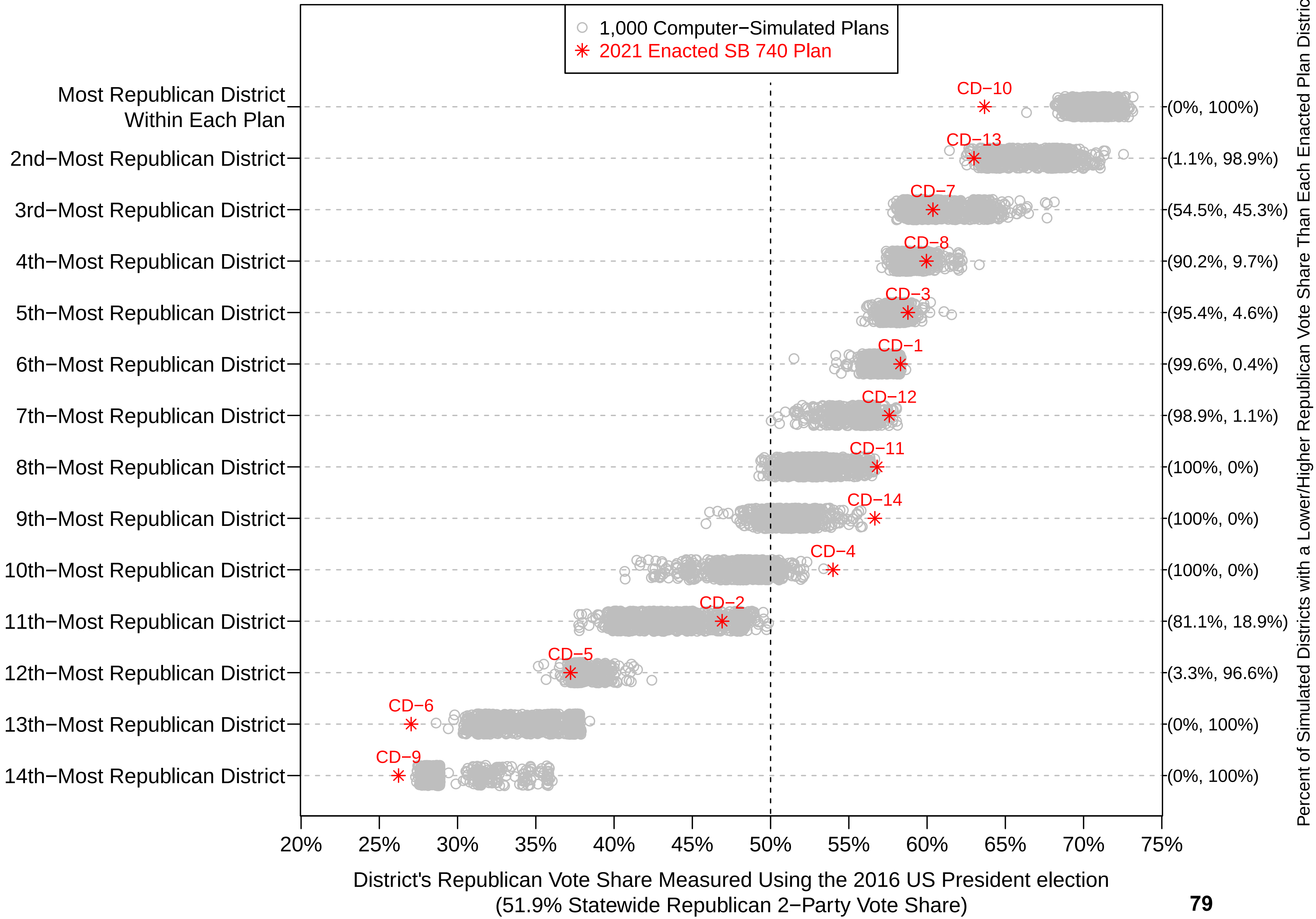


Figure A5: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2016 US Senator Election Results

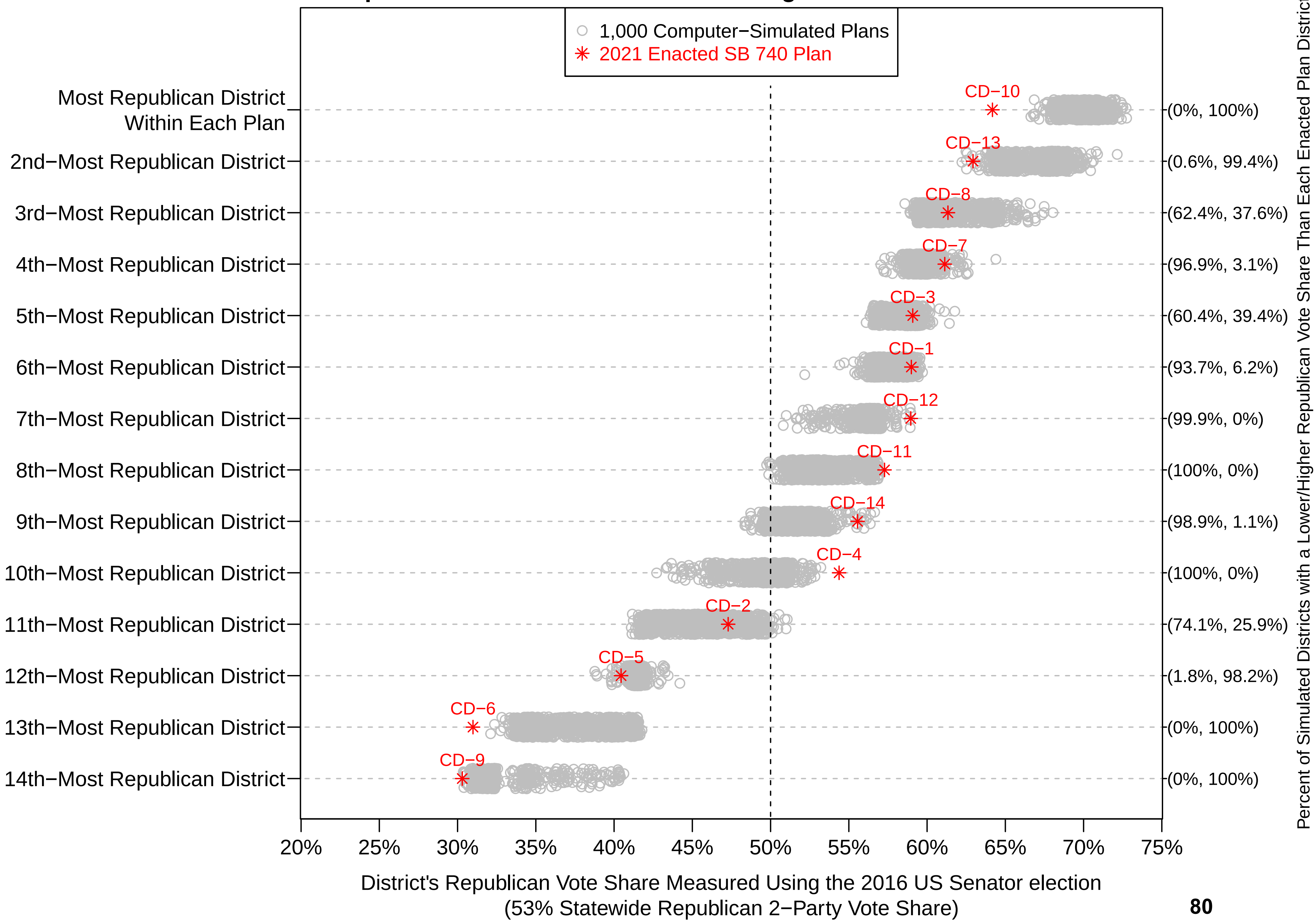


Figure A6: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2020 Attorney General Election Results

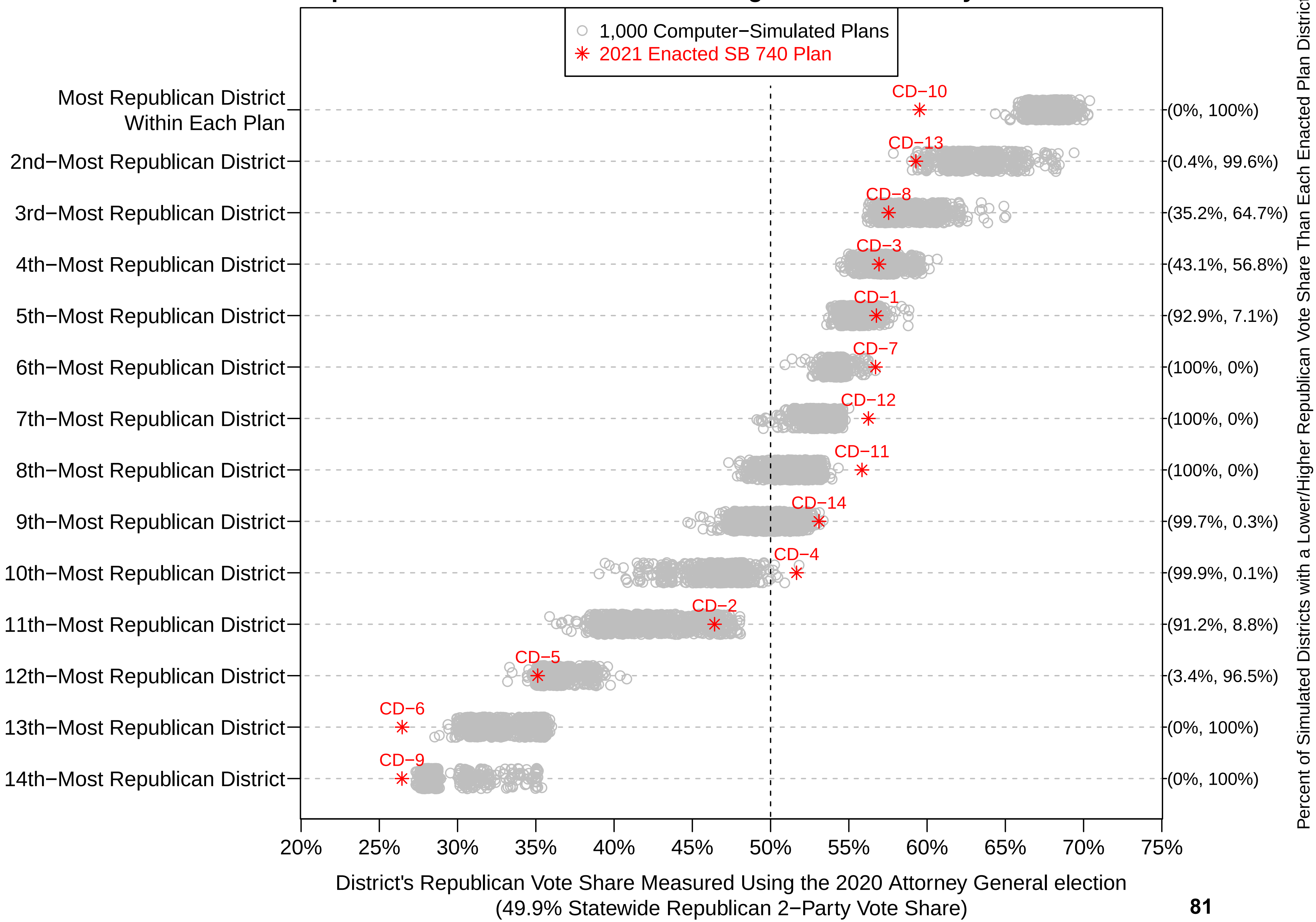


Figure A7: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2020 Governor Election Results

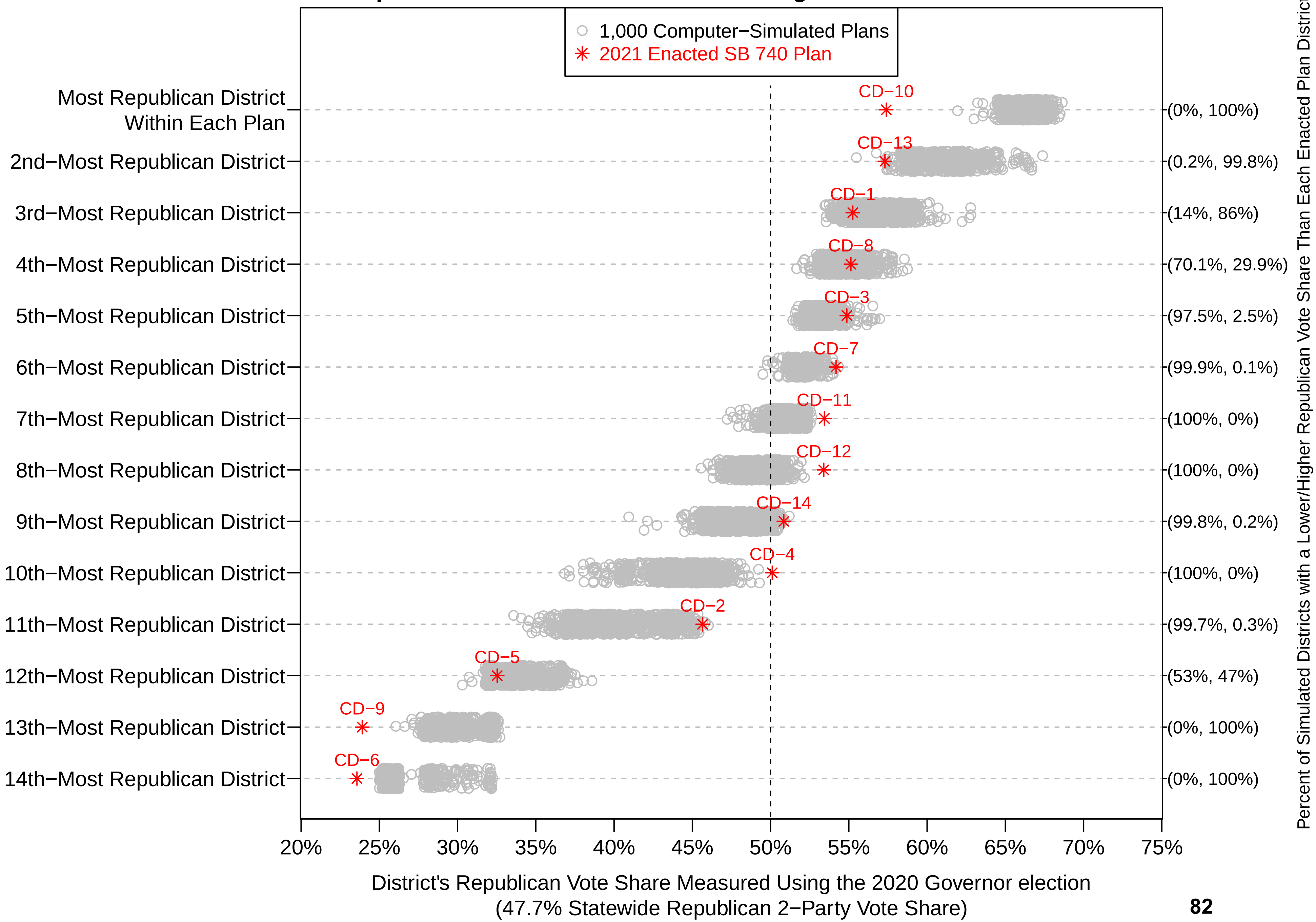


Figure A8: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2020 Lieutenant Governor Election Results

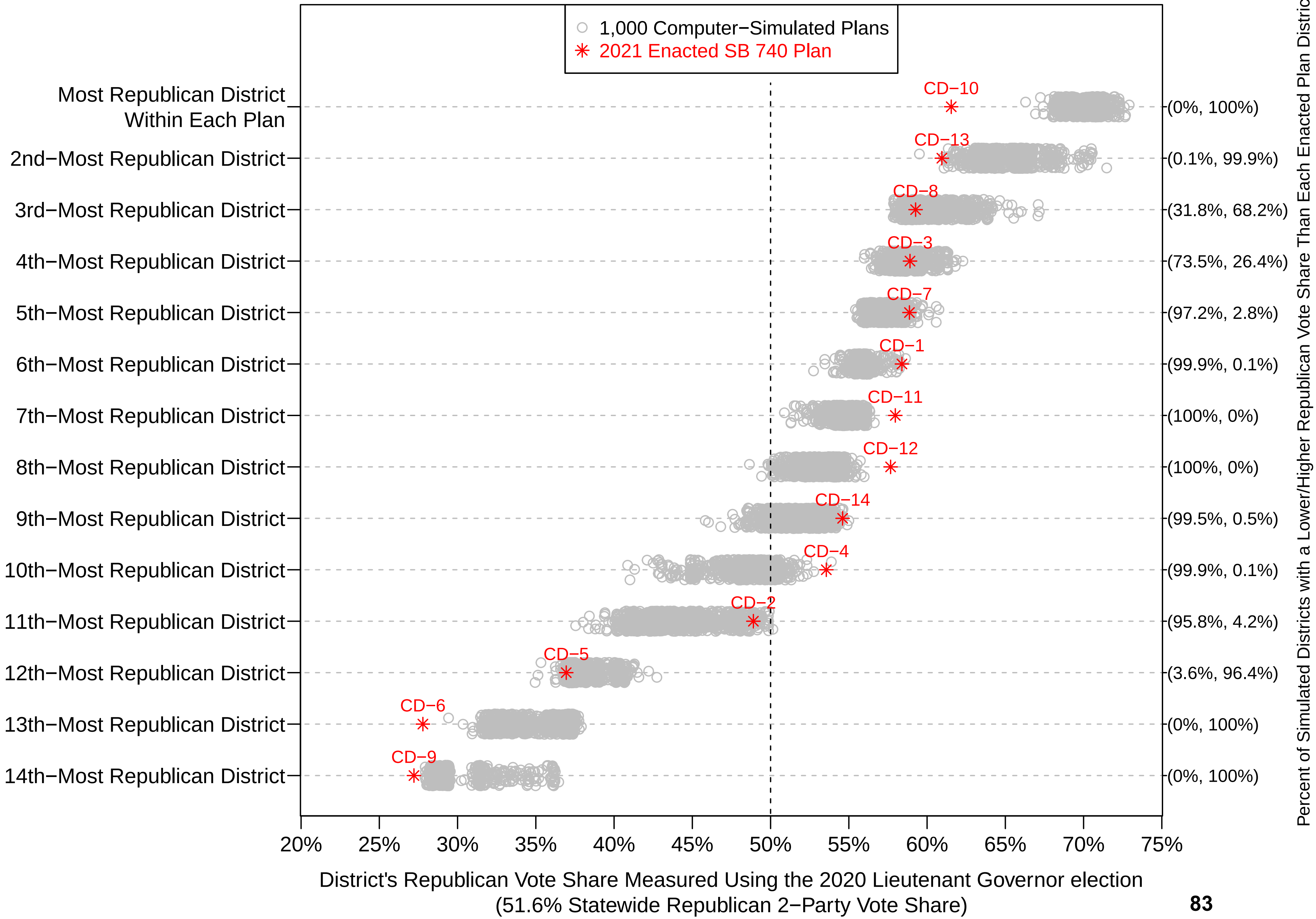


Figure A9: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2020 US President Election Results

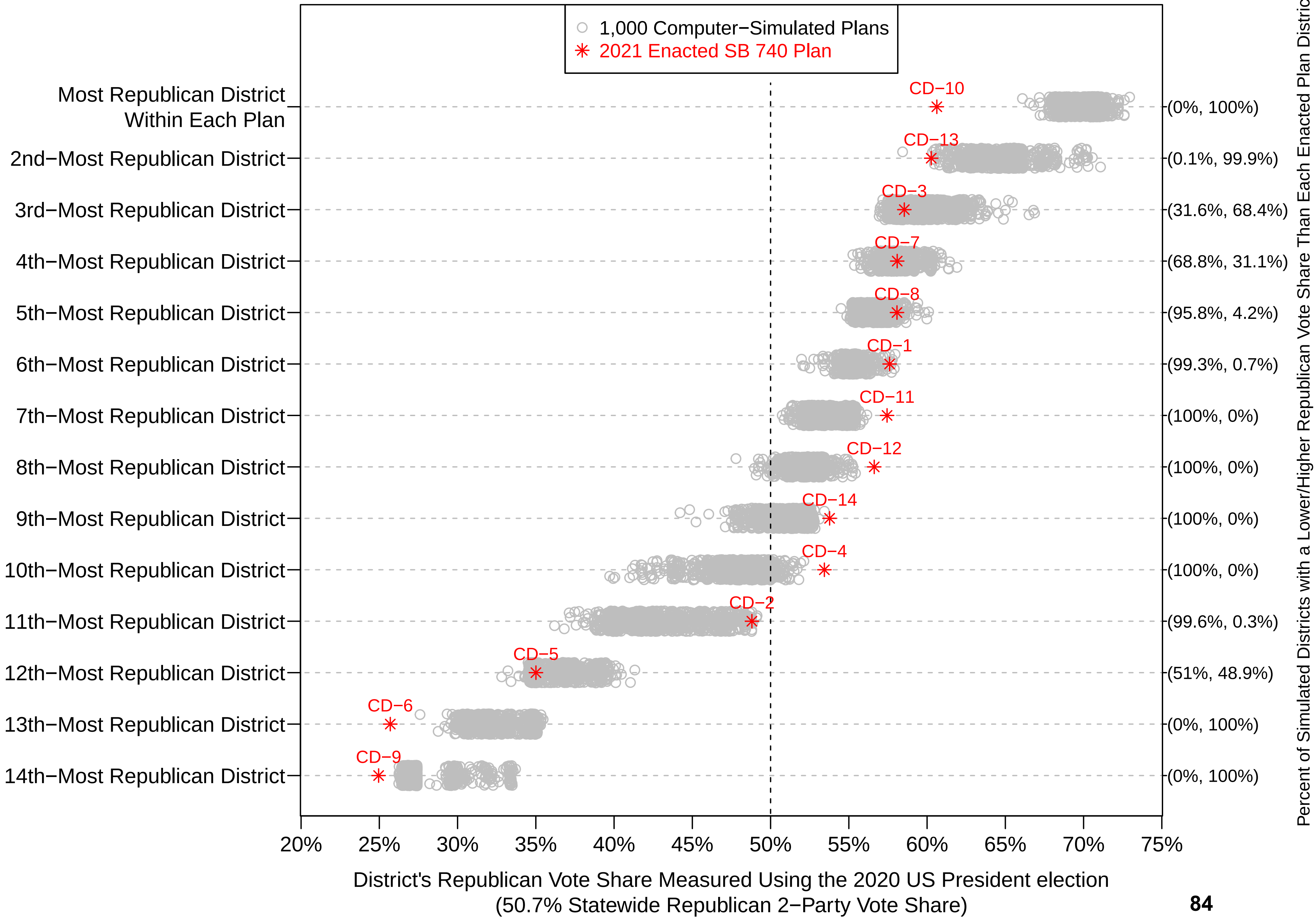


Figure A10: Comparison of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans: Districts' Republican Vote Share Measured Using the 2020 US Senator Election Results

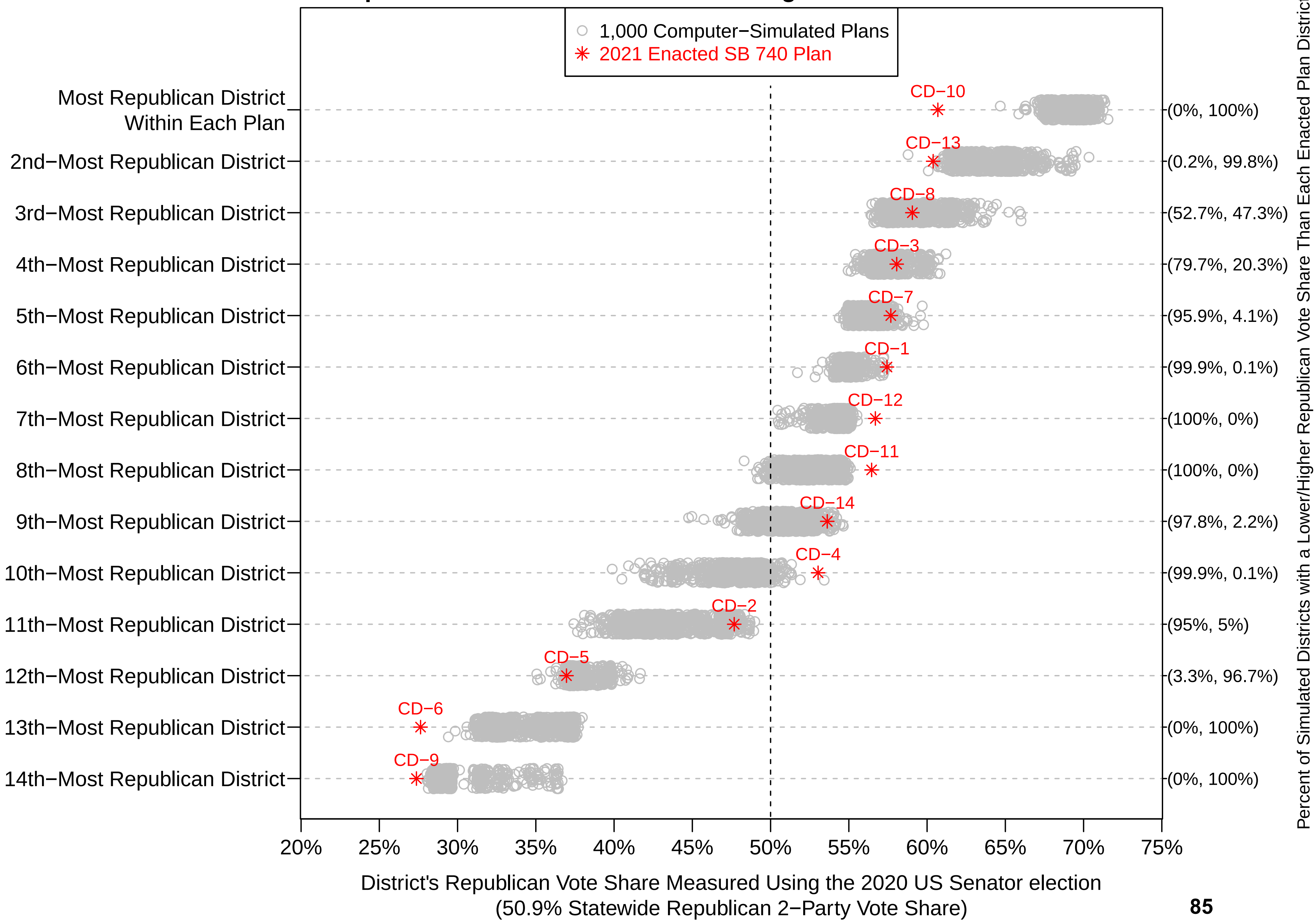


Figure B1: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2016 Attorney General election
(49.7% Statewide Republican 2−Party Vote Share)

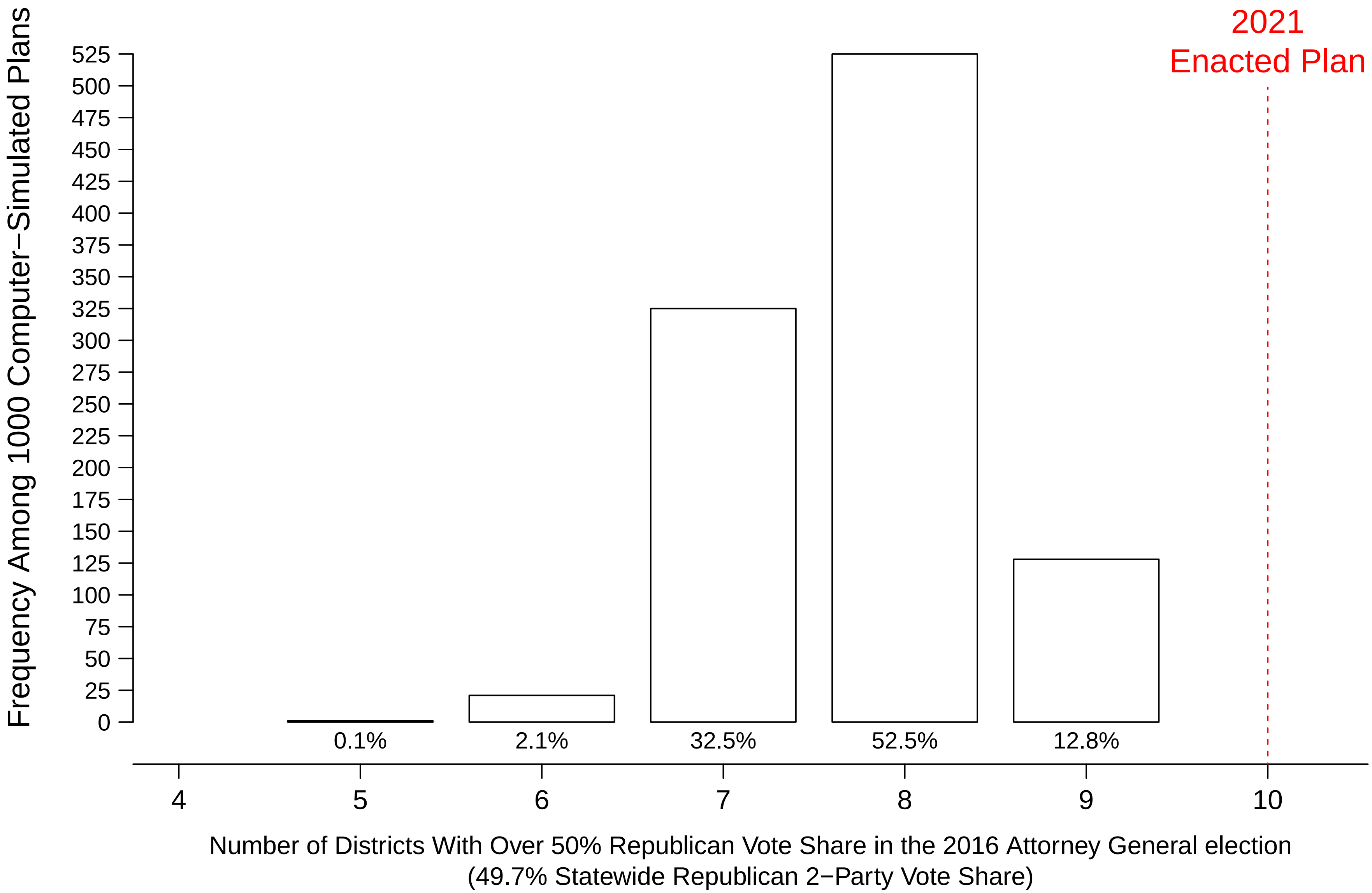


Figure B2: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2016 Governor election
(49.9% Statewide Republican 2−Party Vote Share)

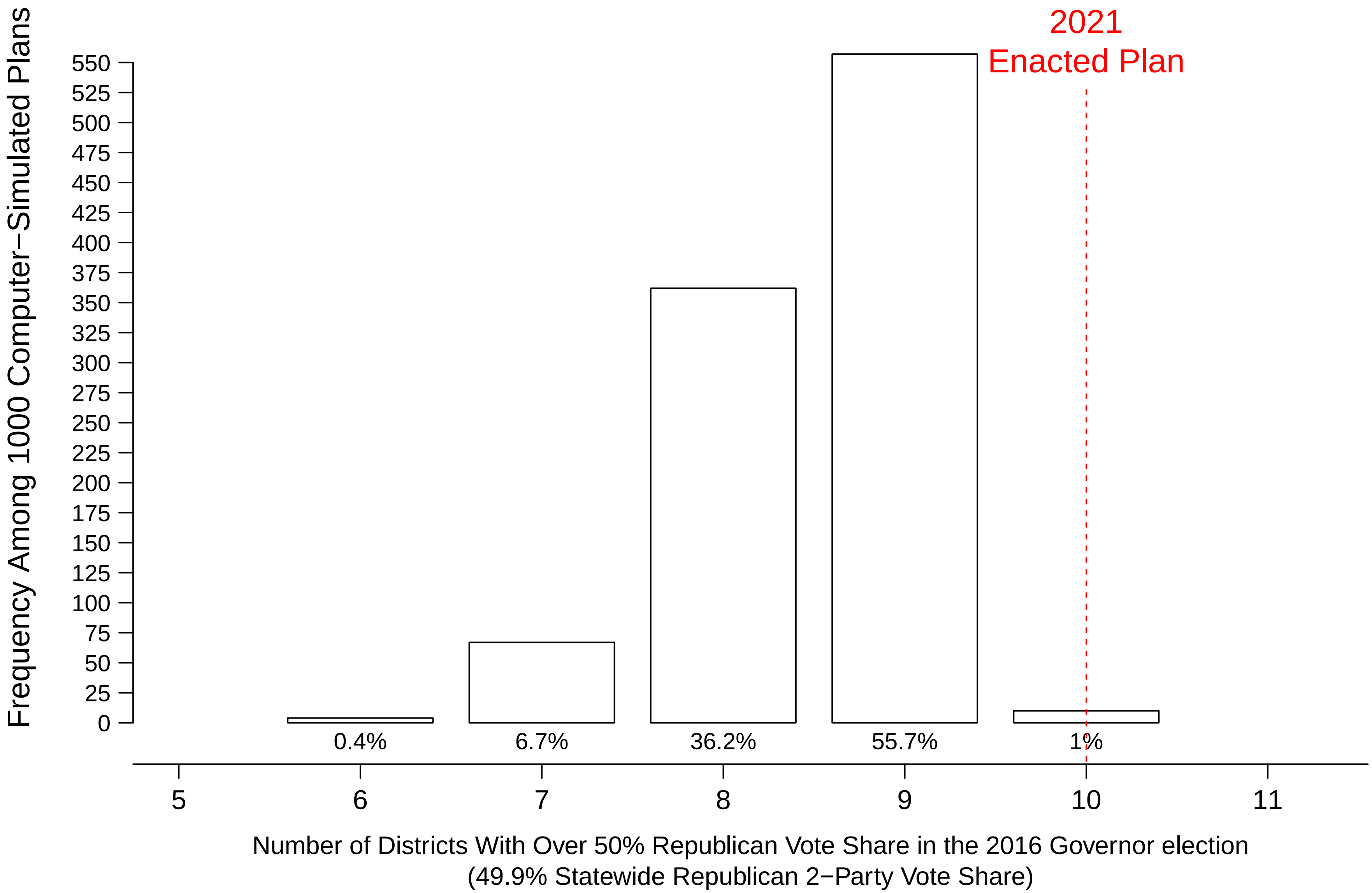


Figure B3: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2016 Lieutenant Governor election
(53.3% Statewide Republican 2−Party Vote Share)

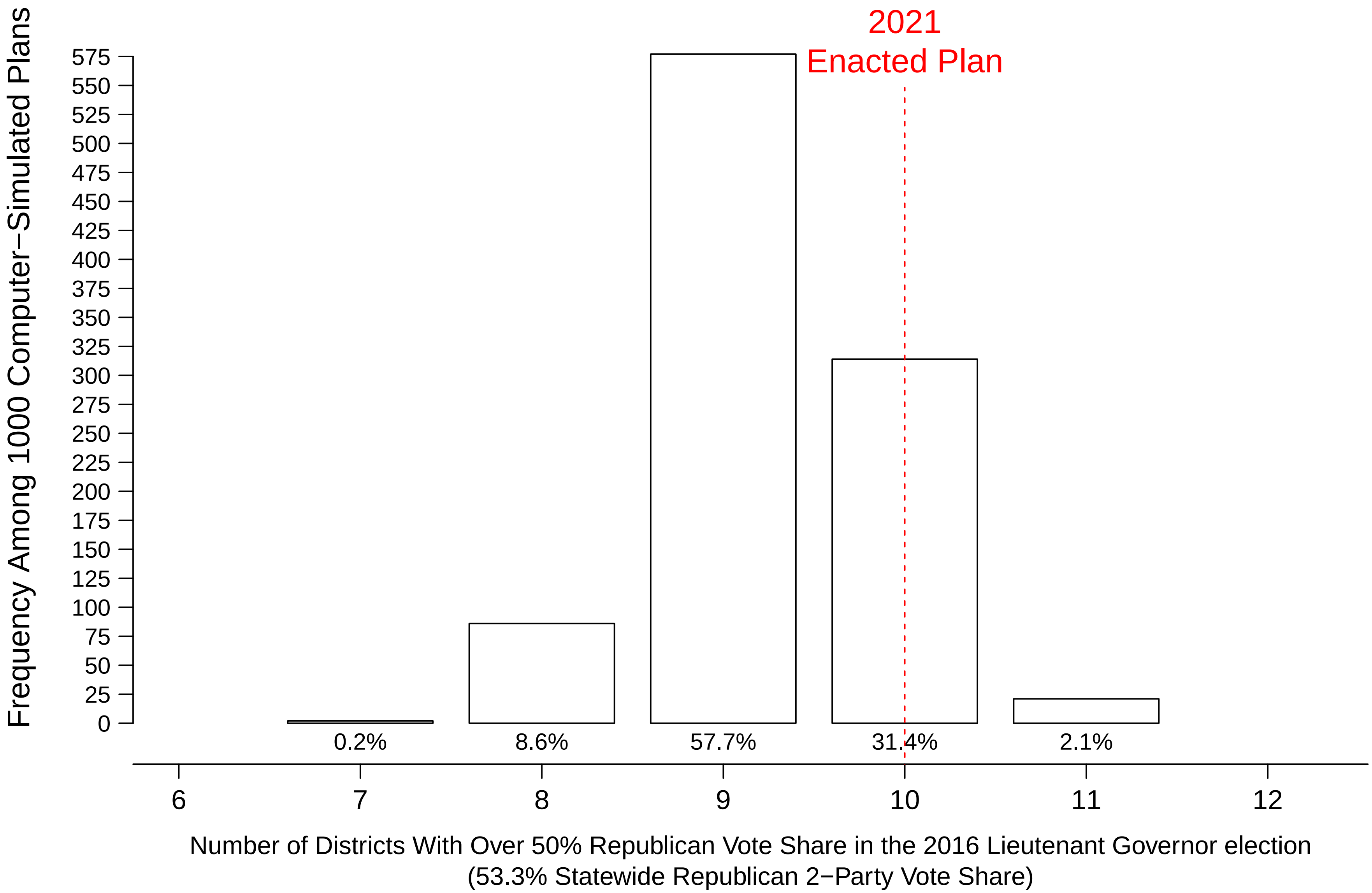


Figure B4: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2016 US President election
(51.9% Statewide Republican 2−Party Vote Share)

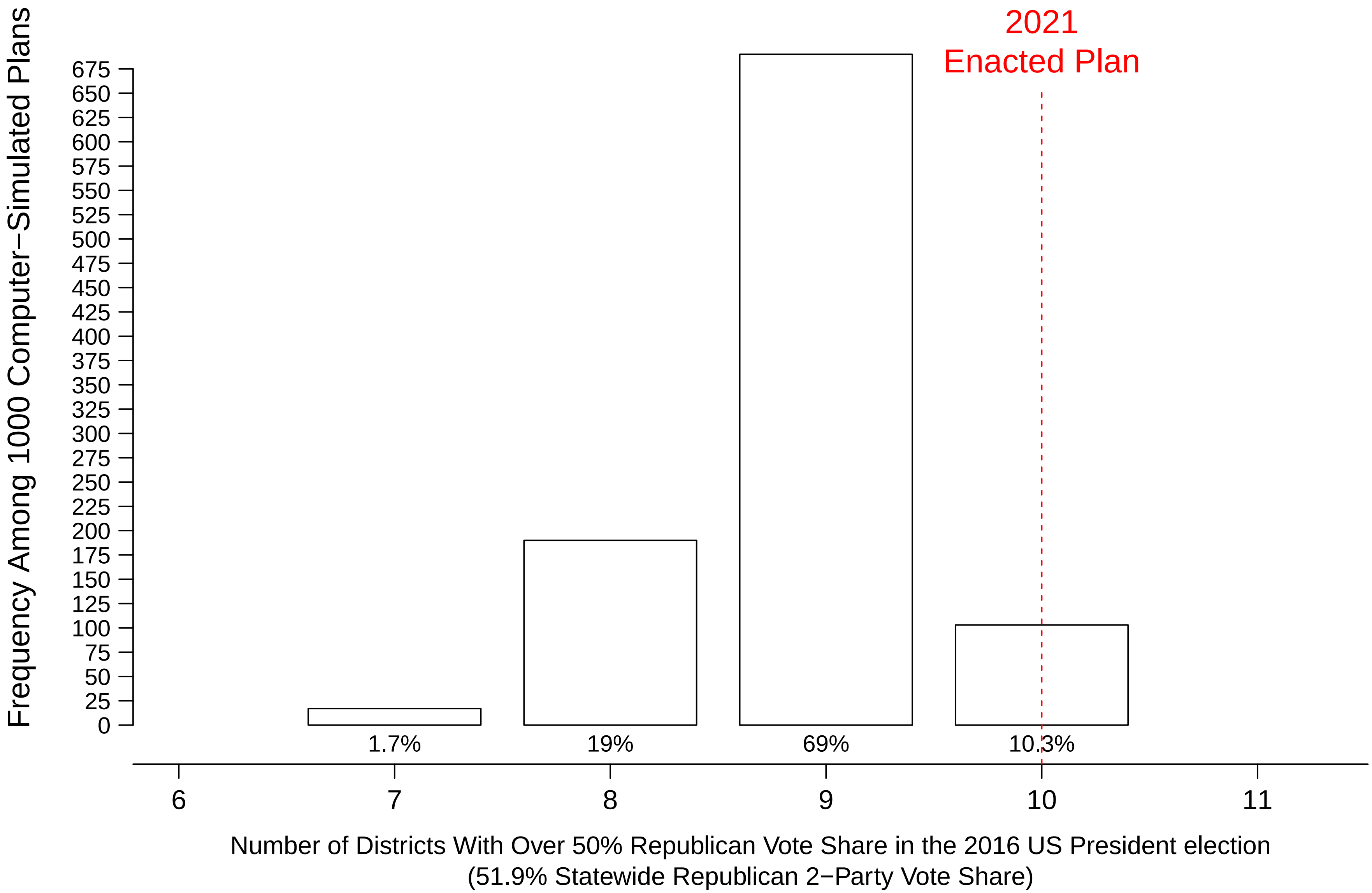


Figure B5: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2016 US Senator election
(53% Statewide Republican 2−Party Vote Share)

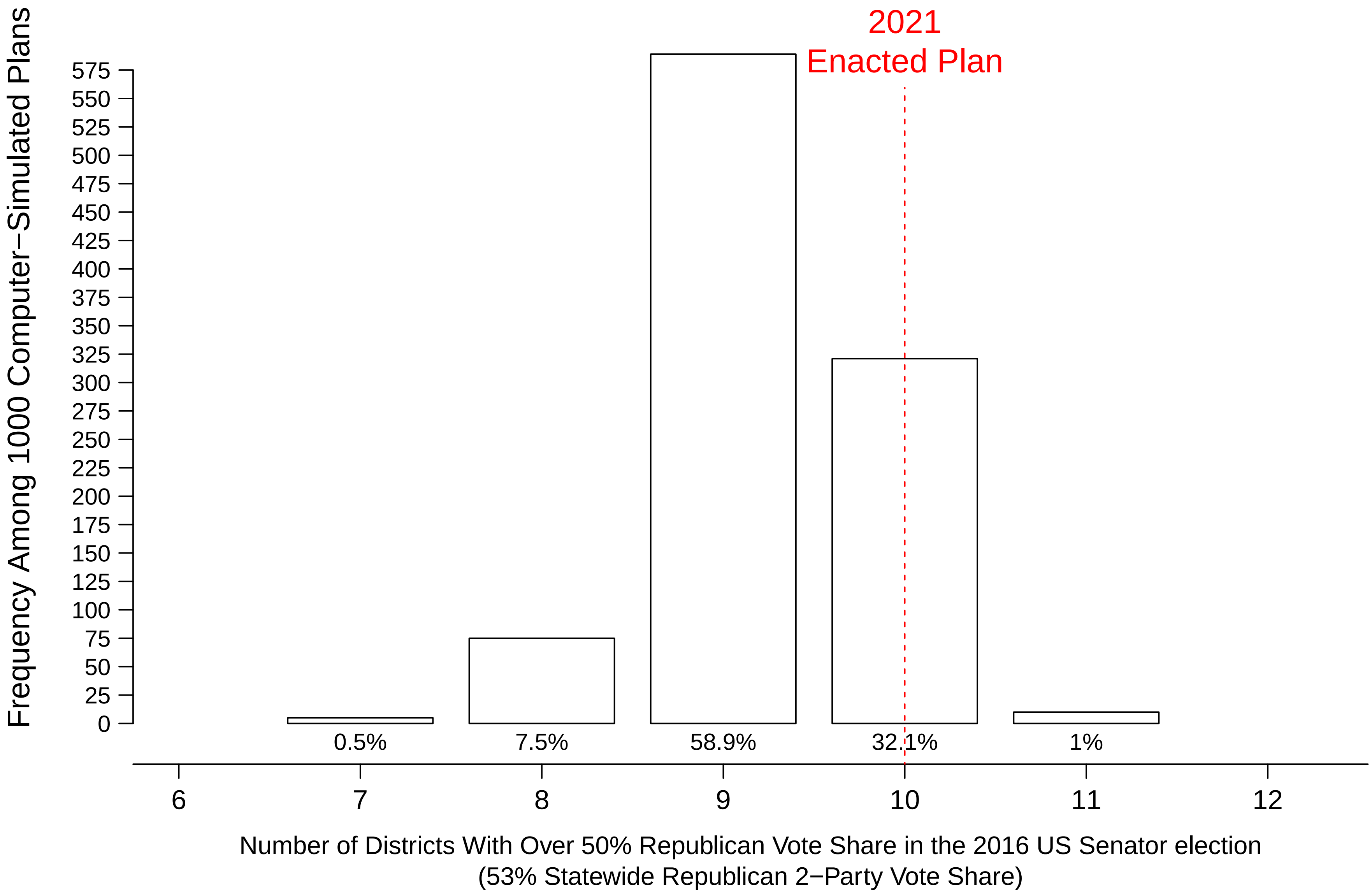


Figure B6: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2020 Attorney General election
(49.9% Statewide Republican 2−Party Vote Share)

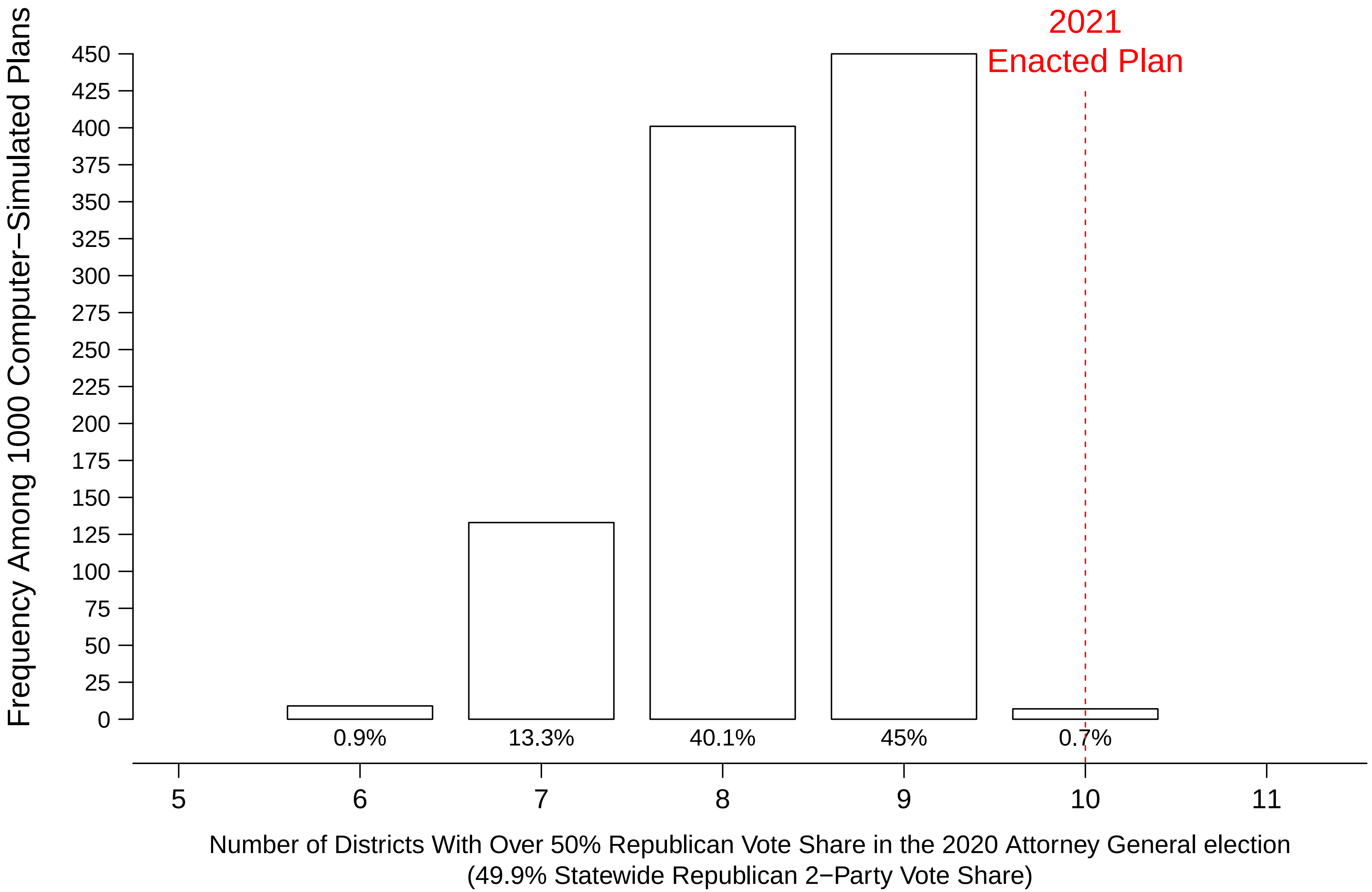


Figure B7: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2020 Governor election
(47.7% Statewide Republican 2−Party Vote Share)

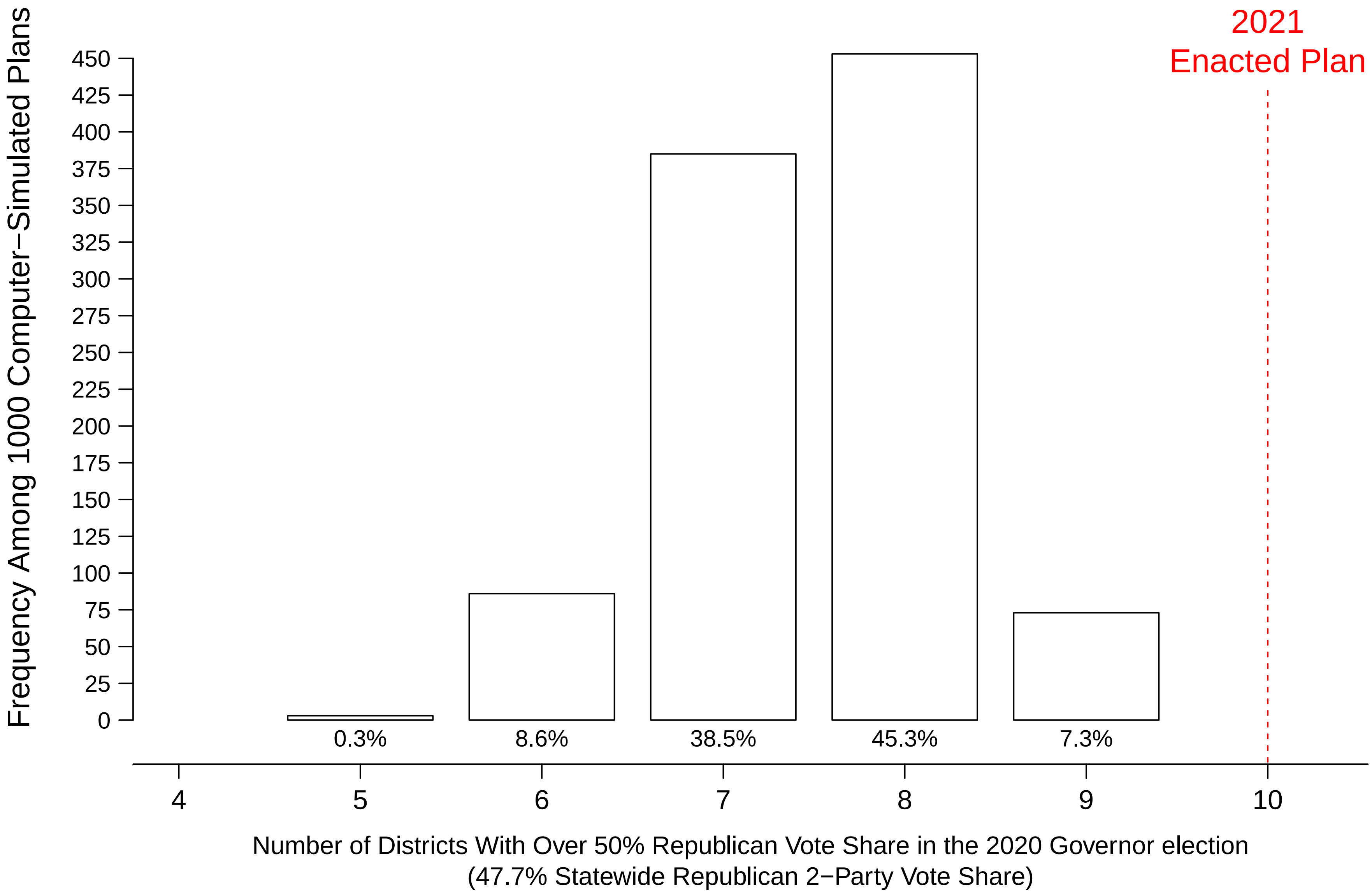


**Figure B8: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2020 Lieutenant Governor election
(51.6% Statewide Republican 2−Party Vote Share)**

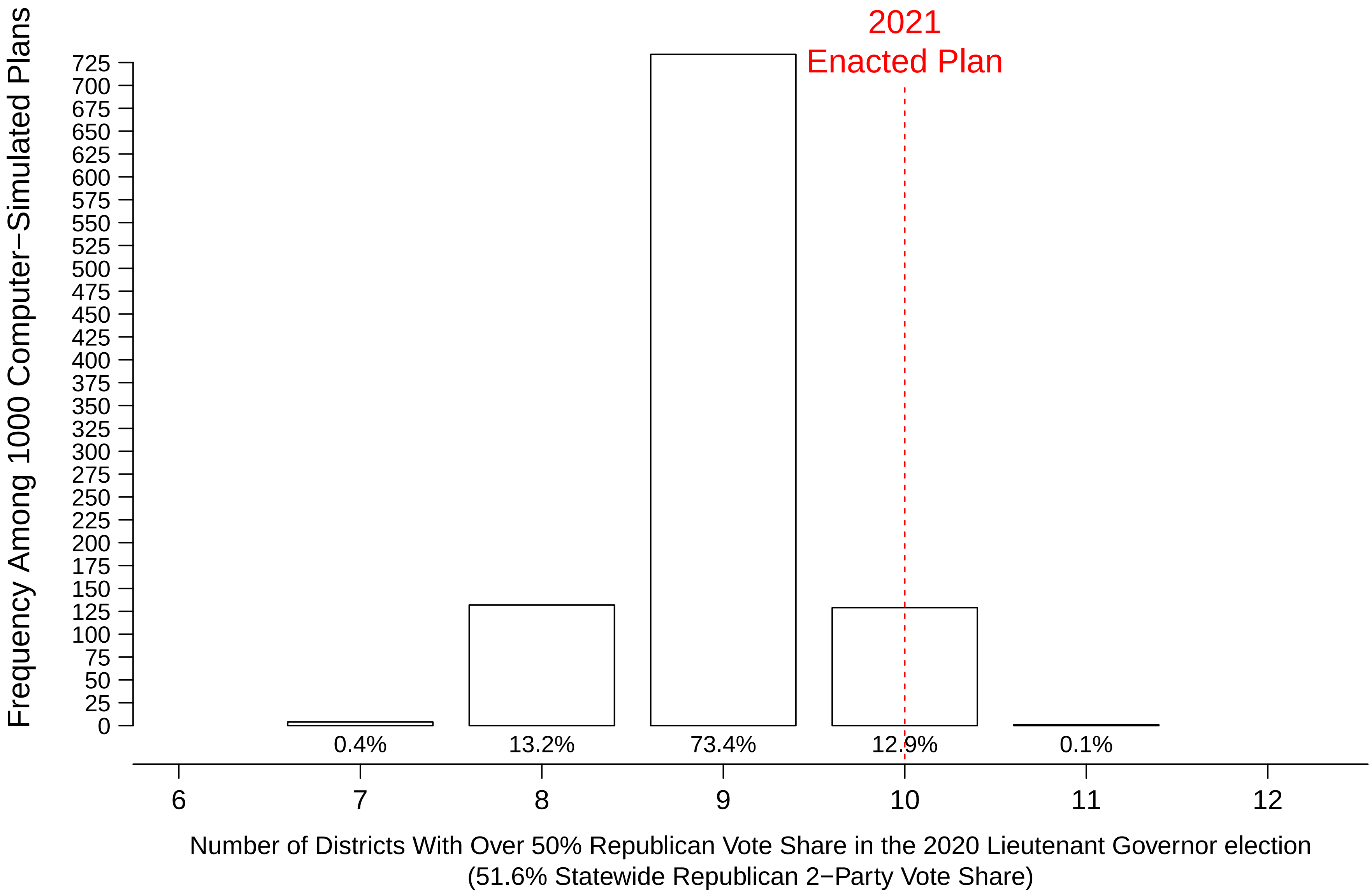


Figure B9: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2020 US President election
(50.7% Statewide Republican 2−Party Vote Share)

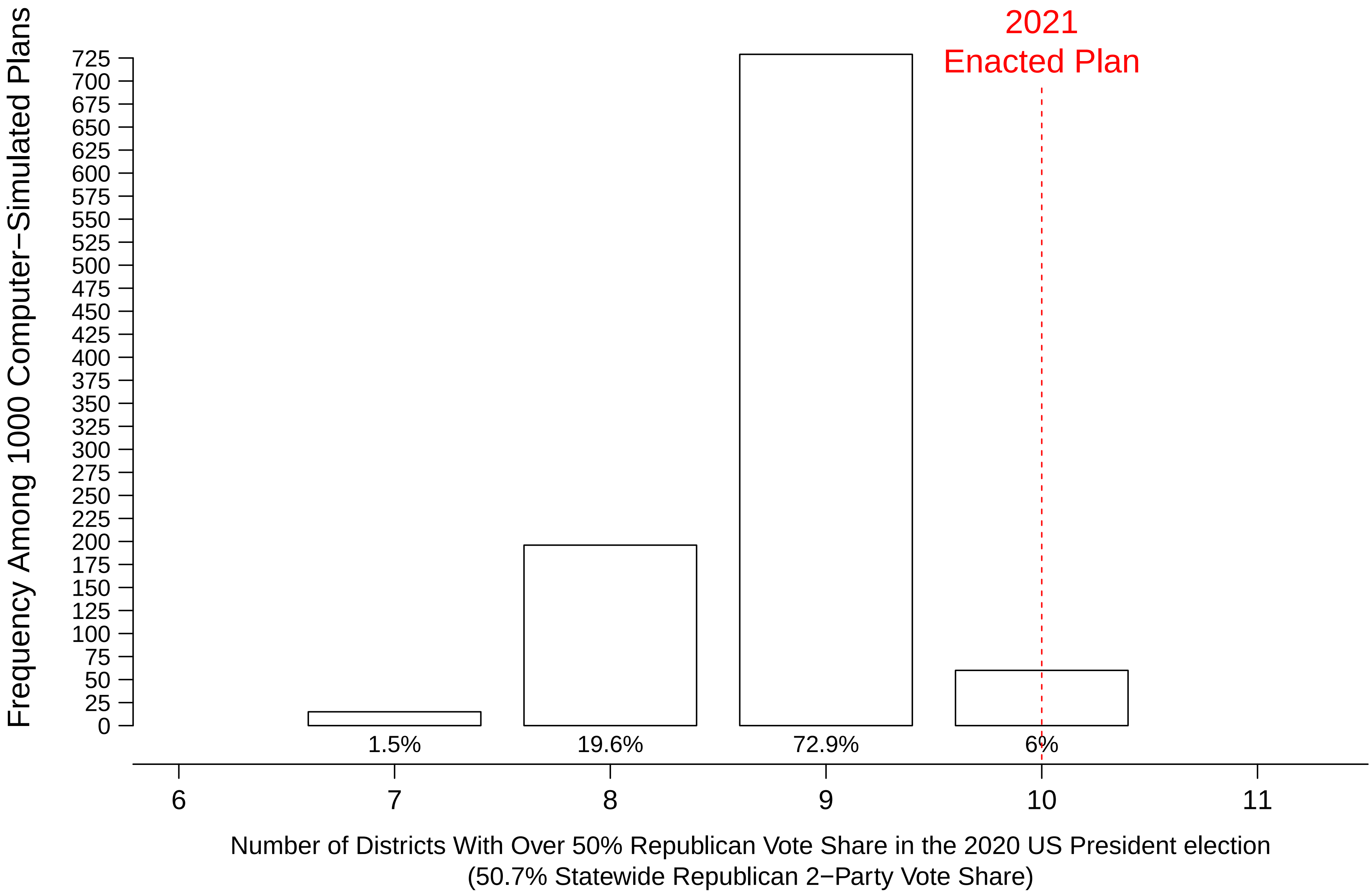
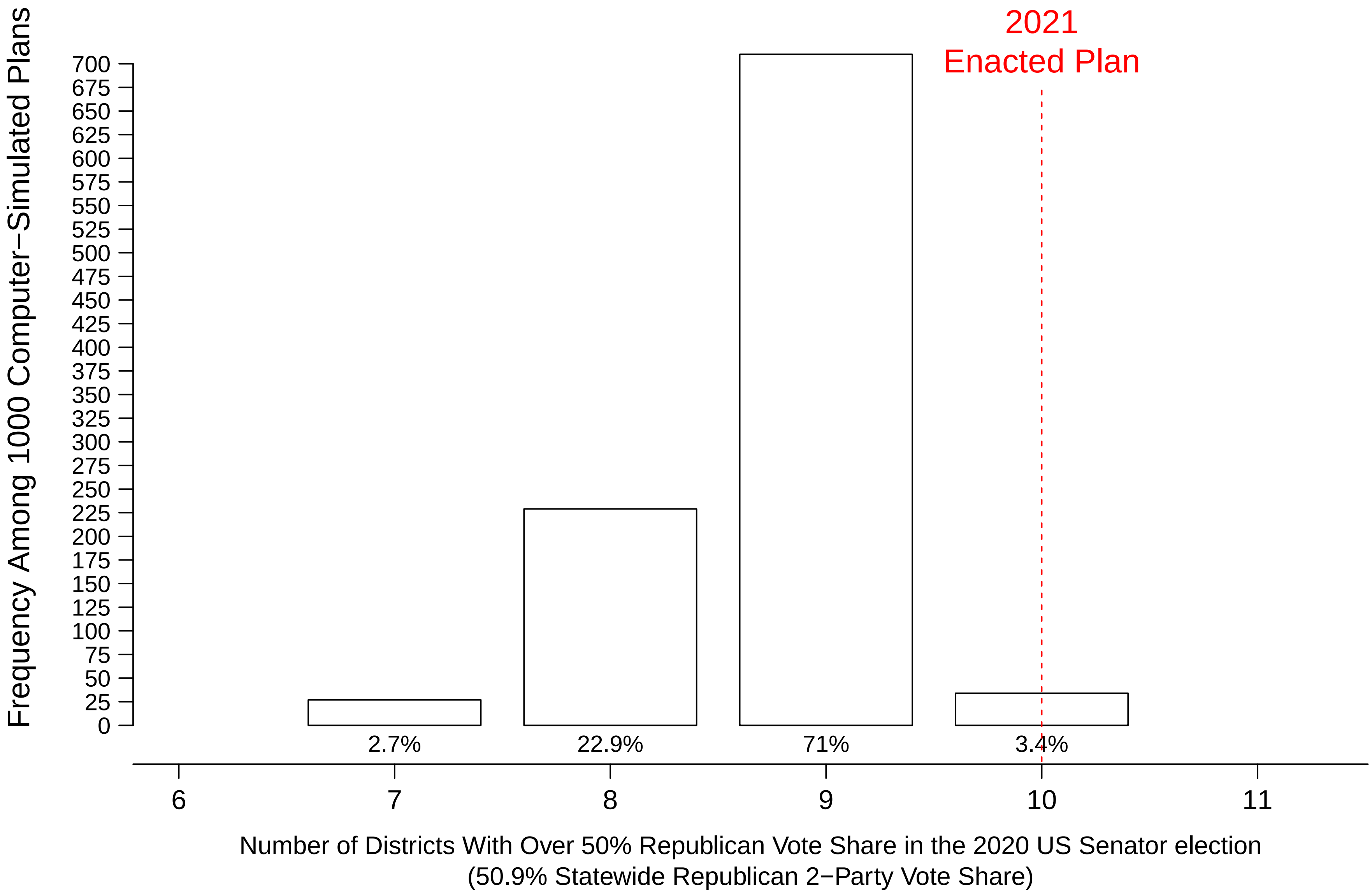


Figure B10: Comparisons of Enacted SB 740 Plan to 1,000 Computer−Simulated Plans
Number of Districts With Over 50% Republican Vote Share in the 2020 US Senator election
(50.9% Statewide Republican 2−Party Vote Share)



– Ex. 10242 –

Joint Meeting of Committees

August 12, 2021

House Committee on Redistricting

Senate Committee on Redistricting and Elections

Criteria Adopted by the Committees

- **Equal Population.** The Committees will use the 2020 federal decennial census data as the sole basis of population for the establishment of districts in the 2021 Congressional, House, and Senate plans. The number of persons in each legislative district shall be within plus or minus 5% of the ideal district population, as determined under the most recent federal decennial census. The number of persons in each congressional district shall be as nearly as equal as practicable, as determined under the most recent federal decennial census.

- **Contiguity.** No point contiguity shall be permitted in any 2021 Congressional, House, and Senate plan. Congressional, House, and Senate districts shall be compromised of contiguous territory. Contiguity by water is sufficient.

- **Counties, Groupings, and Traversals.** The Committees shall draw legislative districts within county groupings as required by *Stephenson v. Bartlett*, 355 N.C. 354, 562 S.E.2d 377 (2002) (*Stephenson I*), *Stephenson v. Bartlett*, 357 N.C. 301, 582 S.E.2d 247 (2003) (*Stephenson II*), *Dickson v. Rucho*, 367 N.C. 542, 766 S.E.2d 238 (2014) (*Dickson I*) and *Dickson v. Rucho*, 368 N.C. 481, 781 S.E. 2d 460 (2015) (*Dickson II*). Within county groupings, county lines shall not be traversed except as authorized by *Stephenson I*, *Stephenson II*, *Dickson I*, and *Dickson II*.

  Division of counties in the 2021 Congressional plan shall only be made for reasons of equalizing population and consideration of double bunking. If a county is of sufficient population size to contain an entire congressional district within the county's boundaries, the Committees shall construct a district entirely within that county.

- **Racial Data.** Data identifying the race of individuals or voters *shall not* be used in the construction or consideration of districts in the 2021 Congressional, House, and Senate plans. The Committees will draw districts that comply with the Voting Rights Act.

- **VTDs.** Voting districts ("VTDs") should be split only when necessary.

- **Compactness.** The Committees shall make reasonable efforts to draw legislative districts in the 2021 Congressional, House and Senate plans that are compact. In doing so, the Committee may use as a guide the minimum Reock ("dispersion") and Polsby-Popper ("permiter") scores identified by Richard H. Pildes and Richard G. Neimi in *Expressive Harms, "Bizarre Districts," and Voting Rights: Evaluating Election-District Appearances After Shaw v. Reno*, 92 Mich. L. Rev. 483 (1993).

- **Municipal Boundaries.** The Committees may consider municipal boundaries when drawing districts in the 2021 Congressional, House, and Senate plans.

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Exhibit #
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12/29/2021 - JNC
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Joint Meeting of Committees

August 12, 2021

House Committee on Redistricting
Senate Committee on Redistricting and Elections

- **Election Data.** Partisan considerations and election results data *shall not* be used in the drawing of districts in the 2021 Congressional, House, and Senate plans.

- **Member Residence.** Member residence may be considered in the formation of legislative and congressional districts.

- **Community Consideration.** So long as a plan complies with the foregoing criteria, local knowledge of the character of communities and connections between communities may be considered in the formation of legislative and congressional districts.

– Ex. 10244 –

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

plan	minpop	maxpop	vtdfiles	spctys	MultSpCtys	spmcds	spmcds.pop	ctyfrags	reockt	polsbyt	EG	UniformRS	RepAvgRshare	DemAvgRshare
1	745670	745671	13	11	2	16	9	113	0.451598	0.368981	0.123858	9	0.576322912	0.385205
2	745670	745671	13	12	1	13	6	113	0.473078	0.369956	0.123206	8	0.573767106	0.389411
3	745670	745671	13	13	0	15	8	113	0.466139	0.40947	0.123595	9	0.576855434	0.383602
4	745670	745671	13	11	2	14	8	113	0.444955	0.356721	-0.00508	7	0.585423012	0.430114
5	745670	745671	13	12	1	19	12	113	0.455168	0.36863	0.12505	9	0.579788901	0.378505
6	745670	745671	13	13	0	17	10	113	0.42915	0.375784	0.052713	8	0.578997141	0.412833
7	745670	745671	13	11	2	15	10	113	0.434711	0.345747	0.120592	9	0.572096291	0.393221
8	745670	745671	13	13	0	14	7	113	0.448675	0.380979	0.051565	8	0.584649886	0.404882
9	745670	745671	13	12	1	17	8	113	0.450791	0.387667	0.061978	7	0.587527791	0.402069
10	745670	745671	13	13	0	20	10	113	0.458141	0.382791	0.124854	9	0.576709477	0.383416
11	745670	745671	13	13	0	18	11	113	0.490733	0.394943	0.122432	9	0.578779984	0.382426
12	745670	745671	13	12	1	15	10	113	0.448991	0.373575	0.123017	9	0.57476168	0.387811
13	745670	745671	13	13	0	19	12	113	0.458365	0.356362	0.058234	7	0.589603032	0.400915
14	745670	745671	13	11	2	18	10	113	0.4503	0.369854	0.124428	9	0.577650996	0.382124
15	745670	745671	13	11	2	14	8	113	0.446939	0.386295	0.124272	8	0.574909271	0.386239
16	745670	745671	13	13	0	12	9	113	0.398264	0.371668	-0.01375	7	0.591666384	0.42437
17	745670	745671	13	12	1	21	11	113	0.444344	0.370409	0.054485	8	0.586838752	0.401075
18	745670	745671	13	12	1	14	9	113	0.467452	0.386806	0.124472	8	0.572034295	0.391641
19	745670	745671	13	11	2	16	9	113	0.471702	0.3817	0.120926	9	0.585903562	0.370406
20	745670	745671	13	12	1	20	10	113	0.465981	0.384393	0.122871	9	0.579341574	0.380626
21	745670	745671	13	12	1	17	7	113	0.449144	0.341959	0.125851	8	0.579878614	0.379971
22	745670	745671	13	13	0	16	10	113	0.432873	0.37709	0.051776	8	0.582871824	0.408624
23	745670	745671	13	12	1	18	10	113	0.472299	0.40113	0.198437	9	0.565878849	0.361159
24	745670	745671	13	11	2	15	7	113	0.451495	0.388321	0.123597	9	0.579298288	0.380828
25	745670	745671	13	12	1	12	8	113	0.479381	0.375814	0.061189	8	0.587468173	0.401943
26	745670	745671	13	13	0	16	10	113	0.457356	0.39312	0.125028	8	0.577114607	0.382849
27	745670	745671	13	11	2	14	10	113	0.463381	0.389102	0.122611	8	0.573586236	0.390718
28	745670	745671	13	12	1	14	10	113	0.485343	0.381439	0.121112	9	0.58074861	0.379848
29	745670	745671	13	13	0	12	9	113	0.452769	0.392097	0.052862	8	0.584169737	0.405136
30	745670	745671	13	12	1	18	11	113	0.429639	0.388181	0.050512	8	0.589462863	0.40304
31	745670	745671	13	13	0	13	9	113	0.452633	0.38089	0.052568	6	0.579059453	0.413415
32	745670	745671	13	10	2	14	9	113	0.470985	0.361236	0.06191	8	0.577716243	0.415797
33	745670	745671	13	13	0	18	9	113	0.437634	0.371785	0.059451	6	0.57584438	0.417614
34	745670	745671	13	11	2	13	10	113	0.460115	0.390439	0.059271	8	0.590751212	0.398589
35	745670	745671	13	11	2	21	11	113	0.498416	0.401525	0.121171	9	0.580742211	0.378724
36	745670	745671	13	11	1	15	9	113	0.47408	0.344794	0.051621	8	0.588467787	0.401933
37	745670	745671	13	11	2	17	9	113	0.48169	0.406629	0.123041	9	0.579349047	0.380269
38	745670	745671	13	12	1	12	6	113	0.461164	0.3856	0.050487	8	0.596226461	0.390969
39	745670	745671	13	12	1	16	8	113	0.479616	0.381897	0.120506	9	0.582169834	0.376489
40	745670	745671	13	12	1	17	10	113	0.456482	0.388028	0.125507	8	0.575338856	0.385815
41	745670	745671	13	11	2	13	8	113	0.494513	0.39027	0.124892	9	0.576184131	0.385383
42	745670	745671	13	11	2	20	11	113	0.450422	0.378294	0.12349	8	0.576496776	0.384103
43	745670	745671	13	11	2	19	11	113	0.486411	0.399242	0.120752	9	0.580141527	0.379823
44	745670	745671	13	11	2	14	8	113	0.455092	0.389074	0.062488	8	0.585804741	0.403019
45	745670	745671	13	12	1	17	9	113	0.460302	0.355968	0.123827	8	0.58094118	0.375879
46	745670	745671	13	13	0	16	11	113	0.475082	0.414792	0.122771	8	0.575715854	0.386091
47	745670	745671	13	12	1	20	10	113	0.465288	0.37099	0.123198	9	0.578563784	0.381008
48	745670	745671	13	11	1	16	10	113	0.448176	0.377506	0.053257	8	0.584671851	0.405165
49	745670	745671	13	13	0	17	8	113	0.449124	0.380929	0.12319	8	0.575065581	0.385724
50	745670	745671	13	11	1	15	8	113	0.439466	0.390797	0.054909	8	0.589565872	0.401074
51	745670	745671	13	12	1	13	9	113	0.467009	0.388952	-0.0132	7	0.607080708	0.406783
52	745670	745671	13	11	2	18	7	113	0.446946	0.39588	0.130049	9	0.583037442	0.374383
53	745670	745671	13	11	2	17	9	113	0.447747	0.382293	0.121676	9	0.581095366	0.378245
54	745670	745671	13	13	0	21	12	113	0.470365	0.381818	0.05006	8	0.584583319	0.405117
55	745670	745671	13	12	1	16	10	113	0.412638	0.369436	0.053962	8	0.577821051	0.415573
56	745670	745671	13	11	2	17	9	113	0.465053	0.378856	0.127811	9	0.573859437	0.387534
57	745670	745671	13	13	0	15	9	113	0.437101	0.377419	0.048809	8	0.588911595	0.39916
58	745670	745671	13	12	1	18	11	113	0.464748	0.388411	0.123261	9	0.57686807	0.384642
59	745670	745671	13	13	0	16	8	113	0.460024	0.394053	0.053289	8	0.58679219	0.403236
60	745670	745671	13	13	0	15	11	113	0.432395	0.360416	0.123617	9	0.576499352	0.385665
61	745670	745671	13	13	0	22	14	113	0.487971	0.400658	0.122297	8	0.5761864	0.384076
62	745670	745671	13	10	3	18	10	113	0.45624	0.383595	0.050392	8	0.583850869	0.406109
63	745670	745671	13	11	2	16	7	113	0.461436	0.389992	0.124117	9	0.579674854	0.38075
64	745670	745671	13	12	1	19	10	113	0.452561	0.368514	0.051995	8	0.583657594	0.407489
65	745670	745671	13	11	2	14	7	113	0.424129	0.369994	0.051578	8	0.580598166	0.412968
66	745670	745671	13	11	2	17	10	113	0.442097	0.385935	0.121727	9	0.576449679	0.386761
67	745670	745671	13	13	0	15	10	113	0.495022	0.40831	0.060699	8	0.585141159	0.404219
68	745670	745671	13	12	1	23	13	113	0.456271	0.3699	0.127249	9	0.578054623	0.380428
69	745670	745671	13	11	2	18	11	113	0.503898	0.39642	0.123296	9	0.580073814	0.378938
70	745670	745671	13	13	0	13	9	113	0.455826	0.388966	-0.0148	7	0.599802916	0.416186
71	745670	745671	13	13	0	19	12	113	0.463677	0.379981	0.060345	8	0.591434118	0.396472
72	745670	745671	13	11	1	17	12	113	0.420705	0.376546	0.123615	8	0.573746675	0.389299
73	745670	745671	13	13	0	17	11	113	0.495735	0.37753	0.123905	8	0.57718091	0.382987
74	745670	745671	13	13	0	16	9	113	0.49013	0.391282	0.060533	8	0.585728015	0.403886
75	745670	745671	13	12	1	16	10	113	0.497628	0.393222	0.122087	9	0.57849926	0.383558
76	745670	745671	13	13	0	20	10	113	0.465352	0.387211	0.199775	7	0.563447219	0.36473
77	745670	745671	13	12	1	14	7	113	0.448912	0.380523	0.064054	7	0.590506962	0.397017
78	745670	745671	13	12	1	16	14	113	0.487288	0.38772	0.121373	8	0.58148434	0.377209
79	745670	745671	13	12	1	20	10	113	0.480785	0.373321	0.194505	9	0.572134438	0.351485
80	745670	745671	13	13	0	14	11	113	0.507466	0.399775	0.121374	8	0.577361472	0.384885
81	745670	745671	13	12	1	13	10	113	0.475872	0.41134	0.058595	8	0.594455513	0.39108
82	745670	745671	13	13	0	17	10	113	0.449896	0.369507	0.125394	9	0.576541334	0.38363
83	745670	745671	13	13	0	14	9	113	0.466033	0.364632	0.123551	8	0.579277793	0.380011
84	745670	745671	13	12	1	15	8	113	0.46442	0.381868	0.122507	8	0.577539225	0.381514
85	745670	745671	13	12	1	12	7	113	0.460785	0.372957	0.121531	7	0.573777236	0.389839
86	745670	745671	13	13	0	18	11	113	0.457358	0.398851	0.122241	9	0.5791577	0.379961
87	745670	745671	13	12	1	17	9	113	0.459274	0.389985	0.124133	7	0.574369117	0.386134
88	745670	745671	13	12	1	12	7	113	0.4551	0.38472	0.123812	9	0.575741591	0.386465
89	745670	745671	13	10	3	13	7	113	0.441896	0.36982	0.124725	9	0.576543291	0.383412
90	745670	745671	13	12	1	18	10	113	0.444239	0.380978	0.051721	8	0.583545644	0.407754
91	745670	745671	13	12	1	16	11	113	0.477051	0.376497	0.196779	9	0.567432581	0.359663
92	745670	745671	13	13	0	17	12	113	0.44311	0.39376	0.057612	8	0.590939615	0.397142
93	745670	745671	13	12	1	19	10	113	0.471822	0.354637	0.120583	9	0.578037664	0.383274

NCLCV v. Hall

21 CVS 15426

LDTX156

Exhibit #

CHEN 5

12/29/2021 - JNC

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

94	745670	745671	13	12	1	18	12	113	0.429327	0.36355	0.121962	8	0.575930239	0.385378
95	745670	745671	13	13	0	20	13	113	0.499463	0.376849	0.121643	9	0.577277312	0.384725
96	745670	745671	13	12	1	18	11	113	0.480896	0.373453	0.123801	8	0.575886998	0.388047
97	745670	745671	13	12	1	18	10	113	0.45747	0.397251	0.194283	10	0.566320682	0.362957
98	745670	745671	13	13	0	18	11	113	0.429722	0.402545	0.05768	8	0.590288198	0.398202
99	745670	745671	13	13	0	14	8	113	0.45132	0.371385	0.127227	7	0.57392208	0.386843
100	745670	745671	13	13	0	17	7	113	0.428701	0.375525	0.12368	9	0.57126668	0.394086
101	745670	745671	13	13	0	16	10	113	0.460368	0.384228	0.062475	7	0.587686011	0.400636
102	745670	745671	13	12	1	15	10	113	0.429454	0.359261	0.052244	8	0.58076674	0.412177
103	745670	745671	13	11	2	18	10	113	0.465424	0.38744	0.120407	8	0.581409143	0.377644
104	745670	745671	13	11	1	13	8	113	0.460876	0.399031	0.058633	8	0.594361513	0.392792
105	745670	745671	13	12	1	20	11	113	0.456183	0.386603	0.124497	8	0.575232029	0.384864
106	745670	745671	13	13	0	13	9	113	0.466484	0.362037	0.124756	9	0.576821371	0.383198
107	745670	745671	13	12	1	20	9	113	0.482322	0.397196	-0.00878	7	0.598324444	0.416614
108	745670	745671	13	11	2	15	10	113	0.456283	0.346618	0.192783	10	0.567845441	0.359532
109	745670	745671	13	13	0	16	10	113	0.487426	0.40014	0.121229	9	0.576797	0.385112
110	745670	745671	13	13	0	17	12	113	0.483376	0.402657	0.122294	9	0.578672629	0.382429
111	745670	745671	13	11	2	20	8	113	0.463703	0.3854	0.120711	9	0.585055068	0.371752
112	745670	745671	13	13	0	14	8	113	0.460863	0.375648	0.123588	9	0.576412998	0.38608
113	745670	745671	13	11	2	16	10	113	0.49101	0.389311	0.063401	8	0.578647942	0.413389
114	745670	745671	13	13	0	19	9	113	0.440912	0.391895	0.123776	9	0.574896798	0.386903
115	745670	745671	13	11	2	13	7	113	0.517116	0.394697	-0.00689	7	0.591775284	0.42323
116	745670	745671	13	12	1	18	9	113	0.496485	0.403019	0.05647	8	0.59042485	0.398597
117	745670	745671	13	12	1	11	7	113	0.496856	0.379188	0.123099	9	0.580407431	0.379296
118	745670	745671	13	12	1	18	8	113	0.475209	0.382935	0.121784	9	0.577314586	0.385171
119	745670	745671	13	13	0	14	9	113	0.500623	0.390103	0.122329	8	0.577836186	0.382305
120	745670	745671	13	13	0	16	12	113	0.473505	0.414056	0.123557	9	0.575855784	0.38523
121	745670	745671	13	11	2	14	9	113	0.49473	0.402864	0.125152	9	0.580257974	0.37751
122	745670	745671	13	12	1	13	9	113	0.450011	0.361801	0.052014	8	0.585857783	0.404869
123	745670	745671	13	12	1	17	7	113	0.457113	0.377069	0.121184	8	0.579655596	0.380908
124	745670	745671	13	12	1	16	8	113	0.443977	0.362548	0.124525	7	0.576647089	0.383396
125	745670	745671	13	13	0	17	9	113	0.477246	0.395172	0.122359	8	0.574639352	0.388006
126	745670	745671	13	12	1	14	9	113	0.456446	0.389391	0.123734	8	0.572328914	0.390328
127	745670	745671	13	13	0	19	10	113	0.463465	0.399441	0.121668	8	0.579543589	0.380083
128	745670	745671	13	12	1	13	5	113	0.447754	0.374131	0.123316	9	0.574036558	0.391672
129	745670	745671	13	11	1	15	9	113	0.443413	0.335629	0.122607	8	0.572154486	0.391621
130	745670	745671	13	12	1	20	9	113	0.467043	0.375882	0.124149	8	0.579405649	0.379922
131	745670	745671	13	11	1	11	8	113	0.406771	0.362881	0.055846	7	0.577728764	0.414375
132	745670	745671	13	10	2	15	8	113	0.451747	0.363921	0.051411	8	0.579428853	0.414714
133	745670	745671	13	12	1	17	9	113	0.457424	0.366433	0.125617	7	0.575272636	0.385319
134	745670	745671	13	10	2	21	12	113	0.496007	0.38978	0.120302	9	0.581789313	0.377463
135	745670	745671	13	10	3	18	9	113	0.476087	0.396879	0.125381	9	0.579789969	0.379025
136	745670	745671	13	12	1	17	8	113	0.442216	0.376041	0.052649	8	0.585068158	0.40574
137	745670	745671	13	12	1	18	10	113	0.441531	0.369221	0.051845	8	0.586355021	0.402343
138	745670	745671	13	12	1	13	8	113	0.450363	0.40792	0.053589	8	0.577948148	0.415028
139	745670	745671	13	13	0	16	8	113	0.477612	0.386891	0.124089	9	0.575247109	0.385165
140	745670	745671	13	12	1	15	6	113	0.465123	0.365589	0.12362	8	0.576387496	0.384168
141	745670	745671	13	12	1	18	10	113	0.509499	0.411811	-0.00889	7	0.606703198	0.407656
142	745670	745671	13	11	2	18	11	113	0.469719	0.383465	0.131953	7	0.579668564	0.380803
143	745670	745671	13	12	1	16	10	113	0.445599	0.389075	-0.0197	7	0.58817504	0.42911
144	745670	745671	13	12	1	21	10	113	0.433327	0.382582	0.194453	9	0.569612074	0.354355
145	745670	745671	13	13	0	15	9	113	0.45142	0.370612	0.053439	8	0.585625268	0.403329
146	745670	745671	13	12	1	13	7	113	0.447816	0.381807	0.046127	8	0.588289904	0.402229
147	745670	745671	13	12	1	16	8	113	0.425005	0.353999	0.130542	8	0.576935263	0.385621
148	745670	745671	13	13	0	18	8	113	0.461832	0.4006	0.123662	8	0.576713553	0.384002
149	745670	745671	13	13	0	18	11	113	0.446233	0.394048	0.125548	8	0.574754797	0.385502
150	745670	745671	13	12	1	14	9	113	0.485445	0.340365	0.065487	8	0.577508187	0.414457
151	745670	745671	13	12	1	17	9	113	0.486459	0.375238	0.126172	8	0.576850796	0.383364
152	745670	745671	13	13	0	16	9	113	0.476657	0.399369	0.124536	8	0.576034639	0.384034
153	745670	745671	13	12	1	14	9	113	0.455908	0.36875	0.120848	9	0.577424578	0.384063
154	745670	745671	13	13	0	20	14	113	0.496023	0.40362	0.122436	8	0.579314773	0.38056
155	745670	745671	13	12	1	18	12	113	0.447193	0.356616	0.052369	7	0.576910605	0.416676
156	745670	745671	13	12	1	15	12	113	0.479827	0.40232	0.053411	7	0.585420107	0.403634
157	745670	745671	13	11	1	14	10	113	0.46971	0.386364	0.05765	8	0.582599631	0.407619
158	745670	745671	13	12	1	18	10	113	0.458861	0.359678	0.123087	8	0.575844143	0.385961
159	745670	745671	13	12	1	13	8	113	0.461676	0.393425	0.050286	8	0.586711464	0.403019
160	745670	745671	13	12	1	15	11	113	0.447101	0.380566	0.118779	9	0.578312917	0.382771
161	745670	745671	13	13	0	20	9	113	0.449932	0.398501	0.120484	9	0.571823654	0.394315
162	745670	745671	13	13	0	16	11	113	0.480767	0.38725	0.122408	9	0.574154485	0.388519
163	745670	745671	13	12	1	15	11	113	0.457775	0.348565	0.123418	9	0.576947125	0.383642
164	745670	745671	13	13	0	14	9	113	0.462684	0.393664	0.049128	8	0.585833556	0.40476
165	745670	745671	13	12	1	16	9	113	0.453114	0.392241	-0.02096	7	0.584674401	0.431574
166	745670	745671	13	11	2	18	9	113	0.45962	0.366986	0.12317	9	0.577637423	0.38292
167	745670	745671	13	12	1	16	12	113	0.466672	0.394697	0.056075	8	0.585113544	0.40506
168	745670	745671	13	12	1	15	10	113	0.448164	0.382639	0.050686	8	0.582046333	0.40905
169	745670	745671	13	12	1	22	9	113	0.472335	0.389705	0.04899	8	0.587064551	0.40302
170	745670	745671	13	12	1	17	8	113	0.484626	0.376604	0.061105	8	0.585777175	0.404015
171	745670	745671	13	11	2	15	7	113	0.429264	0.376531	0.121237	8	0.571421968	0.395194
172	745670	745671	13	13	0	15	9	113	0.470513	0.386243	0.122313	9	0.580704133	0.378494
173	745670	745671	13	13	0	17	10	113	0.452391	0.38817	0.052275	7	0.587881071	0.400337
174	745670	745671	13	13	0	20	11	113	0.483504	0.400684	0.123238	9	0.577207218	0.384037
175	745670	745671	13	13	0	23	11	113	0.466759	0.398246	0.1254	9	0.579723808	0.378592
176	745670	745671	13	11	2	22	14	113	0.495872	0.407318	0.1227	9	0.580609492	0.377852
177	745670	745671	13	13	0	21	9	113	0.453131	0.394132	0.123547	9	0.576127318	0.384536
178	745670	745671	13	13	0	21	11	113	0.470041	0.398052	0.054617	7	0.587688187	0.399995
179	745670	745671	13	12	1	16	8	113	0.470081	0.368327	0.059557	8	0.585730241	0.404405
180	745670	745671	13	11	2	14	9	113	0.457218	0.363544	0.05854	8	0.584335875	0.404487
181	745670	745671	13	13	0	15	10	113	0.500535	0.383706	0.123203	9	0.580208723	0.379209
182	745670	745671	13	13	0	22	16	113	0.444896	0.36946	0.048377	8	0.58897615	0.400891
183	745670	745671	13	12	1	16	10	113	0.418725	0.376654	0.058633	7	0.588835297	0.398887
184	745670	745671	13	12	1	13	8	113	0.453556	0.363803	0.123135	8	0.574507549	0.388761
185	745670	745671	13	12	1	18	10	113	0.4579	0.367358	0.059627	7	0.582597569	0.407022
186	745670	745671	13	11	1	12	6	113	0.440087	0.378193	0.119644	9	0.580175074	0.379729
187	745670	745671	13	13	0	19	8	113	0.49353	0.393621	0.062222	8	0.585364693	0.402474

188	745670	745671	13	9	3	16	7	113	0.474838	0.377578	0.120092	9	0.578762155	0.381938
189	745670	745671	13	13	0	22	9	113	0.461771	0.379695	0.051525	8	0.586582616	0.402097
190	745670	745671	13	12	1	16	9	113	0.457208	0.375199	0.058281	8	0.583281742	0.405876
191	745670	745671	13	12	1	17	10	113	0.489578	0.376494	0.124306	9	0.578170605	0.380349
192	745670	745671	13	12	1	19	11	113	0.452996	0.388669	0.120946	8	0.575994418	0.386064
193	745670	745671	13	13	0	12	7	113	0.495751	0.37404	0.059472	8	0.591534745	0.396323
194	745670	745671	13	13	0	16	11	113	0.442522	0.370136	0.055647	8	0.596555064	0.390414
195	745670	745671	13	12	1	15	9	113	0.447635	0.360723	0.122067	8	0.578503074	0.380839
196	745670	745671	13	12	1	15	8	113	0.453605	0.382489	0.057403	7	0.57925817	0.413618
197	745670	745671	13	11	1	13	6	113	0.411086	0.359691	0.056077	8	0.579840898	0.412204
198	745670	745671	13	13	0	14	7	113	0.454379	0.384495	0.052389	7	0.584797359	0.406045
199	745670	745671	13	13	0	16	8	113	0.464668	0.364558	0.123489	8	0.577875502	0.381884
200	745670	745671	13	11	2	17	8	113	0.451737	0.344105	0.052059	8	0.581171698	0.412486
201	745670	745671	13	13	0	22	8	113	0.455591	0.371318	0.054527	8	0.588047319	0.399376
202	745670	745671	13	13	0	18	11	113	0.465558	0.374637	0.122291	9	0.578707644	0.382238
203	745670	745671	13	11	2	20	8	113	0.456012	0.373566	0.131758	8	0.577170377	0.383666
204	745670	745671	13	12	1	14	7	113	0.440324	0.37132	0.121468	7	0.578824569	0.382447
205	745670	745671	13	11	2	16	11	113	0.473599	0.362705	0.121445	8	0.577880424	0.382331
206	745670	745671	13	13	0	17	7	113	0.440639	0.355322	0.046325	8	0.58296056	0.408934
207	745670	745671	13	10	2	19	10	113	0.46326	0.378725	0.121437	8	0.586830534	0.368248
208	745670	745671	13	12	1	16	10	113	0.464398	0.390526	0.05212	7	0.585560255	0.404226
209	745670	745671	13	12	1	13	7	113	0.442132	0.369008	0.054452	8	0.584305106	0.405104
210	745670	745671	13	11	1	15	8	113	0.47438	0.39985	0.120581	9	0.580531964	0.379808
211	745670	745671	13	13	0	18	9	113	0.430079	0.373899	0.122064	9	0.574659289	0.389086
212	745670	745671	13	11	2	22	10	113	0.489728	0.387863	0.192943	9	0.569156383	0.358763
213	745670	745671	13	13	0	16	10	113	0.483069	0.382653	0.06155	8	0.584923057	0.403578
214	745670	745671	13	13	0	17	9	113	0.448077	0.356649	0.121162	9	0.579257784	0.378937
215	745670	745671	13	12	1	19	10	113	0.472141	0.391722	0.124816	8	0.573139662	0.38999
216	745670	745671	13	12	1	17	8	113	0.479117	0.397358	0.063019	8	0.585210398	0.403269
217	745670	745671	13	13	0	18	10	113	0.418029	0.369511	0.12356	9	0.575783691	0.385087
218	745670	745671	13	12	1	15	10	113	0.487995	0.395712	0.125266	9	0.577973454	0.38282
219	745670	745671	13	13	0	18	10	113	0.487498	0.377105	0.122515	8	0.578706447	0.380344
220	745670	745671	13	12	1	17	8	113	0.461798	0.395722	0.122727	9	0.580010293	0.380195
221	745670	745671	13	13	0	13	8	113	0.481082	0.402591	0.126573	8	0.576533755	0.382666
222	745670	745671	13	12	1	10	7	113	0.454144	0.372924	0.052976	8	0.583700896	0.406112
223	745670	745671	13	12	1	14	8	113	0.468419	0.378431	0.122902	7	0.57462307	0.387965
224	745670	745671	13	11	2	14	9	113	0.451182	0.411429	0.122752	9	0.572767102	0.392306
225	745670	745671	13	11	1	13	5	113	0.427086	0.345012	0.124506	8	0.575641725	0.383867
226	745670	745671	13	13	0	18	12	113	0.465228	0.401375	0.062128	8	0.585812724	0.402347
227	745670	745671	13	12	1	16	9	113	0.457133	0.342532	-0.01077	7	0.597731239	0.417959
228	745670	745671	13	11	1	11	5	113	0.464374	0.39289	0.059192	8	0.594560712	0.393704
229	745670	745671	13	11	2	23	11	113	0.487296	0.388698	0.19558	9	0.571628074	0.351378
230	745670	745671	13	12	1	14	9	113	0.41558	0.349066	0.194647	10	0.567962103	0.358782
231	745670	745671	13	11	2	18	11	113	0.425734	0.381815	0.120944	9	0.582653457	0.376952
232	745670	745671	13	11	2	14	9	113	0.4675	0.381935	0.121343	9	0.580214685	0.379019
233	745670	745671	13	13	0	23	11	113	0.45957	0.358165	0.052787	8	0.585073096	0.405233
234	745670	745671	13	12	1	16	11	113	0.4573	0.353788	0.063365	8	0.583192903	0.405879
235	745670	745671	13	13	0	17	8	113	0.463588	0.38065	0.05449	8	0.590578153	0.399351
236	745670	745671	13	12	1	13	9	113	0.484794	0.364531	0.124043	9	0.580663474	0.377553
237	745670	745671	13	12	1	17	9	113	0.468087	0.386521	0.123086	8	0.577282193	0.38474
238	745670	745671	13	11	2	13	9	113	0.44679	0.374934	0.05345	8	0.580851196	0.412348
239	745670	745671	13	13	0	14	7	113	0.458875	0.362453	0.123693	7	0.568850556	0.398186
240	745670	745671	13	12	1	15	8	113	0.440217	0.36673	0.05582	7	0.587084451	0.402683
241	745670	745671	13	12	1	18	12	113	0.477568	0.37623	0.124694	9	0.576770015	0.385749
242	745670	745671	13	11	2	14	9	113	0.482807	0.376506	0.055924	8	0.587370034	0.401531
243	745670	745671	13	11	1	14	10	113	0.407071	0.348678	0.050233	8	0.58298695	0.409797
244	745670	745671	13	11	2	17	10	113	0.468817	0.391736	0.056631	7	0.58272019	0.407217
245	745670	745671	13	13	0	14	6	113	0.468745	0.389364	0.123911	9	0.579872384	0.378045
246	745670	745671	13	12	1	16	9	113	0.45964	0.407021	0.060319	8	0.586773156	0.401799
247	745670	745671	13	12	1	16	10	113	0.454376	0.380865	0.125732	7	0.573150938	0.389372
248	745670	745671	13	12	1	17	10	113	0.494578	0.410669	0.125118	8	0.576178707	0.384769
249	745670	745671	13	11	2	16	9	113	0.473172	0.379943	0.063431	8	0.588858516	0.399409
250	745670	745671	13	11	2	18	10	113	0.448125	0.369329	0.058899	8	0.581601519	0.409484
251	745670	745671	13	13	0	18	9	113	0.456259	0.352712	0.063454	8	0.580538586	0.409762
252	745670	745671	13	13	0	15	11	113	0.497428	0.390683	0.122845	9	0.577249372	0.384001
253	745670	745671	13	13	0	15	11	113	0.455883	0.381216	-0.01179	6	0.591297565	0.424883
254	745670	745671	13	11	2	14	7	113	0.455047	0.341718	0.123182	9	0.581422066	0.374719
255	745670	745671	13	12	1	20	11	113	0.490526	0.393956	0.123609	8	0.574300474	0.389207
256	745670	745671	13	12	1	13	6	113	0.443559	0.376399	0.061948	7	0.584334984	0.405301
257	745670	745671	13	13	0	19	10	113	0.487024	0.360844	0.1212	9	0.582818139	0.375873
258	745670	745671	13	12	1	17	10	113	0.449105	0.374512	-0.01246	7	0.596027858	0.418921
259	745670	745671	13	12	1	17	10	113	0.463682	0.402591	0.063149	8	0.582669464	0.406155
260	745670	745671	13	12	1	20	10	113	0.457865	0.378153	0.052136	8	0.584208686	0.406485
261	745670	745671	13	13	0	17	8	113	0.469688	0.381823	0.060421	8	0.585102801	0.403874
262	745670	745671	13	13	0	10	8	113	0.42486	0.363984	0.051725	8	0.583620604	0.406609
263	745670	745671	13	12	1	21	10	113	0.432486	0.382044	0.048669	8	0.58371107	0.407064
264	745670	745671	13	13	0	17	12	113	0.446756	0.345066	0.195948	8	0.565650514	0.362578
265	745670	745671	13	13	0	16	9	113	0.47565	0.39714	0.054892	8	0.587047782	0.401516
266	745670	745671	13	12	1	14	8	113	0.442095	0.356017	0.066189	8	0.581501363	0.408546
267	745670	745671	13	13	0	14	9	113	0.464161	0.35732	0.124963	9	0.577582606	0.381784
268	745670	745671	13	12	1	13	9	113	0.476177	0.378542	0.12299	9	0.577728496	0.38365
269	745670	745671	13	13	0	19	9	113	0.470142	0.351176	0.122117	9	0.578206394	0.382853
270	745670	745671	13	13	0	22	11	113	0.464444	0.396331	0.125669	9	0.577673206	0.381316
271	745670	745671	13	11	2	18	9	113	0.443326	0.353042	0.12312	9	0.578290933	0.380479
272	745670	745671	13	10	2	24	11	113	0.482099	0.382711	0.12221	9	0.57767697	0.383016
273	745670	745671	13	11	2	20	11	113	0.481007	0.388666	0.121623	9	0.58452299	0.372479
274	745670	745671	13	11	1	17	8	113	0.473568	0.40058	0.056902	8	0.586637779	0.403547
275	745670	745671	13	12	1	23	12	113	0.481983	0.375819	0.066657	8	0.582156747	0.407391
276	745670	745671	13	11	2	14	8	113	0.457986	0.368343	0.063425	8	0.587668781	0.398784
277	745670	745671	13	13	0	19	9	113	0.474416	0.388277	0.06163	7	0.576214569	0.417483
278	745670	745671	13	10	2	16	11	113	0.481623	0.388317	0.119836	8	0.583750849	0.374145
279	745670	745671	13	10	3	17	9	113	0.454212	0.397468	0.12346	9	0.579824892	0.38119
280	745670	745671	13	12	1	16	9	113	0.472901	0.391796	0.057966	8	0.591546433	0.396459
281	745670	745671	13	12	1	17	9	113	0.484642	0.396541	0.054591	8	0.590537407	0.398208

– Ex. 10247 –

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

282	745670	745671	13	11	1	19	8	113	0.460514	0.375199	0.058731	7	0.58083105	0.410504
283	745670	745671	13	12	1	15	9	113	0.452631	0.362379	0.121505	9	0.577948643	0.382399
284	745670	745671	13	12	1	16	11	113	0.471267	0.385592	0.046501	8	0.576616693	0.418204
285	745670	745671	13	13	0	17	7	113	0.416402	0.352164	0.060479	7	0.581201618	0.408839
286	745670	745671	13	13	0	17	10	113	0.444673	0.406103	0.122942	9	0.583727118	0.372199
287	745670	745671	13	13	0	19	8	113	0.451586	0.371352	0.058803	8	0.591151207	0.396852
288	745670	745671	13	13	0	22	8	113	0.454645	0.3681	0.055576	8	0.586891326	0.403061
289	745670	745671	13	11	2	18	12	113	0.486367	0.406274	0.123192	8	0.581755437	0.37537
290	745670	745671	13	13	0	20	12	113	0.452305	0.361564	0.193886	10	0.569610552	0.35382
291	745670	745671	13	13	0	15	9	113	0.457962	0.383922	0.124551	8	0.575985037	0.383486
292	745670	745671	13	11	2	14	8	113	0.439331	0.380434	0.126473	8	0.572108763	0.389213
293	745670	745671	13	12	1	13	9	113	0.44742	0.370644	0.059275	8	0.579553903	0.412775
294	745670	745671	13	13	0	21	11	113	0.474089	0.396318	0.124853	9	0.577289373	0.382468
295	745670	745671	13	12	1	18	10	113	0.46003	0.363432	0.056413	8	0.591978458	0.396245
296	745670	745671	13	13	0	18	11	113	0.452438	0.386696	0.125116	7	0.576160416	0.384169
297	745670	745671	13	12	1	15	9	113	0.462422	0.391808	0.11976	9	0.571170046	0.396166
298	745670	745671	13	11	2	15	9	113	0.474744	0.396682	0.126081	9	0.580600351	0.377539
299	745670	745671	13	11	2	22	11	113	0.467618	0.383071	0.048883	8	0.585185724	0.406078
300	745670	745671	13	11	2	15	11	113	0.459216	0.363804	0.121132	8	0.570838784	0.397496
301	745670	745671	13	12	1	18	10	113	0.511891	0.380887	0.12151	9	0.584593014	0.372822
302	745670	745671	13	13	0	16	11	113	0.487353	0.410873	0.051937	8	0.585581033	0.404603
303	745670	745671	13	11	1	20	10	113	0.438828	0.36327	0.118071	9	0.581209026	0.37894
304	745670	745671	13	13	0	18	9	113	0.467227	0.398588	0.050452	8	0.584014734	0.407016
305	745670	745671	13	13	0	21	11	113	0.482332	0.413522	0.122537	9	0.57590587	0.386579
306	745670	745671	13	12	1	18	12	113	0.469082	0.395996	0.050399	7	0.581547392	0.409904
307	745670	745671	13	12	1	20	8	113	0.491782	0.361514	0.194168	9	0.571302381	0.351978
308	745670	745671	13	10	3	17	9	113	0.478214	0.398344	0.123789	9	0.579827509	0.380712
309	745670	745671	13	13	0	16	8	113	0.48659	0.394327	0.122834	8	0.575763407	0.385704
310	745670	745671	13	12	1	16	11	113	0.482547	0.377372	0.123455	9	0.581071206	0.375748
311	745670	745671	13	13	0	14	9	113	0.4468	0.389049	0.048715	8	0.585341784	0.405958
312	745670	745671	13	13	0	17	10	113	0.471382	0.394204	0.122832	8	0.576132866	0.385248
313	745670	745671	13	13	0	21	10	113	0.456685	0.405286	0.124368	9	0.575198024	0.385739
314	745670	745671	13	11	2	22	13	113	0.471042	0.384972	0.124932	8	0.575881924	0.387375
315	745670	745671	13	12	1	17	11	113	0.481026	0.392242	0.122657	9	0.575964582	0.385752
316	745670	745671	13	13	0	21	9	113	0.46937	0.39013	0.122611	9	0.575382872	0.386318
317	745670	745671	13	12	1	20	12	113	0.491752	0.396275	0.121043	9	0.58231243	0.37611
318	745670	745671	13	11	2	14	8	113	0.44714	0.384693	0.119811	9	0.577614203	0.385625
319	745670	745671	13	12	1	15	9	113	0.456248	0.381713	0.122665	9	0.575306807	0.388181
320	745670	745671	13	12	1	13	11	113	0.434274	0.375094	0.059899	7	0.578963699	0.413449
321	745670	745671	13	12	1	15	8	113	0.479195	0.379057	0.064251	8	0.585290774	0.402315
322	745670	745671	13	13	0	14	10	113	0.443965	0.350239	0.123473	8	0.573633899	0.389955
323	745670	745671	13	13	0	18	12	113	0.465942	0.399331	-0.01754	7	0.590487896	0.42601
324	745670	745671	13	12	1	17	9	113	0.468994	0.397095	0.12292	8	0.576129044	0.385278
325	745670	745671	13	11	2	14	11	113	0.446625	0.383306	0.120937	8	0.575135945	0.387486
326	745670	745671	13	13	0	15	8	113	0.4601	0.388068	0.12537	9	0.57446026	0.38751
327	745670	745671	13	11	2	20	8	113	0.464329	0.35716	0.195667	9	0.57185553	0.35082
328	745670	745671	13	12	1	18	11	113	0.444117	0.376704	0.12348	8	0.576100539	0.385355
329	745670	745671	13	13	0	15	10	113	0.44014	0.387003	0.125135	8	0.586903509	0.364311
330	745670	745671	13	13	0	16	9	113	0.472809	0.359525	0.123218	9	0.577759654	0.382559
331	745670	745671	13	13	0	17	7	113	0.450171	0.380447	0.123413	9	0.576114424	0.383774
332	745670	745671	13	13	0	21	11	113	0.467173	0.38438	0.121854	8	0.577395103	0.38295
333	745670	745671	13	13	0	17	11	113	0.454308	0.377136	0.121475	7	0.572193359	0.392853
334	745670	745671	13	11	2	13	9	113	0.453198	0.394703	0.052476	8	0.585506282	0.403617
335	745670	745671	13	12	1	11	8	113	0.431688	0.352236	0.057246	8	0.583548794	0.407152
336	745670	745671	13	11	2	17	8	113	0.478693	0.392961	0.059091	8	0.584173848	0.406052
337	745670	745671	13	12	1	17	10	113	0.476866	0.390097	0.123997	8	0.576066457	0.384236
338	745670	745671	13	12	1	16	10	113	0.453904	0.373044	0.121686	9	0.572176024	0.39277
339	745670	745671	13	12	1	21	9	113	0.487988	0.402054	0.123576	9	0.578289842	0.383224
340	745670	745671	13	13	0	16	7	113	0.493682	0.360177	0.051145	8	0.588214404	0.401314
341	745670	745671	13	12	1	19	10	113	0.48517	0.386195	0.058391	8	0.588007302	0.401857
342	745670	745671	13	12	1	18	10	113	0.496423	0.407542	0.051858	7	0.583403172	0.407154
343	745670	745671	13	10	3	19	9	113	0.470462	0.377295	0.123434	9	0.579311688	0.382802
344	745670	745671	13	12	1	16	9	113	0.472367	0.368393	0.121402	9	0.579762061	0.381361
345	745670	745671	13	12	1	16	12	113	0.433459	0.371186	0.122777	8	0.573898732	0.389523
346	745670	745671	13	11	2	12	7	113	0.473951	0.379862	0.059361	8	0.583886094	0.405377
347	745670	745671	13	12	1	16	8	113	0.440625	0.372776	0.123641	9	0.576874961	0.384179
348	745670	745671	13	11	2	15	8	113	0.484661	0.382522	0.12283	9	0.580267414	0.380243
349	745670	745671	13	11	2	12	8	113	0.450669	0.394039	0.125199	9	0.577752962	0.381321
350	745670	745671	13	13	0	15	9	113	0.464391	0.39772	0.061401	8	0.585571236	0.403641
351	745670	745671	13	13	0	16	11	113	0.461943	0.390427	0.122496	9	0.57694979	0.383636
352	745670	745671	13	12	1	20	7	113	0.469074	0.385085	0.126684	9	0.573404132	0.388109
353	745670	745671	13	10	2	13	8	113	0.440138	0.372555	0.122274	8	0.589409666	0.362449
354	745670	745671	13	13	0	17	8	113	0.445956	0.367973	0.061824	8	0.577710382	0.413711
355	745670	745671	13	11	1	12	6	113	0.456477	0.352078	0.120911	8	0.574894926	0.387576
356	745670	745671	13	13	0	19	11	113	0.443141	0.363956	0.056772	8	0.5864365	0.401884
357	745670	745671	13	13	0	19	11	113	0.4811	0.392391	0.122373	9	0.576736481	0.385035
358	745670	745671	13	11	2	16	10	113	0.451123	0.386667	0.125568	9	0.577578689	0.382652
359	745670	745671	13	13	0	17	6	113	0.485754	0.403327	-0.01788	6	0.596092268	0.419549
360	745670	745671	13	11	2	10	5	113	0.486353	0.382504	0.122948	8	0.580149002	0.378624
361	745670	745671	13	13	0	17	11	113	0.4707	0.362857	0.123526	9	0.57546924	0.38647
362	745670	745671	13	13	0	17	13	113	0.430468	0.356811	0.046567	8	0.58490526	0.406241
363	745670	745671	13	11	2	17	8	113	0.42161	0.372442	0.120024	9	0.580154538	0.381592
364	745670	745671	13	12	1	17	10	113	0.471236	0.390924	0.125762	9	0.575629248	0.385625
365	745670	745671	13	13	0	13	9	113	0.506625	0.377443	0.12245	9	0.578441562	0.382682
366	745670	745671	13	13	0	15	9	113	0.466403	0.382218	0.123572	9	0.578594023	0.380673
367	745670	745671	13	12	1	15	11	113	0.470505	0.387432	0.059056	8	0.585071703	0.405463
368	745670	745671	13	12	1	16	9	113	0.469929	0.402701	0.121722	9	0.578169259	0.383878
369	745670	745671	13	12	1	13	7	113	0.435885	0.373237	0.0535	7	0.576299272	0.416788
370	745670	745671	13	13	0	13	9	113	0.473432	0.404846	0.124181	9	0.575801188	0.384886
371	745670	745671	13	12	1	13	7	113	0.419611	0.349144	0.067504	7	0.58118483	0.4075
372	745670	745671	13	11	2	16	11	113	0.453042	0.369931	0.120191	8	0.569525935	0.398664
373	745670	745671	13	13	0	12	8	113	0.436661	0.371521	0.123269	9	0.575378908	0.386677
374	745670	745671	13	13	0	15	8	113	0.453587	0.391836	0.124815	8	0.577435637	0.383368
375	745670	745671	13	11	2	13	10	113	0.465632	0.392471	0.125056	9	0.574624077	0.388509

376	745670	745671	13	13	0	14	10	113	0.454529	0.367349	0.052767	7	0.578451812	0.414044
377	745670	745671	13	13	0	16	12	113	0.493512	0.386468	0.061811	8	0.585172679	0.40356
378	745670	745671	13	13	0	19	12	113	0.481508	0.412063	0.124336	8	0.575677358	0.384534
379	745670	745671	13	12	1	16	7	113	0.447703	0.384854	0.123717	9	0.579546959	0.381143
380	745670	745671	13	11	1	18	10	113	0.446358	0.373715	0.118968	9	0.580702765	0.379622
381	745670	745671	13	12	1	20	12	113	0.466494	0.397877	0.061737	8	0.584953597	0.403624
382	745670	745671	13	11	2	21	12	113	0.469248	0.381011	0.050791	8	0.585953193	0.404241
383	745670	745671	13	13	0	16	9	113	0.489364	0.366442	0.123378	9	0.576287772	0.385701
384	745670	745671	13	10	3	12	6	113	0.476614	0.388756	0.125319	9	0.580383139	0.3791
385	745670	745671	13	13	0	18	7	113	0.454307	0.364968	0.124116	8	0.57932217	0.379306
386	745670	745671	13	12	1	14	7	113	0.425993	0.329378	0.120946	9	0.580635576	0.379583
387	745670	745671	13	11	2	18	12	113	0.476253	0.403393	0.050693	8	0.585687985	0.40392
388	745670	745671	13	12	1	18	11	113	0.472178	0.381358	0.121833	9	0.580245022	0.379694
389	745670	745671	13	11	2	13	8	113	0.485462	0.388705	0.124664	9	0.57932406	0.381412
390	745670	745671	13	12	1	18	7	113	0.477223	0.392929	0.124385	9	0.578674903	0.382849
391	745670	745671	13	13	0	13	8	113	0.478533	0.378686	0.05106	7	0.581407607	0.409909
392	745670	745671	13	13	0	16	9	113	0.464334	0.393146	0.126642	8	0.576145724	0.382686
393	745670	745671	13	12	1	17	10	113	0.443125	0.355123	0.064443	8	0.581456005	0.408053
394	745670	745671	13	13	0	17	11	113	0.503308	0.391423	0.121431	8	0.576473899	0.385601
395	745670	745671	13	11	2	16	9	113	0.483524	0.372611	0.122617	9	0.578946207	0.381504
396	745670	745671	13	12	1	17	9	113	0.420794	0.366661	0.12254	9	0.578803764	0.38185
397	745670	745671	13	13	0	19	10	113	0.463369	0.385367	0.123633	9	0.575445786	0.385867
398	745670	745671	13	12	1	19	12	113	0.472569	0.380162	0.127436	8	0.577629845	0.379933
399	745670	745671	13	13	0	16	11	113	0.435949	0.380788	0.057932	8	0.589520884	0.398362
400	745670	745671	13	12	1	15	8	113	0.434187	0.369691	0.12174	7	0.572617588	0.393011
401	745670	745671	13	12	1	18	9	113	0.458435	0.388209	0.124871	8	0.573230299	0.389797
402	745670	745671	13	12	1	19	9	113	0.438475	0.325785	0.197435	9	0.570371994	0.352567
403	745670	745671	13	12	1	14	9	113	0.4529	0.378954	0.052325	8	0.582044918	0.410176
404	745670	745671	13	12	1	19	12	113	0.453889	0.37706	0.047348	8	0.58376355	0.408026
405	745670	745671	13	12	1	18	8	113	0.450805	0.360522	0.051455	8	0.58382446	0.407032
406	745670	745671	13	12	1	15	8	113	0.472733	0.39334	0.059566	8	0.597640544	0.388732
407	745670	745671	13	11	1	14	8	113	0.402236	0.357333	0.051692	8	0.587639156	0.403071
408	745670	745671	13	12	1	15	7	113	0.45388	0.333317	0.050609	8	0.583731648	0.40913
409	745670	745671	13	12	1	13	8	113	0.492731	0.378257	0.060277	7	0.588789052	0.400802
410	745670	745671	13	13	0	16	8	113	0.470655	0.396647	0.051613	8	0.58187192	0.409794
411	745670	745671	13	11	1	19	7	113	0.434945	0.372267	0.120848	8	0.57842694	0.381583
412	745670	745671	13	10	2	16	7	113	0.47495	0.398674	0.124528	9	0.579134386	0.381941
413	745670	745671	13	12	1	13	10	113	0.424028	0.360496	0.125801	8	0.572079796	0.39289
414	745670	745671	13	13	0	19	10	113	0.469809	0.395965	0.061352	8	0.584615305	0.404789
415	745670	745671	13	13	0	20	12	113	0.448887	0.370632	0.121228	8	0.575500962	0.387715
416	745670	745671	13	12	1	19	9	113	0.447328	0.373764	0.060751	8	0.582773311	0.408115
417	745670	745671	13	12	1	17	12	113	0.45347	0.382932	-0.01328	7	0.597269801	0.418589
418	745670	745671	13	12	1	13	10	113	0.507222	0.391888	0.131592	9	0.575497135	0.385892
419	745670	745671	13	12	1	17	11	113	0.454416	0.3739	-0.01804	7	0.58868185	0.427947
420	745670	745671	13	12	1	13	9	113	0.471321	0.394121	0.120253	9	0.577985824	0.382964
421	745670	745671	13	10	2	17	10	113	0.467709	0.355124	0.059825	8	0.587954973	0.401386
422	745670	745671	13	11	1	14	5	113	0.44304	0.365363	0.118472	9	0.575515113	0.389238
423	745670	745671	13	13	0	18	11	113	0.485019	0.408727	0.053766	7	0.585367463	0.40321
424	745670	745671	13	13	0	21	12	113	0.448221	0.398176	-0.00999	7	0.584453271	0.431353
425	745670	745671	13	12	1	14	8	113	0.477163	0.359313	0.052145	7	0.589078889	0.399406
426	745670	745671	13	12	1	16	9	113	0.476875	0.387584	0.124283	9	0.576258718	0.384554
427	745670	745671	13	11	2	14	9	113	0.452278	0.395588	-0.01258	7	0.598662274	0.417049
428	745670	745671	13	11	1	16	8	113	0.455473	0.393448	0.056766	7	0.581994813	0.407786
429	745670	745671	13	12	1	16	8	113	0.492272	0.391194	0.053061	8	0.585769556	0.40355
430	745670	745671	13	12	1	17	9	113	0.493457	0.396919	0.058279	8	0.591453553	0.398607
431	745670	745671	13	11	2	17	9	113	0.477646	0.371261	0.122256	9	0.585573338	0.369808
432	745670	745671	13	12	1	19	9	113	0.461081	0.371048	0.121647	8	0.575707161	0.385939
433	745670	745671	13	11	2	13	9	113	0.472539	0.392661	0.123549	9	0.57402996	0.391051
434	745670	745671	13	11	2	19	10	113	0.439321	0.354376	0.059973	8	0.57717052	0.415935
435	745670	745671	13	12	1	20	9	113	0.47758	0.396127	0.122262	9	0.584202164	0.372831
436	745670	745671	13	13	0	14	9	113	0.469595	0.378466	0.058312	8	0.588412323	0.401828
437	745670	745671	13	12	1	13	7	113	0.451478	0.371227	-0.01619	7	0.598643224	0.418075
438	745670	745671	13	13	0	18	12	113	0.418504	0.360487	0.131595	8	0.583439702	0.370687
439	745670	745671	13	13	0	13	7	113	0.46397	0.397081	0.060889	8	0.585012874	0.404584
440	745670	745671	13	12	1	21	11	113	0.436908	0.388809	0.121663	8	0.578459801	0.382625
441	745670	745671	13	12	1	19	10	113	0.476732	0.382082	0.12186	9	0.583863082	0.374915
442	745670	745671	13	13	0	17	8	113	0.44696	0.380753	0.122255	9	0.579436322	0.379455
443	745670	745671	13	13	0	20	10	113	0.440268	0.395335	0.05377	8	0.587747748	0.402612
444	745670	745671	13	13	0	15	9	113	0.454624	0.366838	0.124004	9	0.57449	0.387663
445	745670	745671	13	12	1	16	9	113	0.489365	0.392547	0.125267	9	0.571995493	0.392421
446	745670	745671	13	12	1	18	11	113	0.487863	0.405697	0.051546	7	0.588026618	0.401076
447	745670	745671	13	13	0	19	13	113	0.497165	0.391404	0.121335	9	0.576971855	0.385031
448	745670	745671	13	9	4	16	7	113	0.44903	0.379465	0.053885	8	0.585523521	0.406803
449	745670	745671	13	12	1	18	9	113	0.465355	0.386009	0.119902	9	0.56955729	0.398628
450	745670	745671	13	13	0	18	8	113	0.449163	0.351115	0.194732	9	0.567642582	0.359791
451	745670	745671	13	13	0	16	12	113	0.433539	0.339506	0.052343	8	0.585535591	0.405433
452	745670	745671	13	12	1	19	10	113	0.453432	0.381477	0.049941	8	0.584933306	0.406207
453	745670	745671	13	12	1	14	10	113	0.430665	0.352592	0.060378	7	0.584031008	0.407097
454	745670	745671	13	12	1	18	10	113	0.461792	0.362175	0.063984	7	0.588265194	0.399281
455	745670	745671	13	13	0	20	11	113	0.434525	0.389096	0.122951	9	0.575641879	0.385576
456	745670	745671	13	13	0	20	11	113	0.455593	0.379183	0.126432	9	0.577441083	0.381664
457	745670	745671	13	13	0	15	10	113	0.457759	0.384421	0.125681	9	0.576828559	0.382345
458	745670	745671	13	13	0	19	9	113	0.456546	0.371196	0.122334	9	0.577750244	0.383237
459	745670	745671	13	13	0	14	9	113	0.452369	0.380005	0.05819	7	0.582591374	0.407024
460	745670	745671	13	13	0	18	10	113	0.477736	0.391442	0.121652	9	0.578967763	0.38105
461	745670	745671	13	13	0	18	10	113	0.47042	0.387949	0.122585	9	0.575500695	0.387104
462	745670	745671	13	12	1	16	10	113	0.474819	0.369203	0.048064	8	0.587750472	0.403158
463	745670	745671	13	12	1	12	5	113	0.446981	0.378918	0.05194	8	0.585199746	0.406157
464	745670	745671	13	11	2	18	11	113	0.4844	0.402409	0.121722	9	0.580203046	0.37952
465	745670	745671	13	13	0	24	10	113	0.453402	0.365383	0.123308	8	0.577121051	0.384282
466	745670	745671	13	11	2	14	11	113	0.474447	0.369133	0.122465	9	0.579862748	0.380136
467	745670	745671	13	11	2	18	11	113	0.460568	0.377412	0.120717	9	0.576589788	0.386882
468	745670	745671	13	13	0	19	10	113	0.456391	0.385036	0.061086	7	0.585528031	0.405369
469	745670	745671	13	11	2	18	13	113	0.497938	0.404028	0.121493	8	0.576877956	0.385136

470	745670	745671	13	12	1	14	8	113	0.469831	0.386151	0.124705	8	0.575822457	0.38419
471	745670	745671	13	13	0	15	10	113	0.449628	0.403191	0.047558	8	0.583969084	0.406903
472	745670	745671	13	13	0	14	9	113	0.490744	0.408453	0.126659	7	0.574895819	0.384725
473	745670	745671	13	12	1	17	9	113	0.477453	0.358258	0.124959	9	0.581688551	0.376919
474	745670	745671	13	11	2	15	9	113	0.495024	0.406413	0.124844	9	0.58005448	0.379914
475	745670	745671	13	13	0	22	12	113	0.446734	0.365271	0.122579	9	0.576250823	0.386333
476	745670	745671	13	12	1	15	7	113	0.445016	0.371811	0.0552	8	0.583738764	0.406573
477	745670	745671	13	13	0	15	10	113	0.506221	0.413351	0.122404	9	0.578623798	0.382728
478	745670	745671	13	13	0	13	8	113	0.474131	0.359749	0.125067	9	0.577926668	0.381474
479	745670	745671	13	11	2	16	9	113	0.448177	0.376345	0.123526	9	0.575261201	0.387808
480	745670	745671	13	12	1	16	12	113	0.472162	0.389085	0.060065	7	0.582862247	0.407714
481	745670	745671	13	12	1	18	11	113	0.476376	0.369778	0.121753	9	0.577862481	0.383948
482	745670	745671	13	13	0	19	7	113	0.423716	0.351373	0.124487	9	0.575870847	0.385286
483	745670	745671	13	13	0	18	12	113	0.395937	0.351802	0.053895	7	0.583157379	0.405837
484	745670	745671	13	13	0	17	12	113	0.434201	0.390824	0.122338	8	0.575060529	0.386217
485	745670	745671	13	12	1	15	9	113	0.497259	0.400585	-0.0078	7	0.591546261	0.423597
486	745670	745671	13	13	0	19	14	113	0.432367	0.368731	0.123255	8	0.574501279	0.388169
487	745670	745671	13	11	2	15	8	113	0.454106	0.381003	0.13102	7	0.577845165	0.384165
488	745670	745671	13	11	2	12	7	113	0.468335	0.390913	0.121897	9	0.579172982	0.380697
489	745670	745671	13	12	1	16	6	113	0.457709	0.387923	0.049328	7	0.578239306	0.415891
490	745670	745671	13	13	0	19	10	113	0.470516	0.395906	0.058042	8	0.588529454	0.400373
491	745670	745671	13	13	0	19	7	113	0.434339	0.36085	0.048672	8	0.584477363	0.407037
492	745670	745671	13	11	2	14	6	113	0.445591	0.385584	0.122732	9	0.574731095	0.387572
493	745670	745671	13	13	0	20	9	113	0.494541	0.423932	0.052581	8	0.586014843	0.40374
494	745670	745671	13	13	0	13	9	113	0.463397	0.36631	0.122439	9	0.577540675	0.383984
495	745670	745671	13	10	2	16	9	113	0.463133	0.372951	0.120699	9	0.581142548	0.37746
496	745670	745671	13	11	2	15	9	113	0.437943	0.369472	0.120279	9	0.577477153	0.384728
497	745670	745671	13	11	2	18	11	113	0.467694	0.387113	0.05057	8	0.584644779	0.405571
498	745670	745671	13	13	0	17	12	113	0.384046	0.308329	0.052108	8	0.583071082	0.408364
499	745670	745671	13	12	1	18	9	113	0.457211	0.376533	0.122898	9	0.576405429	0.384794
500	745670	745671	13	12	1	14	11	113	0.452082	0.368084	0.049444	7	0.582251902	0.409242
501	745670	745671	13	13	0	18	12	113	0.480719	0.372886	0.122469	9	0.582088604	0.37522
502	745670	745671	13	13	0	13	6	113	0.476025	0.400084	0.126541	9	0.569309079	0.398056
503	745670	745671	13	12	1	16	13	113	0.496591	0.382802	0.059007	8	0.587105774	0.40319
504	745670	745671	13	12	1	19	9	113	0.440183	0.371406	0.122628	8	0.574296103	0.388672
505	745670	745671	13	13	0	17	10	113	0.411346	0.357402	0.125118	9	0.576304755	0.384848
506	745670	745671	13	11	2	20	11	113	0.473713	0.395102	0.120849	8	0.576812926	0.386322
507	745670	745671	13	12	1	15	6	113	0.456478	0.375534	0.051271	8	0.585053268	0.404247
508	745670	745671	13	12	1	19	9	113	0.441008	0.387985	0.121604	9	0.581001331	0.37686
509	745670	745671	13	11	2	14	12	113	0.474536	0.379471	0.197359	9	0.568404804	0.356823
510	745670	745671	13	11	2	14	8	113	0.436554	0.377856	0.122325	9	0.572468631	0.392626
511	745670	745671	13	13	0	20	10	113	0.474814	0.36882	0.122376	9	0.579913021	0.380145
512	745670	745671	13	11	1	14	8	113	0.456742	0.361461	0.119769	9	0.573791424	0.392172
513	745670	745671	13	12	1	14	9	113	0.415855	0.322555	0.061284	8	0.584838674	0.40469
514	745670	745671	13	12	1	19	8	113	0.445869	0.359213	0.053328	8	0.589746341	0.401524
515	745670	745671	13	13	0	15	11	113	0.455464	0.372445	0.121434	8	0.577421536	0.38513
516	745670	745671	13	10	3	12	8	113	0.471948	0.377376	0.051511	8	0.584687503	0.405653
517	745670	745671	13	12	1	13	8	113	0.458612	0.379393	0.124612	9	0.571846214	0.395221
518	745670	745671	13	12	1	15	8	113	0.496883	0.404045	0.124652	9	0.575528073	0.387544
519	745670	745671	13	13	0	12	8	113	0.445686	0.397596	0.127272	8	0.575417682	0.38402
520	745670	745671	13	11	2	17	11	113	0.446254	0.375676	0.050828	8	0.585308644	0.405402
521	745670	745671	13	11	2	15	10	113	0.454958	0.37933	-0.0179	7	0.590049389	0.426377
522	745670	745671	13	12	1	17	9	113	0.448695	0.392643	0.121332	9	0.578448286	0.382377
523	745670	745671	13	13	0	13	9	113	0.476498	0.364699	0.121496	9	0.578054306	0.382945
524	745670	745671	13	13	0	14	9	113	0.436861	0.353065	0.121	9	0.578945693	0.382497
525	745670	745671	13	12	1	14	8	113	0.469048	0.345163	0.193375	9	0.565913682	0.365795
526	745670	745671	13	12	1	15	9	113	0.470844	0.397257	0.053689	8	0.584522446	0.405792
527	745670	745671	13	12	1	15	11	113	0.489109	0.386781	0.121751	9	0.576892944	0.385489
528	745670	745671	13	13	0	19	10	113	0.475436	0.385309	0.123522	8	0.577092884	0.38371
529	745670	745671	13	13	0	18	10	113	0.460484	0.381923	0.121754	9	0.577843142	0.383692
530	745670	745671	13	12	1	19	9	113	0.459184	0.407568	0.122174	9	0.577014625	0.384726
531	745670	745671	13	12	1	19	11	113	0.475134	0.379632	0.122962	9	0.577340235	0.383745
532	745670	745671	13	12	1	16	11	113	0.469468	0.394172	0.06375	8	0.585536371	0.402296
533	745670	745671	13	12	1	17	10	113	0.447721	0.35797	-0.00889	7	0.604507406	0.410069
534	745670	745671	13	12	1	16	7	113	0.446401	0.360245	0.121282	9	0.584125048	0.372863
535	745670	745671	13	12	1	20	9	113	0.473057	0.397456	0.126483	9	0.579752649	0.378319
536	745670	745671	13	13	0	19	11	113	0.484299	0.405829	0.122315	9	0.578346985	0.382874
537	745670	745671	13	12	1	18	10	113	0.462967	0.362213	0.058098	8	0.595664	0.388896
538	745670	745671	13	13	0	18	9	113	0.450791	0.395925	0.125058	9	0.574399273	0.386889
539	745670	745671	13	11	1	12	7	113	0.440693	0.382851	0.058362	8	0.593361958	0.395848
540	745670	745671	13	13	0	16	10	113	0.454435	0.375148	0.126013	9	0.576963823	0.382462
541	745670	745671	13	13	0	14	12	113	0.450877	0.378729	0.12163	9	0.576488834	0.386423
542	745670	745671	13	13	0	17	10	113	0.465262	0.36949	0.121611	9	0.57631828	0.385708
543	745670	745671	13	12	1	23	13	113	0.459817	0.380692	0.121573	9	0.586353557	0.368228
544	745670	745671	13	12	1	21	9	113	0.465424	0.382445	0.124318	7	0.575273334	0.384492
545	745670	745671	13	13	0	16	10	113	0.494398	0.396192	0.12171	8	0.577505966	0.384393
546	745670	745671	13	11	1	20	10	113	0.462969	0.359762	0.128778	9	0.579854339	0.379815
547	745670	745671	13	13	0	19	10	113	0.468388	0.390795	-0.01059	7	0.597919092	0.417761
548	745670	745671	13	13	0	13	3	113	0.439077	0.363154	0.121212	8	0.57588139	0.387444
549	745670	745671	13	12	1	19	12	113	0.455283	0.389463	0.053045	8	0.587978474	0.401451
550	745670	745671	13	13	0	13	9	113	0.465966	0.382858	0.060772	8	0.585324747	0.404261
551	745670	745671	13	12	1	15	9	113	0.486083	0.375616	0.126724	8	0.573089814	0.388275
552	745670	745671	13	12	1	17	10	113	0.495595	0.385997	0.125321	8	0.575478856	0.383956
553	745670	745671	13	11	2	15	5	113	0.461074	0.365604	0.12197	9	0.576947585	0.385075
554	745670	745671	13	12	1	13	10	113	0.502644	0.413602	0.122781	9	0.580182671	0.379881
555	745670	745671	13	12	1	24	11	113	0.469641	0.371861	0.122687	9	0.575149637	0.387385
556	745670	745671	13	12	1	21	9	113	0.46776	0.379729	0.054399	8	0.58526315	0.402802
557	745670	745671	13	11	2	18	11	113	0.454788	0.377126	0.051875	8	0.595682583	0.392076
558	745670	745671	13	12	1	20	12	113	0.460341	0.378602	0.137094	7	0.573925186	0.386248
559	745670	745671	13	13	0	15	9	113	0.481231	0.39052	0.063143	8	0.585554468	0.40178
560	745670	745671	13	13	0	14	8	113	0.480833	0.401307	0.126464	8	0.57625279	0.383321
561	745670	745671	13	12	1	16	8	113	0.447169	0.373177	0.124147	9	0.577479704	0.383223
562	745670	745671	13	13	0	17	11	113	0.481204	0.403839	0.053186	8	0.588101783	0.400986
563	745670	745671	13	12	1	10	8	113	0.445553	0.367432	0.050866	7	0.575960763	0.418796

564	745670	745671	13	11	2	17	12	113	0.445528	0.373523	0.062319	8	0.580818141	0.410801
565	745670	745671	13	12	1	15	10	113	0.476745	0.376398	0.046545	8	0.591085712	0.398007
566	745670	745671	13	12	1	17	11	113	0.469832	0.386075	0.05569	8	0.590974082	0.398751
567	745670	745671	13	12	1	17	7	113	0.447644	0.379592	0.121119	9	0.57952403	0.381646
568	745670	745671	13	12	1	14	8	113	0.452301	0.363232	0.124804	8	0.575696289	0.385224
569	745670	745671	13	10	2	8	6	113	0.42205	0.35984	-0.01388	7	0.596114555	0.420611
570	745670	745671	13	13	0	18	9	113	0.482603	0.37558	0.124287	7	0.575925161	0.384206
571	745670	745671	13	13	0	18	10	113	0.468356	0.395198	0.124163	7	0.575316724	0.385853
572	745670	745671	13	13	0	17	10	113	0.465537	0.370592	0.125095	8	0.57684015	0.382916
573	745670	745671	13	10	3	15	10	113	0.459666	0.353596	0.121804	8	0.580717224	0.37651
574	745670	745671	13	13	0	15	7	113	0.425153	0.371576	0.058214	8	0.592647787	0.395521
575	745670	745671	13	12	1	17	12	113	0.474543	0.372814	0.121439	9	0.577493397	0.385849
576	745670	745671	13	12	1	17	9	113	0.426966	0.370622	0.122187	9	0.576229359	0.386036
577	745670	745671	13	13	0	14	6	113	0.474152	0.362316	0.059212	8	0.589012984	0.400876
578	745670	745671	13	12	1	19	11	113	0.451112	0.332311	0.122985	9	0.578337197	0.381928
579	745670	745671	13	12	1	14	7	113	0.469453	0.378515	0.050294	8	0.585731242	0.40467
580	745670	745671	13	12	1	14	8	113	0.452119	0.387651	0.121448	8	0.571528152	0.393562
581	745670	745671	13	13	0	16	9	113	0.480268	0.362691	0.060139	8	0.588812734	0.400839
582	745670	745671	13	12	1	15	7	113	0.446291	0.376125	0.12103	9	0.57750147	0.383879
583	745670	745671	13	12	1	21	10	113	0.483957	0.397864	0.123143	9	0.575891485	0.385418
584	745670	745671	13	12	1	15	10	113	0.447654	0.338642	0.123998	9	0.579303805	0.378615
585	745670	745671	13	11	2	11	6	113	0.440273	0.362843	0.124868	9	0.575216438	0.38553
586	745670	745671	13	13	0	16	6	113	0.442322	0.358408	0.122361	8	0.573258789	0.390301
587	745670	745671	13	13	0	16	10	113	0.446292	0.374289	0.125047	9	0.576420291	0.383645
588	745670	745671	13	10	3	12	8	113	0.457949	0.386085	0.121864	8	0.5718364	0.391468
589	745670	745671	13	12	1	13	9	113	0.467228	0.379026	0.051035	8	0.588056172	0.400608
590	745670	745671	13	12	1	15	9	113	0.463536	0.3791	0.124542	9	0.575763952	0.384859
591	745670	745671	13	12	1	17	10	113	0.459852	0.375871	0.050306	8	0.590358915	0.399779
592	745670	745671	13	12	1	17	13	113	0.399443	0.332339	0.046727	8	0.582094756	0.410854
593	745670	745671	13	11	2	13	9	113	0.491467	0.39344	0.124553	9	0.581336035	0.375155
594	745670	745671	13	11	1	17	9	113	0.456194	0.394521	0.118999	9	0.578550317	0.382803
595	745670	745671	13	11	2	12	6	113	0.47858	0.37943	0.124604	9	0.576871978	0.38232
596	745670	745671	13	12	1	19	11	113	0.483034	0.39251	0.121478	9	0.581003138	0.377768
597	745670	745671	13	13	0	22	10	113	0.431334	0.364501	0.123017	9	0.586676222	0.366965
598	745670	745671	13	12	1	17	10	113	0.489071	0.375884	0.065151	8	0.584569405	0.403272
599	745670	745671	13	13	0	14	10	113	0.419542	0.363398	-0.02041	7	0.600162144	0.415849
600	745670	745671	13	11	2	15	9	113	0.50456	0.401566	0.122902	9	0.576974213	0.385183
601	745670	745671	13	11	2	13	7	113	0.455754	0.34641	0.057578	8	0.585656161	0.402745
602	745670	745671	13	13	0	13	9	113	0.460908	0.366219	0.123012	8	0.579899441	0.377633
603	745670	745671	13	12	1	19	10	113	0.4846	0.370497	0.122695	9	0.581150467	0.37755
604	745670	745671	13	13	0	14	9	113	0.494124	0.407529	0.062262	8	0.585390279	0.402871
605	745670	745671	13	12	1	17	10	113	0.445822	0.368941	0.196695	9	0.564993626	0.364485
606	745670	745671	13	11	2	17	9	113	0.447201	0.343236	0.120367	9	0.576794805	0.384415
607	745670	745671	13	12	1	17	10	113	0.47429	0.394975	0.121371	9	0.571656759	0.394042
608	745670	745671	13	12	1	19	9	113	0.468569	0.380239	0.124701	9	0.575706227	0.387345
609	745670	745671	13	12	1	20	11	113	0.487468	0.394538	-0.00741	7	0.592703044	0.422161
610	745670	745671	13	11	2	13	10	113	0.490131	0.388641	0.120149	9	0.574810308	0.388313
611	745670	745671	13	13	0	16	10	113	0.502078	0.403834	0.124234	9	0.579082468	0.380227
612	745670	745671	13	13	0	18	7	113	0.472851	0.402142	0.123517	9	0.577110766	0.383565
613	745670	745671	13	12	1	17	12	113	0.483333	0.381127	0.121568	9	0.576334251	0.385101
614	745670	745671	13	12	1	14	6	113	0.415439	0.344327	-0.01005	7	0.591799044	0.423797
615	745670	745671	13	12	1	18	10	113	0.465852	0.383392	0.124336	9	0.57611328	0.384225
616	745670	745671	13	13	0	15	9	113	0.473867	0.397491	0.122529	8	0.575546323	0.386362
617	745670	745671	13	13	0	18	10	113	0.450538	0.367338	0.122788	9	0.578430469	0.382021
618	745670	745671	13	11	2	13	6	113	0.483263	0.405229	0.12203	9	0.580425978	0.378788
619	745670	745671	13	12	1	19	10	113	0.457775	0.375018	0.12244	8	0.574661946	0.390276
620	745670	745671	13	11	2	14	8	113	0.484614	0.397057	-0.00678	7	0.594653599	0.41985
621	745670	745671	13	11	2	13	8	113	0.482154	0.363664	0.122097	9	0.580353259	0.380197
622	745670	745671	13	12	1	16	10	113	0.474845	0.406266	0.053769	8	0.586022675	0.402561
623	745670	745671	13	13	0	13	9	113	0.46202	0.356085	-0.01183	7	0.600035171	0.415318
624	745670	745671	13	11	1	21	11	113	0.41606	0.334526	0.120056	8	0.575926743	0.385236
625	745670	745671	13	13	0	20	12	113	0.461225	0.374193	0.195492	8	0.567831491	0.357376
626	745670	745671	13	12	1	16	11	113	0.441629	0.382405	0.120525	8	0.577706315	0.38307
627	745670	745671	13	13	0	16	11	113	0.487486	0.405213	0.123372	8	0.575485271	0.386031
628	745670	745671	13	11	1	14	8	113	0.427469	0.385585	0.054804	8	0.582301041	0.40768
629	745670	745671	13	11	2	16	7	113	0.445325	0.382408	0.051228	7	0.583013172	0.407713
630	745670	745671	13	12	1	15	8	113	0.493659	0.409601	0.121265	9	0.580258502	0.38023
631	745670	745671	13	12	1	17	13	113	0.465854	0.382469	0.19776	9	0.562273845	0.370805
632	745670	745671	13	13	0	22	10	113	0.459183	0.402363	0.124894	9	0.574977171	0.385632
633	745670	745671	13	12	1	13	7	113	0.457887	0.369214	0.121632	9	0.577122604	0.385157
634	745670	745671	13	13	0	17	10	113	0.486818	0.368364	0.122134	8	0.578364259	0.381653
635	745670	745671	13	12	1	14	9	113	0.499024	0.388737	0.121528	9	0.576836934	0.384708
636	745670	745671	13	12	1	16	9	113	0.464804	0.370943	0.122435	9	0.58680189	0.366591
637	745670	745671	13	13	0	13	8	113	0.454876	0.400173	0.049699	8	0.58411414	0.407257
638	745670	745671	13	13	0	14	9	113	0.46609	0.362489	0.060064	8	0.58806749	0.402083
639	745670	745671	13	12	1	17	13	113	0.484999	0.379348	0.064374	8	0.584001025	0.404589
640	745670	745671	13	13	0	20	10	113	0.468225	0.40056	0.051665	8	0.594580154	0.392
641	745670	745671	13	11	1	15	8	113	0.455223	0.379364	0.121924	9	0.577001116	0.384214
642	745670	745671	13	12	1	9	6	113	0.475329	0.387832	0.060675	8	0.587412672	0.402318
643	745670	745671	13	13	0	20	9	113	0.468052	0.39898	0.125684	9	0.574767881	0.386638
644	745670	745671	13	11	2	13	8	113	0.452436	0.381866	0.049687	8	0.582521367	0.409321
645	745670	745671	13	12	1	14	8	113	0.461442	0.363286	0.124363	8	0.579891352	0.378116
646	745670	745671	13	12	1	14	10	113	0.477121	0.413182	0.051317	8	0.589231397	0.399515
647	745670	745671	13	10	2	17	10	113	0.469318	0.381866	0.120198	9	0.58234227	0.376363
648	745670	745671	13	13	0	17	11	113	0.475229	0.404367	0.130575	8	0.567569673	0.400577
649	745670	745671	13	13	0	12	8	113	0.46377	0.375805	0.051738	8	0.581535801	0.412068
650	745670	745671	13	13	0	15	8	113	0.459514	0.383456	0.053469	8	0.579622656	0.412025
651	745670	745671	13	13	0	20	11	113	0.447122	0.385835	0.127376	9	0.576343613	0.382736
652	745670	745671	13	13	0	11	6	113	0.43685	0.363666	0.121101	9	0.573896723	0.391225
653	745670	745671	13	13	0	22	13	113	0.491869	0.393587	0.123703	8	0.579346486	0.379506
654	745670	745671	13	12	1	16	7	113	0.432133	0.364389	0.124815	8	0.573727648	0.389932
655	745670	745671	13	12	1	17	8	113	0.4825	0.394308	0.123351	9	0.575347333	0.388953
656	745670	745671	13	11	2	15	7	113	0.483073	0.34793	0.120406	9	0.578274208	0.382563
657	745670	745671	13	13	0	19	8	113	0.442515	0.377549	0.063667	8	0.582809768	0.406373

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

658	745670	745671	13	12	1	18	12	113	0.445293	0.380595	0.052396	8	0.584893564	0.403596
659	745670	745671	13	13	0	13	9	113	0.47116	0.391026	0.063721	8	0.584873093	0.403969
660	745670	745671	13	13	0	18	11	113	0.480698	0.382011	0.124331	9	0.57518768	0.38617
661	745670	745671	13	12	1	12	8	113	0.500952	0.401486	0.123348	8	0.577349128	0.383363
662	745670	745671	13	12	1	21	11	113	0.457652	0.359453	0.061942	8	0.593289696	0.393833
663	745670	745671	13	13	0	17	10	113	0.469216	0.412297	0.122895	8	0.575640882	0.386124
664	745670	745671	13	12	1	15	7	113	0.454565	0.385666	0.05192	7	0.583260943	0.407339
665	745670	745671	13	12	1	17	8	113	0.416955	0.372235	-0.00631	7	0.593778987	0.421499
666	745670	745671	13	11	2	15	7	113	0.473391	0.370754	0.121541	9	0.578852904	0.383446
667	745670	745671	13	12	1	16	6	113	0.430144	0.368229	0.124474	9	0.572855262	0.391008
668	745670	745671	13	10	2	13	8	113	0.448148	0.388854	0.124825	8	0.57510519	0.384872
669	745670	745671	13	12	1	15	12	113	0.504764	0.390105	-0.0057	7	0.5922972	0.421891
670	745670	745671	13	12	1	17	9	113	0.459915	0.360722	0.124626	8	0.574417766	0.388234
671	745670	745671	13	13	0	17	8	113	0.458959	0.390247	0.060876	8	0.585192351	0.404518
672	745670	745671	13	12	1	20	13	113	0.497259	0.392559	-0.00745	7	0.592038938	0.422817
673	745670	745671	13	11	2	13	7	113	0.44927	0.364178	0.120487	9	0.573606955	0.391666
674	745670	745671	13	11	2	16	7	113	0.47474	0.382589	0.123742	9	0.580369582	0.380026
675	745670	745671	13	13	0	19	10	113	0.438831	0.344257	0.123257	7	0.570259504	0.39542
676	745670	745671	13	12	1	13	9	113	0.482479	0.402147	0.051873	8	0.584874071	0.405651
677	745670	745671	13	11	1	13	8	113	0.471805	0.406438	0.054317	8	0.58284134	0.408824
678	745670	745671	13	12	1	16	11	113	0.464191	0.393996	0.062877	8	0.582783199	0.406448
679	745670	745671	13	13	0	13	10	113	0.489151	0.377213	0.122947	9	0.578546221	0.381734
680	745670	745671	13	13	0	15	8	113	0.456725	0.373816	0.122153	9	0.57612836	0.384627
681	745670	745671	13	10	2	16	8	113	0.487246	0.369652	0.118837	9	0.578306341	0.384008
682	745670	745671	13	13	0	15	7	113	0.477715	0.39944	0.123729	9	0.575900585	0.385912
683	745670	745671	13	13	0	16	11	113	0.437459	0.369355	0.049706	8	0.585353083	0.405069
684	745670	745671	13	11	1	15	8	113	0.432142	0.379709	0.057462	8	0.579383829	0.412648
685	745670	745671	13	13	0	22	12	113	0.477993	0.405286	0.052392	8	0.586011101	0.403911
686	745670	745671	13	13	0	15	7	113	0.447896	0.377282	0.051916	8	0.583000446	0.407164
687	745670	745671	13	13	0	15	8	113	0.471513	0.368043	0.123111	9	0.57729317	0.383302
688	745670	745671	13	12	1	20	10	113	0.44989	0.37237	0.124756	8	0.57470304	0.38942
689	745670	745671	13	13	0	19	9	113	0.476714	0.372683	0.121188	9	0.581122125	0.378264
690	745670	745671	13	11	2	10	6	113	0.450562	0.374219	0.121501	7	0.572617781	0.392204
691	745670	745671	13	12	1	16	11	113	0.462044	0.395205	0.126206	7	0.568587456	0.397758
692	745670	745671	13	13	0	16	10	113	0.462906	0.379495	0.122879	8	0.575519603	0.386493
693	745670	745671	13	12	1	15	10	113	0.483155	0.397606	0.122734	9	0.575730269	0.38717
694	745670	745671	13	13	0	18	12	113	0.479579	0.404712	0.122588	9	0.575783199	0.38681
695	745670	745671	13	13	0	21	10	113	0.437563	0.366366	0.050082	8	0.586376298	0.404344
696	745670	745671	13	12	1	22	11	113	0.467107	0.386211	-0.01412	7	0.597672231	0.418823
697	745670	745671	13	12	1	18	13	113	0.437162	0.369479	0.051619	8	0.588823697	0.401124
698	745670	745671	13	11	2	20	10	113	0.46585	0.399289	0.123786	9	0.580078661	0.381097
699	745670	745671	13	13	0	14	10	113	0.479533	0.40906	0.122987	9	0.575585367	0.386347
700	745670	745671	13	12	1	14	7	113	0.453499	0.387299	0.123426	9	0.576929913	0.384496
701	745670	745671	13	12	1	18	10	113	0.474913	0.37337	0.121395	9	0.581667484	0.377586
702	745670	745671	13	12	1	12	6	113	0.464806	0.381481	0.12451	9	0.575206415	0.386321
703	745670	745671	13	13	0	19	12	113	0.448588	0.394913	0.059929	8	0.588982065	0.398834
704	745670	745671	13	13	0	12	6	113	0.466879	0.366387	0.121827	9	0.583398468	0.375364
705	745670	745671	13	11	2	15	9	113	0.468849	0.360763	0.122232	9	0.579998647	0.379429
706	745670	745671	13	12	1	15	10	113	0.453561	0.371435	-0.00951	7	0.589382218	0.426672
707	745670	745671	13	13	0	18	11	113	0.45701	0.388381	0.125313	7	0.576068149	0.383796
708	745670	745671	13	12	1	13	7	113	0.455316	0.366695	0.123291	9	0.573925666	0.38983
709	745670	745671	13	13	0	18	10	113	0.462463	0.37954	0.122322	8	0.581182231	0.376318
710	745670	745671	13	11	2	10	7	113	0.477512	0.386911	-0.01046	7	0.591502081	0.425677
711	745670	745671	13	11	2	14	7	113	0.479722	0.381557	0.122121	8	0.580512551	0.379922
712	745670	745671	13	11	1	21	11	113	0.451277	0.377501	0.118906	9	0.579890092	0.380312
713	745670	745671	13	12	1	19	9	113	0.429719	0.346403	0.049398	8	0.590421768	0.400815
714	745670	745671	13	13	0	17	9	113	0.46179	0.417207	0.127709	8	0.575082162	0.383201
715	745670	745671	13	11	2	14	9	113	0.473696	0.407776	0.125233	9	0.579844501	0.379759
716	745670	745671	13	12	1	12	7	113	0.446905	0.353968	0.119887	9	0.58447635	0.374241
717	745670	745671	13	13	0	17	12	113	0.465693	0.397515	0.12519	8	0.57894935	0.380699
718	745670	745671	13	11	2	11	7	113	0.462881	0.34397	0.056172	8	0.590658208	0.39842
719	745670	745671	13	12	1	17	8	113	0.453964	0.399262	0.125202	9	0.580046115	0.378601
720	745670	745671	13	11	2	16	9	113	0.457283	0.38103	0.130409	8	0.575854091	0.385236
721	745670	745671	13	12	1	8	5	113	0.471325	0.398982	0.12313	9	0.579695713	0.379884
722	745670	745671	13	12	1	18	11	113	0.433618	0.375636	0.126611	8	0.577893821	0.38229
723	745670	745671	13	13	0	16	10	113	0.453665	0.387407	0.123228	9	0.575169621	0.386678
724	745670	745671	13	11	2	17	11	113	0.449143	0.395769	0.124395	9	0.584196803	0.372589
725	745670	745671	13	12	1	14	11	113	0.490891	0.390196	0.122719	8	0.577334823	0.384939
726	745670	745671	13	11	2	16	10	113	0.50145	0.40526	0.193914	9	0.571338704	0.353618
727	745670	745671	13	13	0	17	10	113	0.465187	0.387697	0.123866	8	0.575875335	0.384373
728	745670	745671	13	13	0	14	11	113	0.428816	0.352838	-0.08023	6	0.605424988	0.434698
729	745670	745671	13	11	2	14	6	113	0.449485	0.369362	0.121197	9	0.573769663	0.390638
730	745670	745671	13	13	0	17	10	113	0.461176	0.370112	0.058673	8	0.577088177	0.415213
731	745670	745671	13	12	1	19	11	113	0.492715	0.377539	0.122987	8	0.57766268	0.384784
732	745670	745671	13	12	1	15	10	113	0.4832	0.387759	0.121906	8	0.57743851	0.384338
733	745670	745671	13	12	1	20	9	113	0.49259	0.404337	0.12154	9	0.580035449	0.380416
734	745670	745671	13	13	0	21	11	113	0.45175	0.392575	0.063543	7	0.592051135	0.394505
735	745670	745671	13	10	3	19	8	113	0.472339	0.376111	0.123325	9	0.573030281	0.392193
736	745670	745671	13	11	2	18	10	113	0.433825	0.344208	0.123247	9	0.578974163	0.379906
737	745670	745671	13	13	0	19	10	113	0.462262	0.377867	0.050319	8	0.583252727	0.407328
738	745670	745671	13	13	0	15	11	113	0.486642	0.399318	0.122308	9	0.578596655	0.382837
739	745670	745671	13	13	0	22	12	113	0.440827	0.355297	0.053973	8	0.585855768	0.402998
740	745670	745671	13	11	2	17	12	113	0.449651	0.387834	0.12316	9	0.579383865	0.382701
741	745670	745671	13	13	0	24	11	113	0.45292	0.361035	0.122701	9	0.578533943	0.382796
742	745670	745671	13	13	0	22	11	113	0.425516	0.359662	0.121219	8	0.574833665	0.387595
743	745670	745671	13	12	1	17	10	113	0.472433	0.376507	0.060176	8	0.585655581	0.403597
744	745670	745671	13	12	1	19	11	113	0.440261	0.389704	0.124876	8	0.576770675	0.383667
745	745670	745671	13	13	0	18	11	113	0.473965	0.397493	0.063073	8	0.58464198	0.404022
746	745670	745671	13	13	0	21	13	113	0.425957	0.358884	-0.01314	7	0.593961697	0.42054
747	745670	745671	13	12	1	14	9	113	0.445085	0.369317	0.120137	9	0.577981172	0.381457
748	745670	745671	13	13	0	19	13	113	0.472589	0.372902	0.125904	9	0.576526921	0.38382
749	745670	745671	13	12	1	18	11	113	0.458047	0.38457	0.124849	8	0.574749882	0.386893
750	745670	745671	13	12	1	19	9	113	0.405794	0.364048	0.050764	8	0.584013134	0.406838
751	745670	745671	13	13	0	15	7	113	0.463913	0.391806	0.061598	8	0.584796473	0.404645

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

752	745670	745671	13	12	1	16	11	113	0.461559	0.387428	0.123455	9	0.577147582	0.38451
753	745670	745671	13	11	1	18	11	113	0.477979	0.381463	0.12032	9	0.585578416	0.371518
754	745670	745671	13	12	1	16	11	113	0.451431	0.38114	0.124388	7	0.572663212	0.389352
755	745670	745671	13	13	0	15	6	113	0.486323	0.377848	0.122452	8	0.579314216	0.379831
756	745670	745671	13	12	1	11	7	113	0.462073	0.387857	-0.01318	7	0.599305448	0.417251
757	745670	745671	13	13	0	16	7	113	0.47373	0.372669	0.123497	9	0.576168419	0.385636
758	745670	745671	13	13	0	17	9	113	0.423184	0.371182	0.050983	8	0.581995243	0.408732
759	745670	745671	13	12	1	15	11	113	0.447147	0.395391	0.060743	8	0.585403911	0.404133
760	745670	745671	13	11	1	17	8	113	0.454499	0.377072	0.055241	7	0.580657846	0.410508
761	745670	745671	13	11	2	16	10	113	0.467035	0.371058	0.060661	8	0.587007057	0.404148
762	745670	745671	13	13	0	17	11	113	0.451528	0.399717	0.123729	9	0.576945901	0.383308
763	745670	745671	13	13	0	23	11	113	0.447027	0.399213	0.051561	8	0.584921833	0.405444
764	745670	745671	13	13	0	23	15	113	0.468512	0.363657	0.121261	8	0.576737555	0.384534
765	745670	745671	13	12	1	18	11	113	0.463043	0.401852	0.123325	9	0.575465349	0.385953
766	745670	745671	13	12	1	17	12	113	0.470255	0.391021	0.12232	8	0.579078536	0.381197
767	745670	745671	13	13	0	18	11	113	0.483519	0.39991	0.123988	9	0.579337002	0.379271
768	745670	745671	13	11	2	14	9	113	0.4818	0.399139	0.053519	8	0.585341716	0.404258
769	745670	745671	13	13	0	19	9	113	0.467798	0.349423	0.121368	9	0.577524194	0.383456
770	745670	745671	13	12	1	22	10	113	0.451958	0.392056	0.129836	8	0.577322708	0.383847
771	745670	745671	13	12	1	10	6	113	0.428982	0.374823	0.122129	8	0.586912338	0.366659
772	745670	745671	13	13	0	18	9	113	0.465026	0.366296	0.12487	8	0.574938936	0.386781
773	745670	745671	13	13	0	16	8	113	0.455055	0.356461	0.124985	8	0.575390912	0.384637
774	745670	745671	13	13	0	18	12	113	0.472105	0.38443	0.123392	9	0.579280106	0.380487
775	745670	745671	13	11	2	14	7	113	0.460537	0.352729	0.122462	9	0.579492624	0.382555
776	745670	745671	13	12	1	15	11	113	0.436932	0.395186	0.124713	9	0.576145557	0.384494
777	745670	745671	13	12	1	15	11	113	0.472938	0.396712	0.1253	9	0.577200819	0.382789
778	745670	745671	13	13	0	17	9	113	0.4732	0.374758	0.122785	9	0.575919113	0.386032
779	745670	745671	13	13	0	17	10	113	0.431301	0.387377	-0.0189	7	0.587520388	0.429365
780	745670	745671	13	12	1	19	12	113	0.484062	0.397818	0.05067	8	0.586089495	0.403612
781	745670	745671	13	12	1	13	8	113	0.460595	0.366436	0.06481	7	0.580508773	0.411267
782	745670	745671	13	13	0	13	8	113	0.496174	0.403579	0.121504	9	0.577105023	0.385032
783	745670	745671	13	12	1	13	8	113	0.457082	0.356045	0.121256	9	0.577633112	0.384825
784	745670	745671	13	13	0	15	10	113	0.469627	0.38528	0.124385	8	0.575538851	0.385385
785	745670	745671	13	11	1	14	11	113	0.455406	0.382199	0.049873	8	0.583209799	0.40839
786	745670	745671	13	12	1	20	11	113	0.481192	0.372056	0.125097	9	0.578768861	0.382167
787	745670	745671	13	11	2	14	9	113	0.444976	0.377296	0.119469	9	0.575508584	0.38832
788	745670	745671	13	13	0	19	9	113	0.474394	0.378855	0.127103	8	0.577949182	0.3799
789	745670	745671	13	12	1	18	12	113	0.475894	0.404847	0.060854	8	0.597214441	0.389244
790	745670	745671	13	12	1	18	6	113	0.465177	0.357439	0.12413	9	0.578414916	0.382417
791	745670	745671	13	13	0	15	8	113	0.442644	0.38562	0.196257	9	0.569999251	0.353939
792	745670	745671	13	12	1	18	11	113	0.454254	0.395685	0.123236	8	0.577082927	0.38333
793	745670	745671	13	11	1	17	12	113	0.459748	0.409612	0.122754	9	0.574707782	0.386786
794	745670	745671	13	11	2	17	9	113	0.444447	0.366112	-0.00631	7	0.592134453	0.421847
795	745670	745671	13	12	1	13	9	113	0.479417	0.368043	0.120564	9	0.576789812	0.384099
796	745670	745671	13	11	2	13	9	113	0.451988	0.380792	0.124675	9	0.577375407	0.383621
797	745670	745671	13	11	2	17	9	113	0.487391	0.398364	0.124966	9	0.580584405	0.378889
798	745670	745671	13	13	0	20	13	113	0.479516	0.361046	0.059115	8	0.587801704	0.401496
799	745670	745671	13	11	2	21	9	113	0.465762	0.375192	0.125663	8	0.572088739	0.390428
800	745670	745671	13	12	1	19	13	113	0.446323	0.376303	0.049846	8	0.583591066	0.408091
801	745670	745671	13	13	0	16	10	113	0.480499	0.390294	0.126802	8	0.576986281	0.380989
802	745670	745671	13	12	1	16	9	113	0.467402	0.38955	0.124827	9	0.577725602	0.381771
803	745670	745671	13	13	0	12	6	113	0.457591	0.357824	0.123249	8	0.575420978	0.386446
804	745670	745671	13	13	0	16	9	113	0.441129	0.350089	0.06118	8	0.595048177	0.390439
805	745670	745671	13	13	0	20	10	113	0.458818	0.344174	0.122846	9	0.581718712	0.376031
806	745670	745671	13	12	1	20	11	113	0.468798	0.40662	0.06287	8	0.585103172	0.403506
807	745670	745671	13	12	1	20	11	113	0.470072	0.39312	0.060999	8	0.584487145	0.404747
808	745670	745671	13	12	1	13	9	113	0.449447	0.38084	0.123499	9	0.586747104	0.366115
809	745670	745671	13	12	1	16	9	113	0.466625	0.415513	0.0608	8	0.579860159	0.411468
810	745670	745671	13	13	0	17	12	113	0.478343	0.372056	0.125056	8	0.576209716	0.383956
811	745670	745671	13	11	1	16	9	113	0.458384	0.370476	0.05058	8	0.583450501	0.407801
812	745670	745671	13	12	1	11	7	113	0.462806	0.380309	0.123854	9	0.577745755	0.383309
813	745670	745671	13	12	1	13	7	113	0.491621	0.38717	0.122097	9	0.580424344	0.378777
814	745670	745671	13	11	2	15	8	113	0.442189	0.374963	0.121093	9	0.584232111	0.372785
815	745670	745671	13	12	1	14	10	113	0.487227	0.393929	0.124005	9	0.578697585	0.380869
816	745670	745671	13	12	1	13	8	113	0.491437	0.400672	0.123991	9	0.578699215	0.380644
817	745670	745671	13	13	0	17	9	113	0.425892	0.382841	-0.01975	7	0.592061199	0.423817
818	745670	745671	13	11	2	16	9	113	0.470352	0.406873	0.126886	9	0.580855983	0.375959
819	745670	745671	13	13	0	17	12	113	0.493428	0.41516	0.124595	7	0.57543475	0.385
820	745670	745671	13	11	2	13	10	113	0.500869	0.387236	0.122486	9	0.577234287	0.384435
821	745670	745671	13	12	1	20	13	113	0.47199	0.412221	0.120408	8	0.580387274	0.379887
822	745670	745671	13	12	1	18	9	113	0.458278	0.376831	0.121405	7	0.577073335	0.385186
823	745670	745671	13	13	0	18	7	113	0.452542	0.364172	0.12212	8	0.572183719	0.391607
824	745670	745671	13	12	1	16	8	113	0.509647	0.398942	0.055986	8	0.586202134	0.405162
825	745670	745671	13	13	0	21	11	113	0.475537	0.384553	-0.01008	7	0.598287884	0.417421
826	745670	745671	13	11	2	15	8	113	0.475702	0.386664	0.122731	9	0.577770696	0.385271
827	745670	745671	13	13	0	15	10	113	0.468936	0.381594	0.057285	7	0.582961362	0.408278
828	745670	745671	13	12	1	20	12	113	0.453694	0.372437	0.194634	8	0.568689786	0.359148
829	745670	745671	13	11	2	18	10	113	0.476099	0.364574	0.121751	9	0.57820298	0.383485
830	745670	745671	13	12	1	16	10	113	0.474721	0.413238	0.124544	9	0.576742682	0.383948
831	745670	745671	13	11	1	15	10	113	0.477372	0.401045	0.060996	7	0.581705824	0.409156
832	745670	745671	13	11	2	18	10	113	0.467778	0.391618	0.123414	9	0.579178232	0.382024
833	745670	745671	13	12	1	19	12	113	0.473988	0.381337	0.12536	8	0.576637411	0.38515
834	745670	745671	13	12	1	15	8	113	0.458643	0.396033	0.123984	9	0.575878934	0.385737
835	745670	745671	13	13	0	18	12	113	0.474231	0.392752	0.12335	9	0.580080528	0.377965
836	745670	745671	13	11	2	15	10	113	0.456872	0.365883	0.064131	8	0.584284625	0.404202
837	745670	745671	13	13	0	18	11	113	0.462115	0.402899	0.125224	8	0.574871743	0.386293
838	745670	745671	13	11	2	14	10	113	0.458331	0.351398	0.052111	7	0.576423832	0.417236
839	745670	745671	13	12	1	13	9	113	0.484362	0.381079	0.124385	9	0.577752011	0.382214
840	745670	745671	13	12	1	20	11	113	0.48402	0.401677	0.120461	9	0.585514798	0.371161
841	745670	745671	13	11	2	18	10	113	0.446391	0.388001	0.12281	7	0.574197033	0.387952
842	745670	745671	13	11	2	18	7	113	0.443395	0.385281	-0.00656	6	0.595522847	0.420366
843	745670	745671	13	10	2	13	7	113	0.430802	0.360012	0.119759	8	0.57373815	0.389933
844	745670	745671	13	13	0	20	10	113	0.46489	0.403948	-0.01219	7	0.588164975	0.427242
845	745670	745671	13	13	0	18	10	113	0.496867	0.385122	0.059912	8	0.587257882	0.403311

– Ex. 10253 –

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

846	745670	745671	13	12	1	19	10	113	0.45305	0.35115	0.124422	9	0.580772738	0.377317
847	745670	745671	13	13	0	19	11	113	0.463134	0.39203	0.122196	7	0.577828718	0.382771
848	745670	745671	13	11	2	16	7	113	0.464966	0.366362	0.131556	9	0.580551722	0.378115
849	745670	745671	13	11	2	14	8	113	0.459281	0.382139	0.122997	8	0.576120808	0.387644
850	745670	745671	13	12	1	15	9	113	0.461762	0.351752	0.052035	8	0.586070589	0.403818
851	745670	745671	13	13	0	15	8	113	0.446412	0.394786	0.053825	8	0.587379612	0.40244
852	745670	745671	13	12	1	16	11	113	0.456216	0.384427	0.127206	9	0.572961491	0.389028
853	745670	745671	13	13	0	19	9	113	0.474604	0.384174	0.127075	9	0.577562736	0.380967
854	745670	745671	13	13	0	14	8	113	0.469946	0.368679	0.121428	9	0.583398289	0.374286
855	745670	745671	13	12	1	18	13	113	0.450524	0.369596	0.12085	8	0.574714682	0.390317
856	745670	745671	13	12	1	13	4	113	0.434294	0.37071	0.121638	8	0.569728499	0.396021
857	745670	745671	13	13	0	16	12	113	0.461965	0.386322	0.125314	9	0.579142473	0.379952
858	745670	745671	13	12	1	15	9	113	0.451347	0.387216	0.06213	8	0.58395247	0.404316
859	745670	745671	13	13	0	19	8	113	0.465689	0.383042	0.062614	8	0.586032521	0.4026
860	745670	745671	13	13	0	17	9	113	0.477437	0.400126	0.12502	7	0.576272578	0.384448
861	745670	745671	13	12	1	14	8	113	0.456502	0.378349	0.121858	9	0.587752014	0.366515
862	745670	745671	13	12	1	14	6	113	0.440698	0.372521	0.049598	8	0.582666043	0.408392
863	745670	745671	13	12	1	13	8	113	0.471579	0.378934	0.124751	9	0.577790855	0.381638
864	745670	745671	13	11	2	13	9	113	0.456089	0.386934	0.120732	7	0.574861007	0.38849
865	745670	745671	13	11	2	15	8	113	0.452373	0.389033	0.124859	9	0.579895355	0.380109
866	745670	745671	13	13	0	16	11	113	0.494131	0.416635	0.123369	9	0.578675768	0.381525
867	745670	745671	13	13	0	19	12	113	0.476389	0.390525	0.123673	8	0.578441576	0.381073
868	745670	745671	13	13	0	18	11	113	0.44668	0.347471	0.062841	8	0.58486728	0.402958
869	745670	745671	13	10	2	16	10	113	0.479028	0.398369	0.057177	8	0.593599858	0.394996
870	745670	745671	13	12	1	22	12	113	0.436714	0.34507	0.121481	9	0.574715375	0.388224
871	745670	745671	13	11	2	15	8	113	0.45804	0.374697	0.058783	8	0.583548977	0.408435
872	745670	745671	13	13	0	17	9	113	0.448578	0.38682	0.125115	8	0.577286393	0.381261
873	745670	745671	13	12	1	14	9	113	0.449985	0.381543	0.123082	8	0.578171327	0.38213
874	745670	745671	13	13	0	18	10	113	0.462004	0.351833	0.12477	8	0.573423553	0.388263
875	745670	745671	13	12	1	16	9	113	0.416963	0.316546	0.052298	8	0.586025226	0.40455
876	745670	745671	13	12	1	21	10	113	0.449746	0.381345	0.119721	9	0.576652848	0.385526
877	745670	745671	13	13	0	18	9	113	0.482919	0.400804	0.124896	9	0.575553815	0.38503
878	745670	745671	13	13	0	20	10	113	0.473027	0.377209	0.121359	9	0.576975552	0.384135
879	745670	745671	13	12	1	16	10	113	0.466611	0.399437	0.123449	9	0.578091498	0.382248
880	745670	745671	13	10	2	16	10	113	0.477534	0.399656	0.1969	10	0.568011041	0.357099
881	745670	745671	13	12	1	18	12	113	0.475941	0.387772	0.123617	9	0.573278994	0.389953
882	745670	745671	13	13	0	15	9	113	0.428273	0.374669	0.122739	7	0.572784486	0.390693
883	745670	745671	13	12	1	15	10	113	0.457962	0.396762	0.061225	7	0.584116181	0.407423
884	745670	745671	13	12	1	20	10	113	0.468742	0.380285	0.191329	9	0.567182518	0.365578
885	745670	745671	13	11	2	14	6	113	0.43124	0.353971	0.124352	9	0.572231923	0.391601
886	745670	745671	13	11	2	12	9	113	0.491023	0.385315	0.121521	9	0.577320992	0.386044
887	745670	745671	13	13	0	15	9	113	0.467123	0.373688	0.123595	9	0.576343358	0.38504
888	745670	745671	13	11	2	17	11	113	0.47642	0.380359	0.123549	9	0.575015359	0.389719
889	745670	745671	13	13	0	17	9	113	0.453316	0.386752	0.124422	9	0.57672142	0.383535
890	745670	745671	13	10	2	15	8	113	0.464958	0.383701	0.058325	8	0.582829482	0.40686
891	745670	745671	13	13	0	16	11	113	0.427364	0.335358	-0.01027	7	0.598331807	0.416366
892	745670	745671	13	11	2	15	10	113	0.446358	0.402997	0.125192	7	0.572864151	0.390521
893	745670	745671	13	13	0	15	8	113	0.473261	0.398147	0.054366	8	0.587403085	0.402109
894	745670	745671	13	10	3	14	7	113	0.469746	0.397223	0.121764	9	0.576211417	0.387645
895	745670	745671	13	13	0	19	10	113	0.462105	0.377899	0.123117	9	0.578337048	0.38231
896	745670	745671	13	12	1	17	10	113	0.475342	0.383397	0.058506	8	0.586071366	0.403621
897	745670	745671	13	12	1	9	5	113	0.456013	0.37788	0.125094	7	0.571283581	0.39453
898	745670	745671	13	12	1	16	9	113	0.442876	0.335321	-0.01493	7	0.599462315	0.416469
899	745670	745671	13	11	2	10	6	113	0.476004	0.376243	0.061905	8	0.585787701	0.403478
900	745670	745671	13	11	2	13	8	113	0.476516	0.409964	0.121954	9	0.585211076	0.371259
901	745670	745671	13	11	2	15	11	113	0.459743	0.369426	-0.01249	7	0.600601491	0.414947
902	745670	745671	13	13	0	17	9	113	0.468684	0.411711	0.123265	8	0.57518743	0.38667
903	745670	745671	13	11	2	16	8	113	0.443433	0.374772	0.049769	8	0.586844717	0.405912
904	745670	745671	13	11	2	14	9	113	0.473215	0.378752	-0.01184	7	0.607083436	0.406546
905	745670	745671	13	13	0	18	10	113	0.451684	0.390104	-0.01438	7	0.600935417	0.414569
906	745670	745671	13	12	1	14	7	113	0.43602	0.374871	0.136163	8	0.566617291	0.401239
907	745670	745671	13	10	3	15	9	113	0.453048	0.388601	0.123316	9	0.583280075	0.372955
908	745670	745671	13	12	1	18	7	113	0.435183	0.38318	0.121213	7	0.577107007	0.384213
909	745670	745671	13	13	0	12	6	113	0.414518	0.315922	0.120432	9	0.568612014	0.398696
910	745670	745671	13	13	0	14	9	113	0.474097	0.374093	0.05561	7	0.577069719	0.415788
911	745670	745671	13	13	0	20	9	113	0.48351	0.396117	-0.01006	7	0.597929123	0.41709
912	745670	745671	13	13	0	19	11	113	0.439323	0.381722	0.12095	9	0.576966767	0.38478
913	745670	745671	13	13	0	18	12	113	0.436426	0.398388	0.057684	8	0.590428968	0.397914
914	745670	745671	13	12	1	19	10	113	0.446019	0.362239	0.060383	7	0.58661692	0.40492
915	745670	745671	13	12	1	20	12	113	0.468552	0.376989	0.19443	9	0.568343095	0.360139
916	745670	745671	13	11	2	14	7	113	0.439601	0.395478	0.123525	9	0.579521739	0.381551
917	745670	745671	13	11	2	16	10	113	0.431917	0.340229	0.191836	8	0.562555769	0.375702
918	745670	745671	13	12	1	18	10	113	0.470726	0.372146	0.120661	9	0.574575188	0.388299
919	745670	745671	13	12	1	20	11	113	0.476005	0.39947	0.12508	9	0.579938381	0.378335
920	745670	745671	13	12	1	16	9	113	0.452968	0.399073	0.122363	8	0.572098238	0.392692
921	745670	745671	13	13	0	17	9	113	0.478087	0.392334	0.051544	8	0.58744384	0.401764
922	745670	745671	13	13	0	14	9	113	0.457958	0.38929	0.121833	8	0.573068032	0.391609
923	745670	745671	13	13	0	12	9	113	0.40617	0.349521	0.056178	8	0.587836528	0.400805
924	745670	745671	13	13	0	16	8	113	0.433265	0.383082	0.125402	8	0.576221844	0.383969
925	745670	745671	13	12	1	11	4	113	0.45064	0.367101	0.121075	8	0.573580566	0.392361
926	745670	745671	13	13	0	19	12	113	0.50182	0.406767	0.12269	9	0.577237137	0.384151
927	745670	745671	13	12	1	19	10	113	0.459692	0.361221	0.13391	9	0.577937455	0.379637
928	745670	745671	13	11	1	13	8	113	0.421475	0.375066	0.051581	8	0.583012896	0.408784
929	745670	745671	13	12	1	16	10	113	0.448178	0.384102	0.053035	7	0.581339456	0.41095
930	745670	745671	13	12	1	17	9	113	0.458456	0.38628	0.121405	9	0.578847985	0.381561
931	745670	745671	13	13	0	15	10	113	0.487673	0.394349	0.124071	8	0.579450187	0.379224
932	745670	745671	13	12	1	16	9	113	0.457862	0.388757	0.12146	8	0.574883452	0.388439
933	745670	745671	13	12	1	16	9	113	0.437955	0.360753	0.133531	8	0.570545874	0.394001
934	745670	745671	13	13	0	14	9	113	0.454826	0.397469	0.124447	9	0.575944484	0.384324
935	745670	745671	13	11	2	11	8	113	0.447942	0.35767	0.059007	6	0.588516453	0.399052
936	745670	745671	13	13	0	16	8	113	0.466768	0.393899	0.125663	9	0.578004771	0.381441
937	745670	745671	13	12	1	18	9	113	0.484253	0.403966	0.124025	9	0.580349425	0.380143
938	745670	745671	13	11	2	17	11	113	0.432492	0.357306	0.130856	8	0.579043845	0.379108
939	745670	745671	13	11	2	13	5	113	0.445252	0.389417	0.12374	9	0.579531857	0.381445

UNDERLYING DATA MARKED CONFIDENTIAL PURSUANT TO PROTECTIVE ORDER

940	745670	745671	13	13	0	17	9	113	0.465146	0.39573	0.124166	8	0.575587048	0.384942
941	745670	745671	13	13	0	16	8	113	0.456993	0.389705	0.121639	6	0.569369839	0.39777
942	745670	745671	13	12	1	15	10	113	0.426231	0.348708	0.048197	8	0.583627758	0.407951
943	745670	745671	13	13	0	15	8	113	0.467525	0.389288	0.060394	8	0.585275903	0.403984
944	745670	745671	13	13	0	15	9	113	0.494472	0.388235	-0.00748	7	0.594213026	0.420422
945	745670	745671	13	12	1	16	10	113	0.470896	0.380829	0.123336	8	0.575336745	0.386122
946	745670	745671	13	10	3	13	7	113	0.455251	0.383716	0.124395	9	0.580024847	0.37955
947	745670	745671	13	12	1	15	10	113	0.45993	0.395088	0.123191	8	0.578980989	0.378889
948	745670	745671	13	13	0	13	10	113	0.485431	0.379708	0.121487	9	0.577261459	0.384909
949	745670	745671	13	12	1	19	10	113	0.438852	0.364482	0.051455	8	0.584288996	0.405058
950	745670	745671	13	11	2	17	10	113	0.438115	0.367412	0.120307	8	0.574642602	0.39014
951	745670	745671	13	11	2	16	7	113	0.463271	0.348125	0.123378	9	0.579835994	0.3809
952	745670	745671	13	12	1	16	7	113	0.462964	0.391137	0.125269	9	0.577870437	0.381747
953	745670	745671	13	11	2	15	6	113	0.465943	0.403217	0.123687	9	0.579691868	0.381108
954	745670	745671	13	9	3	10	6	113	0.430669	0.370621	-0.01349	7	0.589953437	0.426671
955	745670	745671	13	13	0	18	12	113	0.470474	0.391423	0.122683	9	0.577449139	0.383119
956	745670	745671	13	12	1	14	10	113	0.447857	0.353695	0.121988	8	0.578762959	0.381778
957	745670	745671	13	12	1	12	8	113	0.496269	0.403919	0.058192	8	0.579495445	0.413405
958	745670	745671	13	12	1	14	10	113	0.455333	0.376347	0.061685	7	0.584597118	0.402965
959	745670	745671	13	12	1	13	8	113	0.487439	0.372119	0.062082	8	0.582250979	0.407554
960	745670	745671	13	12	1	16	9	113	0.490496	0.403423	0.122476	8	0.578228028	0.382418
961	745670	745671	13	13	0	22	9	113	0.44387	0.405261	0.123443	8	0.577365804	0.38214
962	745670	745671	13	13	0	19	10	113	0.485731	0.396864	0.121571	9	0.576870138	0.384693
963	745670	745671	13	13	0	14	9	113	0.454226	0.368329	0.059341	8	0.593414481	0.393553
964	745670	745671	13	13	0	19	8	113	0.472386	0.405705	0.052938	8	0.586699997	0.402623
965	745670	745671	13	11	2	18	10	113	0.436325	0.380877	0.124279	9	0.566666494	0.401027
966	745670	745671	13	11	2	15	9	113	0.441372	0.380674	0.051062	8	0.580411436	0.410842
967	745670	745671	13	11	2	15	8	113	0.439746	0.353944	0.054485	8	0.58406488	0.406821
968	745670	745671	13	10	2	17	10	113	0.481399	0.391495	0.05029	8	0.58481001	0.404808
969	745670	745671	13	11	2	20	9	113	0.446742	0.37352	0.126937	8	0.57045024	0.394998
970	745670	745671	13	12	1	16	10	113	0.480611	0.382401	0.122124	8	0.578615737	0.382034
971	745670	745671	13	11	2	17	10	113	0.466746	0.380787	0.123665	9	0.579907441	0.381098
972	745670	745671	13	13	0	13	9	113	0.436973	0.316605	0.051332	8	0.582143756	0.407781
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974	745670	745671	13	13	0	16	10	113	0.481307	0.403601	0.122816	9	0.575815227	0.386652
975	745670	745671	13	13	0	26	14	113	0.44983	0.399	0.127666	8	0.577408096	0.379655
976	745670	745671	13	13	0	19	9	113	0.455694	0.354377	0.123674	8	0.585057297	0.36922
977	745670	745671	13	12	1	16	9	113	0.467209	0.36618	0.060019	8	0.581414143	0.410248
978	745670	745671	13	10	3	16	10	113	0.442185	0.382969	0.062498	8	0.587060531	0.401144
979	745670	745671	13	12	1	14	9	113	0.471834	0.409419	0.121935	9	0.580060353	0.379001
980	745670	745671	13	12	1	20	11	113	0.458949	0.386245	0.194406	9	0.56784993	0.360936
981	745670	745671	13	12	1	13	8	113	0.482233	0.405164	0.122194	9	0.585412757	0.370695
982	745670	745671	13	13	0	17	11	113	0.449342	0.38226	0.126616	7	0.576975816	0.381347
983	745670	745671	13	12	1	20	11	113	0.508014	0.412248	0.193648	9	0.569772232	0.357228
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986	745670	745671	13	12	1	16	10	113	0.431184	0.370035	0.049784	8	0.589110247	0.399801
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988	745670	745671	13	11	2	13	8	113	0.429081	0.350482	-0.0079	6	0.583070332	0.432725
989	745670	745671	13	13	0	17	11	113	0.460198	0.396041	0.125572	8	0.576096144	0.383748
990	745670	745671	13	12	1	17	10	113	0.439534	0.378302	0.047319	8	0.582671708	0.408109
991	745670	745671	13	12	1	15	9	113	0.456686	0.385709	0.051931	8	0.583112092	0.407207
992	745670	745671	13	12	1	16	7	113	0.480008	0.380321	0.060056	8	0.58535975	0.405032
993	745670	745671	13	11	2	21	12	113	0.472653	0.399515	0.058847	8	0.590662005	0.398925
994	745670	745671	13	13	0	18	7	113	0.479847	0.385462	0.061511	7	0.584448647	0.404704
995	745670	745671	13	12	1	20	9	113	0.46465	0.382642	0.125278	9	0.57530622	0.386717
996	745670	745671	13	12	1	15	10	113	0.460196	0.371861	0.120869	8	0.578135861	0.383609
997	745670	745671	13	13	0	15	11	113	0.507935	0.400508	0.121441	9	0.577282123	0.384884
998	745670	745671	13	12	1	21	14	113	0.456133	0.38073	0.121319	9	0.578921182	0.381032
999	745670	745671	13	12	1	18	10	113	0.456268	0.362425	0.128195	8	0.567496349	0.39858
1000	745670	745671	13	13	0	12	9	113	0.464761	0.3909	0.061243	8	0.582543184	0.408063

21 CV 015426

FILED

STATE OF NORTH CAROLINA

COUNTY OF WAKE

2021 NOV 16 P 4:2?

WAKE CO., C.S.C.

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
__ CVS ____

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR.; DANDRIELLE LEWIS; TIMOTHY CHARTIER; TALIA FERNÓS; KATHERINE NEWHALL; JASON PARSLEY; EDNA SCOTT; ROBERTA SCOTT; YVETTE ROBERTS; JEREANN KING JOHNSON; REVEREND REGINALD WELLS; YARBROUGH WILLIAMS, JR.; REVEREND DELORIS L. JERMAN; VIOLA RYALS FIGUEROA; and COSMOS GEORGE,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting; SENATOR WARREN DANIEL, in his official capacity as Co-Chair of the Senate Standing Committee on Redistricting and Elections; SENATOR RALPH E. HISE, JR., in his official capacity as Co-Chair of the Senate Standing Committee on Redistricting and Elections; SENATOR PAUL NEWTON, in his official capacity as Co-Chair of the Senate Standing Committee on Redistricting and Elections; REPRESENTATIVE TIMOTHY K. MOORE, in his official capacity as Speaker of the North Carolina House of Representatives; SENATOR PHILIP E. BERGER, in his official capacity as President Pro Tempore of the North Carolina Senate; THE STATE OF NORTH CAROLINA; THE NORTH CAROLINA STATE BOARD OF ELECTIONS; DAMON CIRCOSTA, in his official capacity as Chairman of the North Carolina State Board of Elections; STELLA ANDERSON, in her official capacity as Secretary of the North Carolina State Board of Elections; JEFF CARMON III, in his official capacity as Member of the North Carolina State Board of Elections; STACY EGGERS IV, in his official capacity as Member of the North Carolina State Board of Elections; TOMMY TUCKER, in his official capacity as Member of the North Carolina State Board of Elections; and KAREN BRINSON BELL, in her official capacity as Executive Director of the North Carolina State Board of Elections,

Defendants.

AFFIDAVIT OF DR. MOON DUCHIN

NCLCV v. Hall
21 CVS 15426
LDTX157

Exhibit #
Duchin 1
12/30/21 - SL
exhibitsticker.com

I, Dr. Moon, Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.

2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.

4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.

5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.

6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.

7. A current copy of my full CV is attached to this report.

8. I am compensated at the rate of $400 per hour.

Analysis of 2021 enacted redistricting plans in North Carolina

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

November 16, 2021

1 Introduction

On November 4, 2021, the North Carolina General Assembly enacted three districting plans: maps of 14 U.S. Congressional districts, 50 state Senate districts, and 120 state House districts. This affidavit contains a brief summary of my evaluation of the properties of these plans. My focus will be on the egregious partisan imbalance in the enacted plans, following a brief review of the traditional districting principles.

Because redistricting inevitably involves complex interactions of rules, which can create intricate tradeoffs, it will be useful to employ a direct comparison to an alternative set of plans. These demonstrative plans illustrate that it is possible to *simultaneously maintain or improve* metrics for all of the most important redistricting principles that are operative in North Carolina's constitution and state and federal law. Crucially, this shows that nothing about the state's political geography compels us to draw a plan with a massive and entrenched partisan skew.

To this end, I will be comparing the following plans: the enacted plans SL-174, SL-173, and SL-175 and a corresponding set of alternative plans labeled NCLCV-Cong, NCLCV-Sen, and NCLCV-House (proposed by plaintiffs who include the North Carolina League of Conservation Voters).

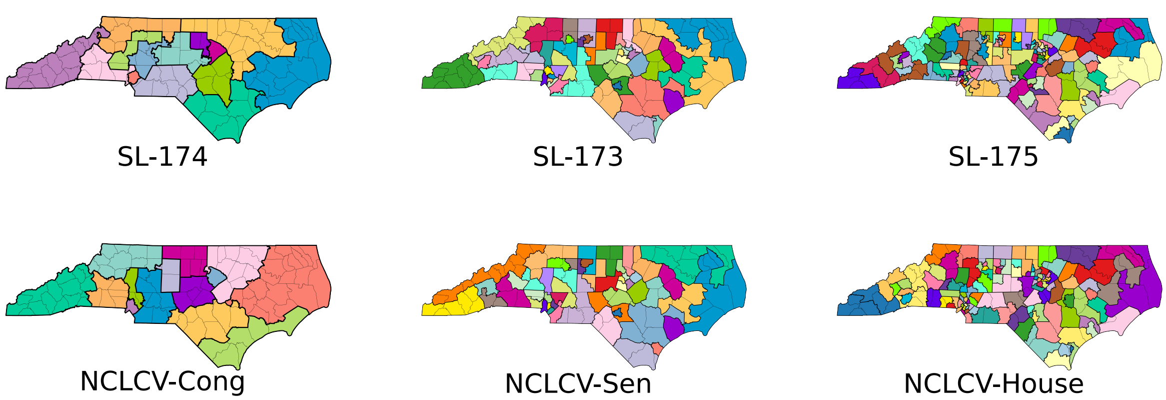


Figure 1: The six plans under discussion in this affidavit.

2 Traditional districting principles

Principles that are relevant to North Carolina redistricting include the following.

- **Population balance.** The standard interpretation of *One Person, One Vote* for Congressional districts is that districts should be fine-tuned so that their total Census population deviates by no more than one person from any district to any other.

  There is more latitude with legislative districts; they typically vary top-to-bottom by no more than 10% of ideal district size. In North Carolina, the Whole County Provisions make it very explicit that 5% deviation must be tolerated if it means preserving more counties intact.

  All six plans have acceptable population balance.

Population deviation

	Max Positive Deviation	District	Max Negative Deviation	District
SL-174	0	(eight districts)	−1	(six districts)
NCLCV-Cong	0	(eight districts)	−1	(six districts)
SL-173	10,355 (4.960%)	5	−10,434 (4.997%)	13,18
NCLCV-Sen	10,355 (4.960%)	5	−10,427 (4.994%)	15
SL-175	4250 (4.885%)	18	−4189 (4.815%)	112
NCLCV-House	4341 (4.990%)	82	−4323 (4.969%)	87

Table 1: Deviations are calculated with respect to the rounded ideal district populations of 745,671 for Congress, 208,788 for Senate, and 86,995 for House.

- **Minority electoral opportunity.** Minority groups' opportunity to elect candidates of choice is protected by both state and federal law. A detailed assessment of opportunity must hinge not on the demographics of the districts but on electoral history and an assessment of polarization patterns. That is not the focus of the current affidavit. Instead we make the brief note that it is important to avoid the conflation of *majority-minority districts* with *effective districts* for a minority group. An involved analysis of voting patterns—necessarily incorporating both primary and general elections to ensure that candidates of choice can be successfully nominated and elected—will frequently reveal that districts can be effective at demographic levels well below 50% of voting-age population or citizen voting-age population (VAP and CVAP, respectively). For instance, in [3], my co-authors and I drew an illustrative plan for Texas congressional districting in which some parts of the state had districts that were shown to reliably elect Black candidates of choice with BCVAP as low as 28.6%; by contrast, there are other parts of Texas where a 40% BCVAP district is less consistently effective. In a Louisiana case study, we found somewhat different patterns of human and political geography, producing numerous examples of Congressional-sized districts with 55% BCVAP in some parts of the state that are nonetheless marginal in terms of opportunity for Black voters to elect candidates of choice.

  In North Carolina, taking the crossover voting patterns of White, Latino, and Asian voters into account, I note that a district with BCVAP in the low to mid 30s can often be effective for Black voters—but there is no demographic shortcut to a full examination of primary and general election history.

- **Contiguity.** All six plans are contiguous; for each district, it is possible to transit from any part of the district to any other part through a sequence of census blocks that share boundary segments of positive length. As is traditional in North Carolina, contiguity through water is accepted.

- **Compactness.** The two compactness metrics most commonly appearing in litigation are the *Polsby-Popper score* and the *Reock score*. Polsby-Popper is the name given in redistricting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula $4\pi A/P^2$. Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1. Reock is a different measurement of how much a shape differs from a circle: it is computed as the ratio of a region's area to that of its circumcircle, defined as the smallest circle in which the region can be circumscribed. From this definition, it is clear that it too is optimized at a value of 1, which is achieved only by circles.

  These scores depend on the contours of a district and have been criticized as being too dependent on map projections or on cartographic resolution [1, 2]. Recently, some mathematicians have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built. The most commonly cited discrete score for districts is the *cut edges score*, which counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations. This score improves on the contour-based scores by better controlling for factors like coastline and other natural boundaries, and by focusing on the units actually available to redistricters rather than treating districts like free-form Rorschach blots.

  The alternative plans are significantly more compact than the enacted plans in all three compactness metrics.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
SL-174	5194	0.303	0.381
NCLCV-Cong	4124	0.383	0.444
SL-173	9702	0.342	0.402
NCLCV-Sen	9249	0.369	0.423
SL-175	16,182	0.351	0.419
NCLCV-House	13,963	0.414	0.456

Table 2: Comparing compactness scores via one discrete and two contour-based metrics.

- **Respect for political subdivisions.** For legislative redistricting, North Carolina has one of the strongest requirements for county consideration of any state in the nation. In my understanding, courts have interpreted the Whole County Provisions as follows.

  - First, if any county is divisible into a whole number of districts that will be within ±5% of ideal population, then it must be subdivided accordingly without districts crossing into other counties.
  - Next, seek any contiguous grouping of two counties that is similarly divisible into a whole number of districts.
  - Repeat for groupings of three, and so on, until all counties are accounted for.

  A complete set of solutions is described in detail in the white paper of Mattingly et al.—though with the important caveat that the work "does not reflect... compliance with the Voting Rights Act" [4]. Absent a VRA conflict, the 2020 Decennial Census population data dictates that the North Carolina Senate plan must be decomposed into ten single-district fixed clusters and seven multi-district fixed clusters (comprising 2, 2, 3, 3, 4, 6, and 6 districts, respectively). It has four more areas in which there is a choice of groupings. In all, there are sixteen different possible clusterings for Senate, each comprising 26 county clusters. The House likewise has 11 single-district fixed clusters and 22 multi-district fixed clusters (with two to thirteen districts per cluster), together with three more areas with a choice of groupings. In all, the House has only eight acceptable clusterings, each comprising 40 county clusters. Again, it is important to note that VRA compliance may present a compelling reason to select some clusterings and reject others.

Once clusters have been formed, there are more rules about respecting county lines within clusters. The legal language is again explicit: "[T]he resulting interior county lines created by any such groupings may be crossed or traversed in the creation of districts within said multi-county grouping but only to the extent necessary" to meet the ±5% population standard for districts. To address this, I have counted the *county traversals* in each plan, i.e., the number of times a district crosses between adjacent counties within a grouping.

Table 3 reflects the county integrity metric that is most relevant at each level: the enacted congressional plan splits 11 counties into 25 pieces while the alternative plan splits 13, but splits no county three ways. (The enacted plans unnecessarily split three counties into three pieces.) In the legislative plans, the law specifies traversals as the fundamental integrity statistic.

The alternative plans are comparable to the enacted plans, or sometimes far superior, in each of these key metrics regarding preservation of political boundaries.

County and municipality preservation

	# county pieces
SL-174	25
NCLCV-Cong	26

	# traversals
SL-173	97
NCLCV-Sen	89
SL-175	69
NCLCV-House	66

	# municipal pieces
SL-174	90
NCLCV-Cong	58
SL-173	152
NCLCV-Sen	125
SL-175	292
NCLCV-House	201

Table 3: Comparing the plans’ conformance to political boundaries.

I will briefly mention several additional redistricting principles.

- **Communities of interest.** In North Carolina, there was no sustained effort by the state or by community groups to formally collect community of interest (COI) maps, to my knowledge. Without this, it is difficult to produce a suitable metric.
- **Cores of prior districts.** In some states, there is statutory guidance to seek districting plans that preserve the cores of prior districts. In North Carolina, this is not a factor in the constitution, in statute, or in case law. In addition, attention to core preservation would be prohibitively difficult in the Senate and House because of the primacy of the Whole County Provisions, which forces major changes to the districts simply as a consequence of fresh population numbers.

- **Incumbent pairing.** In 2017, the North Carolina legislative redistricting committee listed "incumbency protection" as a goal in their itemization of principles. In 2021, this was softened to the statement that "Member residence may be considered" in the drawing of districts. I have counted the districts in each plan that contain more than one incumbent address; these are sometimes colorfully called "double-bunked" districts. For this statistic, it is not entirely clear whether a high or low number is preferable. When a plan remediates a gerrymandered predecessor, we should not be surprised if it ends up pairing numerous incumbents.

Double-bunking

	# districts pairing incumbents
SL-174	3
NCLCV-Cong	1
SL-173	6
NCLCV-Sen	9
SL-175	7
NCLCV-House	15

Table 4: For Congress and Senate, the enacted and alternative plans are comparable; at the House level, the alternative plan has more double-bunking. *Note: These numbers were calculated using the most accurate incumbent addresses that have been provided to me.*

3 Partisan fairness

3.1 Abstract partisan fairness

There are many notions of partisan fairness that can be found in the scholarly literature and in redistricting practitioner guides and software. Most of them are numerical, in the sense that they address *how a certain share of the vote should be translated to a share of the seats* in a state legislature or Congressional delegation.

The numerical notions of partisan fairness all tend to agree on one central point: an electoral climate with a 50-50 split in partisan preference should produce a roughly 50-50 representational split. North Carolina voting has displayed a partisan split staying consistently close to even between the two major parties over the last ten years, but the plans released by the General Assembly after the 2010 census were very far from realizing the ideal of converting even voting to even representation. This time, with a 14th seat added to North Carolina's apportionment, an exactly even seat outcome is possible. But the new enacted plans, like the plans from ten years ago, are not conducive to even representation.

3.2 Geography and fairness

However, some scholars have argued that this ideal (that even vote preferences should translate to even representation) ignores the crucial *political geography*—the location of votes for each party, and not just the aggregate preferences, has a major impact on redistricting outcomes. In [5], my co-authors and I gave a vivid demonstration of the impacts of political geography in Massachusetts: we showed that for a ten-year span of observed voting patterns, even though Republicans tended to get over one-third of the statewide vote, it was impossible to draw a single Congressional district with a Republican majority. That is, the geography of Massachusetts Republicans locked them out of Congressional representation. It is therefore not reasonable to charge the Massachusetts legislature with gerrymandering for having produced maps which yielded all-Democratic delegations; they could not have done otherwise.

In North Carolina, this is not the case. The alternative plans demonstrate that it is possible to produce maps that give the two major parties a roughly equal opportunity to elect their candidates. These plans are just examples among many thousands of plausible maps that convert voter preferences to far more even representation by party. In Congressional redistricting, the geography is easily conducive to a seat share squarely in line with the vote share. In Senate and House plans, even following the strict detail of the Whole County Provisions, there are likewise many alternatives giving a seat share for each party that falls, in aggregate, within a few percentage points of the vote share across a large set of elections.

The clear conclusion is that the political geography of North Carolina today does not obstruct the selection of a map that treats the parties equally and fairly.

3.3 Translating votes to seats

The enacted plans behave as though they are built to resiliently safeguard electoral advantage for Republican candidates. We can examine this effect without invoking assumptions like "uniform partisan swing" that impose counterfactual voting conditions; instead, we will use the rich observed dataset of 52 statewide party-ID general elections in North Carolina in the last ten years. 29 of these are elections for Council of State (ten offices elected three times, with the Attorney General race uncontested in 2012), three presidential races, three for U.S. Senate, and 17 judicial races since mid-decade, when those became partisan contests. See Table 6 for more detail on the election dataset.

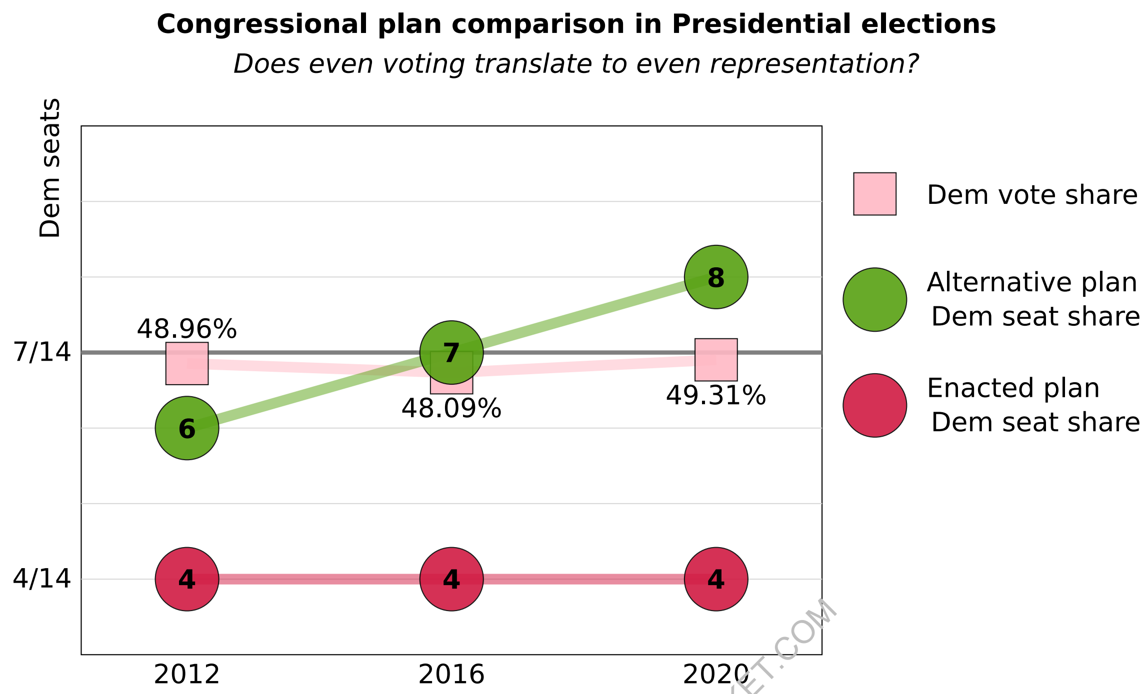
I will sometimes focus on the smaller set of better-known "up-ballot" races: in order, the first five to appear on the ballot are the contests for President, U.S. Senator, Governor, Lieutenant Governor, and Attorney General. Together these occurred 14 times in the last Census cycle.

	Up-ballot generals (14)		**All generals (52)**	
	D vote share	D seat share	D vote share	D seat share
SL-174	.4883	.2908	.4911	.3118
NCLCV-Cong		.4796		.4931
SL-173	.4883	.3957	.4911	.4065
NCLCV-Sen		.4557		.4592
SL-175	.4883	.3994	.4911	.4080
NCLCV-House		.4649		.4684

Table 5: Comparing overall fidelity of representation to the voting preferences of the electorate. Vote shares are reported with respect to the major-party vote total.

To understand how the enacted plans create major shortfalls for Democratic representation, we will overlay the plans with voting patterns from individual elections in the past Census cycle. As we will see, the enacted Congressional plan (SL-174) shows a remarkable lack of responsiveness, giving 10–4 partisan outcomes across a wide range of recent electoral conditions, meaning that 10 Republicans and only 4 Democrats would represent North Carolina in Congress. The alternative plan (NCLCV-Cong) is far more faithful to the vote share, far more responsive, and tends to award more seats to the party with more votes.

The top of Figure 2 shows this dynamic in the three Presidential contests in the last Census cycle, with a Democratic vote share (pink box) between 48% and 50% of the major-party total each time. For a contest that is so evenly divided, we would expect a fair map to have 6, 7, or 8 out of 14 districts favoring each party. The alternative Congressional map NCLCV-Cong does just that, while the enacted plan SL-174 has just 4 out of 14 Democratic-majority districts each time (green and maroon circles). The alternative plan is far more successful at reflecting the even split of voter preferences. Below the initial explainer, simplified versions of the same type of graphic are presented for all five up-ballot races. Figure 3 compares legislative maps in the same fashion. Next, Figure 4 returns to the full 52-election dataset to give the big picture of entrenched partisan advantage in the enacted plans.



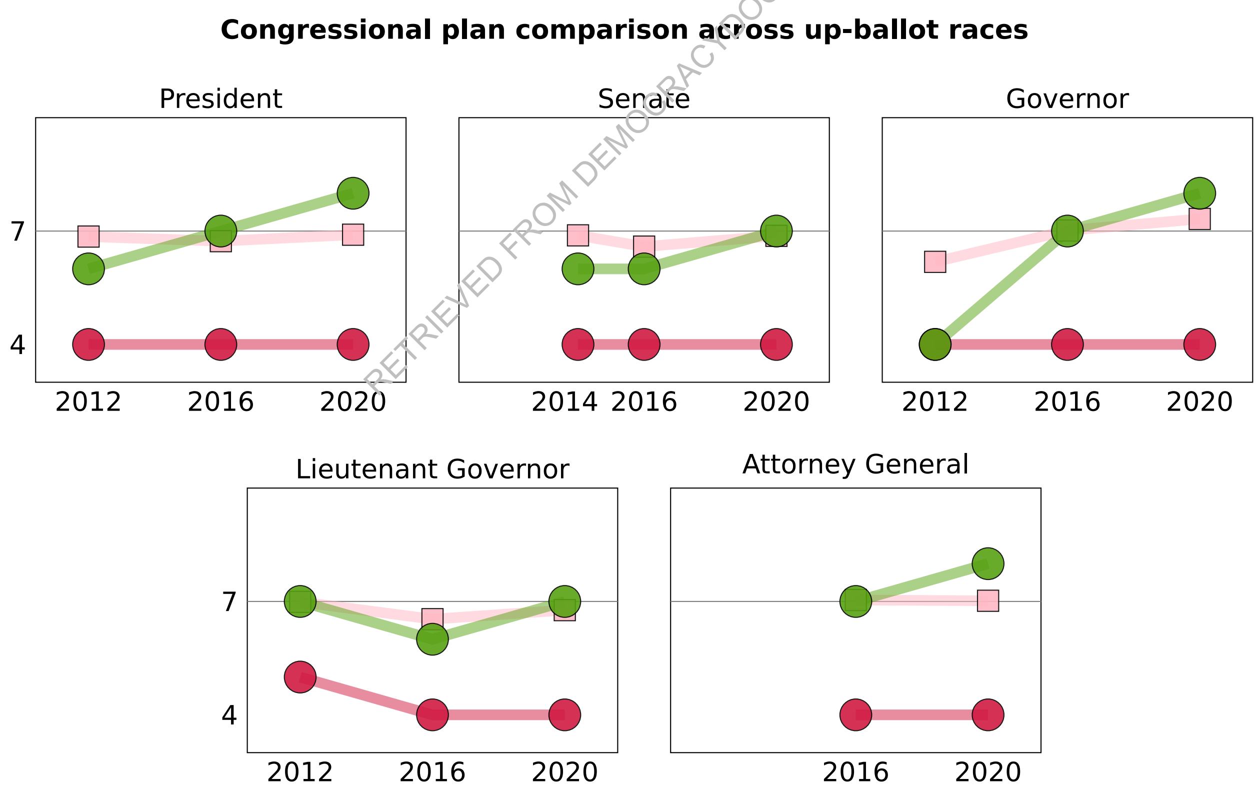
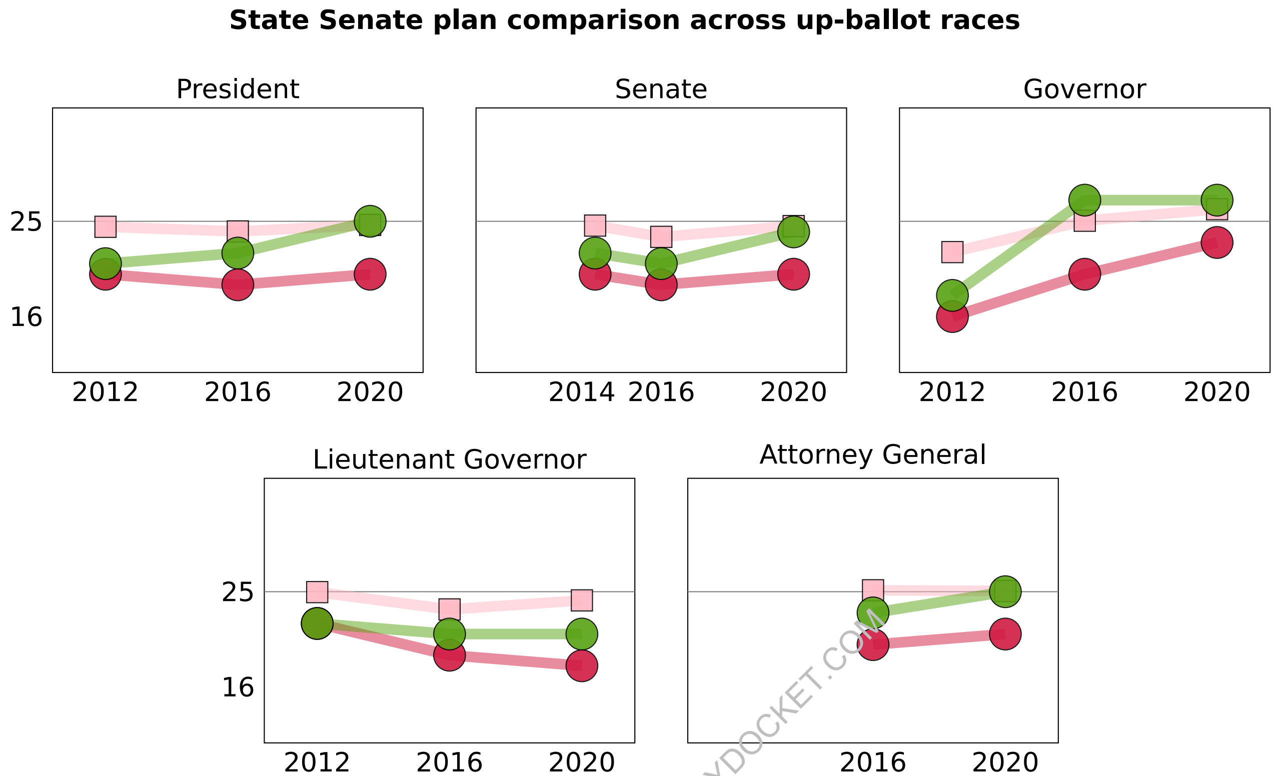


Figure 2: For up-ballot general election contests across the previous Census cycle, we can compare the seat share under the enacted Congressional plan SL-174 (maroon) and the seat share under the alternative Congressional plan NCLCV-Cong (green) to the vote share (pink) for Democratic candidates. At top is a detailed look at the presidential contests; this is repeated below, alongside the other four up-ballot offices. The 50% line is marked each time.



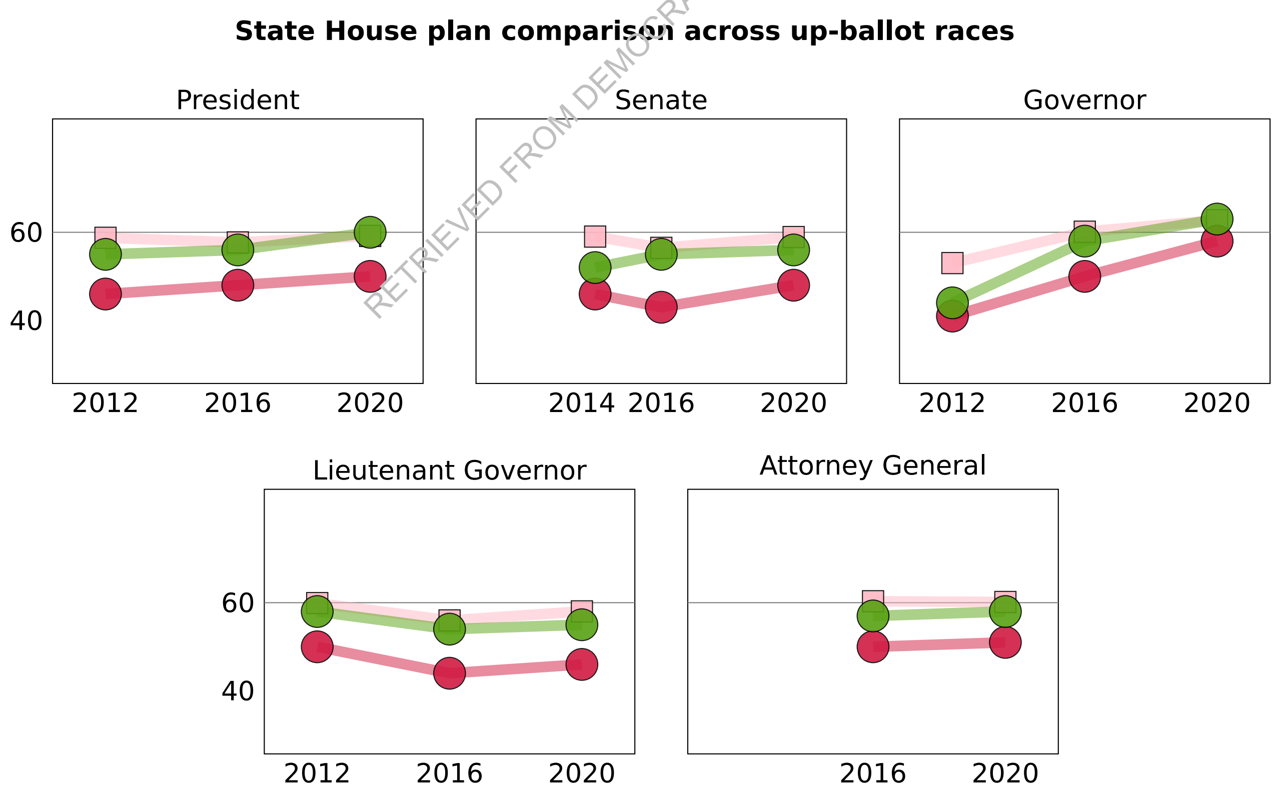
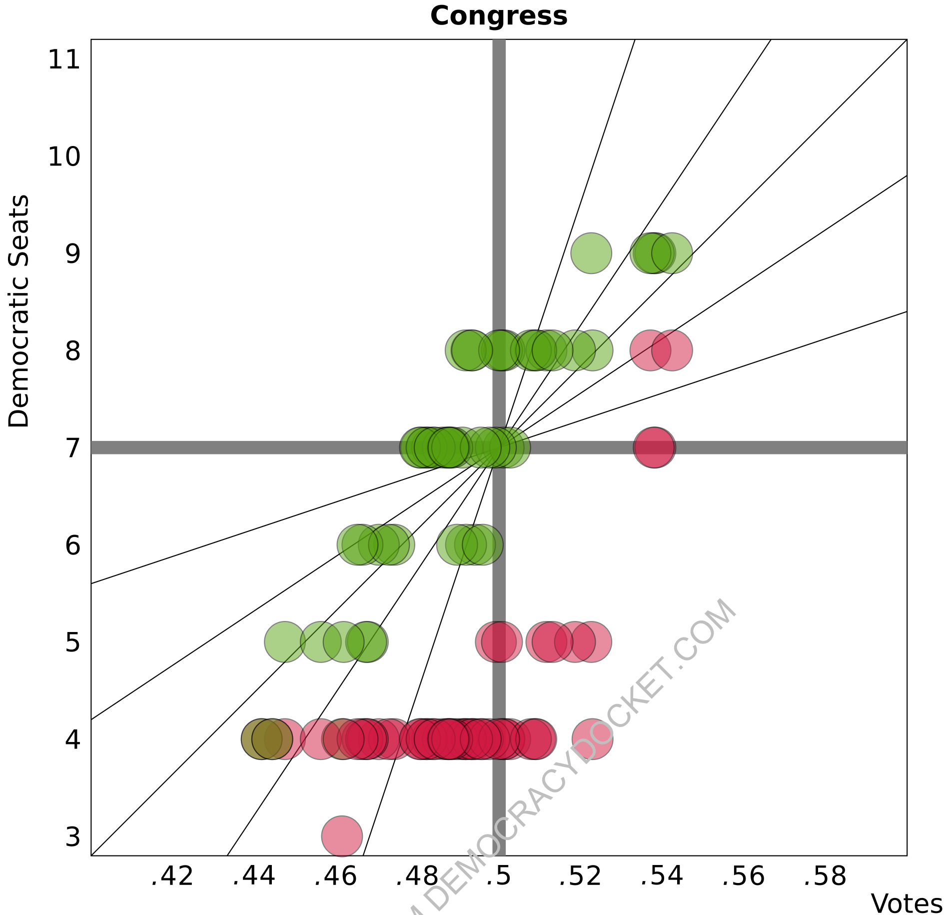
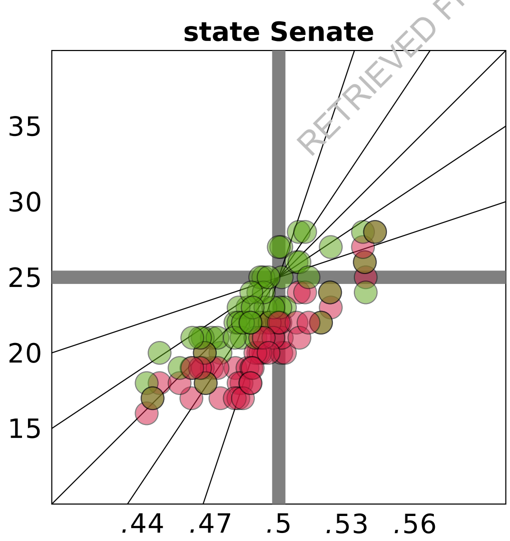


Figure 3: Legislative plans tested against voting patterns from up-ballot elections. The enacted plans SL-173 and SL-175 are shown in maroon. The alternative plans NCLCV-Sen and NCLCV-House, in green, have seat shares tracking much closer to the nearly even voting preferences.





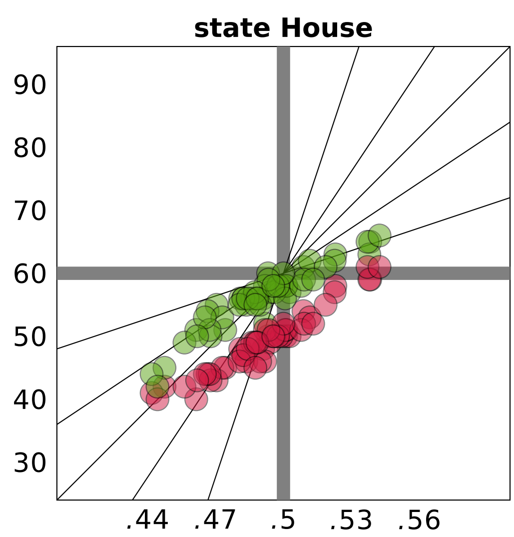


Figure 4: On a seats-vs.-votes plot, the election results for the six maps are shown for 52 general election contests in the last decade; each colored dot is plotted as the coordinate pair (vote share, seat share). The diagonals show various lines of *responsiveness* that pivot around the central point of fairness: half of the votes securing half of the seats. The Congressional comparison is at top, followed by Senate and House. The enacted plans are shown in maroon and the alternative plans in green.

3.4 Swing districts and competitive contests

Another way to understand the electoral properties of districting plans is to investigate how many districts always give the same partisan result over a suite of observed electoral conditions, and how many districts can "swing" between the parties. Figure 5 compares the six plans across the up-ballot elections. The enacted plans lock in large numbers of always-Republican seats. In the Senate and House, nearly half the seats are locked down for Republicans. In the Congressional plan, it's well *over* half. This provides another view from which the NCLCV plans provide attractive alternatives.

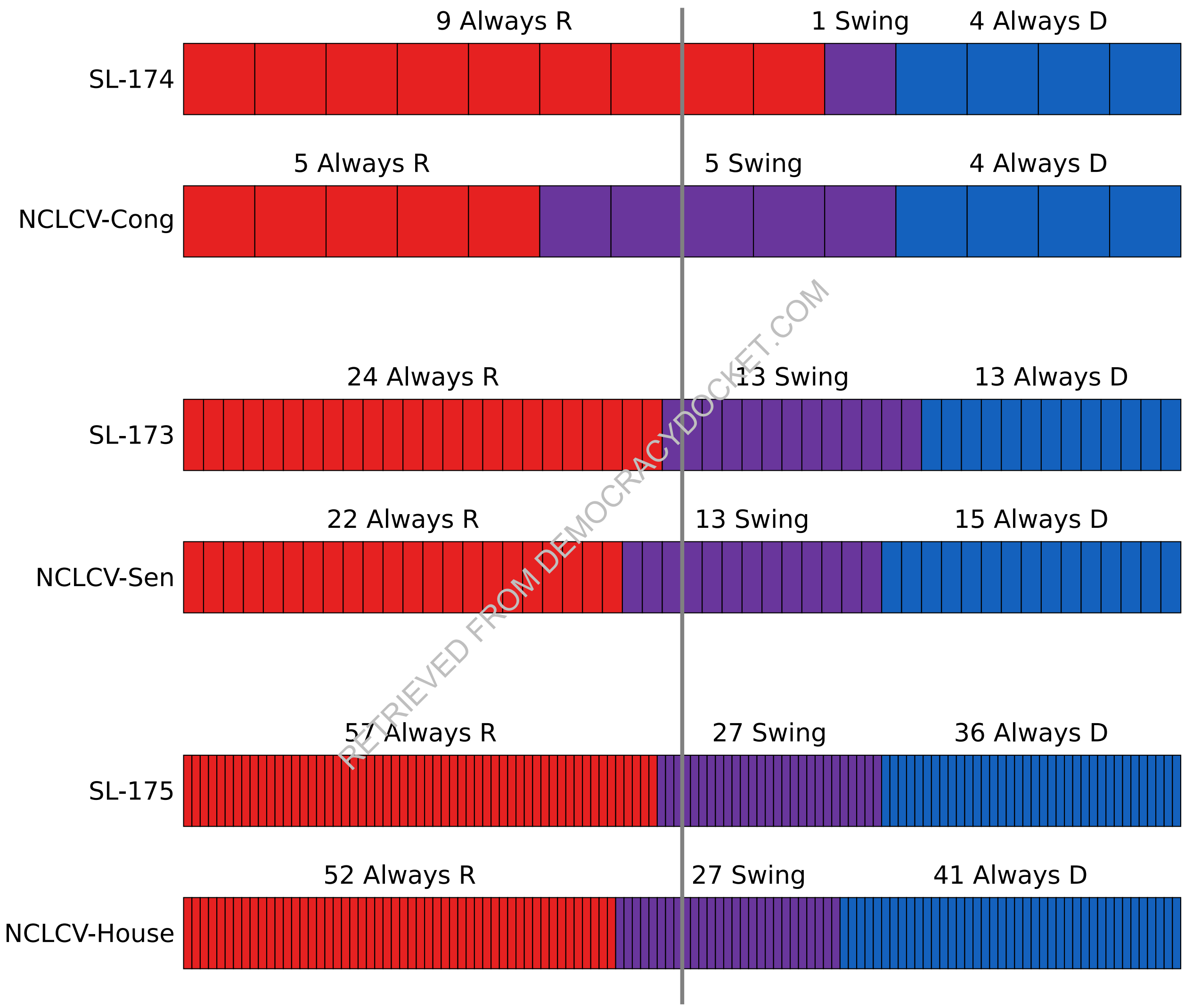


Figure 5: These visuals show the breakdown of seats that always have a Republican winner, always have a Democratic winner, or are sometimes led by each party across the 14 up-ballot elections over the previous Census cycle. The 50-50 split is marked.

One more measure of partisan fairness, frequently referenced in the public discourse, is the tendency of a districting plan to promote close or competitive contests. We close with a comparison of the enacted and alternative plans that displays the number of times across the full dataset of 52 elections that a contest had a partisan margin of closer than 10 points, 6 points, or 2 points, respectively. This can occur up to $14 \cdot 52 = 728$ times in Congressional maps, $50 \cdot 52 = 2600$ times in state Senate maps, and $120 \cdot 52 = 6240$ times in state House maps. The figures below show horizontal rules at every 10% interval of the total number of possible competitive contests; we can see, for instance, that the alternative Congressional plan has contests within a 10-point margin more than 40% of the time.

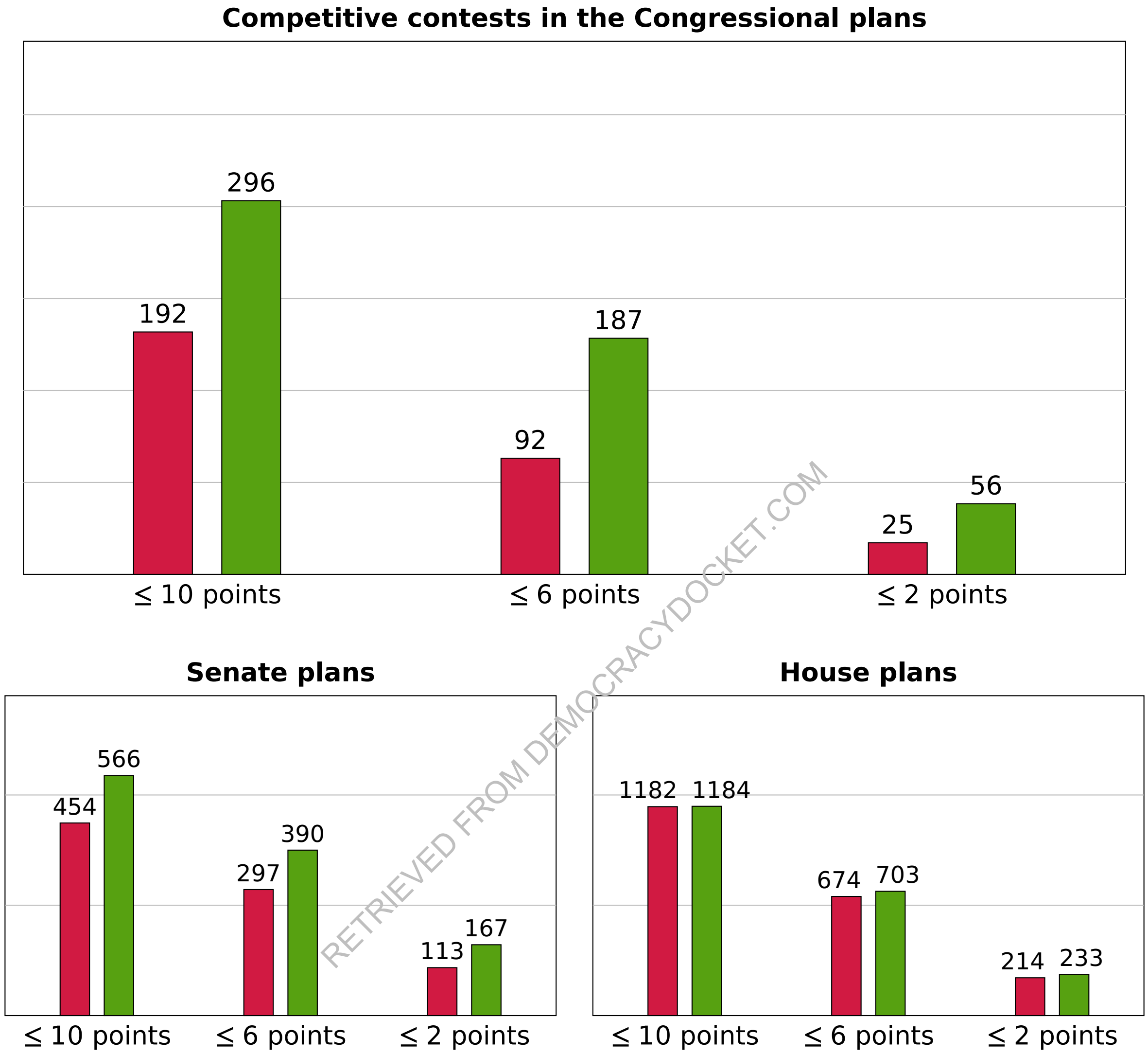


Figure 6: These bar graphs show the number of competitive contests for the enacted plans (maroon) and the alternative plans (green). In each plot, we consider increasingly restrictive definitions of "competitive" from left to right, counting districts in which the major-party vote split is closer than 45-55, 47-53, and 49-51, respectively.

4 Conclusion

North Carolina is a very "purple" state. In 38 out of the 52 contests in our dataset, the statewide partisan outcome is within a 6-point margin: 47-53 or closer. We can make a striking observation by laying our six plans over the vote patterns.

	D Vote Share	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
GOV12	0.4418	4	4	16	18	41	44
AGC16	0.4444	4	4	17	17	40	42
LAC16	0.4475	4	5	18	20	42	45
JHU16	0.4563	4	5	18	19	42	49
AGC20	0.4615	3	4	17	19	40	51
JZA16	0.4619	4	5	19	21	43	50
JDI16	0.4653	4	6	19	21	44	53
LTG16	0.4665	4	6	19	21	44	54
LAC12	0.4674	4	5	20	20	44	51
AGC12	0.4678	4	5	18	18	43	50
SEN16	0.4705	4	6	19	21	43	55
TRS16	0.4730	4	6	19	21	45	53
TRS20	0.4743	4	6	17	20	45	51
JA620	0.4806	4	7	17	21	46	55
PRS16	0.4809	4	7	19	22	48	56
JA420	0.4822	4	7	17	22	47	56
INC20	0.4823	4	7	18	23	47	56
LTG20	0.4836	4	7	18	21	46	55
JA720	0.4842	4	7	17	22	48	56
SUP20	0.4862	4	7	19	23	49	56
JA520	0.4874	4	7	18	22	49	57
JA218	0.4876	4	7	18	22	45	55
JS420	0.4879	4	7	19	24	49	56
J1320	0.4885	4	7	19	23	49	56
PRS12	0.4897	4	6	20	21	46	55
SEN20	0.4910	4	7	20	24	48	56
LAC20	0.4918	4	8	21	25	51	58
SEN14	0.4919	4	6	20	22	46	52
PRS20	0.4932	4	8	20	25	50	60
JS220	0.4934	4	8	21	24	51	59
SUP16	0.4941	4	6	22	23	49	57
JS118	0.4955	4	7	20	25	50	58
INC16	0.4960	4	6	22	22	50	57
JST16	0.4976	4	7	21	23	50	58
LTG12	0.4992	5	7	22	22	50	58
JS120	0.5000	4	8	22	27	52	60
AUD16	0.5007	5	8	22	23	51	56
GOV16	0.5011	4	7	20	27	50	58
ATG20	0.5013	4	8	21	25	51	58
ATG16	0.5027	4	7	20	23	50	57
JA118	0.5078	4	8	22	26	51	58
AUD20	0.5088	4	8	24	28	54	61
JA318	0.5091	4	8	21	26	52	59
SOS20	0.5116	5	8	24	28	53	62
JGE16	0.5131	5	8	22	25	52	59
INC12	0.5186	5	8	22	22	55	61
SOS16	0.5226	5	9	24	24	57	62
GOV20	0.5229	4	8	23	27	58	63
AUD12	0.5371	8	9	27	28	61	65
SOS12	0.5379	7	9	26	26	59	63
TRS12	0.5383	7	9	25	24	59	65
SUP12	0.5424	8	9	28	28	61	66

Table 6: 52 general elections, sorted from lowest to highest Democratic share. Election codes have a three-character prefix and a two-digit suffix designating the office and the election year, respectively. AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; PRS = President; SEN = Senator; SOS = Secretary of State; SUP = Superintendent of Schools; TRS -=Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), those beginning with JS* refer to elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals.

The three enacted plans combine with those 38 relatively even vote patterns to produce 114 outcomes. Every single pairing of an enacted plan with a close statewide contest—a complete sweep of 114 opportunities—gives an *outright Republican majority* of seats. All three enacted plans will lock in an extreme, resilient, and unnecessary advantage for one party.

By every measure considered above that corresponds to a clear legal or good-government redistricting goal or value, the alternative plans meet or exceed the performance of the enacted plans. It is therefore demonstrated to be possible, without any cost to the redistricting principles in play, to select maps that are far fairer to the voters of North Carolina.

References

[1] Assaf Bar-Natan, Lorenzo Najt, and Zachary Schutzmann, *The gerrymandering jumble: map projections permute districts' compactness scores.* Cartography and Geographic Information Science, Volume 47, Issue 4, 2020, 321–335.

[2] Richard Barnes and Justin Solomon, *Gerrymandering and Compactness: Implementation Flexibility and Abuse.* Political Analysis, Volume 29, Issue 4, October 2021, 448–466.

[3] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, *Computational redistricting and the Voting Rights Act.* Election Law Journal.
Available at https://www.liebertpub.com/doi/epdf/10.1089/elj.2020.0704

[4] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, *NC General Assembly County Clusterings from the 2020 Census.*
https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf

[5] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, *Locating the Representational Baseline: Republicans in Massachusetts.* Election Law Journal, Volume 18, Number 4, 2019, 388–401.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 16th day of November, 2021.

Moon Duchin

Moon Duchin

Sworn and subscribed before me

this the 16 of November, 2021.

Notary Public

Name:

GREGORY J. AGNEW
Notary Public
COMMONWEALTH OF MASSACHUSETTS
My Commission Expires
June 28, 2024

My Commission Expires:

Moon Duchin

moon.duchin@tufts.edu - mduchin.math.tufts.edu
Mathematics · STS · Tisch College of Civic Life | Tufts University

Education

University of Chicago MS 1999, PhD 2005
Mathematics
Advisor: Alex Eskin Dissertation: *Geodesics track random walks in Teichmüller space*

Harvard University BA 1998
Mathematics and Women's Studies

Appointments

Tufts University
Professor of Mathematics 2021—
Assistant Professor, Associate Professor 2011–2021

Director | Program in Science, Technology, & Society 2015–2021
(on leave 2018–2019)

Principal Investigator | MGGG Redistricting Lab 2017—

Senior Fellow | Tisch College of Civic Life 2017—

University of Michigan
Assistant Professor (postdoctoral) 2008–2011

University of California, Davis
NSF VIGRE Postdoctoral Fellow 2005–2008

Research Interests

Data science for civil rights, computation and governance, elections, geometry and redistricting.
Science, technology, and society, science policy, technology and law.
Random walks and Markov chains, random groups, random constructions in geometry.
Large-scale geometry, metric geometry, isoperimetric inequalities.
Geometric group theory, growth of groups, nilpotent groups, dynamics of group actions.
Geometric topology, hyperbolicity, Teichmüller theory.

Awards & Distinctions

Research Professor - MSRI Program in Analysis and Geometry of Random Spaces Spring 2022
Guggenheim Fellow 2018
Radcliffe Fellow - Evelyn Green Davis Fellowship 2018–2019
Fellow of the American Mathematical Society elected 2017
NSF C-ACCEL (PI) - Harnessing the Data Revolution: Network science of Census data 2019–2020
NSF grants (PI) - CAREER grant and three standard Topology grants 2009–2022
Professor of the Year, Tufts Math Society 2012–2013
AAUW Dissertation Fellowship 2004–2005
NSF Graduate Fellowship 1998–2002
Lawrence and Josephine Graves Prize for Excellence in Teaching (U Chicago) 2002
Robert Fletcher Rogers Prize (Harvard Mathematics) 1995–1996

Mathematics Publications & Preprints

The (homological) persistence of gerrymandering
Foundations of Data Science, online first. (with Thomas Needham and Thomas Weighill)

You can hear the shape of a billiard table: Symbolic dynamics and rigidity for flat surfaces
Commentarii Mathematici Helvetici, to appear. arXiv:1804.05690
(with Viveka Erlandsson, Christopher Leininger, and Chandrika Sadanand)

Conjugation curvature for Cayley graphs
Journal of Topology and Analysis, online first. (with Assaf Bar-Natan and Robert Kropholler)

A reversible recombination chain for graph partitions
Preprint. (with Sarah Cannon, Dana Randall, and Parker Rule)

Recombination: A family of Markov chains for redistricting
Harvard Data Science Review. Issue 3.1, Winter 2021. online. (with Daryl DeFord and Justin Solomon)

Census TopDown: The impact of differential privacy on redistricting
2nd Symposium on Foundations of Responsible Computing (FORC 2021), 5:1–5:22. online.
(with Aloni Cohen, JN Matthews, and Bhushan Suwal)

Stars at infinity in Teichmüller space
Geometriae Dedicata, Volume 213, 531–545 (2021). (with Nate Fisher) arXiv:2004.04321

Random walks and redistricting: New applications of Markov chain Monte Carlo
(with Daryl DeFord) For edited volume, Political Geometry. Under contract with Birkhäuser.

Mathematics of nested districts: The case of Alaska
Statistics and Public Policy. Vol 7, No 1 (2020), 39–51. (w/ Sophia Caldera, Daryl DeFord, Sam Gutekunst, & Cara Nix)

A computational approach to measuring vote elasticity and competitiveness
Statistics and Public Policy. Vol 7, No 1 (2020), 69–86. (with Daryl DeFord and Justin Solomon)

The Heisenberg group is pan-rational
Advances in Mathematics **346** (2019), 219–263. (with Michael Shapiro)

Random nilpotent groups I
IMRN, Vol 2018, Issue 7 (2018), 1921–1953. (with Matthew Cordes, Yen Duong, Meng-Che Ho, and Ayla Sánchez)

Hyperbolic groups
chapter in *Office Hours with a Geometric Group Theorist*, eds. M.Clay,D.Margalit, Princeton U Press (2017), 177–203.

Counting in groups: Fine asymptotic geometry
Notices of the American Mathematical Society **63**, No. 8 (2016), 871–874.

A sharper threshold for random groups at density one-half
Groups, Geometry, and Dynamics **10**, No. 3 (2016), 985–1005.
(with Katarzyna Jankiewicz, Shelby Kilmer, Samuel Lelièvre, John M. Mackay, and Ayla Sánchez)

Equations in nilpotent groups
Proceedings of the American Mathematical Society **143** (2015), 4723–4731. (with Hao Liang and Michael Shapiro)

Statistical hyperbolicity in Teichmüller space
Geometric and Functional Analysis, Volume 24, Issue 3 (2014), 748–795. (with Howard Masur and Spencer Dowdall)

Fine asymptotic geometry of the Heisenberg group
Indiana University Mathematics Journal 63 No. 3 (2014), 885–916. (with Christopher Mooney)

Pushing fillings in right-angled Artin groups
Journal of the LMS, Vol 87, Issue 3 (2013), 663–688. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

Spheres in the curve complex
In the Tradition of Ahlfors and Bers VI, Contemp. Math. **590** (2013), 1–8. (with Howard Masur and Spencer Dowdall)

The sprawl conjecture for convex bodies
Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122. (with Samuel Lelièvre and Christopher Mooney)

Filling loops at infinity in the mapping class group
Michigan Math. J., Vol 61, Issue 4 (2012), 867–874. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

The geometry of spheres in free abelian groups
Geometriae Dedicata, Volume 161, Issue 1 (2012), 169–187. (with Samuel Lelièvre and Christopher Mooney)

Statistical hyperbolicity in groups
Algebraic and Geometric Topology **12** (2012) 1–18. (with Samuel Lelièvre and Christopher Mooney)

Length spectra and degeneration of flat metrics
Inventiones Mathematicae, Volume 182, Issue 2 (2010), 231–277. (with Christopher Leininger and Kasra Rafi)

Divergence of geodesics in Teichmüller space and the mapping class group
Geometric and Functional Analysis, Volume 19, Issue 3 (2009), 722–742. (with Kasra Rafi)

Curvature, stretchiness, and dynamics
In the Tradition of Ahlfors and Bers IV, Contemp. Math. **432** (2007), 19–30.

Geodesics track random walks in Teichmüller space
PhD Dissertation, University of Chicago 2005.

Science, Technology, Law, and Policy Publications & Preprints

Models, Race, and the Law
Yale Law Journal Forum, Vol. 130 (March 2021). Available online. (with Doug Spencer)

Computational Redistricting and the Voting Rights Act
Election Law Journal, Available online. (with Amariah Becker, Dara Gold, and Sam Hirsch)

Discrete geometry for electoral geography
Preprint. (with Bridget Eileen Tenner) arXiv:1808.05860

Implementing partisan symmetry: Problems and paradoxes
Political Analysis, to appear. (with Daryl DeFord, Natasha Dhamankar, Mackenzie McPike, Gabe Schoenbach, and Ki-Wan Sim) arXiv:2008:06930

Clustering propensity: A mathematical framework for measuring segregation
Preprint. (with Emilia Alvarez, Everett Meike, and Marshall Mueller; appendix by Tyler Piazza)

Locating the representational baseline: Republicans in Massachusetts
Election Law Journal, Volume 18, Number 4, 2019, 388–401.
(with Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen)

Redistricting reform in Virginia: Districting criteria in context
Virginia Policy Review, Volume XII, Issue II, Spring 2019, 120–146. (with Daryl DeFord)

Geometry v. Gerrymandering
The Best Writing on Mathematics 2019, ed. Mircea Pitici. Princeton University Press.
reprinted from Scientific American, November 2018, 48–53.

Gerrymandering metrics: How to measure? What's the baseline?
Bulletin of the American Academy for Arts and Sciences, Vol. LXII, No. 2 (Winter 2018), 54–58.

Rebooting the mathematics of gerrymandering: How can geometry track with our political values?
The Conversation (online magazine), October 2017. (with Peter Levine)

A formula goes to court: Partisan gerrymandering and the efficiency gap
Notices of the American Mathematical Society **64** No. 9 (2017), 1020–1024. (with Mira Bernstein)

International mobility and U.S. mathematics
Notices of the American Mathematical Society **64**, No. 7 (2017), 682–683.

Graduate Advising in Mathematics

Nate Fisher (PhD 2021), Sunrose Shrestha (PhD 2020), Ayla Sánchez (PhD 2017), Kevin Buckles (PhD 2015), Mai Mansouri (MS 2014)

Outside committee member for Chris Coscia (PhD 2020), Dartmouth College

Postdoctoral Advising in Mathematics

Principal supervisor Thomas Weighill (2019–2020)

Co-supervisor Daryl DeFord (MIT 2018–2020), Rob Kropholler (2017–2020), Hao Liang (2013–2016)

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Reading Lab: Mathematical Models in Social Context | sites.tufts.edu/models
One hr/wk discussion seminar of short but close reading on topics in mathematical modeling, including history of psychometrics; algorithmic bias; philosophy of statistics; problems of model explanation and interpretation.

Geometric Literacy
Module-based graduate topics course. Modules have included: p-adic numbers, hyperbolic geometry, nilpotent geometry, Lie groups, convex geometry and analysis, the complex of curves, ergodic theory, the Gauss circle problem.

Markov Chains (graduate topics course)
Teichmüller Theory (graduate topics course)
Fuchsian Groups (graduate topics course)
Continued Fractions and Geometric Coding (undergraduate topics course)
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- Science, Technology, and Society Lunch Seminar

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Distinguished Plenary Lecture June 2021
75th Anniversary Meeting of Canadian Mathematical Society, Ottawa, Ontario *online (COVID)*

BMC/BAMC Public Lecture April 2021
Joint British Mathematics/Applied Mathematics Colloquium, Glasgow, Scotland *online (COVID)*

AMS Einstein Public Lecture in Mathematics [March 2020]
Southeastern Sectional Meeting of the AMS, Charlottesville, VA *postponed*

Gerald and Judith Porter Public Lecture
AMS-MAA-SIAM, Joint Mathematics Meetings, San Diego, CA January 2018

Mathematical Association of America Distinguished Lecture
MAA Carriage House, Washington, DC October 2016

American Mathematical Society Invited Address
AMS Eastern Sectional Meeting, Brunswick, ME September 2016

Named University Lectures

- Parsons Lecture | UNC Asheville October 2020
- Loeb Lectures in Mathematics | Washington University in St. Louis [March 2020]
- Math, Stats, CS, and Society | Macalester College October 2019
- MRC Public Lecture | Stanford University May 2019
- Freedman Memorial Colloquium | Boston University March 2019
- Julian Clancy Frazier Colloquium Lecture | U.S. Naval Academy January 2019
- Barnett Lecture | University of Cincinnati October 2018
- School of Science Colloquium Series | The College of New Jersey March 2018
- Kieval Lecture | Cornell University February 2018
- G. Milton Wing Lectures | University of Rochester October 2017
- Norman Johnson Lecture | Wheaton College September 2017
- Dan E. Christie Lecture | Bowdoin College September 2017

Math/Computer Science Department Colloquia

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- Georgetown (CS) Sept 2020
- Santa Fe Institute July 2020
- UC Berkeley Sept 2018
- Brandeis-Harvard-MIT-NEU Mar 2018
- Northwestern University Oct 2017
- University of Illinois Sept 2017
- University of Utah Aug 2017
- Wesleyan Dec 2016
- Worcester Polytechnic Inst. Dec 2016
- Université de Neuchâtel Jun 2016
- Brandeis University Mar 2016
- Swarthmore College Oct 2015
- Bowling Green May 2015
- City College of New York Feb 2015
- Indiana University Nov 2014
- the Technion Oct 2014
- Wisconsin–Madison Sept 2014
- Stony Brook March 2013

Minicourses

- Integer programming and combinatorial optimization (two talks) | Georgia Tech — May 2021
- Workshop in geometric topology (main speaker, three talks) | Provo, UT — June 2017
- Growth in groups (two talks) | MSRI, Berkeley, CA — August 2016
- Hyperbolicity in Teichmüller space (three talks) | Université de Grenoble — May 2016
- Counting and growth (four talks) | IAS Women's Program, Princeton — May 2016
- Nilpotent groups (three talks) | Seoul National University — October 2014
- Sub-Finsler geometry of nilpotent groups (five talks) | Galatasaray Univ., Istanbul — April 2014

Science, Technology, and Society

- The Mathematics of Accountability | Sawyer Seminar, Anthropology, Johns Hopkins — February 2020
- STS Circle | Harvard Kennedy School of Government — September 2019
- Data, Classification, and Everyday Life Symposium | Rutgers Center for Cultural Analysis — January 2019
- Science Studies Colloquium | UC San Diego — January 2019
- Arthur Miller Lecture on Science and Ethics | MIT Program in Science, Tech, and Society — November 2018

Data Science, Computer Science, Quantitative Social Science

- Data Science for Social Good Workshop (DS4SG) | Georgia Tech (virtual) — November 2020
- Privacy Tools Project Retreat | Harvard (virtual) — May 2020
- Women in Data Science Conference | Microsoft Research New England — March 2020
- Quantitative Research Methods Workshop | Yale Center for the Study of American Politics — February 2020
- Societal Concerns in Algorithms and Data Analysis | Weizmann Institute — December 2018
- Quantitative Collaborative | University of Virginia — March 2018
- Quantitative Social Science | Dartmouth College — September 2017
- Data for Black Lives Conference | MIT — November 2017

Political Science, Geography, Law, Democracy, Fairness

- The Long 19th Amendment: Women, Voting, and American Democracy | Radcliffe Institute — Nov–Dec 2020
- "The New Math" for Civil Rights | Social Justice Speaker Series, Davidson College — November 2020
- Math, Law, and Racial Fairness | Justice Speaker Series, University of South Carolina — November 2020
- Voting Rights Conference | Northeastern Public Interest Law Program — September 2020
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- Geography of Redistricting Conference Keynote | Harvard Center for Geographic Analysis — May 2019
- Political Analytics Conference | Harvard University — November 2018
- Cyber Security, Law, and Society Alliance | Boston University — September 2018
- Clough Center for the Study of Constitutional Democracy | Boston College — November 2017
- Tech/Law Colloquium Series | Cornell Tech — November 2017
- Constitution Day Lecture | Rockefeller Center for Public Policy, Dartmouth College — September 2017

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Supreme Court of the United States, in Rucho v. Common Cause - cited in dissent

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American Mathematical Society

Program Committee 2020–2021
Symposium on Foundations of Responsible Computing

Presenter on Public Mapping, Statistical Modeling 2019, 2020
National Conference of State Legislatures

Committee on the Human Rights of Mathematicians 2016–2019
American Mathematical Society

Committee on The Future of Voting: Accessible, Reliable, Verifiable Technology 2017–2018
National Academies of Science, Engineering, and Medicine

Visiting Positions and Residential Fellowships

Visiting Professor Department of Mathematics — Fall 2021
Boston College | Chestnut Hill, MA

Fellow Radcliffe Institute for Advanced Study — 2018–19
Harvard University | Cambridge, MA

Member Center of Mathematical Sciences and Applications — 2018–19
Harvard University | Cambridge, MA

Visitor Microsoft Research Lab — 2018–19
MSR New England | Cambridge, MA

Research Member Geometric Group Theory program — Fall 2016
Mathematical Sciences Research Institute | Berkeley, CA

Research Member Random Walks and Asymptotic Geometry of Groups program — Spring 2014
Institut Henri Poincaré | Paris, France

Research Member Low-dimensional Topology, Geometry, and Dynamics program — Fall 2013
Institute for Computational and Experimental Research in Mathematics | Providence, RI

Research Member Geometric and Analytic Aspects of Group Theory program — May 2012
Institut Mittag-Leffler | Stockholm, Sweden

Research Member Quantitative Geometry program — Fall 2011
Mathematical Sciences Research Institute | Berkeley, CA

Postdoctoral Fellow Teichmüller "project blanc" — Spring 2009
Agence Nationale de la Recherche (Collège de France) | Paris, France

STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,

REBECCA HARPER, et al.,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

AFFIDAVIT OF PROFESSOR MOON DUCHIN

NCLCV v. Hall
21 CVS 15426
LDTX158

Exhibit #
Duchin 2
12/30/21 - SL
exhibitsticker.com

I, Dr. Moon, Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.
2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.
4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.
5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.
6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.
7. A current copy of my full CV is attached to this report.
8. I am compensated at the rate of $400 per hour.

Analysis of 2021 enacted redistricting plans in North Carolina

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

December 23, 2021

1 Introduction

On November 4, 2021, the North Carolina General Assembly enacted three districting plans: maps of 14 U.S. Congressional districts, 50 state Senate districts, and 120 state House districts. This affidavit contains a brief summary of my evaluation of the properties of these plans. My focus will be on the egregious partisan imbalance and racial vote dilution in the enacted plans, following a brief review of the traditional districting principles.

Because redistricting inevitably involves complex interactions of rules, which can create intricate tradeoffs, it will be useful to employ a direct comparison to an alternative set of plans. These demonstrative plans illustrate that it is possible to *simultaneously maintain or improve* metrics for all of the most important redistricting principles that are operative in North Carolina's constitution and state and federal law. Crucially, this shows that nothing about the state's political geography compels us to draw a plan with a massive and entrenched partisan skew or a significant dilutive effect on Black voters.

To this end, I will be comparing the following plans: the enacted plans SL-174, SL-173, and SL-175 and a corresponding set of alternative plans labeled NCLCV-Cong, NCLCV-Sen, and NCLCV-House (proposed by plaintiffs who include the North Carolina League of Conservation Voters). The accompanying block assignment files are Appendices A1, A2, A3 to this affidavit, and I understand that they will be provided to the court in native format.

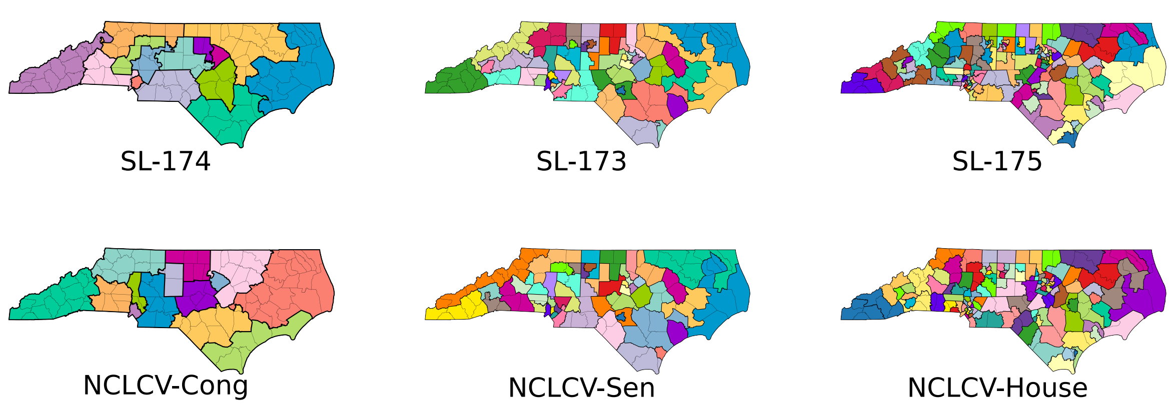

Figure 1: The six plans under discussion in this affidavit.

2 Partisan gerrymandering

2.1 Abstract partisan fairness

There are many notions of partisan fairness that can be found in the scholarly literature and in redistricting practitioner guides and software. Most of them are numerical, in the sense that they address *how a certain quantitative share of the vote should be translated to a quantitative share of the seats* in a state legislature or Congressional delegation.

The numerical notions of partisan fairness all tend to agree on one central point: an electoral climate with a roughly 50-50 split in partisan preference should produce a roughly 50-50 representational split. I will call this the *Close-Votes-Close-Seats* principle. North Carolina voting has displayed a partisan split staying consistently close to even between the two major parties over the last ten years, but the plans released by the General Assembly after the 2010 census were very far from realizing the ideal of converting even voting to even representation. This time, with a 14th seat added to North Carolina's apportionment, an exactly even seat outcome is possible. But the new enacted plans, like the plans from ten years ago, are decidedly not conducive to even representation.

Importantly, Close-Votes-Close-Seats is not tantamount to a requirement for proportionality. Rather, it is closely related to the principle of *Majority Rule*: a party or group with more than half of the votes should be able to secure more than half of the seats. In fact, Close-Votes-Close-Seats is essentially a corollary (or byproduct) of Majority Rule. It is not practicable to design a map that *always* attains these properties, but by contrast a map that *consistently thwarts* them should be closely scrutinized and usually rejected.

Unlike proportionality, neither Close-Votes-Close-Seats nor Majority Rule has any bearing on the preferred representational outcome when one party has a significant voting advantage: these principles are silent about whether 70% vote share should secure 70% of the seats, as proportionality would dictate, or 90% of the seats, as supporters of the efficiency gap would prefer. The size of the "winner's bonus" is not at all prescribed by a Close-Votes-Close-Seats norm.

2.2 Geography and fairness

Some scholars have argued that all numerical ideals, including Close-Votes-Close-Seats, ignore the crucial *political geography*—this school of thought reminds us that the location of votes for each party, and not just the aggregate preferences, has a major impact on redistricting outcomes. In [5], my co-authors and I gave a vivid demonstration of the impacts of political geography in Massachusetts: we showed that for a ten-year span of observed voting patterns, even though Republicans tended to get over one-third of the statewide vote, it was impossible to draw a single Congressional district with a Republican majority. That is, the geography of Massachusetts Republicans locked them out of Congressional representation. It is therefore not reasonable to charge the Massachusetts legislature with gerrymandering for having produced maps which yielded all-Democratic delegations; they could not have done otherwise.

In North Carolina, this is not the case. The alternative plans demonstrate that it is possible to produce maps that give the two major parties a roughly equal opportunity to elect their candidates. These plans are just examples among many thousands of plausible maps that convert voter preferences to far more even representation by party. In Congressional redistricting, present-day North Carolina geography is easily conducive to a seat share squarely in line with the vote share. In Senate and House plans, even following the strict detail of the Whole County Provisions, there are likewise many alternatives converting nearly even voting patterns to nearly even representation, across a large set of recent elections.

The clear conclusion is that the political geography of North Carolina today does not obstruct the selection of a map that treats Democratic and Republican voters fairly and evenhandedly.

2.3 Overlaying elections and plans

The enacted plans behave as though they are built to resiliently safeguard electoral advantage for Republican candidates. We can examine this effect without invoking any predictions or assumptions about future voting behavior by using a standard technique in election analysis: pairing proposed plans with actual recent elections. This method works by overlaying (or superimposing) the districting plans on a series of observed voting patterns from the recent past; this lets us take advantage of the rich dataset of real electoral outcomes in North Carolina in the last ten years to avoid speculative or predictive modeling about voting trends in the future.[1]

The overlay method works best when there is a large set of statewide elections to apply, which is certainly true in North Carolina. Of the 52 statewide party-ID general elections from the last cycle, 29 are elections for Council of State (ten offices elected three times, with the Attorney General race uncontested in 2012), three are presidential races, three are for U.S. Senate, and 17 are judicial races since mid-decade, when those became partisan contests. See Table 1 for more detail on the election dataset.

2.4 Partisanship outcomes

North Carolina is a very "purple" state. In 38 out of the 52 contests in our dataset, the statewide partisan outcome is within a 6-point margin: 47-53 or closer.

To understand how the enacted plans create major shortfalls for Democratic representation, we will overlay the plans with voting patterns from individual elections in the past Census cycle. We can make a striking observation by laying our six plans over the vote patterns, shown in Table 1. This reveals that the enacted Congressional plan (SL-174) shows a remarkable lack of responsiveness, giving 10–4 partisan outcomes across a wide range of recent electoral conditions, meaning that 10 Republicans and only 4 Democrats would represent North Carolina in Congress. The alternative plan (NCLCV-Cong) is far more faithful to the vote share, far more responsive, and tends to award more seats to the party with more votes—usually upholding both basic small-d-democratic principles of Majority Rules and Close-Votes-Close-Seats, which are violated by the enacted plan.

The same patterns are visible at the Senate and House level. Overall, the three enacted plans combine with those 38 relatively even vote patterns to produce 114 outcomes. Every single pairing of an enacted plan with a close statewide contest—a complete sweep of 114 opportunities—gives an *outright Republican majority* of seats. All three enacted plans will lock in an extreme, resilient, and unnecessary advantage for one party.

By every measure considered above that corresponds to a clear legal or good-government redistricting goal or value, the alternative plans meet or exceed the performance of the enacted plans. This demonstrates that it is possible, without any cost to the redistricting principles in play, to select maps that are far fairer to the voters of North Carolina.

Below, the outcomes of overlaying the plans on the elections will be presented in a series of tables and figures. First, Table 1 overviews the overlays with numbers.[2] Then, Figure 2 offers a visualization to depict the same big picture of entrenched partisan advantage in the enacted plans with the full 52-election dataset. The diagonals show various lines of *responsiveness* that pivot around the central point of fairness: half of the votes securing half of the seats.

Finally, we will restrict to a smaller set of the 14 "up-ballot" races and consider the comparison for one office at a time in Figures 3-5.

[1] Many authors have used this technique of overlaying "exogenous" statewide elections rather than using statistical regressions and other modeling to manipulate "endogenous" districted elections. For instance this can be found in peer-reviewed work and expert reports of scholar-practitioners such as Bernard Grofman and Steven Ansolabehere.

[2] The backup data supporting Table 1 is attached to this report as Appendix C and I understand that it will be provided to the court in native format.

Do close votes translate to close seats?

The table records the number of districts in each plan with a Democratic win. This shows that the enacted maps systematically violate the principles of Close-Votes-Close-Seats and Majority Rule.

	D Vote Share	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
GOV12	0.4418	4	4	16	18	41	44
AGC16	0.4444	4	4	17	17	40	42
LAC16	0.4475	4	5	18	20	42	45
JHU16	0.4563	4	5	18	19	42	49
AGC20	0.4615	3	4	17	19	40	51
JZA16	0.4619	4	5	19	21	43	50
JDI16	0.4653	4	6	19	21	44	53
LTG16	0.4665	4	6	19	21	44	54
LAC12	0.4674	4	5	20	20	44	51
AGC12	0.4678	4	5	18	18	43	50
SEN16	0.4705	4	6	19	21	43	55
TRS16	0.4730	4	6	19	21	45	53
TRS20	0.4743	4	6	17	20	45	51
JA620	0.4806	4	7	17	21	46	55
PRS16	0.4809	4	7	19	22	48	56
JA420	0.4822	4	7	17	22	47	56
INC20	0.4823	4	7	18	23	47	56
LTG20	0.4836	4	7	18	21	46	55
JA720	0.4842	4	7	17	22	48	56
SUP20	0.4862	4	7	19	23	49	56
JA520	0.4874	4	7	18	22	49	57
JA218	0.4876	4	7	18	22	45	55
JS420	0.4879	4	7	19	24	49	56
J1320	0.4885	4	7	19	23	49	56
PRS12	0.4897	4	6	20	21	46	55
SEN20	0.4910	4	7	20	24	48	56
LAC20	0.4918	4	8	21	25	51	58
SEN14	0.4919	4	6	20	22	46	52
PRS20	0.4932	4	8	20	25	50	60
JS220	0.4934	4	8	21	24	51	59
SUP16	0.4941	4	6	22	23	49	57
JS118	0.4955	4	7	20	25	50	58
INC16	0.4960	4	6	22	22	50	57
JST16	0.4976	4	7	21	23	50	58
LTG12	0.4992	5	7	22	22	50	58
JS120	0.5000	4	8	22	27	52	60
AUD16	0.5007	5	8	22	23	51	56
GOV16	0.5011	4	7	20	27	50	58
ATG20	0.5013	4	8	21	25	51	58
ATG16	0.5027	4	7	20	23	50	57
JA118	0.5078	4	8	22	26	51	58
AUD20	0.5088	4	8	24	28	54	61
JA318	0.5091	4	8	21	26	52	59
SOS20	0.5116	5	8	24	28	53	62
JGE16	0.5131	5	8	22	25	52	59
INC12	0.5186	5	8	22	22	55	61
SOS16	0.5226	5	9	24	24	57	62
GOV20	0.5229	4	8	23	27	58	63
AUD12	0.5371	8	9	27	28	61	65
SOS12	0.5379	7	9	26	26	59	63
TRS12	0.5383	7	9	25	24	59	65
SUP12	0.5424	8	9	28	28	61	66

53 – 47 or closer

AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; LTG = Lieutenant Governor; PRS = President; SEN = Senator; SOS = Secretary of State; SUP = Superintendent of Public Instruction; TRS = Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), JS* are elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals. Where there was more than one judicial candidate from a given party on the ballot, they were combined for this analysis. The two-digit suffix designates the election year.

Table 1: 52 general elections, sorted from lowest to highest Democratic share.

Seats vs. Votes

Majority Rule says that outcomes should tend to fall in the Northeast and Southwest quadrants, avoiding the Southeast and Northwest. Close-Votes-Close-Seats says that points should not miss the bulls-eye near the center by systematically deviating to the North or the South. These principles are clearly upheld by the alternative plans (**green**) and violated by the enacted plans (**maroon**).

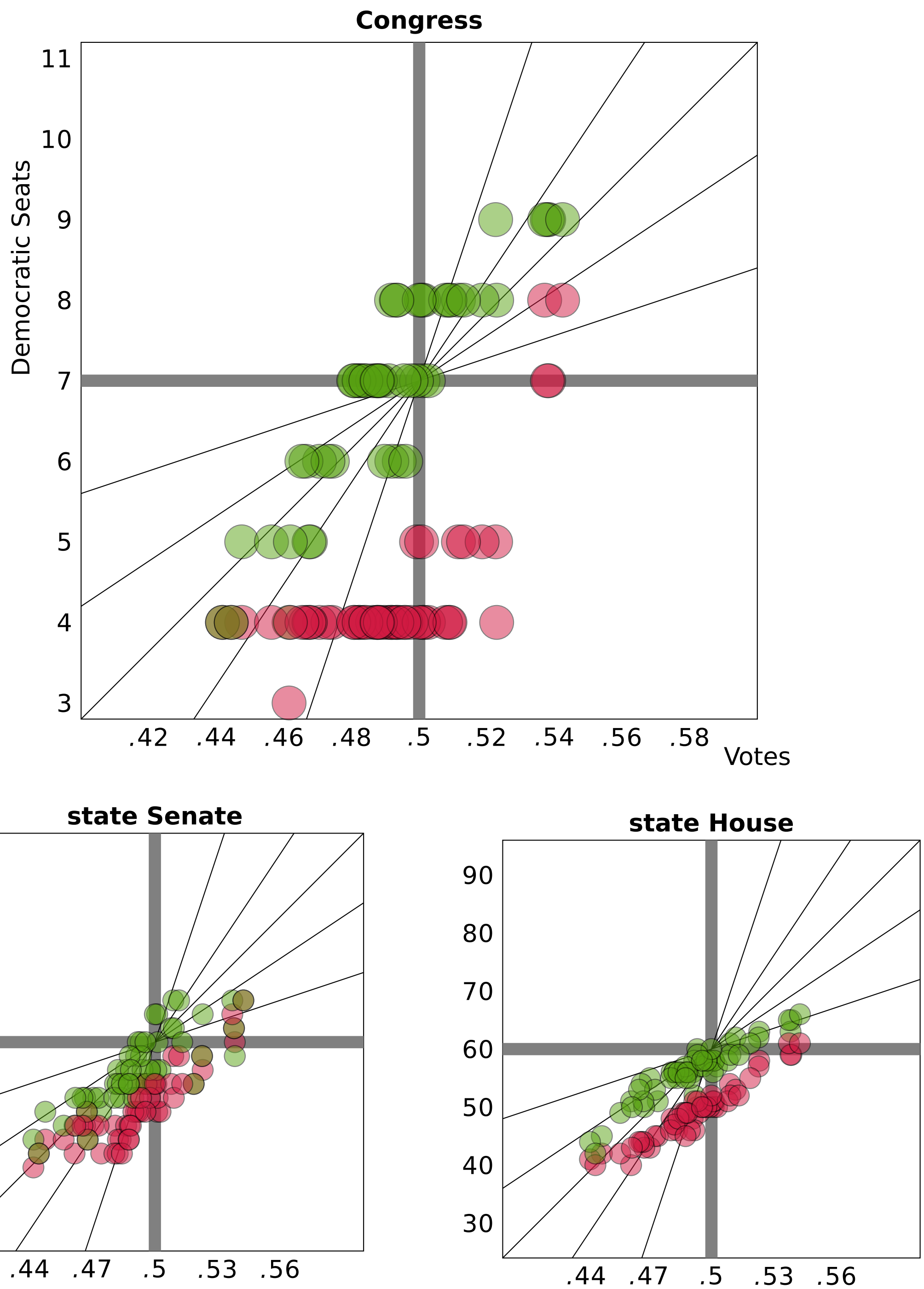

Figure 2: On these seats-vs.-votes plots, we see the election results when overlaying the six maps on the 52 general election contests in the last decade; each colored dot is plotted as the coordinate pair (vote share, seat share).

2.5 Up-ballot races

The same patterns are apparent if we narrow our focus to the smaller set of better-known "up-ballot" races: in order, the first five to appear on the ballot are the contests for President, U.S. Senator, Governor, Lieutenant Governor, and Attorney General. Together these occurred 14 times in the last Census cycle.

	Up-ballot generals (14)		**All generals (52)**	
	D vote share	D seat share	D vote share	D seat share
SL-174	.4883	.2908	.4911	.3118
NCLCV-Cong		.4796		.4931
SL-173	.4883	.3957	.4911	.4065
NCLCV-Sen		.4557		.4592
SL-175	.4883	.3994	.4911	.4080
NCLCV-House		.4649		.4684

Table 2: Comparing overall fidelity of representation to the voting preferences of the electorate. Vote shares are computed with respect to the major-party vote total.

Figure 3 shows the performance of the Congressional maps in the three Presidential contests in the last Census cycle, where the Democratic vote share (pink box) was between 48% and 50% of the major-party total each time. For a contest that is so evenly divided, we would expect a fair map to have 6, 7, or 8 out of 14 districts favoring each party. The alternative Congressional map NCLCV-Cong does just that, while the enacted plan SL-174 has just 4 out of 14 Democratic-majority districts each time (green and maroon circles). The alternative plan is far more successful at reflecting the even split of voter preferences.

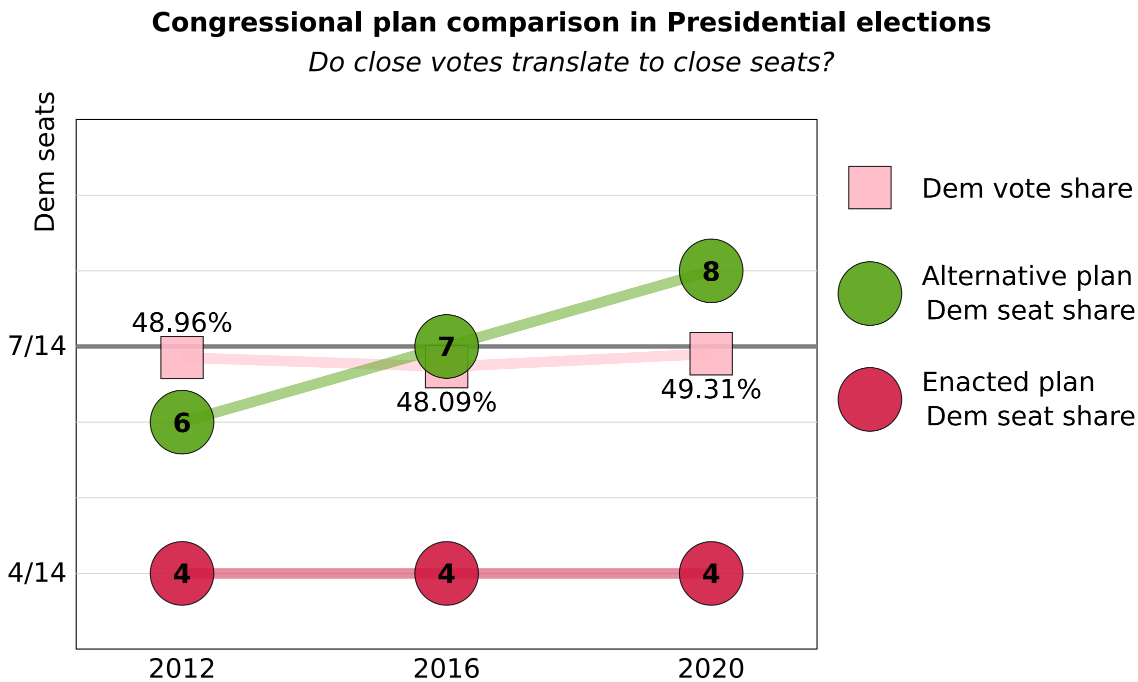

Figure 3: When Presidential voting is overlaid on the plans, we can compare the Democratic seat share in the enacted Congressional plan SL-174 (**maroon**) and the alternative Congressional plan NCLCV-Cong (**green**) to the vote share (**pink**) for Democratic candidates. The 50% line is marked.

Next, simplified versions of the same type of graphic are presented for all five up-ballot offices. Figure 4 compares Congressional maps, and Figure 5 compares legislative maps in the same fashion.

In these figures, we can view whether the plans display a tendency to uphold the Close-Votes-Close-Seats norm, for one office at a time. The pink squares are the vote share. If they are close to the 50-50 mark, then a fair map would also produce seat shares that are close to that mark. This is consistently true for the alternative plans and consistently false for the enacted plans.

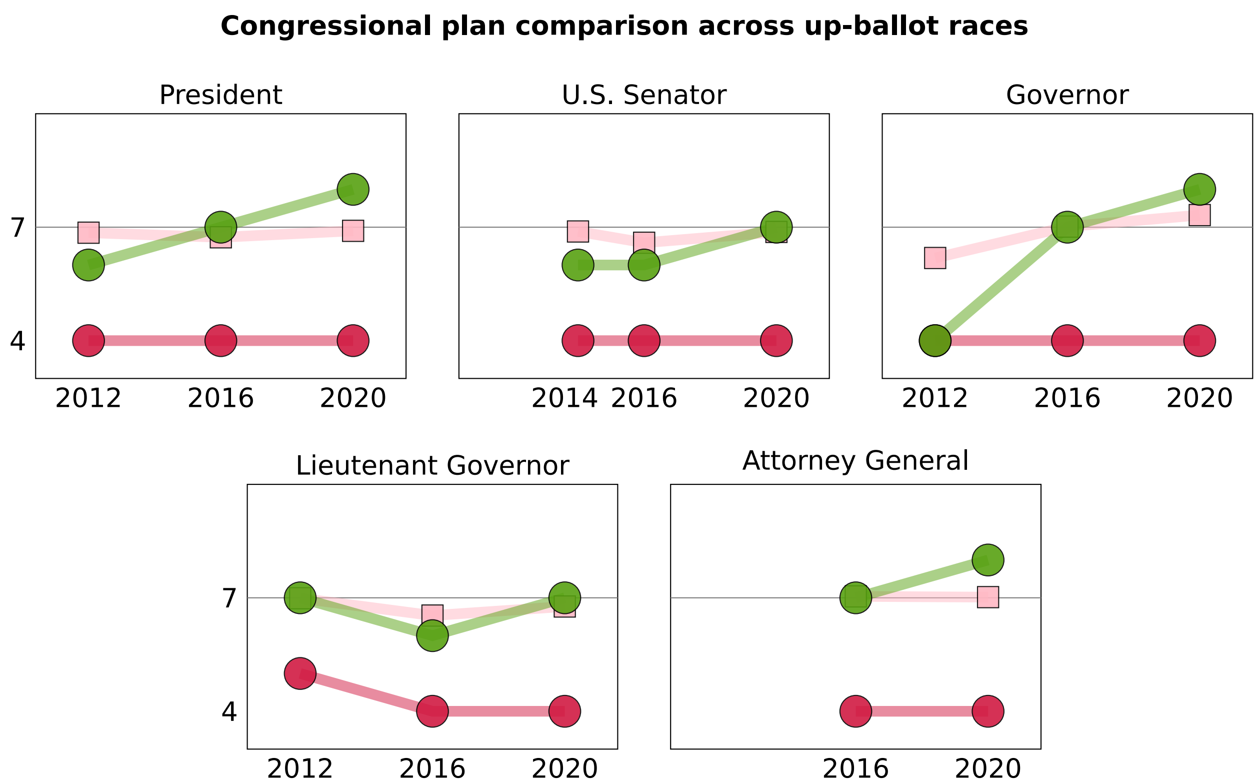

Figure 4: For up-ballot general election contests across the previous Census cycle, we can compare the seat share under the enacted Congressional plan SL-174 (**maroon**) and the seat share under the alternative Congressional plan NCLCV-Cong (**green**) to the vote share (**pink**) for Democratic candidates. The presidential comparison from the previous figure is repeated here, alongside the other four up-ballot offices. The 50% line is marked each time.

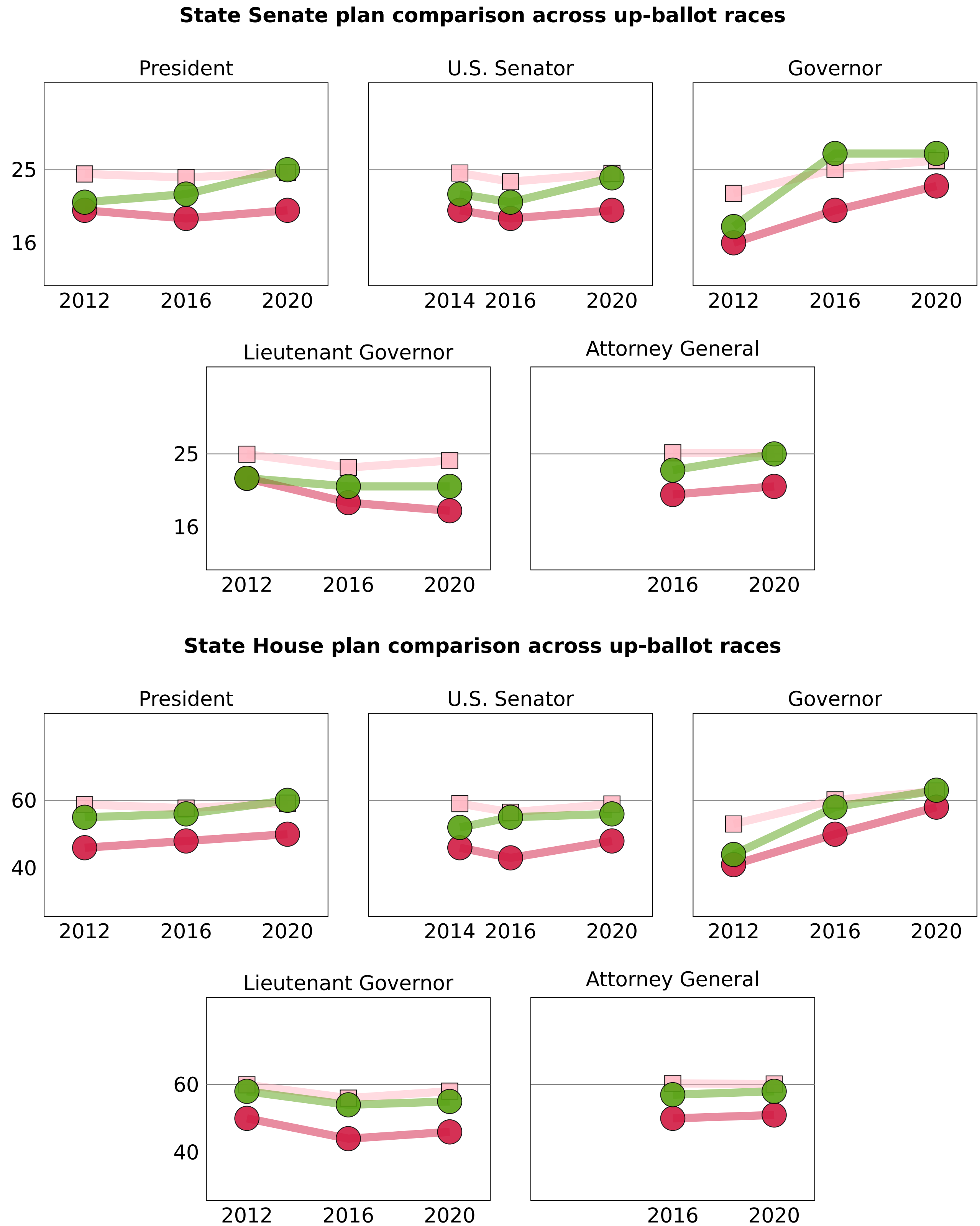

Figure 5: Legislative plans overlaid with voting patterns from up-ballot elections. The enacted plans SL-173 and SL-175 are shown in **maroon**. The alternative plans NCLCV-Sen and NCLCV-House, in **green**, have seat shares tracking much closer to the nearly even voting preferences.

3 Racial vote dilution

North Carolina has a large minority of Black-identified residents. Over two million North Carolinians—2,107,526 out of 10,439,388 to be precise, or about 20.2%—were identified as non-Hispanic Black-alone on the Census. Within the voting-age population, the numbers shift to 1,620,569 out of 8,155,099, or about 19.9%. Increasing numbers of Americans identify as Black in combination with other races and/or Hispanic ethnicity. Passing to this more expansive definition of Black voting age population raises the numbers to 1,743,052 out of 8,155,099, or 21.4%.

Minority groups' opportunity to elect candidates of choice is protected by both state and federal law. A detailed assessment of opportunity must not primarily hinge on the demographics of the districts, but must also rely on electoral history and an assessment of polarization patterns.[3]

I have used industry-leading techniques to study the racial polarization patterns in North Carolina general and primary elections from the last decade. They indicate a consistent pattern of polarization in statewide general elections, such that White voters are estimated to support the Republican candidate at a rate of over 61% in every general election, and Black voters are estimated to support the Democratic candidate at a rate of over 94% each time. Polarization is present in many Democratic primary elections as well, particularly in elections in which there is a Black Democratic candidate. I have designated a selection of eight elections—four generals and four primaries—chosen to be particularly informative in determining whether Black voters have an opportunity to elect their candidates of choice.

Democratic Primaries

- Sutton preferred over Mangrum in the 2020 Superintendent primary;
- Smith preferred over Wadsworth in the 2020 Ag. Commissioner primary;
- Williams preferred over Stein in the 2016 Attorney General primary;
- Coleman preferred over the field in the 2016 Lieutenant Governor primary.

General Elections

- Holley preferred over Robinson in the 2020 Lieutenant Governor election;
- Cunningham preferred over Tillis in the 2020 U.S. Senate election;
- Coleman preferred over Forest in the 2016 Lieutenant Governor election;
- Blue preferred over Folwell in the 2016 Treasurer election.

These eight contests were chosen by a combination of factors that combine to make an election particularly informative with respect to the preferences of Black voters. Namely: I prioritized elections that are more recent, that have a Black candidate on the ballot, that are clearly polarized, and that are close enough to produce variation at the district level.[4]

The electoral alignment score derived from these elections is a value from 0 to 8. I consider a district in which the Black candidate of choice prevails in at least 6 of these 8 contests to be aligned with Black voting preferences in the state.[5] If, in addition, at least 25% of the voting age population is Black, then I label the district to be effective for Black voters.

I note that the use of electoral history is not just cosmetic: there are House-sized districts with 35-39% BVAP that are nonetheless not labeled effective in these lists because they fall short of the standard of inclining to the Black candidate of choice in at least six out of the eight chosen elections.

[3] A detailed discussion of the inadequacy of using demographics alone as a proxy can be found in [3].

[4] Of the candidates above, Sutton, Williams, Coleman, Colley, and Blue are themselves Black-identified.

[5] I have used statewide ecological inference ("EI") runs to determine the candidate of choice for Black voters. I note that it is also possible to run EI on smaller geographies (such as counties or county clusters) to detect regional candidates of choice rather than statewide candidates of choice; in most cases, these will be the same, but in some cases, regional effects may be meaningful and could affect these results at the margin.

At all three levels, the NCLCV alternative maps provide more effective opportunity-to-elect districts for Black voters than the corresponding enacted plans.

Effective districts for Black voters
Out of 14 Congressional districts, SL-174 has 2 effective districts, while NCLCV-Cong has 4.
Out of 50 Senate districts, SL-173 has 8 effective districts, while NCLCV-Sen has 12.
Out of 120 House districts, SL-175 has 24 effective districts, while NCLCV-House has 36.

effective districts in state plan	effective districts in alternative plan
CD2, 9	CD2, 4, 9, 11
SD5, 11, 14, 19, 28, 38, 39, 40	SD1, 5, 11, 14, 18, 19, 26, 27, 32, 38, 39, 40
HD8, 23, 24, 25, 27, 32, 38, 39, 42, 44, 48, 57, 58, 60, 66, 71, 92, 99, 100, 101, 102, 106, 107, 112	HD2, 8, 9, 10, 23, 24, 25, 27, 31, 32, 33, 38, 39, 40, 42, 43, 44, 45, 48, 57, 58, 59, 60, 61, 63, 66, 71, 88, 92, 99, 100, 101, 102, 106, 107, 112

4 Detailed plan comparison

Detailed maps showing how the district lines cut through the patterns of Democratic and Republican support, and how they cut through the demographic location of Black voting age population, can be found in Appendix B.

4.1 Traditional districting principles

Principles that are relevant to North Carolina redistricting include the following.

- **Population balance.** The standard interpretation of *One Person, One Vote* for Congressional districts is that districts should be fine-tuned so that their total Census population deviates by no more than one person from any district to any other.

  There is more latitude with legislative districts; they typically vary top-to-bottom by no more than 10% of ideal district size. In North Carolina, the Whole County Provisions make it very explicit that 5% deviation must be tolerated if it means preserving more counties intact.

  All six plans have acceptable population balance.

Population deviation

	Max Positive Deviation	District	Max Negative Deviation	District
SL-174	0	(eight districts)	−1	(six districts)
NCLCV-Cong	0	(eight districts)	−1	(six districts)
SL-173	10,355 (4.960%)	5	−10,434 (4.997%)	13,18
NCLCV-Sen	10,355 (4.960%)	5	−10,427 (4.994%)	15
SL-175	4250 (4.885%)	18	−4189 (4.815%)	112
NCLCV-House	4341 (4.990%)	82	−4323 (4.969%)	87

Table 3: Deviations are calculated with respect to the rounded ideal district populations of 745,671 for Congress, 208,788 for Senate, and 86,995 for House.

- **Contiguity.** All six plans are contiguous; for each district, it is possible to transit from any part of the district to any other part through a sequence of census blocks that share boundary segments of positive length. As is traditional in North Carolina, contiguity through water is accepted.

- **Compactness.** The two compactness metrics most commonly appearing in litigation are the *Polsby-Popper score* and the *Reock score*. Polsby-Popper is the name given in redistricting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula $4\pi A/P^2$. Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1. Reock is a different measurement of how much a shape differs from a circle: it is computed as the ratio of a region's area to that of its circumcircle, defined as the smallest circle in which the region can be circumscribed. From this definition, it is clear that it too is optimized at a value of 1, which is achieved only by circles.

  These scores depend on the contours of a district and have been criticized as being too dependent on map projections or on cartographic resolution [1, 2]. Recently, some mathematicians have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built. The most commonly cited discrete score for districts is the *cut edges score*, which counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations. This score improves on the contour-based scores by better controlling for factors like coastline and other natural boundaries, and by focusing on the units actually available to redistricters rather than treating districts like free-form Rorschach blots.

  The alternative plans are significantly more compact than the enacted plans in all three compactness metrics.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
SL-174	5194	0.303	0.417
NCLCV-Cong	4124	0.383	0.470
SL-173	9702	0.342	0.416
NCLCV-Sen	9249	0.369	0.428
SL-175	16,182	0.351	0.437
NCLCV-House	13,963	0.414	0.465

Table 4: Comparing compactness scores via one discrete and two contour-based metrics. These scores were computed using dissolved districts based on the census blocks that were assigned in the plans under discussion.

District-by-district compactness scores for the contour-based metrics are shown in Tables 5-7.

	Reock		Polsby-Popper	
CD	SL-174	NCLCV-Cong	SL-174	NCLCV-Cong
1	0.517	0.534	0.324	0.403
2	0.303	0.47	0.278	0.323
3	0.484	0.212	0.331	0.228
4	0.487	0.412	0.39	0.304
5	0.468	0.582	0.347	0.514
6	0.418	0.472	0.231	0.483
7	0.424	0.664	0.199	0.434
8	0.472	0.523	0.532	0.398
9	0.678	0.579	0.469	0.43
10	0.41	0.285	0.197	0.254
11	0.282	0.553	0.207	0.532
12	0.247	0.388	0.243	0.368
13	0.41	0.558	0.266	0.379
14	0.232	0.354	0.221	0.313

Table 5: Compactness scores by district for the Congressional plans.

	Reock		Polsby-Popper	
SD	SL-173	NCLCV-Sen	SL-173	NCLCV-Sen
1	0.263	0.297	0.213	0.174
2	0.231	0.397	0.105	0.178
3	0.409	0.409	0.179	0.179
4	0.564	0.564	0.406	0.406
5	0.403	0.403	0.335	0.335
6	0.616	0.616	0.595	0.595
7	0.213	0.553	0.219	0.411
8	0.446	0.457	0.439	0.478
9	0.443	0.441	0.217	0.226
10	0.618	0.618	0.614	0.614
11	0.464	0.464	0.376	0.376
12	0.42	0.388	0.395	0.404
13	0.284	0.357	0.257	0.4
14	0.399	0.523	0.247	0.45
15	0.397	0.52	0.231	0.398
16	0.619	0.51	0.473	0.388
17	0.488	0.54	0.361	0.505
18	0.376	0.644	0.309	0.514
19	0.53	0.53	0.34	0.34
20	0.384	0.387	0.363	0.344
21	0.218	0.218	0.137	0.137
22	0.473	0.459	0.471	0.517
23	0.498	0.498	0.529	0.529
24	0.52	0.52	0.452	0.452
25	0.283	0.325	0.271	0.276
26	0.451	0.397	0.301	0.331
27	0.541	0.364	0.437	0.321
28	0.444	0.544	0.248	0.457
29	0.317	0.378	0.202	0.252
30	0.4	0.4	0.456	0.456
31	0.482	0.429	0.344	0.355
32	0.62	0.455	0.422	0.354
33	0.322	0.322	0.294	0.294
34	0.49	0.477	0.523	0.489
35	0.375	0.342	0.225	0.348
36	0.463	0.314	0.411	0.294
37	0.401	0.397	0.421	0.437
38	0.523	0.566	0.334	0.444
39	0.356	0.391	0.295	0.368
40	0.381	0.453	0.382	0.538
41	0.287	0.519	0.294	0.531
42	0.429	0.397	0.273	0.469
43	0.533	0.341	0.522	0.274
44	0.386	0.425	0.46	0.357
45	0.343	0.391	0.25	0.3
46	0.229	0.249	0.184	0.213
47	0.186	0.116	0.127	0.113
48	0.404	0.373	0.38	0.264
49	0.479	0.424	0.358	0.22
50	0.422	0.312	0.441	0.335

Table 6: Compactness scores by district for the Senate plans.

	Reock		Polsby-Popper	
HD	SL-175	NCLCV-House	SL-175	NCLCV-House
1	0.413	0.393	0.213	0.168
2	0.316	0.404	0.326	0.468
3	0.377	0.448	0.298	0.329
4	0.482	0.337	0.448	0.237
5	0.28	0.28	0.3	0.3
6	0.389	0.539	0.479	0.549
7	0.476	0.442	0.44	0.403
8	0.394	0.437	0.327	0.314
9	0.587	0.698	0.411	0.425
10	0.589	0.606	0.567	0.398
11	0.359	0.654	0.246	0.473
12	0.312	0.312	0.291	0.291
13	0.379	0.367	0.425	0.488
14	0.384	0.305	0.291	0.204
15	0.546	0.468	0.371	0.395
16	0.404	0.483	0.242	0.388
17	0.416	0.668	0.227	0.473
18	0.589	0.336	0.37	0.374
19	0.462	0.482	0.285	0.359
20	0.463	0.172	0.557	0.173
21	0.45	0.591	0.206	0.469
22	0.528	0.528	0.361	0.361
23	0.453	0.453	0.359	0.359
24	0.463	0.554	0.538	0.638
25	0.463	0.402	0.511	0.455
26	0.45	0.474	0.4	0.412
27	0.433	0.433	0.353	0.353
28	0.573	0.411	0.498	0.43
29	0.36	0.519	0.333	0.645
30	0.381	0.306	0.356	0.389
31	0.415	0.476	0.323	0.533
32	0.534	0.528	0.587	0.543
33	0.491	0.254	0.289	0.252
34	0.414	0.383	0.289	0.349
35	0.28	0.528	0.292	0.464
36	0.586	0.396	0.532	0.443
37	0.417	0.372	0.369	0.379
38	0.377	0.522	0.247	0.383
39	0.649	0.399	0.519	0.245
40	0.413	0.342	0.336	0.242
41	0.521	0.581	0.423	0.498
42	0.537	0.402	0.395	0.258
43	0.52	0.415	0.281	0.372
44	0.587	0.564	0.419	0.564
45	0.248	0.555	0.274	0.495
46	0.316	0.432	0.239	0.275
47	0.604	0.535	0.498	0.453
48	0.479	0.479	0.442	0.442
49	0.447	0.555	0.358	0.604
50	0.375	0.384	0.343	0.388
51	0.48	0.427	0.283	0.262
52	0.352	0.468	0.214	0.28
53	0.322	0.597	0.256	0.449
54	0.459	0.486	0.376	0.442
55	0.458	0.534	0.312	0.399
56	0.502	0.652	0.37	0.691
57	0.436	0.589	0.368	0.475
58	0.397	0.521	0.257	0.432
59	0.455	0.463	0.334	0.56
60	0.383	0.361	0.261	0.407
61	0.388	0.356	0.294	0.346
62	0.318	0.651	0.312	0.589
63	0.56	0.596	0.353	0.533
64	0.329	0.48	0.257	0.459
65	0.594	0.594	0.764	0.764
66	0.457	0.46	0.264	0.293
67	0.444	0.444	0.486	0.486
68	0.45	0.577	0.305	0.502
69	0.539	0.49	0.346	0.364
70	0.542	0.638	0.535	0.65
71	0.267	0.488	0.275	0.509
72	0.521	0.495	0.27	0.398
73	0.487	0.46	0.421	0.612
74	0.367	0.548	0.299	0.425
75	0.388	0.468	0.266	0.53
76	0.43	0.43	0.497	0.497
77	0.408	0.408	0.297	0.297
78	0.341	0.479	0.204	0.447
79	0.523	0.353	0.36	0.2
80	0.285	0.413	0.319	0.359
81	0.481	0.434	0.312	0.359
82	0.311	0.444	0.32	0.477
83	0.474	0.473	0.328	0.342
84	0.498	0.57	0.515	0.645
85	0.501	0.493	0.315	0.299
86	0.49	0.49	0.437	0.437
87	0.538	0.512	0.437	0.526
88	0.233	0.367	0.211	0.364
89	0.304	0.462	0.291	0.338
90	0.508	0.431	0.349	0.381
91	0.541	0.563	0.522	0.583
92	0.28	0.399	0.244	0.455
93	0.317	0.33	0.288	0.319
94	0.507	0.496	0.348	0.371
95	0.616	0.49	0.596	0.516
96	0.358	0.316	0.351	0.33
97	0.321	0.321	0.515	0.515
98	0.593	0.574	0.576	0.589
99	0.469	0.471	0.322	0.443
100	0.537	0.359	0.333	0.312
101	0.488	0.518	0.31	0.515
102	0.392	0.621	0.23	0.36
103	0.278	0.546	0.349	0.479
104	0.573	0.432	0.32	0.313
105	0.395	0.437	0.419	0.391
106	0.599	0.485	0.419	0.503
107	0.304	0.529	0.183	0.556
108	0.374	0.402	0.24	0.288
109	0.466	0.485	0.421	0.522
110	0.355	0.514	0.277	0.39
111	0.348	0.641	0.24	0.436
112	0.58	0.266	0.397	0.229
113	0.392	0.368	0.224	0.186
114	0.307	0.549	0.182	0.46
115	0.559	0.308	0.349	0.289
116	0.401	0.532	0.159	0.332
117	0.422	0.581	0.271	0.393
118	0.412	0.412	0.247	0.247
119	0.276	0.276	0.22	0.22
120	0.4	0.4	0.367	0.367

Table 7: Compactness scores by district for the House plans.

- **Respect for political subdivisions.** For legislative redistricting, North Carolina has one of the strongest requirements for county consideration of any state in the nation. In my understanding, courts have interpreted the Whole County Provisions as follows.[6]
  - First, if any county is divisible into a whole number of districts that will be within ±5% of ideal population, then it must be subdivided accordingly without districts crossing into other counties.
  - Next, seek any contiguous grouping of two counties that is similarly divisible into a whole number of districts.
  - Repeat for groupings of three, and so on, until all counties are accounted for.

  Once clusters have been formed, there are more rules about respecting county lines within clusters. The legal language is again explicit: "[T]he resulting interior county lines created by any such groupings may be crossed or traversed in the creation of districts within said multi-county grouping but only to the extent necessary" to meet the ±5% population standard for districts. To address this, I have counted the *county traversals* in each plan, i.e., the number of times a district crosses between adjacent counties within a grouping.

  Table 8 reflects the county integrity metric that is most relevant at each level: the enacted congressional plan splits 11 counties into 25 pieces while the alternative plan splits 13, but splits no county three ways. (The enacted plans unnecessarily split three counties into three pieces.) In the legislative plans, the law specifies traversals as the fundamental integrity statistic.

County and municipality preservation

	# county pieces
SL-174	25
NCLCV-Cong	26

	# traversals
SL-173	97
NCLCV-Sen	89
SL-175	69
NCLCV-House	66

	# municipal pieces (considering all blocks)	# municipal pieces (considering populated blocks)
SL-174	90	50
NCLCV-Cong	58	41
SL-173	152	91
NCLCV-Sen	125	100
SL-175	292	222
NCLCV-House	201	173

Table 8: Comparing the plans' conformance to political boundaries.

[6] A complete set of solutions is described in detail in the white paper of Mattingly et al.—though with the important caveat that the work "does not reflect... compliance with the Voting Rights Act" [4]. Absent a VRA conflict, the 2020 Decennial Census population data dictates that the North Carolina Senate plan must be decomposed into ten single-district fixed clusters and seven multi-district fixed clusters (comprising 2, 2, 3, 3, 4, 6, and 6 districts, respectively). It has four more areas in which there is a choice of groupings. In all, there are sixteen different possible clusterings for Senate, each comprising 26 county clusters. The House likewise has 11 single-district fixed clusters and 22 multi-district fixed clusters (with two to thirteen districts per cluster), together with three more areas with a choice of groupings. In all, the House has only eight acceptable clusterings, each comprising 40 county clusters. Again, it is important to note that VRA compliance may present a compelling reason to select some clusterings and reject others.

The alternative plans are comparable to the enacted plans, and often superior, in each of these key metrics regarding preservation of political boundaries. This remains true whether splits of municipalities are counted by the division of any of their census blocks, or only by the division of populated census blocks.

I will briefly mention several additional redistricting principles.

- **Communities of interest.** In North Carolina, there was no sustained effort by the state or by community groups to formally collect community of interest (COI) maps, to my knowledge. Without this, it is difficult to produce a suitable metric.
- **Cores of prior districts.** In some states, there is statutory guidance to seek districting plans that preserve the cores of prior districts. In North Carolina, this is not a factor in the constitution, in statute, or in case law. In addition, attention to core preservation would be prohibitively difficult in the Senate and House because of the primacy of the Whole County Provisions, which forces major changes to the districts simply as a consequence of fresh population numbers.
- **Incumbent pairing.** In 2017, the North Carolina legislative redistricting committee listed "incumbency protection" as a goal in their itemization of principles. In 2021, this was softened to the statement that "Member residence may be considered" in the drawing of districts. I have counted the districts in each plan that contain more than one incumbent address; these are sometimes colorfully called "double-bunked" districts. For this statistic, it is not entirely clear whether a high or low number is preferable. When a plan remediates a gerrymandered predecessor, we should not be surprised if it ends up pairing numerous incumbents.

Double-bunking

	# districts pairing incumbents
SL-174	3
NCLCV-Cong	1
SL-173	5
NCLCV-Sen	9
SL-175	6
NCLCV-House	16

Table 9: For Congress and Senate, the enacted and alternative plans are comparable; at the House level, the alternative plan has more double-bunking. *Note: These numbers were calculated using incumbent addresses that I understand were provided by the Legislative Defendants.*

4.2 Swing districts and competitive contests

Another way to understand the electoral properties of districting plans is to investigate how many districts always give the same partisan result over a suite of observed electoral conditions, and how many districts can "swing" between the parties. Figure 6 compares the six plans across the up-ballot elections. The enacted plans lock in large numbers of always-Republican seats. In the Senate and House, nearly half the seats are locked down for Republicans. In the Congressional plan, it's well *over* half. This provides another view from which the NCLCV plans provide attractive alternatives.

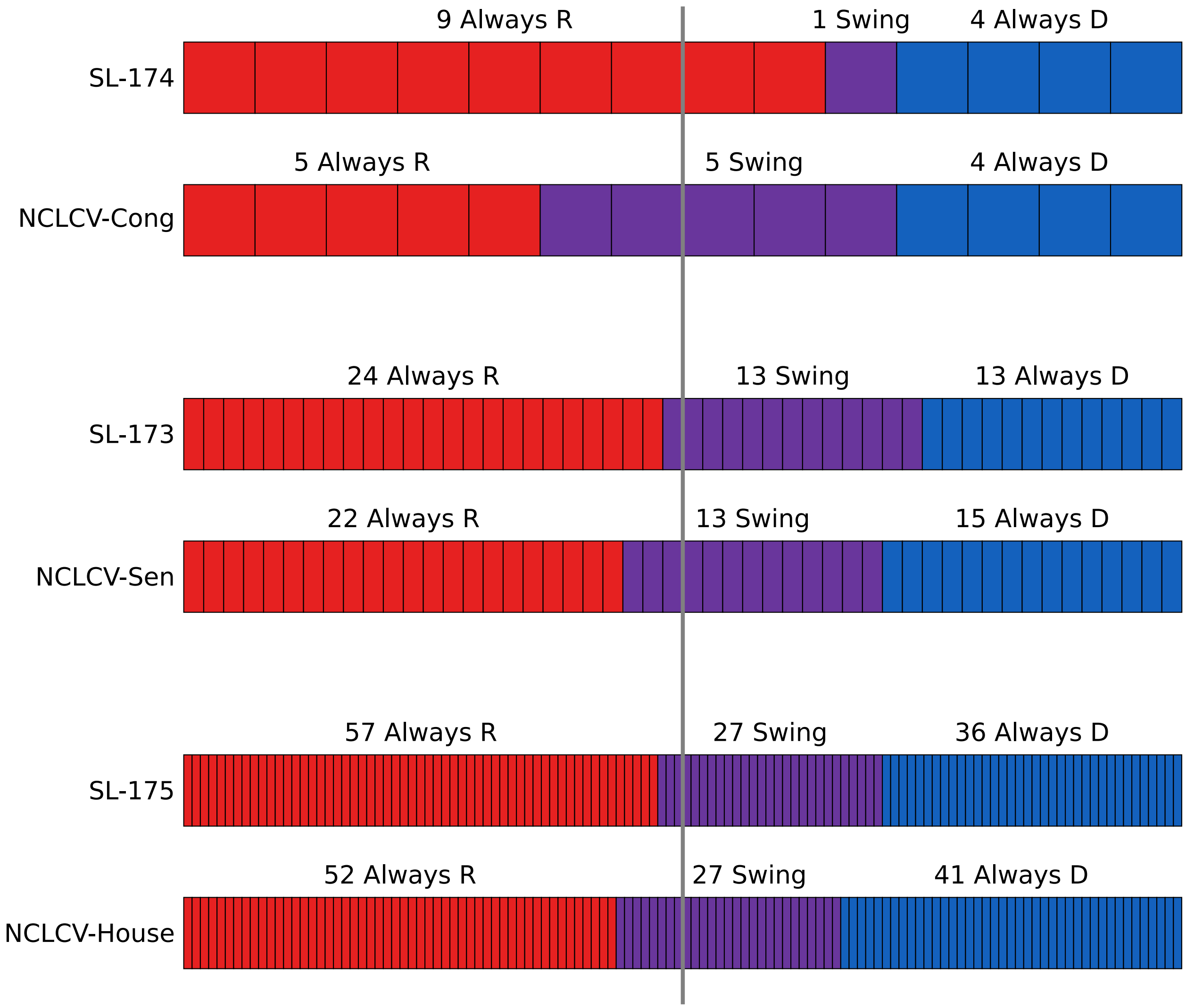

Figure 6: These visuals show the breakdown of seats that always have a Republican winner, always have a Democratic winner, or are sometimes led by each party across the 14 up-ballot elections over the previous Census cycle. The 50-50 split is marked.

In interpreting this visualization, note that this is consistent with the discussion elsewhere of entrenched Republican majorities in the enacted maps. These Always-Republican districts provide a *floor* for Republican performance from the viewpoint of these up-ballot contests.

One more measure of partisan fairness, frequently referenced in the public discourse, is the tendency of a districting plan to promote close or competitive contests. We close with a comparison of the enacted and alternative plans that displays the number of times across the full dataset of 52 elections that a contest had a partisan margin of closer than 10 points, 6 points, or 2 points, respectively. This can occur up to $14 \cdot 52 = 728$ times in Congressional maps, $50 \cdot 52 = 2600$ times in state Senate maps, and $120 \cdot 52 = 6240$ times in state House maps. The figures below show horizontal rules at every 10% interval of the total number of possible competitive contests; we can see, for instance, that the alternative Congressional plan has contests within a 10-point margin more than 40% of the time.

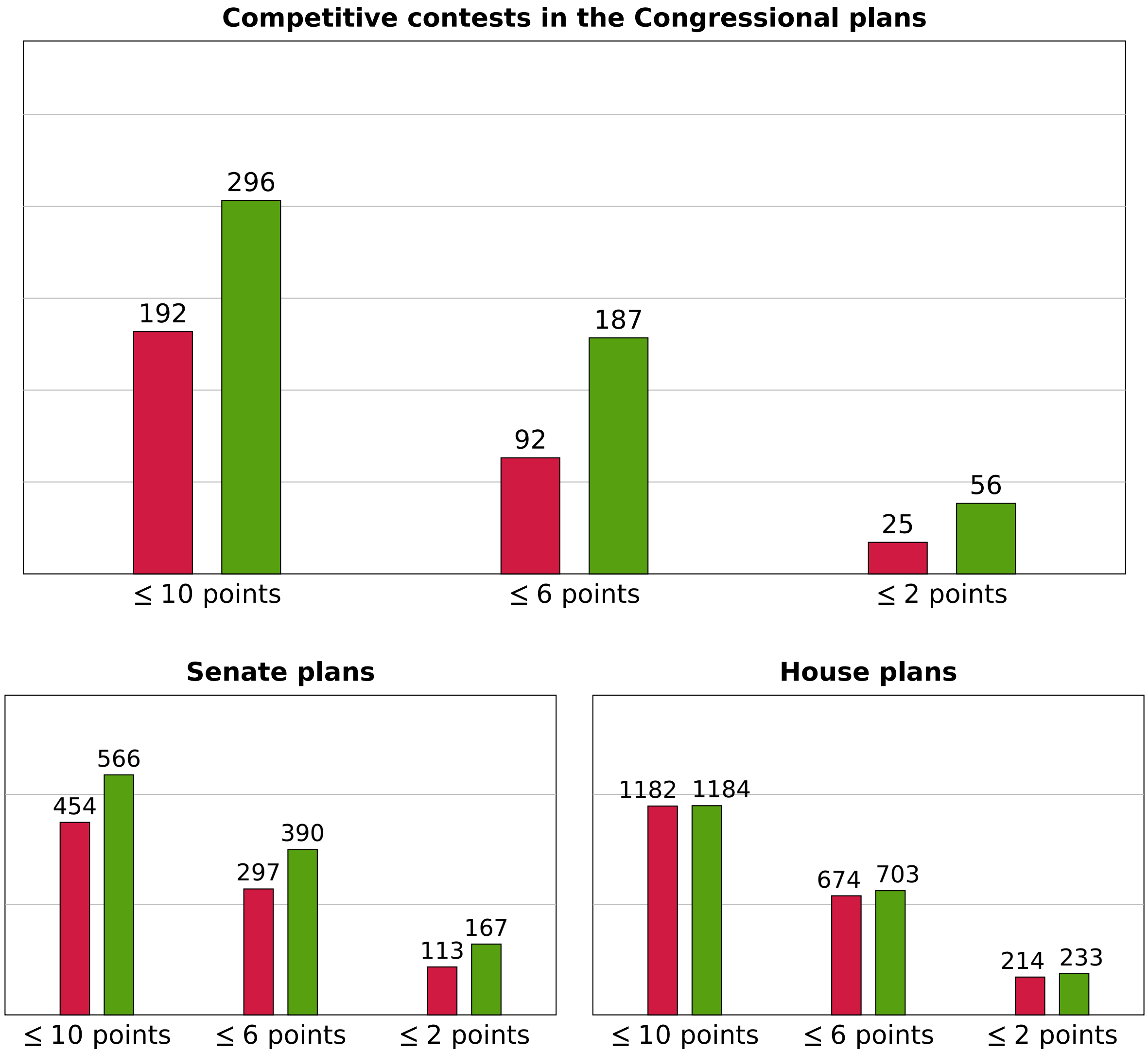

Figure 7: These bar graphs show the number of competitive contests for the enacted plans (**maroon**) and the alternative plans (**green**). In each plot, we consider increasingly restrictive definitions of "competitive" from left to right, counting districts in which the major-party vote split is closer than 45-55, 47-53, and 49-51, respectively.

5 Location-specific comparison of electoral opportunity

I received information reflecting the residential locations of 147 individuals, who come from either of two groups:

- plaintiffs in the NCLCV v. Hall case; or
- registered voters belonging to the NCLCV membership who are Black and/or are registered as Democrats.

In Table 10 below, I summarize the impact on the identified individuals in terms of electoral opportunity if the enacted maps are compared to the alternative maps.

Subsequently, Figures 8 and 9 provide a visualization that pinpoints the geographical sites where the alternative plans improve electoral opportunities for plaintiffs and NCLCV members—that is, places where the identified individuals (as Democrats and/or Black voters) have measurably greater ability to elect their candidates of choice under the alternative plans than under the existing plans.

This is backed up by the data in Tables 11-13 below, which identify the district numbers in the six enacted and alternative plans for each of these identified individuals. The district numbers were computed using census block information to specify the locations, but the table reports the locations by larger units (VTDs) in order to protect privacy.

Lost opportunity for Democratic and Black voters

	greater Democratic opportunity in alternative plan than enacted plan
Congress	51 individuals
Senate	37 individuals
House	39 individuals

	resides in effective district in alternative plan but not enacted plan
Congress	28 Black voters
Senate	21 Black voters
House	21 Black voters

Table 10: Of the 147 identified individuals, how many saw a change in their opportunity for Democratic representation? How many Black voters saw a change in their opportunity to elect Black candidates of choice?

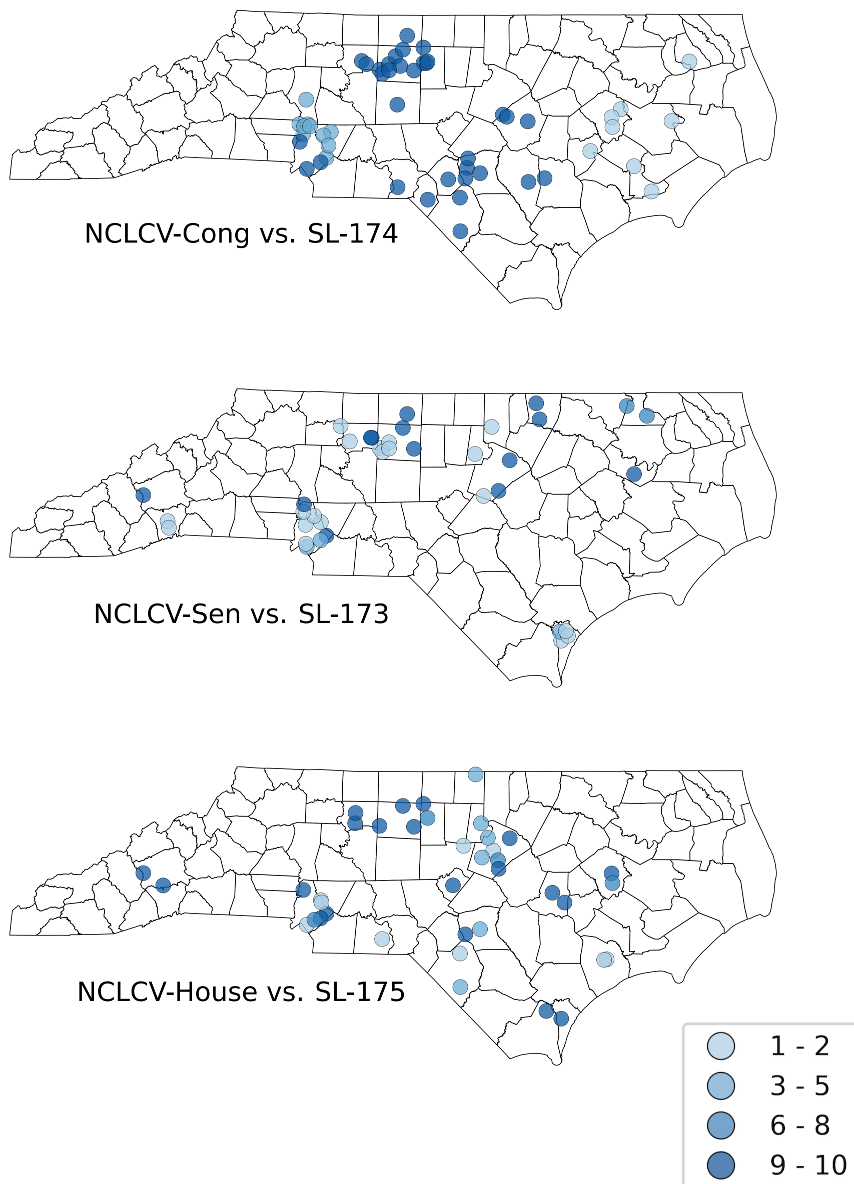

Figure 8: Locations where identified individuals have less opportunity to be represented by a Democrat in Congress, state Senate, and state House under the enacted plans. The shading indicates the drop in Democratic wins across the 14 up-ballot races in the enacted map relative to the alternative map. There are 51 such individuals in the Congressional maps, 37 in the Senate maps, and 31 in the House maps.

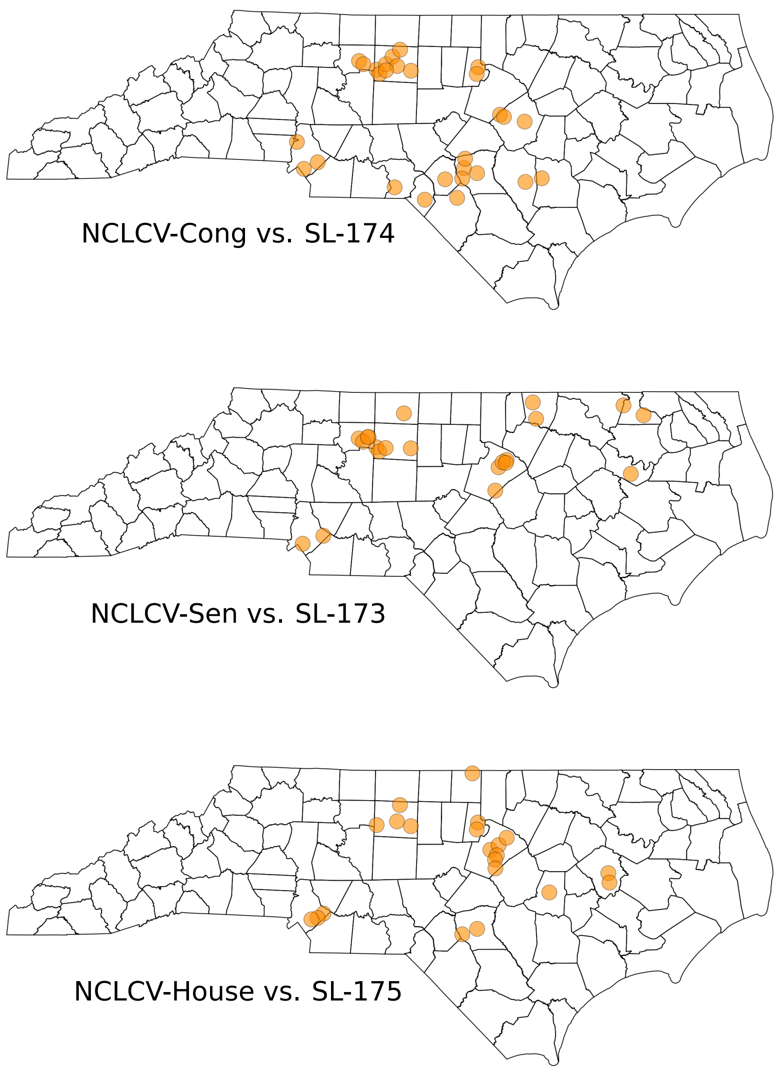

Figure 9: Locations where Black voters from the identified individuals list would be in a district that provides effective electoral opportunity under the alternative plan, but not under the enacted plan. There are 28 such voters at the Congressional level and 21 at each of the Senate and House level.

VTD Census ID	VTD/Precinct Name	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
37025001-07	01-07	10	10	34	34	73	73
37025012-03	12-03	10	10	34	34	82	82
37025002-07	02-07	10	10	34	34	83	73
37009000002	CLIFTON	11	12	47	47	93	93
37063000029	GLENN ELEMENTARY	6	2	22	22	2	2
37063000043	FOREST VIEW ELEMENTARY	6	6	22	20	30	30
37063000052	EVANGEL ASSEMBLY OF GOD	6	2	22	22	31	31
37063055-11	055-11	6	6	20	22	29	29
37071000012	FLINT GROVES	13	13	43	43	108	108
37071000004	FOREST HEIGHTS	13	13	43	43	109	109
37057000076	THOMASVILLE 10 76	7	8	30	30	80	80
371350000EF	EFLAND	6	6	23	23	50	50
371050000A2	A2	7	7	12	12	51	54
37131NEWTOW	NEWTOWN	2	2	1	1	27	27
371350000CF	CEDAR FALLS	6	6	23	23	56	56
37081000H25	H25	10	11	27	27	62	60
37093000061	RAEFORD 1	8	4	24	24	48	48
37081000RC2	RC2	7	11	26	26	59	59
3712700P15A	OAK LEVEL	2	2	11	11	25	25
3707700TYHO	00TYHO	2	2	13	13	32	32
370910000CO	COFIELD	2	1	1	1	5	5
37057000038	EASTSIDE 38	7	8	30	30	81	81
370210021.1	HAW CREEK ELEMENTARY SCHOOL	14	14	49	49	115	114
37019000015	GRISSETTOWN	3	3	8	8	17	19
37047000P15	TATUM	3	3	8	8	46	46
37019000002	LELAND	3	3	8	8	17	17
370450CASAR	CASAR	13	13	44	44	110	111
370210007.1	KENILWORTH PRESBYTERIAN CHURCH	14	14	49	49	114	115
370210053.1	LEICESTER 2 - COMMUNITY CENTER	14	14	46	49	116	116
370210054.2	LUTHERAN CHURCH OF THE NATIVITY	14	14	49	49	116	115
37193000108	FAIRPLAINS	11	12	36	36	94	94
37173000BC2	BC2	14	14	50	47	119	119
37119000054	54	9	9	40	42	102	112
37119000108	108	9	9	40	40	100	100
37119000208	208	13	10	37	38	98	98
371190204.1	204.1	9	10	40	40	99	106
37119000097	97	9	9	42	39	112	105
37119000222	222	9	9	38	39	101	101
37097000ST6	STATESVILLE 6	12	10	37	37	84	84
370970DV1-B	DAVIDSON 1-B	10	10	37	37	95	95
37119000048	48	9	9	42	42	88	104
37119000216	216	8	9	41	41	103	99
37081000G27	G27	11	11	28	28	57	57
37081000G43	G43	11	11	27	28	58	62
37153000006	WOLF PIT 3	8	4	29	29	52	52
371570000MS	MOSS STREET	11	6	26	26	65	65
3716300ROWA	ROWAN	4	4	9	9	22	22
3719500PRWI	WILSON I	2	2	4	4	24	24
37119000206	206	13	10	37	37	98	98
37119000236	236	8	10	41	40	103	99

Table 11: Locations of identified individuals, Part 1 of 3. For each location, the district numbers are given for the six plans discussed here. VTDs are listed rather than the more precise census block in order to protect privacy. Rows highlighted **blue** indicate individuals who lose Democratic opportunity in at least one of the enacted plans, relative to the alternative plans. Rows highlighted **orange** indicate Black voters who lose the opportunity to be in an effective district for Black candidates of choice in at least one level. (As it turns out, every instance of lost opportunity for Black voters is also an instance of lost Democratic opportunity.)

VTD Census ID	VTD/Precinct Name	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
37119000142	142	13	10	38	38	98	112
37081000G65	G65	11	11	27	27	58	58
37081000G70	G70	11	11	28	26	61	61
3708100H19A	H19A	10	11	27	27	60	60
3708100MON3	MON3	11	11	26	28	59	57
37183015-01	15-01	5	7	17	14	37	38
37183019-17	19-17	5	5	18	18	39	66
37183001-31	01-31	5	5	15	15	11	33
37183012-02	12-02	7	7	17	17	37	37
37119000087	87	8	9	41	41	105	105
37119000068	68	9	9	42	41	104	100
371190223.1	223.1	13	9	39	39	101	101
37119000081	81	9	9	39	39	92	101
37119000237	237	9	10	38	40	106	106
37119000127	127	13	10	37	37	98	98
37191000014	14	2	1	4	4	4	10
37183005-01	05-01	6	7	16	16	41	41
37183020-09	20-09	6	7	16	17	36	36
37183004-18	04-18	6	7	16	16	49	11
37191000010	10	2	1	4	4	10	10
37183019-21	19-21	5	5	13	18	35	66
37183001-46	01-46	5	5	18	18	34	40
37183001-50	01-50	5	5	14	14	33	38
37183016-05	16-05	5	5	14	14	21	38
37119000145	145	9	10	38	38	107	107
37183008-03	08-03	5	5	15	15	40	49
37183017-05	17-05	5	5	14	18	38	40
37183013-09	13-09	5	5	18	18	66	66
370490000N2	FORT TOTTEN	1	1	3	3	3	3
37049000002	HAVELOCK	1	1	3	3	13	13
37001000004	MORTON	7	6	25	25	64	63
37001000126	BURLINGTON 6	7	6	25	25	63	64
3700100003N	NORTH BOONE	7	6	25	25	64	64
37001000124	BURLINGTON 4	7	6	25	25	63	63
37165001-16	01-16/01	8	4	24	24	48	48
37067000063	CASH ELEMENTARY SCHOOL	12	12	31	32	75	75
37067000074	MEADOWLARK MIDDLE SCHOOL	12	12	31	31	74	74
37067000709	WARD ELEMENTARY SCHOOL	12	12	32	31	74	71
37067000065	KERNERSVILLE 7TH DAY ADVENTIST CHURCH	12	12	31	32	75	75
37067000507	SEDGE GARDEN REC CTR	12	11	32	32	71	75
371510000AE	ASHEBORO EAST	7	11	29	29	70	70
37067000905	BETHABARA MORAVIAN CH	12	12	32	31	91	72
37067000402	FOURTEENTH STREET REC	12	11	32	32	72	72
370890000FR	FLAT ROCK	14	14	48	48	113	117
3708900HV-1	HENDERSONVILLE-1	14	14	48	48	117	117
37023000039	MORGANTON 09	13	13	46	46	86	86
3710900LB34	LABORATORY	12	13	44	46	97	97
3706100WARS	WARSAW	3	4	9	9	4	4
3712900CF01	CF01	3	3	8	7	18	17
370130BELHV	BELHAVEN	1	1	3	3	79	1

Table 12: Locations of identified individuals, Part 2 of 3. For each location, the district numbers are given for the six plans discussed here. VTDs are listed rather than the more precise census block in order to protect privacy. Rows highlighted **blue** indicate individuals who lose Democratic opportunity in at least one of the enacted plans, relative to the alternative plans. Rows highlighted **orange** indicate Black voters who lose the opportunity to be in an effective district for Black candidates of choice in at least one level. (As it turns out, every instance of lost opportunity for Black voters is also an instance of lost Democratic opportunity.)

VTD Census ID	VTD/Precinct Name	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
37037NWM117	NORTH WILLIAMS	7	7	20	20	54	54
3714100CL05	COLUMBIA	3	3	9	9	16	16
3713300BM08	BRYNN MARR	1	3	6	6	14	15
3713300NR02	NEW RIVER	1	3	6	6	15	15
37051SL78-3	Spring Lake 3	4	4	21	21	42	44
3705100G10A	STONEY POINT 2-G10	4	4	19	19	45	45
37051000G1A	CROSS CREEK 02-G1	4	4	19	19	43	42
37035000035	SWEETWATER	12	13	45	45	96	96
37035000032	SOUTH NEWTON	12	13	45	45	89	89
3705100CC32	CROSS CREEK 32	4	4	19	19	44	44
37059000007	JERUSALEM	10	8	30	30	77	77
3708500PR01	ANDERSON CREEK	4	7	12	12	6	6
3708500PR07	BARBECUE	4	7	12	12	6	6
371070000K8	KINSTON-8	1	1	3	3	12	12
37189000009	ELK	14	12	47	47	87	93
371170000BG	BEAR GRASS	2	1	2	1	23	23
371010PR12B	NORTH CLEVELAND 2	4	2	10	10	26	26
371010PR31B	SOUTHWEST CLEVELAND	4	2	10	10	53	53
3710100PR24	EAST SELMA	4	2	10	10	28	28
3714701102A	SIMPSON A	1	1	5	5	9	8
37167000003	ALBEMARLE NUMBER 3	8	8	33	33	67	67
3700700LILE	LILESVILLE	8	8	29	29	55	55
3704500KM-N	KM N	13	13	44	44	111	110
37143BETHEL	BETHEL	1	1	1	2	1	1
37147000601	CHICOD	1	1	5	5	9	9
37147001201	PACTOLUS	1	1	5	5	8	8
37159000040	NORTH WARD	10	8	33	33	76	76
3712900FP04	FP04	3	3	7	8	19	20
37129000W16	W16	3	3	7	7	20	18
37129000H11	H11	3	3	7	7	18	20
37129000H02	H02	3	3	7	7	20	20
37159000036	SOUTH WARD	10	8	33	33	76	76
37125000DHR	DEEP RIVER/HIGH FALLS/RITTER	8	7	21	21	78	51
37069000015	EAST FRANKLINTON	2	2	11	11	7	7
3719908-CRA	CRABTREE	14	14	47	47	85	85
3719700EBND	EAST BEND	12	12	36	31	77	77
37171000018	MT AIRY 8	11	12	36	36	90	90
3708700WS-2	WAYNESVILLE SOUTH 2	14	14	50	50	118	118
3715500005A	FAIRMONT	3	4	24	24	46	47
37155000028	RENNERT	3	4	24	24	47	47
37113000011	SMITHBRIDGE	14	14	50	50	120	120
3714500WDSD	WOODSDALE	2	6	23	23	2	2
3717900029A	SHILOH ELEMENTARY SCHOOL	8	8	35	35	68	69
3717900037A	NEXT LEVEL CHURCH	8	8	35	35	69	69
37169000017	WEST WALNUT COVE	11	12	31	36	91	91
37185000007	SHOCCO	2	2	2	1	27	27
37185000013	NORLINA	2	2	2	1	27	27

Table 13: Locations of identified individuals, Part 3 of 3. For each location, the district numbers are given for the six plans discussed here. VTDs are listed rather than the more precise census block in order to protect privacy. Rows highlighted **blue** indicate individuals who lose Democratic opportunity in at least one of the enacted plans, relative to the alternative plans. Rows highlighted **orange** indicate Black voters who lose the opportunity to be in an effective district for Black candidates of choice in at least one level. (As it turns out, every instance of lost opportunity for Black voters is also an instance of lost Democratic opportunity.)

References

[1] Assaf Bar-Natan, Lorenzo Najt, and Zachary Schutzmann, *The gerrymandering jumble: map projections permute districts' compactness scores*. Cartography and Geographic Information Science, Volume 47, Issue 4, 2020, 321–335.

[2] Richard Barnes and Justin Solomon, *Gerrymandering and Compactness: Implementation Flexibility and Abuse.* Political Analysis, Volume 29, Issue 4, October 2021, 448–466.

[3] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, *Computational redistricting and the Voting Rights Act.* Election Law Journal.
Available at https://www.liebertpub.com/doi/epdf/10.1089/elj.2020.0704

[4] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, *NC General Assembly County Clusterings from the 2020 Census.*
https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf

[5] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, *Locating the Representational Baseline: Republicans in Massachusetts.* Election Law Journal, Volume 18, Number 4, 2019, 388–401.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 23 day of December, 2021.

Professor Moon Duchin

Sworn and subscribed before me
this the 23rd of December, 2021

SHANNON C PETERSON
Notary Public
State of Colorado
Notary ID # 20214018369
My Commission Expires 05-10-2025

Notary Public

Name: Shannon C Peterson

My commission expires: 05/10/2025

Moon Duchin

moon.duchin@tufts.edu - mduchin.math.tufts.edu
Mathematics · STS · Tisch College of Civic Life | Tufts University

Education

University of Chicago MS 1999, PhD 2005
Mathematics
Advisor: Alex Eskin Dissertation: *Geodesics track random walks in Teichmüller space*

Harvard University BA 1998
Mathematics and Women's Studies

Appointments

Tufts University
Professor of Mathematics 2021—
Assistant Professor, Associate Professor 2011–2021

Director | Program in Science, Technology, & Society 2015–2021
(on leave 2018–2019)

Principal Investigator | MGGG Redistricting Lab 2017—

Senior Fellow | Tisch College of Civic Life 2017—

University of Michigan
Assistant Professor (postdoctoral) 2008–2011

University of California, Davis
NSF VIGRE Postdoctoral Fellow 2005–2008

Research Interests

Data science for civil rights, computation and governance, elections, geometry and redistricting.
Science, technology, and society, science policy, technology and law.
Random walks and Markov chains, random groups, random constructions in geometry.
Large-scale geometry, metric geometry, isoperimetric inequalities.
Geometric group theory, growth of groups, nilpotent groups, dynamics of group actions.
Geometric topology, hyperbolicity, Teichmüller theory.

Awards & Distinctions

Research Professor - MSRI Program in Analysis and Geometry of Random Spaces Spring 2022
Guggenheim Fellow 2018
Radcliffe Fellow - Evelyn Green Davis Fellowship 2018–2019
Fellow of the American Mathematical Society elected 2017
NSF C-ACCEL (PI) - Harnessing the Data Revolution: Network science of Census data 2019–2020
NSF grants (PI) - CAREER grant and three standard Topology grants 2009–2022
Professor of the Year, Tufts Math Society 2012–2013
AAUW Dissertation Fellowship 2004–2005
NSF Graduate Fellowship 1998–2002
Lawrence and Josephine Graves Prize for Excellence in Teaching (U Chicago) 2002
Robert Fletcher Rogers Prize (Harvard Mathematics) 1995–1996

Mathematics Publications & Preprints

The (homological) persistence of gerrymandering
Foundations of Data Science, online first. (with Thomas Needham and Thomas Weighill)

You can hear the shape of a billiard table: Symbolic dynamics and rigidity for flat surfaces
Commentarii Mathematici Helvetici, to appear. arXiv:1804.05690
(with Viveka Erlandsson, Christopher Leininger, and Chandrika Sadanand)

Conjugation curvature for Cayley graphs
Journal of Topology and Analysis, online first. (with Assaf Bar-Natan and Robert Kropholler)

A reversible recombination chain for graph partitions
Preprint. (with Sarah Cannon, Dana Randall, and Parker Rule)

Recombination: A family of Markov chains for redistricting
Harvard Data Science Review. Issue 3.1, Winter 2021. online. (with Daryl DeFord and Justin Solomon)

Census TopDown: The impact of differential privacy on redistricting
2nd Symposium on Foundations of Responsible Computing (FORC 2021), 5:1–5:22. online.
(with Aloni Cohen, JN Matthews, and Bhushan Suwal)

Stars at infinity in Teichmüller space
Geometriae Dedicata, Volume 213, 531–545 (2021). (with Nate Fisher) arXiv:2004.04321

Random walks and redistricting: New applications of Markov chain Monte Carlo
(with Daryl DeFord) For edited volume, Political Geometry. Under contract with Birkhäuser.

Mathematics of nested districts: The case of Alaska
Statistics and Public Policy. Vol 7, No 1 (2020), 39–51. (w/ Sophia Caldera, Daryl DeFord, Sam Gutekunst, & Cara Nix)

A computational approach to measuring vote elasticity and competitiveness
Statistics and Public Policy. Vol 7, No 1 (2020), 69–86. (with Daryl DeFord and Justin Solomon)

The Heisenberg group is pan-rational
Advances in Mathematics **346** (2019), 219–263. (with Michael Shapiro)

Random nilpotent groups I
IMRN, Vol 2018, Issue 7 (2018), 1921–1953. (with Matthew Cordes, Yen Duong, Meng-Che Ho, and Ayla Sánchez)

Hyperbolic groups
chapter in *Office Hours with a Geometric Group Theorist*, eds. M.Clay,D.Margalit, Princeton U Press (2017), 177–203.

Counting in groups: Fine asymptotic geometry
Notices of the American Mathematical Society **63**, No. 8 (2016), 871–874.

A sharper threshold for random groups at density one-half
Groups, Geometry, and Dynamics **10**, No. 3 (2016), 985–1005.
(with Katarzyna Jankiewicz, Shelby Kilmer, Samuel Lelièvre, John M. Mackay, and Ayla Sánchez)

Equations in nilpotent groups
Proceedings of the American Mathematical Society **143** (2015), 4723–4731. (with Hao Liang and Michael Shapiro)

Statistical hyperbolicity in Teichmüller space
Geometric and Functional Analysis, Volume 24, Issue 3 (2014), 748–795. (with Howard Masur and Spencer Dowdall)

Fine asymptotic geometry of the Heisenberg group
Indiana University Mathematics Journal 63 No. 3 (2014), 885–916. (with Christopher Mooney)

Pushing fillings in right-angled Artin groups
Journal of the LMS, Vol 87, Issue 3 (2013), 663–688. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

Spheres in the curve complex
In the Tradition of Ahlfors and Bers VI, Contemp. Math. **590** (2013), 1–8. (with Howard Masur and Spencer Dowdall)

The sprawl conjecture for convex bodies
Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122. (with Samuel Lelièvre and Christopher Mooney)

Filling loops at infinity in the mapping class group
Michigan Math. J., Vol 61, Issue 4 (2012), 867–874. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

The geometry of spheres in free abelian groups
Geometriae Dedicata, Volume 161, Issue 1 (2012), 169–187. (with Samuel Lelièvre and Christopher Mooney)

Statistical hyperbolicity in groups
Algebraic and Geometric Topology **12** (2012) 1–18. (with Samuel Lelièvre and Christopher Mooney)

Length spectra and degeneration of flat metrics
Inventiones Mathematicae, Volume 182, Issue 2 (2010), 231–277. (with Christopher Leininger and Kasra Rafi)

Divergence of geodesics in Teichmüller space and the mapping class group
Geometric and Functional Analysis, Volume 19, Issue 3 (2009), 722–742. (with Kasra Rafi)

Curvature, stretchiness, and dynamics
In the Tradition of Ahlfors and Bers IV, Contemp. Math. **432** (2007), 19–30.

Geodesics track random walks in Teichmüller space
PhD Dissertation, University of Chicago 2005.

Science, Technology, Law, and Policy Publications & Preprints

Models, Race, and the Law
Yale Law Journal Forum, Vol. 130 (March 2021). Available online. (with Doug Spencer)

Computational Redistricting and the Voting Rights Act
Election Law Journal, Available online. (with Amariah Becker, Dara Gold, and Sam Hirsch)

Discrete geometry for electoral geography
Preprint. (with Bridget Eileen Tenner) arXiv:1808.05860

Implementing partisan symmetry: Problems and paradoxes
Political Analysis, to appear. (with Daryl DeFord, Natasha Dhamankar, Mackenzie McPike, Gabe Schoenbach, and Ki-Wan Sim) arXiv:2008:06930

Clustering propensity: A mathematical framework for measuring segregation
Preprint. (with Emilia Alvarez, Everett Meike, and Marshall Mueller; appendix by Tyler Piazza)

Locating the representational baseline: Republicans in Massachusetts
Election Law Journal, Volume 18, Number 4, 2019, 388–401.
(with Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen)

Redistricting reform in Virginia: Districting criteria in context
Virginia Policy Review, Volume XII, Issue II, Spring 2019, 120–146. (with Daryl DeFord)

Geometry v. Gerrymandering
The Best Writing on Mathematics 2019, ed. Mircea Pitici. Princeton University Press.
reprinted from Scientific American, November 2018, 48–53.

Gerrymandering metrics: How to measure? What's the baseline?
Bulletin of the American Academy for Arts and Sciences, Vol. LXII, No. 2 (Winter 2018), 54–58.

Rebooting the mathematics of gerrymandering: How can geometry track with our political values?
The Conversation (online magazine), October 2017. (with Peter Levine)

A formula goes to court: Partisan gerrymandering and the efficiency gap
Notices of the American Mathematical Society **64** No. 9 (2017), 1020–1024. (with Mira Bernstein)

International mobility and U.S. mathematics
Notices of the American Mathematical Society **64**, No. 7 (2017), 682–683.

Graduate Advising in Mathematics

Nate Fisher (PhD 2021), Sunrose Shrestha (PhD 2020), Ayla Sánchez (PhD 2017), Kevin Buckles (PhD 2015), Mai Mansouri (MS 2014)

Outside committee member for Chris Coscia (PhD 2020), Dartmouth College

Postdoctoral Advising in Mathematics

Principal supervisor Thomas Weighill (2019–2020)

Co-supervisor Daryl DeFord (MIT 2018–2020), Rob Kropholler (2017–2020), Hao Liang (2013–2016)

Teaching

Courses Developed or Customized

Mathematics of Social Choice | sites.tufts.edu/socialchoice
Voting theory, impossibility theorems, redistricting, theory of representative democracy, metrics of fairness.

History of Mathematics | sites.tufts.edu/histmath
Social history of mathematics, organized around episodes from antiquity to present. Themes include materials and technologies of creation and dissemination, axioms, authority, credibility, and professionalization. In-depth treatment of mathematical content from numeration to cardinal arithmetic to Galois theory.

Reading Lab: Mathematical Models in Social Context | sites.tufts.edu/models
One hr/wk discussion seminar of short but close reading on topics in mathematical modeling, including history of psychometrics; algorithmic bias; philosophy of statistics; problems of model explanation and interpretation.

Geometric Literacy
Module-based graduate topics course. Modules have included: p-adic numbers, hyperbolic geometry, nilpotent geometry, Lie groups, convex geometry and analysis, the complex of curves, ergodic theory, the Gauss circle problem.

Markov Chains (graduate topics course)
Teichmüller Theory (graduate topics course)
Fuchsian Groups (graduate topics course)
Continued Fractions and Geometric Coding (undergraduate topics course)
Mathematics for Elementary School Teachers

Standard Courses

Discrete Mathematics, Calculus I-II-III, Intro to Proofs, Linear Algebra, Complex Analysis, Differential Geometry, Abstract Algebra, Graduate Real Analysis, Mathematical Modeling and Computation

Weekly Seminars Organized

- Geometric Group Theory and Topology
- Science, Technology, and Society Lunch Seminar

Selected Talks and Lectures

Distinguished Plenary Lecture June 2021
75th Anniversary Meeting of Canadian Mathematical Society, Ottawa, Ontario *online (COVID)*

BMC/BAMC Public Lecture April 2021
Joint British Mathematics/Applied Mathematics Colloquium, Glasgow, Scotland *online (COVID)*

AMS Einstein Public Lecture in Mathematics [March 2020]
Southeastern Sectional Meeting of the AMS, Charlottesville, VA *postponed*

Gerald and Judith Porter Public Lecture
AMS-MAA-SIAM, Joint Mathematics Meetings, San Diego, CA January 2018

Mathematical Association of America Distinguished Lecture
MAA Carriage House, Washington, DC October 2016

American Mathematical Society Invited Address
AMS Eastern Sectional Meeting, Brunswick, ME September 2016

Named University Lectures

- Parsons Lecture | UNC Asheville — October 2020
- Loeb Lectures in Mathematics | Washington University in St. Louis — [March 2020]
- Math, Stats, CS, and Society | Macalester College — October 2019
- MRC Public Lecture | Stanford University — May 2019
- Freedman Memorial Colloquium | Boston University — March 2019
- Julian Clancy Frazier Colloquium Lecture | U.S. Naval Academy — January 2019
- Barnett Lecture | University of Cincinnati — October 2018
- School of Science Colloquium Series | The College of New Jersey — March 2018
- Kieval Lecture | Cornell University — February 2018
- G. Milton Wing Lectures | University of Rochester — October 2017
- Norman Johnson Lecture | Wheaton College — September 2017
- Dan E. Christie Lecture | Bowdoin College — September 2017

Math/Computer Science Department Colloquia

- Reed College — Dec 2020
- Georgetown (CS) — Sept 2020
- Santa Fe Institute — July 2020
- UC Berkeley — Sept 2018
- Brandeis-Harvard-MIT-NEU — Mar 2018
- Northwestern University — Oct 2017
- University of Illinois — Sept 2017
- University of Utah — Aug 2017
- Wesleyan — Dec 2016
- Worcester Polytechnic Inst. — Dec 2016
- Université de Neuchâtel — Jun 2016
- Brandeis University — Mar 2016
- Swarthmore College — Oct 2015
- Bowling Green — May 2015
- City College of New York — Feb 2015
- Indiana University — Nov 2014
- the Technion — Oct 2014
- Wisconsin–Madison — Sept 2014
- Stony Brook — March 2013

Minicourses

- Integer programming and combinatorial optimization (two talks) | Georgia Tech — May 2021
- Workshop in geometric topology (main speaker, three talks) | Provo, UT — June 2017
- Growth in groups (two talks) | MSRI, Berkeley, CA — August 2016
- Hyperbolicity in Teichmüller space (three talks) | Université de Grenoble — May 2016
- Counting and growth (four talks) | IAS Women's Program, Princeton — May 2016
- Nilpotent groups (three talks) | Seoul National University — October 2014
- Sub-Finsler geometry of nilpotent groups (five talks) | Galatasaray Univ., Istanbul — April 2014

Science, Technology, and Society

- The Mathematics of Accountability | Sawyer Seminar, Anthropology, Johns Hopkins — February 2020
- STS Circle | Harvard Kennedy School of Government — September 2019
- Data, Classification, and Everyday Life Symposium | Rutgers Center for Cultural Analysis — January 2019
- Science Studies Colloquium | UC San Diego — January 2019
- Arthur Miller Lecture on Science and Ethics | MIT Program in Science, Tech, and Society — November 2018

Data Science, Computer Science, Quantitative Social Science

- Data Science for Social Good Workshop (DS4SG) | Georgia Tech (virtual) — November 2020
- Privacy Tools Project Retreat | Harvard (virtual) — May 2020
- Women in Data Science Conference | Microsoft Research New England — March 2020
- Quantitative Research Methods Workshop | Yale Center for the Study of American Politics — February 2020
- Societal Concerns in Algorithms and Data Analysis | Weizmann Institute — December 2018
- Quantitative Collaborative | University of Virginia — March 2018
- Quantitative Social Science | Dartmouth College — September 2017
- Data for Black Lives Conference | MIT — November 2017

Political Science, Geography, Law, Democracy, Fairness

- The Long 19th Amendment: Women, Voting, and American Democracy | Radcliffe Institute — Nov–Dec 2020
- "The New Math" for Civil Rights | Social Justice Speaker Series, Davidson College — November 2020
- Math, Law, and Racial Fairness | Justice Speaker Series, University of South Carolina — November 2020
- Voting Rights Conference | Northeastern Public Interest Law Program — September 2020
- Political Analysis Workshop | Indiana University — November 2019
- Program in Public Law Panel | Duke Law School — October 2019
- Redistricting 2021 Seminar | University of Chicago Institute of Politics — May 2019
- Geography of Redistricting Conference Keynote | Harvard Center for Geographic Analysis — May 2019
- Political Analytics Conference | Harvard University — November 2018
- Cyber Security, Law, and Society Alliance | Boston University — September 2018
- Clough Center for the Study of Constitutional Democracy | Boston College — November 2017
- Tech/Law Colloquium Series | Cornell Tech — November 2017
- Constitution Day Lecture | Rockefeller Center for Public Policy, Dartmouth College — September 2017

Editorial Boards

Harvard Data Science Review
Associate Editor — since 2019

Advances in Mathematics
Member, Editorial Board — since 2018

Selected Professional and Public Service

Amicus Brief of Mathematicians, Law Professors, and Students 2019
principal co-authors: Guy-Uriel Charles and Moon Duchin
Supreme Court of the United States, in Rucho v. Common Cause - cited in dissent

Committee on Science Policy 2020–2023
American Mathematical Society

Program Committee 2020–2021
Symposium on Foundations of Responsible Computing

Presenter on Public Mapping, Statistical Modeling 2019, 2020
National Conference of State Legislatures

Committee on the Human Rights of Mathematicians 2016–2019
American Mathematical Society

Committee on The Future of Voting: Accessible, Reliable, Verifiable Technology 2017–2018
National Academies of Science, Engineering, and Medicine

Visiting Positions and Residential Fellowships

Visiting Professor Department of Mathematics Fall 2021
Boston College | Chestnut Hill, MA

Fellow Radcliffe Institute for Advanced Study 2018–19
Harvard University | Cambridge, MA

Member Center of Mathematical Sciences and Applications 2018–19
Harvard University | Cambridge, MA

Visitor Microsoft Research Lab 2018–19
MSR New England | Cambridge, MA

Research Member Geometric Group Theory program Fall 2016
Mathematical Sciences Research Institute | Berkeley, CA

Research Member Random Walks and Asymptotic Geometry of Groups program Spring 2014
Institut Henri Poincaré | Paris, France

Research Member Low-dimensional Topology, Geometry, and Dynamics program Fall 2013
Institute for Computational and Experimental Research in Mathematics | Providence, RI

Research Member Geometric and Analytic Aspects of Group Theory program May 2012
Institut Mittag-Leffler | Stockholm, Sweden

Research Member Quantitative Geometry program Fall 2011
Mathematical Sciences Research Institute | Berkeley, CA

Postdoctoral Fellow Teichmüller "project blanc" Spring 2009
Agence Nationale de la Recherche (Collège de France) | Paris, France

STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,

REBECCA HARPER, et al.,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

AFFIDAVIT OF PROFESSOR MOON DUCHIN

NCLCV v. Hall
21 CVS 15426
LDTX159

Exhibit #
Duchin 3
12/30/21 - SL
exhibitsticker.com

I, Dr. Moon Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.
2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.
4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.
5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.
6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.
7. A current copy of my full CV is attached to this report.
8. I am compensated at the rate of $400 per hour.

Rebuttal Report

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

December 28, 2021

1 Background and Introduction

I have previously submitted expert reports in NCLCV vs. Hall. I have been asked by counsel to respond to the report of Dr. Michael Barber, examining his study design and his conclusions.

1.1 Summary of Barber report

In Dr. Barber's report, he uses a new statistical sampling method called Sequential Monte Carlo (SMC) to produce a large collection (called an *ensemble*) of alternative districting plans for both bodies of the North Carolina state legislature—state Senate and state House. SMC is a method based on ideas developed in my research group,[1] but which has not been supported by any peer-reviewed publications.

Dr. Barber proceeds to build ensembles of districting plans for the purposes of comparison, but primarily does so individually on small pieces of the state: groups of counties (often called "county clusters") that correspond to groupings in the Senate and House plans recently enacted in North Carolina (SL-173 and SL-175).

- For legislative redistricting, the Barber report discusses the clusters only on an individual basis, neglecting to assemble them into the big picture for the whole state.
- Dr. Barber omits an ensemble comparison for the enacted Congressional plan, SL-174.

1.2 Summary of findings

- When assembling the statistics from Dr. Barber's own ensembles—completely granting him all methodological choices for algorithm selection and specifications—the enacted House plan is shown to be a major partisan outlier, while the NCLCV alternative plans are not (Figure 6).
- In exactly the same way, the enacted Senate plan is likewise shown to be a major partisan outlier, while the NCLCV alternative plans are not (Figure 5).
- Finally, I was able to run Barber's code to create a Congressional ensemble in the same fashion as his legislative ensembles. Here, too, the enacted plan is a significant outlier in a direction of partisan advantage that is not justified by any good-government goal (Figure 3).

[1] The McCartan–Imai article introducing SMC [5] acknowledges Deford–Duchin–Solomon [3] for "pioneer[ing] the spanning tree-based proposal used in the merge-split algorithm."

2 Ensembles and outliers

Today, the dominant method in computational redistricting analysis is to employ Markov chains to generate ensembles of thousands or millions of alternative valid redistricting plans against which to compare a given proposed plan. When a quantity of interest is measured over the ensemble, it frequently forms a "bell curve" of values, and we can then examine whether the proposed plan falls in the thick of the observed values or whether it is an extreme outlier, falling in one of the tails. If this exercise is carried out with respect to each party's representation, a telltale sign of a partisan gerrymander is when the seat share for a proposed plan falls (a) far from the corresponding vote share, and (b) far to the side of advantage for the party that controlled the line-drawing process. This is particularly problematic in a politically competitive "purple" state like North Carolina.

It is important to note that outlier status is a flag of intentionality, but not necessarily a smoking gun of wrongdoing. Being in a tails of a distribution that was created around certain design principles can often provide persuasive evidence that other principles or agendas were in play. For example, a map might be an outlier as the most compact, or the map that gives minority groups the greatest chance to elect their candidates of choice—these kinds of outlier status would not be marks of a bad plan. But being an outlier can indeed be a sign of problems, as when a plan systematically converts close voting to lopsided seat shares for the party that controls the process.

2.1 Barber methods

The creation and use of districting ensembles in the Barber report can be summarized as follows.

Step 1 *Fix a set of clusters.* Barber focuses on the county clustering found in the enacted plan, not exhaustively considering the dozens of other possibilities.

Step 2 *Partition each cluster.* Split each multi-district cluster into the corresponding number of districts using Sequential Monte Carlo sampling. Create 50,000 partitions (i.e., districting plans) for each cluster.

Step 3 *Winnow.* Selectively discard some of the partitions. Barber uses two statistics from the enacted plan (average Polsby-Popper score and county traversals) as the cutoff for inclusion.

Step 4 *Create an election index.* Barber blends the 11 up-ballot elections since 2014 into a single vote index rather than considering them one at a time. In particular, he sums the votes over all elections before taking shares, which does not control for turnout differences across elections.

Step 5 *Plot histograms and declare outliers.* Barber forms histograms counting "Democratic-leaning districts" for individual clusters, and does not present an overall compilation. His non-standard definition of "outlier" includes a full 50% of the ensemble.

In my opinion, better and more reliable results would have been obtained if several of the choices required in this study design were executed differently.

One glaring omission from Barber's methods is any consideration of the State's obligations under the Voting Rights Act of 1965, which could impact the partisan bottom line.[2] A non-exhaustive list of other potential flaws in Dr. Barber's methods includes the following.

- *Failure to consider all alternative clusterings.*
  North Carolina law dictates that districts be drawn within groupings or clusters of counties from which several districts will be formed. Sometimes, however, the General Assembly has a choice and can pick multiple groupings consistent with North Carolina law. Dr. Barber only gives cursory attention to alternative clusterings.
- *Use of sampling methodology not vetted by peer review.*
  Even when an idea is promising, peer review is an essential component of vetting. A method may appear promising in concept, but not work in practice. A method may work at small tasks—like the 34-map dataset used for testing in [5]—but not scale well to the enormous sizes needed for realistic problems. Peer review helps surface those issues, which is why the scientific community regards peer review as a mark of reliability.
- *Use of bright-line thresholds for compactness and traversals.*
  Dr. Barber's code already samples with a preference for compactness, and is fully capable of handling traversals in a similar manner.[3] Imposing sharp cutoffs for these at the level of the enacted plan creates highly misleading results.[4]
- *Use of election data in a blended rather than serial fashion.*
  If Barber records a Democratic share of 49% in his outputs, that is likely to reflect a Democratic win in some of the 11 elections and a Republican win in others—this is obscured when the results are blended to a single number. By the same token, a Democratic share of 45% in the blended election index might downplay a map that favors Republicans 11 out of 11 times, which entrenches an advantage.[5]
- *Employing a highly unconventional use of the "outlier" label.*
  As Dr. Barber himself puts it, "I consider a plan to be a partisan outlier if the number of Democratic districts generated by the plan falls outside the middle 50% of simulation results [sic]. This is a conservative definition of an outlier. In the social sciences, medicine, and other disciplines it is traditional to consider something an outlier if it falls outside the middle 95% or 90% of the comparison distribution." As I will show below in my whole-state comparisons, the enacted plans are outliers at any of these levels of significance, while the NCLCV alternative plans are not.

I will discuss the thresholding question further in §2.3. For the remainder of the report, I will set aside the other concerns and will simply assess Dr. Barber's outputs within his own methodological framework.

[2] Robust VRA consideration is fully compatible with computational redistricting, as is shown in [1].

[3] A preference for compactness is coded in the `smc_redist` parameterization in `house_clusters.R`, lines 354–356 and `senate_clusters.R`, lines 349–351.

[4] The imposition of cutoffs, which Dr. Barber calls "culling," occurs in two stages. Stage 1 (country traversals) is found in `house_clusters.R`, lines 531–536 and `senate_clusters.R`, lines 539–544. Stage 2 (average Polsby-Popper) is found in `house_clusters.R`, line 543–564 and `senate_clusters.R`, lines 552–573. An ad hoc adjustment in the Duplin and Wayne House County Grouping is found in lines 566-568 of the House code.

[5] The 49% Democratic lean occurs, for instance, in the NCLCV alternative maps in the Onslow/Pender House cluster. Vote averaging is found in the Barber replication materials in `house_clusters.R` lines 18-28 and `senate_clusters.R` lines 18-29.

2.2 Analysis methods

Reading Dr. Barber's report, it is striking that he only reported that the enacted plan often performed within the middle 50% of each small comparison while never evaluating how the individual choices aggregate at the level of the map as a whole. After all, if moderate partisan advantage is secured over and over again, it may well accrue to extreme advantage overall. In the context of a state legislature, the overall results are crucial: they determine who controls the chamber. Pursuing this in the Barber materials, I found that this is exactly what happens.

First, I was able to extract Dr. Barber's raw statistical outputs for legislative runs from his materials obtained by counsel.[6] With those, I was able to assemble his ensembles for individual clusters into a compiled ensemble for the entire state. The histogram of Senate outcomes can be found in Figure 6 and the histogram of House outcomes can be found in Figure 5. Second, I was able to run Dr. Barber's code to create an ensemble of alternative Congressional plans with exactly the algorithm and with similar specifications to those he used for his legislative demonstrations.[7] A corresponding plot of Congressional outcomes can be found in Figure 3. For all phases of analysis, Dr. Barber pulled electoral data from a free webapp called Dave's Redistricting App (`davesredistricting.org`). In replicating his analysis, I used the same data source in the same manner.

2.3 Filtered and unfiltered results

As I described above, Dr. Barber took his raw districting plan samples (50,000 maps created for each of 12 Senate cluster ensembles and 26 House cluster ensembles) and aggressively filtered them, applying a cutoff that sometimes left under ten maps out of the original set of 50,000. In fact, when Dr. Barber's filtering rule was applied in the Duplin and Wayne House County Grouping (§6.6 on p.58 of Barber Report), *zero* maps were left, because none of the randomly constructed maps had an average compactness score to match the enacted plan in that cluster. Since this is blatantly unworkable for comparison purposes, Dr. Barber made the ad hoc decision to loosen the rule to retain 2704 maps. Other cluster ensembles were filtered down to leave only 4, 6, or 2 out of 50,000 alternatives and did not receive an adjustment. The "outlier" label was then applied to these tiny sets.

To illustrate why this is methodologically unreasonable, consider JaVale McGee, a basketball center who recently signed with the Phoenix Suns of the NBA on a one-year, $5 million contract. If McGee wanted to argue that he is not unusually wealthy, he could choose to restrict the universe of comparison to Americans at least as tall as he is. Since he is 7 feet tall, this would greatly restrict the comparison pool to a relatively tiny group that also includes Mo Bamba (Orlando Magic), Joel Embiid (Philadelphia 76ers), and Brook Lopez (Milwaukee Bucks), all of whom make more money than he does. Not satisfied with this comparison, he could keep increasing the requirements by insisting on comparing to people who don't speak any more languages than he does, are no older than he is, and have lived in at least as many different cities. Eventually he will narrow the pool enough that he doesn't look like an outlier anymore.

Dr. Barber's filtering skews his sample in a similar way, because he effectively insists that maps have a statistic matching or exceeding the enacted map in every cluster—and then uses that pool to compare the enacted map. Overall, this reduces the number of plans under consideration by a factor of over 500 trillion. And it excludes options that may be better than the enacted plan overall but are less compact or have more traversals in a particular cluster.

Generally, if you are trying to argue that you look typical of a range of alternatives, it is obviously unreasonable to first require the alternatives to look like you in dozens of independent ways (i.e., in each cluster individually).

[6] His materials include the numerical outputs from his runs, but as far as I can determine he does not seem to have saved the district assignments for the individual plans in the ensemble.

[7] To be precise, the ensemble was generated at the state level for Congress, since the concept of county clusters is not applicable, and without the compactness and traversal thresholds. I ran the code exactly as Dr. Barber did, except tightening the allowed population deviation to 1% from ideal instead of 5% as in legislative maps. All other choices are identical. My congressional ensemble includes 20,000 maps rather than 50,000 just because of time limitations.

3 Findings

In this section, I will present the full histograms (or "bell curves") of all the results from Dr. Barber's methodology, compiled to the state level and shown without filtering. (Filtered ensembles can be seen in Appendix A, for comparison purposes.)

By Dr. Barber's own constructs, all three levels of districting show that **the enacted plans are partisan outliers and the NCLCV alternative plans are not**.

In the House, the enacted map is in the most extreme 0.00133 fraction of the Barber ensemble—well under 1 percent of sampled House plans are as extreme as SL-175. By contrast, the NCLCV alternative plan is in the upper .2516 share of the ensemble, not an outlier even by the Barber standard.

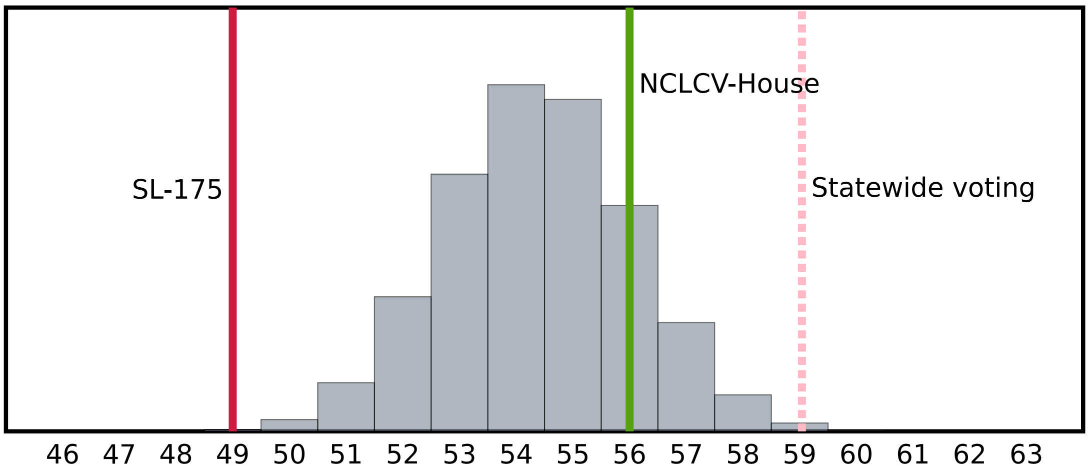

Figure 1: "Democratic-leaning seats" in Dr. Barber's House district ensemble.

At the Senate level, the enacted map is in the most extreme .007 fraction of the Barber ensemble—again, less than 1 percent of sampled plans are as extreme as SL-173. By contrast, the NCLCV alternative map is in the upper .2787 share of ensemble, not an outlier even by the Barber standard.

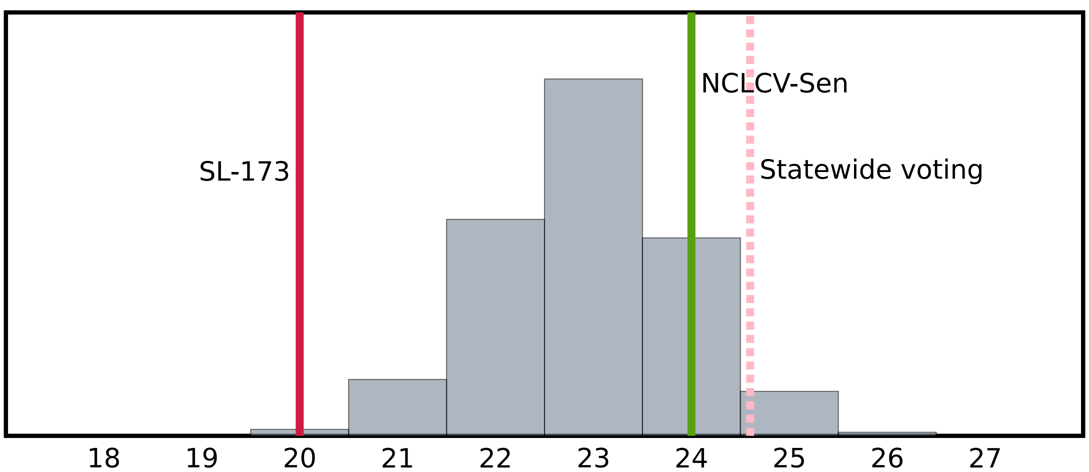

Figure 2: "Democratic-leaning seats" in Dr. Barber's Senate district ensemble.

The Congressional picture, omitted from the Barber report, is likewise crystal clear. The enacted plan is in the most extreme 0.0056 fraction of this Barber-style ensemble, while the NCLCV alternative map is very near the ensemble center—0.5620 share of the ensemble (more than half of randomly constructed maps) has an equal or greater Democratic lean.

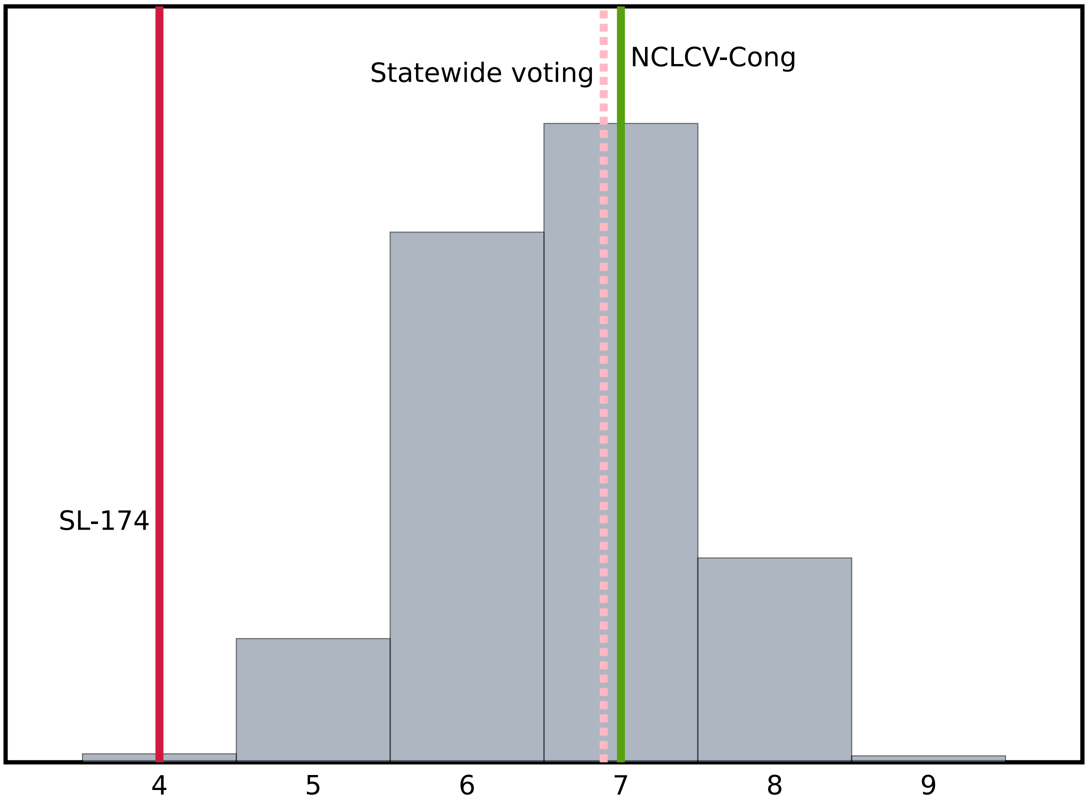

Figure 3: "Democratic-leaning seats" in a Congressional ensemble created with Dr. Barber's code, following his specifications.

4 Conclusion

Granting Dr. Barber all of his methodological choices, the enacted maps are extreme partisan outliers at all three levels, while the NCLCV alternative maps are not.

References

[1] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, *Computational redistricting and the Voting Rights Act.* Election Law Journal. Available online.

[2] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, *NC General Assembly County Clusterings from the 2020 Census.* sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf

[3] Daryl DeFord, Moon Duchin, and Justin Solomon, *Recombination: A Family of Markov Chains for Redistricting*, Harvard Data Science Review. Issue 3.1, Winter 2021. Available online.

[4] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, *Locating the Representational Baseline: Republicans in Massachusetts.* Election Law Journal, Volume 18, Number 4, 2019, 388–401. Available online.

[5] Cory McCartan and Kosuke Imai, *Sequential Monte Carlo for Sampling Balanced and Compact Redistricting Plans*, preprint. Available at arxiv.org/abs/2008.06131.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 28 day of December, 2021.

Professor Moon Duchin

Sworn and subscribed before me
this the 28 of December, 2021

Notary Public

Name: Simarjit Manhas

My commission expires: 02/02/2024

A notary public or other officer completing this certificate verifies only the identity of the individual who signed the document to which this certificate is attached, and not the truthfulness, accuracy, or validity of that document.

State of California, County of Alameda
Subscribed and sworn to (or affirmed) before me
on this 28 day of December, 20 21,
by Moon Duchin,
proved to me on the basis of satisfactory evidence
to be the person(~~s~~) who appeared before me.
Signature:

SIMARJIT MANHAS
COMM. #2317737
Notary Public - California
Contra Costa County
My Comm. Expires Feb. 2, 2024
NR01

Appendix A: Filtering comparison

To illustrate the skewing effects of the thresholds applied by Dr. Barber, consider a single example: the Pitt House County Cluster, where the number of Democratic-leaning seats in the sample is either 1 or 2. By thresholding compactness and traversals at the level of the enacted map, Dr. Barber is able to drop the frequency of the 2-seats outcome from roughly 25% of the sample to just 9%.

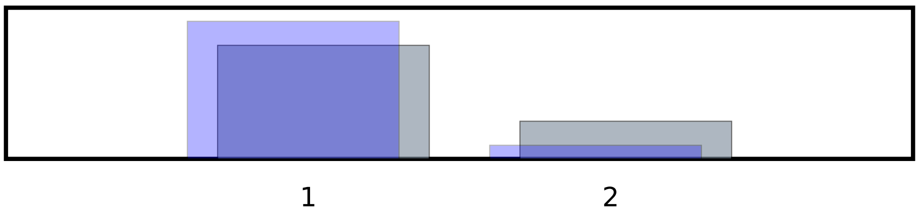

Figure 4: Just focusing on the Pitt House County Cluster (Barber report, p.42), we see that the filtering changes the outcome of 2 "Democratic-leaning seats" from occurring in roughly 25% of the full set of sampled maps (gray) to only occurring in 9% of the reduced sample (blue).

The effects of this cluster-by-cluster restriction do not wash out when aggregated to the full state, but instead add up to a noticeable shift toward the enacted plan, as demonstrated in the House and Senate figures below.

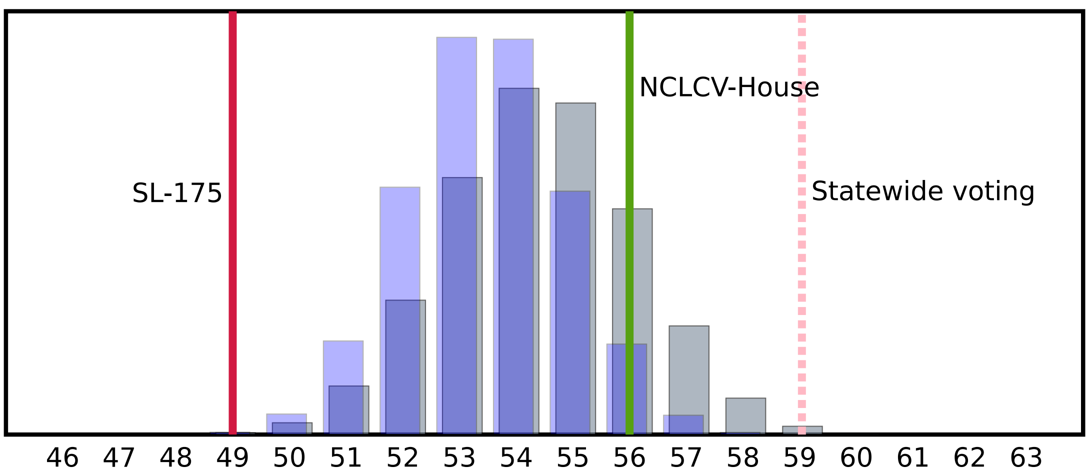

Figure 5: "Democratic-leaning seats" in Dr. Barber's House district ensemble. The unfiltered ensemble (gray) includes $50,000^{26} \approx 1.5 \cdot 10^{122}$ maps; the filtered ensemble (blue) is smaller by a factor of octillions.

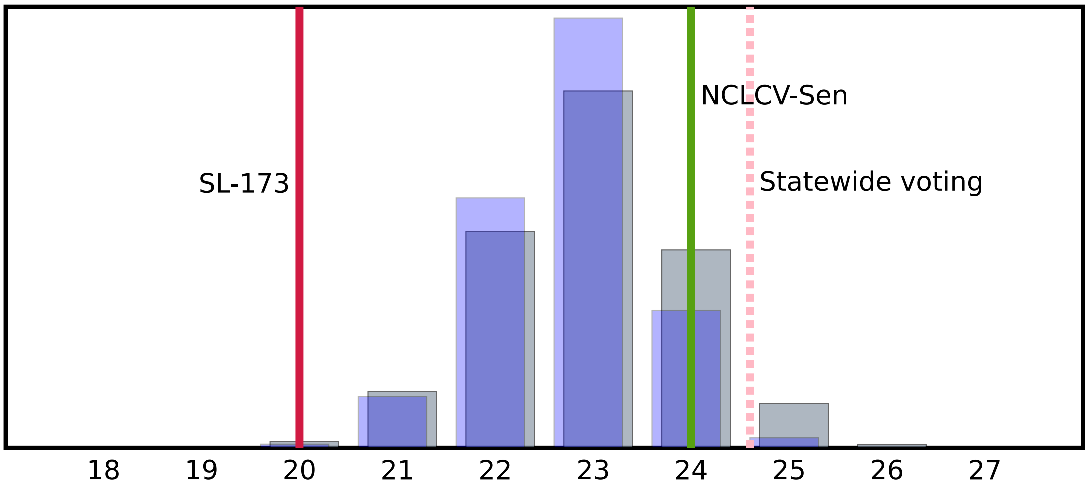

Figure 6: "Democratic-leaning seats" in Dr. Barber's Senate district ensemble. The unfiltered ensemble (gray) includes $50,000^{12} \approx 2.4 \cdot 10^{56}$ maps; the filtered ensemble (blue) is smaller by a factor of trillions.

Significantly, even the subsets of alternative plans that have been heavily limited by the cluster-by-cluster thresholds—that is, the blue bell curves instead of the gray—still show the enacted plans to be extreme outliers, while the NCLCV alternative plans are both far less extreme and comport with statewide voting.

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Education

University of Chicago MS 1999, PhD 2005
Mathematics
Advisor: Alex Eskin Dissertation: *Geodesics track random walks in Teichmüller space*

Harvard University BA 1998
Mathematics and Women's Studies

Appointments

Tufts University
Professor of Mathematics 2021—
Assistant Professor, Associate Professor 2011–2021

Director | Program in Science, Technology, & Society 2015–2021
(on leave 2018–2019)

Principal Investigator | MGGG Redistricting Lab 2017—

Senior Fellow | Tisch College of Civic Life 2017—

University of Michigan
Assistant Professor (postdoctoral) 2008–2011

University of California, Davis
NSF VIGRE Postdoctoral Fellow 2005–2008

Research Interests

Data science for civil rights, computation and governance, elections, geometry and redistricting.
Science, technology, and society, science policy, technology and law.
Random walks and Markov chains, random groups, random constructions in geometry.
Large-scale geometry, metric geometry, isoperimetric inequalities.
Geometric group theory, growth of groups, nilpotent groups, dynamics of group actions.
Geometric topology, hyperbolicity, Teichmüller theory.

Awards & Distinctions

Research Professor - MSRI Program in Analysis and Geometry of Random Spaces Spring 2022
Guggenheim Fellow 2018
Radcliffe Fellow - Evelyn Green Davis Fellowship 2018–2019
Fellow of the American Mathematical Society elected 2017
NSF C-ACCEL (PI) - Harnessing the Data Revolution: Network science of Census data 2019–2020
NSF grants (PI) - CAREER grant and three standard Topology grants 2009–2022
Professor of the Year, Tufts Math Society 2012–2013
AAUW Dissertation Fellowship 2004–2005
NSF Graduate Fellowship 1998–2002
Lawrence and Josephine Graves Prize for Excellence in Teaching (U Chicago) 2002
Robert Fletcher Rogers Prize (Harvard Mathematics) 1995–1996

Mathematics Publications & Preprints

The (homological) persistence of gerrymandering
Foundations of Data Science, online first. (with Thomas Needham and Thomas Weighill)

You can hear the shape of a billiard table: Symbolic dynamics and rigidity for flat surfaces
Commentarii Mathematici Helvetici, to appear. arXiv:1804.05690
(with Viveka Erlandsson, Christopher Leininger, and Chandrika Sadanand)

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Journal of Topology and Analysis, online first. (with Assaf Bar-Natan and Robert Kropholler)

A reversible recombination chain for graph partitions
Preprint. (with Sarah Cannon, Dana Randall, and Parker Rule)

Recombination: A family of Markov chains for redistricting
Harvard Data Science Review. Issue 3.1, Winter 2021. online. (with Daryl DeFord and Justin Solomon)

Census TopDown: The impact of differential privacy on redistricting
2nd Symposium on Foundations of Responsible Computing (FORC 2021), 5:1–5:22. online.
(with Aloni Cohen, JN Matthews, and Bhushan Suwal)

Stars at infinity in Teichmüller space
Geometriae Dedicata, Volume 213, 531–545 (2021). (with Nate Fisher) arXiv:2004.04321

Random walks and redistricting: New applications of Markov chain Monte Carlo
(with Daryl DeFord) For edited volume, Political Geometry. Under contract with Birkhäuser.

Mathematics of nested districts: The case of Alaska
Statistics and Public Policy. Vol 7, No 1 (2020), 39–51. (w/ Sophia Caldera, Daryl DeFord, Sam Gutekunst, & Cara Nix)

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Statistics and Public Policy. Vol 7, No 1 (2020), 69–86. (with Daryl DeFord and Justin Solomon)

The Heisenberg group is pan-rational
Advances in Mathematics **346** (2019), 219–263. (with Michael Shapiro)

Random nilpotent groups I
IMRN, Vol 2018, Issue 7 (2018), 1921–1953. (with Matthew Cordes, Yen Duong, Meng-Che Ho, and Ayla Sánchez)

Hyperbolic groups
chapter in *Office Hours with a Geometric Group Theorist*, eds. M.Clay,D.Margalit, Princeton U Press (2017), 177–203.

Counting in groups: Fine asymptotic geometry
Notices of the American Mathematical Society **63**, No. 8 (2016), 871–874.

A sharper threshold for random groups at density one-half
Groups, Geometry, and Dynamics **10**, No. 3 (2016), 985–1005.
(with Katarzyna Jankiewicz, Shelby Kilmer, Samuel Lelièvre, John M. Mackay, and Ayla Sánchez)

Equations in nilpotent groups
Proceedings of the American Mathematical Society **143** (2015), 4723–4731. (with Hao Liang and Michael Shapiro)

Statistical hyperbolicity in Teichmüller space
Geometric and Functional Analysis, Volume 24, Issue 3 (2014), 748–795. (with Howard Masur and Spencer Dowdall)

Fine asymptotic geometry of the Heisenberg group
Indiana University Mathematics Journal 63 No. 3 (2014), 885–916. (with Christopher Mooney)

Pushing fillings in right-angled Artin groups
Journal of the LMS, Vol 87, Issue 3 (2013), 663–688. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

Spheres in the curve complex
In the Tradition of Ahlfors and Bers VI, Contemp. Math. **590** (2013), 1–8. (with Howard Masur and Spencer Dowdall)

The sprawl conjecture for convex bodies
Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122. (with Samuel Lelièvre and Christopher Mooney)

Filling loops at infinity in the mapping class group
Michigan Math. J., Vol 61, Issue 4 (2012), 867–874. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

The geometry of spheres in free abelian groups
Geometriae Dedicata, Volume 161, Issue 1 (2012), 169–187. (with Samuel Lelièvre and Christopher Mooney)

Statistical hyperbolicity in groups
Algebraic and Geometric Topology **12** (2012) 1–18. (with Samuel Lelièvre and Christopher Mooney)

Length spectra and degeneration of flat metrics
Inventiones Mathematicae, Volume 182, Issue 2 (2010), 231–277. (with Christopher Leininger and Kasra Rafi)

Divergence of geodesics in Teichmüller space and the mapping class group
Geometric and Functional Analysis, Volume 19, Issue 3 (2009), 722–742. (with Kasra Rafi)

Curvature, stretchiness, and dynamics
In the Tradition of Ahlfors and Bers IV, Contemp. Math. **432** (2007), 19–30.

Geodesics track random walks in Teichmüller space
PhD Dissertation, University of Chicago 2005.

Science, Technology, Law, and Policy Publications & Preprints

Models, Race, and the Law
Yale Law Journal Forum, Vol. 130 (March 2021). Available online. (with Doug Spencer)

Computational Redistricting and the Voting Rights Act
Election Law Journal, Available online. (with Amariah Becker, Dara Gold, and Sam Hirsch)

Discrete geometry for electoral geography
Preprint. (with Bridget Eileen Tenner) arXiv:1808.05860

Implementing partisan symmetry: Problems and paradoxes
Political Analysis, to appear. (with Daryl DeFord, Natasha Dhamankar, Mackenzie McPike, Gabe Schoenbach, and Ki-Wan Sim) arXiv:2008:06930

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Preprint. (with Emilia Alvarez, Everett Meike, and Marshall Mueller; appendix by Tyler Piazza)

Locating the representational baseline: Republicans in Massachusetts
Election Law Journal, Volume 18, Number 4, 2019, 388–401.
(with Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen)

Redistricting reform in Virginia: Districting criteria in context
Virginia Policy Review, Volume XII, Issue II, Spring 2019, 120–146. (with Daryl DeFord)

Geometry v. Gerrymandering
The Best Writing on Mathematics 2019, ed. Mircea Pitici. Princeton University Press.
reprinted from Scientific American, November 2018, 48–53.

Gerrymandering metrics: How to measure? What's the baseline?
Bulletin of the American Academy for Arts and Sciences, Vol. LXII, No. 2 (Winter 2018), 54–58.

Rebooting the mathematics of gerrymandering: How can geometry track with our political values?
The Conversation (online magazine), October 2017. (with Peter Levine)

A formula goes to court: Partisan gerrymandering and the efficiency gap
Notices of the American Mathematical Society **64** No. 9 (2017), 1020–1024. (with Mira Bernstein)

International mobility and U.S. mathematics
Notices of the American Mathematical Society **64**, No. 7 (2017), 682–683.

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Outside committee member for Chris Coscia (PhD 2020), Dartmouth College

Postdoctoral Advising in Mathematics

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Co-supervisor Daryl DeFord (MIT 2018–2020), Rob Kropholler (2017–2020), Hao Liang (2013–2016)

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Fuchsian Groups (graduate topics course)
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- Science, Technology, and Society Lunch Seminar

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75th Anniversary Meeting of Canadian Mathematical Society, Ottawa, Ontario — *online (COVID)*

BMC/BAMC Public Lecture — April 2021
Joint British Mathematics/Applied Mathematics Colloquium, Glasgow, Scotland — *online (COVID)*

AMS Einstein Public Lecture in Mathematics — [March 2020]
Southeastern Sectional Meeting of the AMS, Charlottesville, VA — *postponed*

Gerald and Judith Porter Public Lecture
AMS-MAA-SIAM, Joint Mathematics Meetings, San Diego, CA — January 2018

Mathematical Association of America Distinguished Lecture
MAA Carriage House, Washington, DC — October 2016

American Mathematical Society Invited Address
AMS Eastern Sectional Meeting, Brunswick, ME — September 2016

Named University Lectures

- Parsons Lecture | UNC Asheville — October 2020
- Loeb Lectures in Mathematics | Washington University in St. Louis — [March 2020]
- Math, Stats, CS, and Society | Macalester College — October 2019
- MRC Public Lecture | Stanford University — May 2019
- Freedman Memorial Colloquium | Boston University — March 2019
- Julian Clancy Frazier Colloquium Lecture | U.S. Naval Academy — January 2019
- Barnett Lecture | University of Cincinnati — October 2018
- School of Science Colloquium Series | The College of New Jersey — March 2018
- Kieval Lecture | Cornell University — February 2018
- G. Milton Wing Lectures | University of Rochester — October 2017
- Norman Johnson Lecture | Wheaton College — September 2017
- Dan E. Christie Lecture | Bowdoin College — September 2017

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- Reed College — Dec 2020
- Georgetown (CS) — Sept 2020
- Santa Fe Institute — July 2020
- UC Berkeley — Sept 2018
- Brandeis-Harvard-MIT-NEU — Mar 2018
- Northwestern University — Oct 2017
- University of Illinois — Sept 2017
- University of Utah — Aug 2017
- Wesleyan — Dec 2016
- Worcester Polytechnic Inst. — Dec 2016
- Université de Neuchâtel — Jun 2016
- Brandeis University — Mar 2016
- Swarthmore College — Oct 2015
- Bowling Green — May 2015
- City College of New York — Feb 2015
- Indiana University — Nov 2014
- the Technion — Oct 2014
- Wisconsin–Madison — Sept 2014
- Stony Brook — March 2013

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- Workshop in geometric topology (main speaker, three talks) | Provo, UT — June 2017
- Growth in groups (two talks) | MSRI, Berkeley, CA — August 2016
- Hyperbolicity in Teichmüller space (three talks) | Université de Grenoble — May 2016
- Counting and growth (four talks) | IAS Women's Program, Princeton — May 2016
- Nilpotent groups (three talks) | Seoul National University — October 2014
- Sub-Finsler geometry of nilpotent groups (five talks) | Galatasaray Univ., Istanbul — April 2014

Science, Technology, and Society

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- Science Studies Colloquium | UC San Diego — January 2019
- Arthur Miller Lecture on Science and Ethics | MIT Program in Science, Tech, and Society — November 2018

Data Science, Computer Science, Quantitative Social Science

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Research Member Random Walks and Asymptotic Geometry of Groups program Spring 2014
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Computational Redistricting and the Voting Rights Act

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ABSTRACT

In recent years, computers have been used to generate *ensembles* of districting plans: collections of large numbers of electoral maps that are used to assess a proposed map in the context of valid alternatives. Ensemble-based outlier analysis has played a central role in recent redistricting disputes, especially regarding partisan gerrymandering. Until now, methods for generating these ensembles have enforced districting rules that are relatively simple to assess, such as population equality, but have not contended with more complex ones, such as the prohibitions against racial gerrymandering and minority vote dilution that flow from the Constitution and the Voting Rights Act (VRA). We take up the task of building ensembles of plans that respect those legal constraints. Rather than relying on demographic data alone, our method uses precinct-level returns from a large collection of recent primary and general elections. With this electoral history, we build *effectiveness scores* that identify districts where members of minority groups have had realistic opportunities to nominate and elect their preferred candidates. In a case study of Texas congressional districts, we find that detailed election data is indispensable to assessing a map's effectiveness for minority voters. Purely demographic targets, such as demanding some specific number of majority-minority districts, not only raise constitutional concerns but also are inadequate proxies for empirical effectiveness. Beyond the primary task of building VRA-conscious ensembles for comparison, we also repurpose the same algorithmic search methods to find plans that dramatically increase minority electoral opportunities. In Texas, for example, the current enacted 36-district congressional plan has perhaps 11 to 13 districts that are effective for Latino voters, Black voters, or both. We find that better mapmaking could raise that number to at least 16 without sacrificing traditional principles such as contiguity and compactness. This would nearly eliminate the historic underrepresentation of both groups throughout the state.

Keywords: redistricting, gerrymandering, Voting Rights Act, algorithmic ensembles

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Racial identification of candidates was made possible by a dataset purchased from Carl Klarner (klarnerpolitics.org).
This article is dedicated to the memory of Rice University sociology professor Chandler Davidson (1936–2021), who fought successfully for a half century to protect Latino and Black voting rights and to expand minority electoral opportunities in Texas and throughout the United States.

1. INTRODUCTION

Today, only 107 representatives in congress—fewer than a quarter of all House members—belong to a racial or language minority group.[1] If those groups were represented in proportion to their share of the nation's adult citizen population, that number would increase to 144 Representatives.[2] And this sub-proportional representation is not confined to Congress, but is replicated today in 47 of the 50 state legislatures.[3] There are two strands of conventional wisdom on the causes of this shortfall in minority representation. Either districters simply are not trying hard enough, or entrenched patterns of racial polarization in housing and voting make proportionality impossible to attain.

This article explores a third option: perhaps better tools can bring better results. Our algorithmically generated *ensembles*—collections of thousands or millions of alternative maps—show that better-designed redistricting plans could close much (though not all) of that gap and ensure that the House of Representatives and state legislatures "look more like America" than at any time in our history.

The tools to study this issue comprehensively did not exist as recently as a decade ago, when the 50 states last redistricted. Since then, algorithmic innovation and steadily improving computational power have revolutionized our ability to understand the variety of redistricting plans that could plausibly be enacted. It is now possible to generate a multitude of diverse, valid plans on a laptop overnight—and to describe how they are distributed in the universe of all possibilities. That in turn allows any plan, including one proposed for adoption, to be compared meaningfully to the available alternatives.

Not surprisingly, work in this direction has come to dominate some types of redistricting litigation in the last few years, especially lawsuits claiming that a districting plan is excessively partisan. But until now, ensemble methods have not seriously grappled with issues of race in redistricting. And these tend to be the most heavily litigated issues in the field, due to the demands imposed by the Voting Rights Act (VRA) and the Constitution's Equal Protection Clause. The legal rules addressing race in redistricting are much more complex than, say, the "one person, one vote" doctrine in federal constitutional law, or the contiguity requirements in state constitutional law. Modeling the racial rules is far from straightforward.

This article takes up that task. First, we develop methods that incorporate the legal rules involving the consideration of race in redistricting into the algorithms that generate redistricting ensembles. The main applications of these VRA-conscious ensembles would be to study the normal range of attributes of lawful plans, for instance to assess claims of partisan gerrymandering. Second, we show that the methods used to accomplish that task can also be used to draw maps that increase opportunities for minority groups to elect candidates of their choice. As it turns out, there is the potential to provide much more opportunity, at least in some states, than was previously recognized. In short, the algorithmic creation of redistricting ensembles holds the promise of not only sharpening our understanding of redistricting choices and tradeoffs, but also better fostering the aims of the Voting Rights Act, "a statute meant to hasten the waning of racism in American politics" (*Johnson* v. *De Grandy* 1994, 1020).

To that end, one of our strongest findings deserves particular emphasis. In the past, the dominant method of looking for effective minority electoral opportunity has been to use district demographics as a proxy, such as by seeking majority-Black districts to secure effective electoral opportunities for Black voters. But in our case studies, demographic share alone is a poor proxy for effectiveness; relying too heavily on demographics could inadvertently disempower minority citizens by packing them into too few districts.

Our methods will be most helpful for proactive legislatures and commissions that wish to draw legally defensible maps that will prove effective for racial and language minority groups while upholding other criteria simultaneously. The tools described here will generate examples of maps with valuable properties and will help elucidate the cost in minority electoral opportunity, if any, that results from strict application of lower-ranked criteria. Although these tools also may be helpful to

[1]Bialik (2019). This figure refers to the 116th Congress (2019–2021).

[2]This number is based on 2019 one-year American Community Survey (ACS) data, U.S. Bureau of the Census (2019a), figured as the share of citizen voting-age population comprising those who are either Hispanic/Latino or from a non-white racial group.

[3]*See* U.S. Bureau of the Census (2019b); National Conference of State Legislatures (2020). Putting those sources together, the three exceptions are Arizona (34.4% minority citizen voting-age population vs. 38% minority legislators), Hawaii (73.2% vs. 76%), and Ohio (16.7% vs. 18%).

plaintiffs who wish to challenge existing maps under the VRA, that use is not our main focus.

We will use three main elements: a Markov chain procedure that proposes successive modifications to districting plans, an ecological-inference procedure that identifies minority-preferred candidates based on precinct-level historical election data matched to demographics, and a benchmark plan from which we can establish a presumptively acceptable number of effective districts.

Below, for our proof of concept, we will use a spanning-tree recombination procedure for the first element, a hierarchical Bayesian model for the second, and an enacted plan that has survived VRA scrutiny for the third[4]—but we emphasize that the main contribution of the current article is the overarching protocol, which is designed to be *modular*, letting users substitute in other alternatives to play these three roles. Combining these elements, our protocol defines *effective* districts for minority groups at any given threshold of confidence.

Article Outline. We begin in section 2 with a review of the burgeoning science of redistricting ensembles. Section 3 summarizes the legal rules governing the consideration of race and racial data in redistricting. Section 4 sets forth our VRA-conscious ensemble protocol, relying on recent election data to generate effectiveness scores that rate each district's likelihood of nominating and electing minority-preferred candidates. Section 5 applies this protocol to congressional redistricting in Texas, where both Latino and Black residents are numerous enough to require VRA attention. Section 6 applies techniques from statistics and machine learning to the Texas results to show the importance of using detailed electoral data. And section 7 concludes with a clear proof of concept showing that the long-standing underrepresentation of minority voters in Texas, far from being an immutable fact, can be addressed through proactive mapmaking.

Finally, we have made the corresponding software tools available for public use in our GitHub (MGGG Redistricting Lab 2020a) and through a user-friendly portal at districtr.org/VRA.

2. ENSEMBLE METHODS: ALGORITHMS FOR CREATING DISTRICTING PLANS

As Justice Kagan explained in her dissent in *Rucho v. Common Cause* (2019, 2517–23), a computer equipped with an algorithm that generates a huge number of redistricting plans could potentially create a baseline to help answer questions like:

- What is an extreme, or unfair, number of Republican (or Democratic) districts, given the partisan composition and political geography of the state's voters? or,
- What would be a typical number of competitive districts, given those same parameters? or,
- Given the new census data, can a plan comply with the "one person, one vote" principle without pairing two incumbents' homes in the same district?

And as we will soon demonstrate, an ensemble approach also can help us address questions like:

- What is a fair map for Latino and Black voters?

2.1. *Illustrative example: Iowa*

To see the power of redistricting ensembles, let's consider the case of Iowa. According to the 2010 census, Iowa's 99 counties contained 216,007 census blocks and 3,046,355 residents—enough for four congressional districts. Iowa's constitution simplifies the redistricting problem by mandating that "no county shall be divided in forming a congressional district," so drawing our four districts requires assigning only the 99 counties (Iowa Const. art. III, § 37). We might hope to approach the task of finding fair plans by first building all possible plans, and comparing a particular plan to the full set.

But even this modest problem of dividing 99 counties into four connected parts (four contiguous districts) is currently out of reach: no one has yet been able to find a precise answer for this problem by computer, even with a clever enumeration algorithm and a month of computing time.[5]

This problem is only compounded in most states, which build their districts from census blocks

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[4]As described below, we use an implementation called GerryChain for plan generation, we use eiPack for ecological inference, and we use the current enacted Texas congressional map as our Voting Rights Act (VRA) benchmark.

[5]Indeed, even the simpler problem of partitioning a 9×9 grid into nine districts of nine units each has 706,152,947,468,301 solutions.

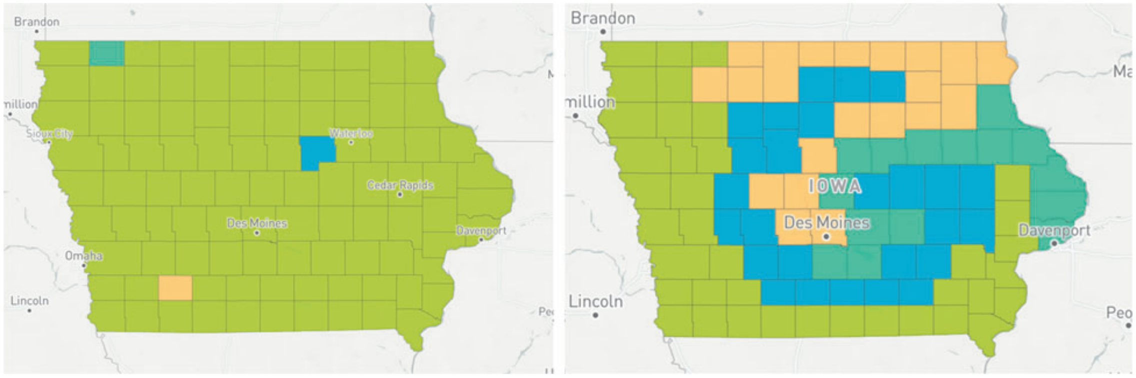

FIG. 1. These two partitions of Iowa into four connected pieces are not plausible for adoption as districting plans. The first has nearly all the state's population in a single large (*green*) district. The second more closely balances each district's population, but would likely violate Iowa law's compactness requirement.

(on average, there are more than 2,000 blocks per county). The full enumeration is subject to what is called *combinatorial explosion*, and the associated counting problem has forbidding complexity. This means not only that we lack the computing power to enumerate all plans today, but that computers likely will never be able to do so.

A second issue is that most plans in a complete enumeration would be irrelevant to the practical problem of redistricting because they would be blatantly unlawful. This is illustrated in Figure 1. The plan on the left, in which the biggest district has more than 750 times the population of the smallest one, would patently violate the federal Constitution's "one person, one vote" doctrine.[6] This means that districting plans with large population inequalities are of no practical interest, so a useful ensemble should exclude them.

The map on the right has much better population balance, but it also falls outside the plausible zone for plans. Its blue G-shaped district ("G" for gerrymandering) flaunts the mapmaker's disrespect for the traditional districting principle of compactness, which Iowa law explicitly safeguards (Iowa Code § 42.4.4).

Good ensemble methods allow us to draw a *representative sample* of compact, contiguous, population-balanced plans from the full space of possibilities—that is, a sample distributed in a known way that is suited to the law. By appealing to this sample, we can hope to address questions of partisan fairness, competitiveness, racial fairness, and all the other concerns and values we bring to bear on redistricting. To illustrate this methodology, we generated a sample of 100,000 valid Iowa congressional maps by the recombination method explained below in section 4.2, without taking partisan data into account.[7] This lets us compare the enacted plan against these alternatives in terms of votes cast for president in the November 2016 election, say. In our ensemble of compact, contiguous, population-balanced plans, nearly 75% have one safe Republican seat and three competitive seats (using a 55% majority as the line between competitive and safe). The current enacted plan has one heavily Trump-favoring district and three competitive ones, putting it in the largest category. This does not tell us by any stretch that the current plan is ideal or fair, but it does tell us that this plan is not an outlier by this way of measuring partisanship. This illustrates an elementary use of ensembles to benchmark partisan lean and competitiveness.

Similarly, ensembles can help us study how plans made without regard to race might tend to distribute a state's minority populations across districts, merely as a function of human geography. This

[6]A district-to-district population difference greater than 10% of the ideal district size is presumptively unconstitutional under the Fourteenth Amendment; for congressional districts, the standard is far stricter, under Article I of the Constitution (*Brown* v. *Thomson* 1983, 842–48; *Karcher* v. *Daggett* 1983, 730–44). The malapportioned plan in Figure 1 has top-to-bottom deviation nearly as large as the whole state, or close to 400% of ideal district size.

[7]ReCom always produces contiguous, balanced districts, and favors compact districts for reasons explained below in section 4.2.

racial baseline has been studied in a range of reports and papers, including MGGG Redistricting Lab (2018d, 2018a, 2019b, 2019a); DeFord and Duchin (2019); Duchin and Spencer (2021). But exploring the distribution of racial-group members in an ensemble is a different task from building an ensemble that takes VRA compliance into account. We will turn to that task shortly.

2.2. Building ensembles

Ensemble methods backed by powerful computers have proliferated in the last decade. Large ensembles of alternative plans proved critically important in federal-court cases invalidating extreme partisan gerrymanders in Ohio and Michigan (before the Supreme Court in *Rucho* held these claims nonjusticiable in federal courts) and more recently in similar state-court cases in Pennsylvania and North Carolina (*Rucho* v. *Common Cause* 2019, 2493–508; *League of Women Voters of Mich.* v. *Benson* 2019, 893–908; *Ohio A. Philip Randolph Institute* v. *Householder* 2019, 1025–62, 1082–85; *League of Women Voters* v. *Commonwealth* 2018, 770–81; *Common Cause* v. *Lewis* 2019, 17–43, 80–96).

Past ensemble methods used in litigation have focused on generating plans while controlling population balance, contiguity, compactness, and sometimes county and municipality integrity. Generating large ensembles while accounting in some way for these legitimate districting criteria helped judges decide whether one political party's disproportionate successes were due to the state's geographic features and the distribution of its voters—or to partisan manipulation of district lines. But in building their ensembles, the experts who testified in these cases did not seriously grapple with the legal requirements involving the consideration of race in redistricting.

In the Wisconsin case, for example, Democratic plaintiffs brought partisan-gerrymandering claims against a state Assembly plan that had resulted in Republicans winning 60 or more of the 99 seats, even in elections where Democratic candidates collectively received more votes than their Republican counterparts. In work prepared for the litigation and described in a subsequent article (Chen 2017), political scientist Jowei Chen built an ensemble of alternative Assembly plans to help evaluate the enacted plan and to demonstrate that the heavy advantage that Republicans enjoyed under that plan did not result inevitably from the political geography of the state's voters. Chen generated an ensemble of plans that altered boundaries for 92 of the 99 districts, while "freezing" seven heavily minority districts in and around Milwaukee, one of which had been ordered into effect to remedy a VRA violation.

Likewise, in the North Carolina cases, the experts' ensembles relied on proxies for districts' effectiveness for minority voters. For example, consider the work of one plaintiffs' expert, mathematician Jonathan Mattingly, as described in a subsequent article by his research group (Herschlag et al. 2020). Mattingly's work in North Carolina used demographic targets of 44.48% and 36.20% Black population for two congressional districts—the precise levels found in the enacted plan that the plaintiffs were challenging. He then built an ensemble by iterating a random step biased to favor plans that hit those demographic targets.[8] In addition to the effects of this tilted search, he discarded plans that fell short of those targets from the final ensemble presented in court, so that the prescribed population levels served as a minimum for all included plans.

In the context of these mid-decade partisan-gerrymandering cases, the experts' decisions to de-emphasize VRA complexities were understandable. The litigation, after all, focused on party, not race, and lawful VRA-compliant districts were already in place. But at the beginning of a new decade, with fresh census results available, that option will be foreclosed, as the minority districts from the previous map will have become either over- or under-populated due to population shifts and will thus violate "one person, one vote." So the minority districts (like all other districts) will have to be redrawn to accommodate the new census data. When generating alternative plans to create a baseline for comparison, redistricters will need to account for the delicate legal requirements imposed by the VRA and the Constitution.

For techniques that have been implemented to build VRA requirements into redistricting ensembles,

[8]Mattingly's method used a search procedure weighted to favor plans with better scores, based on a combination of population balance, compactness, county integrity, and nearness to his demographic targets for Black population.

the literature review is brief. In a new *Yale Law Journal* article called “The Race-Blind Future of Voting Rights” (Chen and Stephanopoulos 2021), Jowei Chen and legal scholar Nick Stephanopoulos take the problem of identifying suitable VRA districts head-on, defining a minority opportunity district by using a combination of partisan data (returns from the 2012 presidential general election) and demographic data (voting-age population from the 2010 census). In particular, they define a minority opportunity district to be one in which (1) the candidate of choice (typically Obama) carried the district in the general election and (2) most of the candidate’s support is estimated to have come from minority voters. This is somewhat closer in spirit to the method proposed here, though this article draws dramatically different conclusions from theirs.[9]

Our method for measuring district effectiveness, described in section 4 below, will draw on a much larger collection of recent elections, pairing a primary with each general. The outcomes from these elections are the essential components of our effectiveness scores. And in section 6 we will show that the scores we develop cannot be well approximated by considering only a district’s partisan lean and demographics.

2.3. *Using ensembles*

As we develop techniques for building VRA-conscious ensembles, there are two important general caveats about how and how not to use these ensembles.

Comparison, not selection. Our protocol is not designed to simulate the nuanced judgment of a seasoned voting-rights attorney. Rather, as we generate a chain of thousands of maps, we need a fast and reliable rough cut for VRA compliance. Our protocol uses a random iterative process in which districting plans are proposed, weighed, and potentially accepted into our ensemble of plans. We will be designing an in-or-out criterion that can be assessed in a fraction of a second. It is too much to expect perfection in excluding all unlawful maps and including all lawful ones, partly because the law itself is hardly a bright-line field. For example, even what seems like a rule with a clear threshold, such as the constitutional prohibition against state-legislative plans with population deviations greater than 10%, has exceptions in case law (*Cox* v. *Larios* 2004; *Unger* v. *Manchin* 2002). Nonetheless, an ensemble that includes most of the lawful maps that are proposed in the chain and rejects most of the unlawful ones will suffice for our goals of comparison and benchmarking. Ensembles should not be regarded as supplies of plans ready for immediate adoption; they are not likely to be good plans without extensive human vetting and adaptation.

Normal range, not ideal. We advocate using redistricting ensembles to learn a normal range for metrics and measures under the constraints of a set of stated redistricting rules and priorities. Ensembles allow us to justify statements such as *Plan X is an outlier in its partisan lean, taking all relevant rules into account*. While talking about normal ranges and outliers, we should avoid the temptation to valorize the top of the bell curve (or its center of mass, or any other value) as an ideal. By analogy, we can talk about people who are unusually tall or short without believing that any height is most desirable or ideal. If the 50th percentile height for American women is 5’4” and the 99th percentile height is 5’10,” we can conclude that a woman who is six feet tall is unusual, and we can look for reasons (family history, diet, and so on) to explain her height. But it would be quite strange to decide that a woman who is 5’4” is a “better” height than one who is 5’5.”

Justice Kagan’s *Rucho* dissent skirted the edge of this temptation. She mostly reasoned from ensembles just as we will recommend here, envisioning a bell curve (in that case, of partisan advantage) and describing plans far from the bulk of the curve as presumptively impermissible: “The further out on the tail, the more extreme the partisan distortion and the more significant the vote dilution” (*Rucho* v. *Common Cause* 2019, 2518). But in the course of describing the outlier logic, she implied that plans “at or near the median” are the best of all. An outcome “smack dab in the center” (in Justice Kagan’s words) may not be in any sense the most fair, however. For instance, turning to the November 2012 Obama-Romney election as a touchpoint, Obama received nearly 53% of the major-party vote in Iowa. Even if just over half

[9]For their method’s details, see the full description in Chen and Stephanopoulos (2021). For a critique of their definition of minority opportunity districts and its application, see Duchin and Spencer (2021).

the congressional plans in our ensemble have three Obama-favoring districts out of four (making that the median outcome), we might still reasonably consider a map with two Obama-favoring and two Romney-favoring districts to have at least as strong a claim on fairness, given the nearly even vote split.

Likewise, there would be no reason to prefer a map that preserves intact a *median* number of whole counties or municipalities. Indeed, some states' redistricting laws expressly demand keeping the greatest practicable number of counties or municipalities intact.

The same warning, to be wary of the magnetic attraction to the middle of a bell curve, surely applies as well to racial fairness. If a state's Latino, Black, Asian American, and Native American residents have historically been (and currently remain) underrepresented, we should gravitate toward solutions that fix the shortfall rather than perpetuate it. Fortunately, federal law pushes redistricters in the right direction.

3. THE LAW OF RACE AND REDISTRICTING

The rules regarding the consideration of race in redistricting flow primarily from two sources of federal law: the Fourteenth Amendment's Equal Protection Clause and Section 2 of the Voting Rights Act, which Congress, exercising its power to enforce the Fifteenth Amendment, enacted in 1965 and significantly revised in 1982.

3.1. The Voting Rights Act prohibits minority vote dilution

Section 2 of the VRA prohibits a redistricting plan that abridges any citizen's right to vote "on account of race or color [or membership in a language-minority group]" (VRA §§ 10301(a), 10301(f)(2)). Minority plaintiffs can establish a violation of amended Section 2 by showing, "based on the totality of circumstances," that members of their racial or language-minority group "have less opportunity than other members of the electorate" to "nominat[e]" and "elect representatives of their choice" (VRA § 10301(b)).

In assessing whether a redistricting plan provides equal electoral opportunity under amended Section 2, Congress expressly permitted state redistricters and federal judges alike to consider recent election outcomes, namely "[t]he extent to which members of a protected class have been elected to office" (VRA § 10301(b)). Nothing in Section 2, however, "establishes a right to have members of a protected class elected in numbers equal to their proportion in the population." While electoral success for minority candidates is important, even more important under Section 2 is that the candidate be the "chosen representative" of a particular racial or language-minority group, regardless of the candidate's race or ethnicity (*Thornburg* v. *Gingles* 1986, 68 (plurality opinion)). And Section 2's lodestar is "equality of opportunity, not a guarantee of electoral success for minority-preferred candidates of whatever race" (*Johnson* v. *De Grandy* 1994, 1014 n.11). As the Supreme Court has explained, "minority citizens are not immune from the obligation to pull, haul, and trade to find common political ground, the virtue of which is not to be slighted in applying a statute meant to hasten the waning of racism in American politics" (*Johnson* v. *De Grandy* 1994, 1020).

In redistricting cases "the ultimate question [under Section 2] is whether a districting decision dilutes the votes of minority voters" (*Abbott* v. *Perez* 2018, 2332). District lines can dilute the voting strength of politically cohesive minority-group members either by "cracking," or dispersing, them among multiple districts where they are routinely outvoted by a bloc-voting majority, or by "packing," or concentrating, them into too few districts, wasting votes that could have mattered in neighboring districts (*Johnson* v. *De Grandy* 1994, 1007). Section 2 prohibits both cracking and packing whenever district lines combine with social and historical conditions to impair the minority group's ability to elect its preferred candidates "on an equal basis with other voters" (*Voinovich* v. *Quilter* 1993, 153).

In jurisdictions where all sizable demographic groups (majority and minority alike) consistently favor the same candidates, a redistricting plan cannot dilute minority citizens' voting strength, so Section 2 plays no role (*Thornburg* v. *Gingles* 1986, 51). But in most states, where voting is in varying degrees racially polarized, Section 2 can require replacing one or more districts that elect candidates preferred by the majority (usually, a white majority) with districts that would elect candidates preferred by one or more minority groups (*Johnson* v. *De Grandy* 1994, 1008). To prevail, Section 2 plaintiffs must prove that, under the challenged plan, a bloc-voting majority usually will defeat "candidates supported by a politically cohesive, geographically insular minority group" (*Thornburg* v. *Gingles* 1986, 49). But even with such proof, plaintiffs' challenge to a state districting plan ordinarily will fail if the plan provides effective opportunities to nominate and elect minority-preferred candidates in a number of districts *roughly proportional* to the minority group's share of the state's citizen voting-age population, or CVAP (*LULAC* v. *Perry* 2006, 436–38; *Johnson* v. *De Grandy* 1994, 1000).

One particularly useful—and simple—method for assessing minority electoral opportunities under a districting plan is to add up the votes cast for each candidate in recent *statewide* primary and general elections by district, to learn which districts gave more votes to the minority-preferred candidate than to any other candidate (*LULAC v. Perry* 2006, 428 (majority opinion), 493–94, 499–501 (Roberts, C.J., dissenting in part); *Session v. Perry* 2004, 499–501). This approach is particularly straightforward if each precinct is kept intact within a single district: simply adding up the votes for each candidate in all of a district's precincts shows, for each election, which candidate carried the district. The most difficult part of these analyses, especially in primaries, is identifying the candidate who was minority-preferred in each election, which is typically performed by a statistical-inference procedure comparing demographic patterns to voting patterns (King 1997; King, Rosen, and Tanner 1999; Elmendorf, Quinn, and Abrajano 2016). But we will take care to place actual electoral history at the center of our assessment of district effectiveness, keeping the role of statistical inference to a minimum.

3.2. *The Equal Protection Clause prohibits excessive attention to race*

Regardless of what techniques are used to assess minority electoral opportunities, compliance with Section 2 necessarily requires detailed consideration of race and racial data. But a state's consideration of race is constrained by the Fourteenth Amendment mandate that "[n]o State shall … deny to any person within its jurisdiction the equal protection of the laws" (U.S. Const. amend. XIV; see *Bethune-Hill* v. *Virginia State Bd. of Elections* 2017, 802). Starting in the 1990s in its *Shaw* line of cases, the Supreme Court has identified at least two ways that the excessive use of race can give rise to a presumptively unconstitutional *racial gerrymander* under the Equal Protection Clause (*Miller* v. *Johnson* 1995, 904–05, 910–17; *Shaw* v. *Reno* 1993).

First, a bizarrely noncompact district is subject to strict scrutiny under that Clause if the district's boundary is "so irrational on its face that it can be understood only as an effort to segregate voters into separate voting districts because of their race" (*Shaw* v. *Reno* 1993, 658). This type of racial predominance most often arises where a district's perimeter is defined not by the boundaries of intact precincts, for which electoral data exists, but by the boundaries of (much smaller) census blocks that have been conspicuously sorted into or out of districts according to their racial composition (Hebert et al. 2010, 66–68 & n.21; *Alabama Legislative Black Caucus* v. *Alabama* 2015, 274).

Second, although only a minority of justices have stated that the intentional creation of a majority-minority district should always be presumptively unconstitutional, a majority of the Court has held that districts violated the Equal Protection Clause because they were drawn to "maintain a particular numerical minority percentage" or to meet arbitrary or "mechanical racial targets." The Court has thus rejected a bald mandate that certain districts must have at least a 50% or a 55% Black voting-age population regardless of whether that percentage was actually shown to be necessary for the district to nominate and elect minority-preferred candidates (*Cooper* v. *Harris* 2017, 1469; *Bethune-Hill* v. *Virginia State Bd. of Elections* 2017, 799, 801–02; *Alabama Legislative Black Caucus* v. *Alabama* 2015, 267, 275; *Bush* v. *Vera* 1996, 969–72).

3.3. *Implications for redistricting ensembles*

These legal points have major implications for an ensemble-creation protocol keyed to compliance with the VRA and the Constitution. As an initial matter, recalling the earlier point about ensembles being far more useful for comparison than for selection, the focus here is on drawing a collection of maps that would be relatively safe from challenges under VRA Section 2, rather than on crafting a map for plaintiffs to propose when suing the state.

As a gatekeeping function before ultimately assessing the "totality of circumstances," courts generally require Section 2 plaintiffs to present an illustrative map showing that the minority group in question could constitute a literal arithmetic majority of the voting-age population (VAP) in a proposed district.[10] The Supreme Court has noted, however, that a district that falls short of the 50% threshold yet can still nominate and elect minority-preferred candidates "can … [and] should" count as a minority-effective district when assessing a state's compliance with Section 2 (*Bartlett* v. *Strickland* 2009, 24 (plurality opinion); see also *Cooper* v. *Harris* 2017, 1470). So actual electoral opportunity for minority groups—a track record of effectiveness in elections—is what matters when defending a map against a VRA challenge. Taken together, the legal points elucidated above in sections 3.1 and 3.2 suggest three crucial design principles for a VRA-conscious ensemble protocol.

(1) *Ensure effectiveness in both primaries and generals.* Aiming to weed out of an ensemble plans that violate Section 2, while retaining plans that comply, a protocol must assess whether particular districts will or will not be effective for minority-preferred candidates seeking both nomination (in primaries) and election (in generals). This assessment requires attention to both demographic data and actual election results, including precinct-level returns from primary and general elections.

(2) A*void* a priori *demographic targets.* Threshold decisions about the composition of districts should not be based on purely demographic targets—for example, requiring a certain number of districts that are at least, say, 55% Latino or 50% Black. That approach not only could lead to false positives or false negatives for district effectiveness, but could leave the methodology vulnerable to constitutional attack for excessive race-consciousness.

(3) *Maintain reasonable compactness.* To further reduce constitutional exposure, the ensemble-generating technique should admit few or no plans with bizarre district shapes.

We note that both the first and the third principles recommend the use of precincts, rather than the much smaller census blocks, when assembling districts. Precinct-based plans promote compactness and facilitate more accurate assessment of electoral history, which is fundamental to evaluating district effectiveness. And though they may not achieve perfect population equality, that fact usually should not present significant constitutional concerns.[11]

4. DESIGN OF A VRA-CONSCIOUS ENSEMBLE PROTOCOL

In this section, we will describe the design of a protocol for generating redistricting plans that comply with not only the criteria of population equality, contiguity, and reasonable compactness, but also the race-related rules mandated by the VRA and the Equal Protection Clause. The protocol begins with data preparation and culminates in the use of a constrained recombination algorithm for generating plans that meet VRA-related requirements. We propose this as a sound and detailed *VRA-conscious algorithm*, but not as *the authoritative VRA algorithm*. There may well be other ways to incorporate the legal requirements around race, and to do it well. But the methods laid out in this section come closer to the big-picture goal—building a representative sample of lawful maps—than any previous work we know. We believe that this elaborated example of one concrete, reasonable way to take account of race and the law helps illuminate some key decisions.

We recall from above that the protocol is modular with respect to three ingredients: a procedure for iteratively modifying districting plans (here, spanning-tree recombination), a procedure

[10] *See Bartlett* v. *Strickland* (2009, 6, 9–11, 20, 24–25, 26 (plurality opinion)). *Bartlett* also may be satisfied with a majority of the proposed district's *citizen* voting-age population (CVAP). And *Bartlett*'s 50% rule may not apply if the defendant drew the challenged districts with discriminatory intent, as might well be the case when a state dismantles an existing minority-effective district.

[11] Using whole precincts will rarely raise "one person, one vote" concerns for state-legislative maps. However, the Constitution imposes stricter population-equality standards for congressional maps (*Karcher* v. *Daggett* 1983, 740-41). Although the most common current practice is to draw congressional plans so that the largest and smallest districts differ by only one person, the Supreme Court has upheld plans with significantly larger deviations (*Tennant* v. *Jefferson County Comm'n* 2012, 762, 764–65; *Abrams* v. *Johnson* 1997, 99–100). In any event, a map built from whole precincts can usually be readily modified into a map with a minimal deviation by swapping a limited number of census blocks between adjacent districts.

for identifying minority-preferred candidates (here, a Bayesian hierarchical model of ecological inference), and a benchmark that prescribes a threshold number of effective districts for each minority group (here, an enacted plan that has evaded or withstood VRA scrutiny). Our choices can be swapped out for others as new methods or special circumstances warrant, leaving the overall structure intact.

4.1. Preparing data

4.1.1. Electoral and demographic data. We will require a cleaned precinct *shapefile* for the state, with election returns and demographic data joined to those precincts.[12] This can be difficult to obtain because precincts change from year to year and a longitudinal precinct shapefile is needed for the span of years covered by the election dataset. Furthermore, we may need to clean the precinct shapes to get suitable topology: to be usable as building blocks for plans, precincts must tile the state, with every resident located in one and only one precinct.[13]

The shapefile allows us to match reported vote totals to geographic units and to record which pairs of precincts are adjacent, which will be needed to ensure that districts are contiguous. For each precinct, we have joined data on total population from the 2010 decennial census, adult citizen population by race and ethnicity from the American Community Survey (ACS) five-year rolling estimates ending in each election year, and counts of votes received by each candidate for statewide election in a large set of primary and general elections.

Although our modeling concern is with districted elections for Congress and state legislatures, our analysis is based primarily on statewide (exogenous) contests. This is because the choices facing voters in districted elections vary across the state: in any given election year, some districts are uncontested, some have strong incumbents or other idiosyncrasies. When district boundaries are moved to create alternative plans, the newly proposed districts will be composed of voters who faced completely different candidate choices. It is not clear how votes for one candidate would translate to votes for a different candidate. By contrast, statewide elections allow us to make apples-to-apples comparisons across different parts of the state, since the same set of candidates competed everywhere. Ideally, we would include all statewide contests for the last ten years, but this is not always possible because of data availability and precinct instability. As we will discuss further below, this protocol is not intended for use with fewer than five general elections, grouped with the primaries (and, where applicable, primary runoffs) that preceded them.

Because our main concern here is whether minority-preferred candidates are ultimately elected to office, we *link* the primary (and primary runoff) for a given office in a given year to the general election for that same office that same year, and define success by whether the candidate who was minority-preferred in the primary succeeded at all stages of the electoral process.

We use a simplified set of racial groups: every person who identified as Hispanic/Latino on the census or ACS is classified as *Latino*. We use the term *Black* for non-Hispanic respondents who selected Black as their single racial category, and we use *White* similarly. All other respondents (those non-Hispanic persons selecting two or more races, Asian American, Native American, and so on) are grouped together and designated as *Other*. In a state with only one sizable minority group, all other minority groups may be merged into the Other category for purposes of this VRA protocol. Citizen voting-age population is denoted by CVAP, and we use HCVAP, BCVAP, WCVAP, and OCVAP to denote Hispanic/Latino, Black, White, and Other CVAP. We focus on Latino and Black voters as minority groups because our main case study involves congressional redistricting in Texas. In other states, like California, Hawaii, or Alaska, or in certain local districting projects, we might specify different racial groups for analysis.

Importantly, we make no prior assumptions about whether the voting behavior of Latino, Black, White, or Other groups will align. This is a case-by-case empirical question addressed with statistical inference.

4.1.2. Candidates of choice. As explained above, the linchpin of a vote-dilution claim under

[12]Shapefiles store data about the position and attributes of a geographic unit, such as a precinct.

[13]Cleaned and vetted shapefiles that are suitable for longitudinal data are easier to create in some states than others. For instance, the Louisiana shapefile used in this study required hundreds of person-hours of data preparation from members of the MGGG Redistricting Lab. It would be extremely difficult to obtain an analogous data product in Mississippi, for example.

the VRA is the right to replace districts where minority-preferred candidates usually lose with districts where they have a realistic opportunity to win (*Johnson* v. *De Grandy* 1994, 1020). To assess whether a district falls into the former category or the latter requires determining which candidates are preferred by members of each sizable minority group.

Because vote totals are not reported by racial group, we cannot directly determine which candidates are minority-preferred. Instead, this effort falls under the umbrella of *ecological inference* (EI). Voting preferences are never monolithic, but techniques for measuring racial polarization have been refined for decades, and they can help us estimate the degree of bloc voting. The techniques in the ecological-inference family, like all statistical-inference methods in the presence of missing data, give imperfect and uncertain answers (Elmendorf, Quinn, and Abrajano 2016). It is fundamentally important to estimate the error that is produced by techniques and keep track of how it compounds or cancels out in our high-level conclusions. As much as possible, we will opt to make gradated and not bright-line determinations from the outputs of EI.

Our VRA-conscious ensemble protocol requires identifying the candidate who was preferred by each sizable minority group in each election, together with confidence measures that these preferred candidates are correctly identified. To perform the check for minority control of a district, as well as to identify district-wide candidates of choice for newly proposed districts, we make use of not only statewide but also precinct-level vote estimates by race for each candidate (with variance estimates). Users can employ various methods to generate these estimates (e.g., using King's EI, Ecological Regression, exit polls, or voter files). Notably, this allows our protocol to immediately incorporate any future advances in inference techniques.

In the implementation described here, we generate estimates using a version of King's EI, specifically the ei.MD.bayes function from eiPack (Lau, Moore, and Kellermann 2020) which is based on the Bayesian hierarchical Multinomial Dirichlet model for $R \times C$ tables proposed in King, Rosen, and Tanner (1999).[14] For each election we run EI at the statewide level, using precinct-level input tables. The inputs for each precinct are the row and column *sums* for the $R \times C$ table of vote counts. The row sums correspond to the precinct's estimated number of adult citizens in each racial group (HCVAP, BCVAP, WCVAP, and OCVAP). The column sums are the precinct's vote totals for each candidate as well as a *None* count, which is the sum of the four CVAP figures minus the sum of the recorded vote totals for all candidates, estimating the number of nonvoters. EI then infers values for the internal cells of these tables, i.e., estimated vote counts by racial group and candidate. Inclusion of the *None* column allows the underlying model to estimate differential turnout by race; without this, EI would rely on the unrealistic assumption that adult citizens from all demographic groups were equally likely to have cast a ballot.

Each EI run generates a large random sample of estimated precinct vote counts; we can sum these across the entire state to get statewide estimates. For each racial group, the candidate with the highest average estimated vote total for a given election is identified as the group's "candidate of choice." For a measure of confidence that Candidate X was the candidate of choice for a racial group in a given election, we first take repeated draws from the EI distribution and record the frequency with which X receives the most votes from that group. We then transform this to a confidence score.[15]

[14]Here, $R \times C$ stands for the number of rows (or racial groups) R and columns (or candidates) C.

[15]Let p be the frequency in a batch of trials with which X is observed to be the preferred candidate. We logistically transform this to a confidence score using $C(p) = 1/(1 + \exp(18 - 26p))$ to weight the election in the compound score of district effectiveness (see Table 1 below). The parameters 18 and 26 were chosen so that an election in which the draws have Candidate X ahead only 50% of the time should receive almost no weight (because it is a toss-up); but if Candidate X comes out ahead in, say, 85% of trials, the confidence should be nearly 100%. It is certainly possible to use other parameters, to skip this step and just use $C(p) = p$ as a measure of confidence, or even to forgo confidence altogether. Without some factor of this kind, however, the resulting score will have more noise due to cases where the candidate of choice is uncertain. If we do not strongly down-weight the uncertain elections, we risk a situation in which just rerunning the EI with identical settings could produce a significantly different answer. We discuss this and other robustness checks in footnote 31.

4.2. Building new plans by recombination

The science of representative sampling has advanced greatly in the past few years as ensemble methods for redistricting have matured. Using a technique known as *Markov chain Monte Carlo* (MCMC), it is now possible to efficiently create an ensemble of thousands or millions, even billions, of plausible maps. We can even sample while keeping control of the weighting that makes some kinds of plans appear more often than others. For example, we can be sure that a preference for more compact plans is designed to depend *only* on a prescribed score of compactness and on no hidden factors.[16]

The engine of our district-generation process is a Markov chain known as recombination, abbreviated ReCom, whose central idea of using spanning trees to split districts is fast becoming the standard in the field (DeFord, Duchin, and Solomon 2021; Autrey et al. 2021; McCartan and Imai 2021). We will apply it to plans built from whole precincts, the smallest geographic units for which we have accurate, detailed electoral data. Earlier MCMC methods for redistricting reassigned a single geographic unit (such as a precinct) from District A into adjacent District B at each step, creating a new plan that agreed with its predecessor on the assignment of every unit except one. (If Texas, for example, had 9,000 precincts, 8,999 would stay in their districts at each step.) By contrast, ReCom typically proposes a much larger change: at each step, two entire (adjacent) districts are merged and then re-split in a new way that is completely independent of the division in the previous plan. This means that a single ReCom step can reassign hundreds of precincts at a time. (Each of Texas's 36 congressional districts, for instance, has roughly 9,000/36, or 250, precincts, so each recombination step performs a random division of roughly 500 precincts into two new districts.) By iterating this transformation hundreds of times per minute, the map soon loses any resemblance to its starting configuration.

A ReCom step merges a random pair of adjacent districts and splits the region in a new way. Under the hood, each ReCom step uses a *spanning tree*, which is a kind of "skeleton" of the double-district created by the random merger, and then searches for a place to cut that tree to leave behind two population-balanced, connected pieces. So, by construction, all plans proposed by recombination are contiguous and maintain the desired population balance. What is less obvious is that ReCom's use of spanning trees also places an automatic priority on districts that have more internal adjacencies: so *compactness*, or a preference for plump, regular forms over thin necks or stringy appendages, is also a structural feature of the algorithm (see Figure 2) and does not have to be set as a manual choice by the programmer (DeFord, Duchin, and Solomon 2021). In fact, when the district boundaries of a plan generated by ReCom look ragged to the eye, it is often because the building-block units themselves (such as precincts) have jagged edges.[17]

Over thousands or millions of iterations, this simple method can undertake far-reaching exploration of the universe of possible plans subject to population balance, contiguity, and reasonable compactness. We will call a set of plans collected in a recombination chain an *ensemble* of plans.

Additional features and constraints can be incorporated into ReCom either with hard thresholds (i.e., validity checks) or by using probabilistic acceptance. To illustrate this, consider the traditional districting principle that counties should be kept intact when practicable. We could enforce a maximum allowable number of county splits by adding an instruction to automatically reject as invalid any proposed plan that exceeds some level of county-splitting, creating a *constrained* ensemble. A different option would be to impose a bias to the probability of acceptance, essentially flipping a weighted coin each time a proposal is generated that makes it rare but not impossible to accept plans with a large number of county splits. This would create a *biased* (or *tilted*) ensemble favoring fewer county splits.

When a proposed plan is rejected, a new plan is proposed by merging and re-splitting a freshly

[16]To be precise, the recombination algorithm used here approximately targets a known distribution called the *spanning-tree distribution*, where the probability of selecting a particular plan is proportional to a certain measure of compactness. A modified algorithm called *reversible recombination* exactly targets that steady state. See DeFord, Duchin, and Solomon 2021; Duchin and Tenner 2018; Sarah Cannon, Moon Duchin, Dana Randall, and Parker Rule 2020. "A Reversible Recombination Chain for Redistricting." On file with authors.

[17]The reasons spanning-tree partition methods produce compact districts are explored in Duchin and Tenner (2018) and DeFord, Duchin and Solomon (2021).

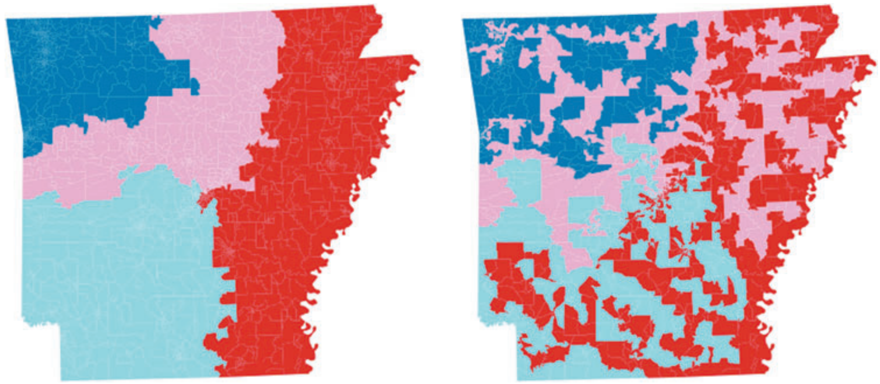

FIG. 2. If all contiguous, population-balanced plans were made equally likely, the compact plans (*left*) would be enormously outnumbered by bizarrely noncompact ones (*right*). The ReCom algorithm prefers the compact one, with a relative weight dictated *only* by its compactness score.

chosen pair of adjacent districts. This continues until some proposed plan passes the necessary tests to be accepted, at which point it is added to our ensemble. The next step proceeds from this newly accepted map, and so on until the Markov chain reaches its stopping condition (such as by collecting a prescribed number of plans). Our ensembles contain every valid plan rather than *subsampling*, or thinning out by accepting only every 1,000th or 10,000th plan as previous authors have done (Herschlag et al. 2020; Fifield et al. 2020). The long-range statistical properties are the same whether we use continuous sampling or sub-sampling, and we employ standard convergence heuristics from the scientific computing literature to provide evidence that our chains are run long enough for the statistics we collect to approach stationarity.[18] For more information about spanning-tree recombination and for comparisons to other methods, see DeFord, Duchin, and Solomon (2021); Becker and Solomon (2021); DeFord and Duchin (2020); McCartan and Imai (2021); and Autrey et al. (2021).[19]

Below, we will refer to district-level as well as statewide EI estimates as we build scores of district effectiveness. The district-level procedure requires some thought because of the computational cost of any calculation that occurs while the algorithm runs, rather than being performed in advance. It is not feasible to rerun EI to determine district-level candidate preferences with each newly proposed plan in a ReCom chain. We need a highly efficient calculation to retrieve both a point estimate and an estimated confidence level when a new district is formed. To handle this, we make use of the hierarchical structure of EI. The EI algorithm generates large random samples for each precinct from the distribution of possibilities produced by the underlying Bayesian model. This means that we can store outputs for each precinct in the state. Ideally, we would save the full *detailed histogram* describing the frequency with which various vote counts were estimated for each candidate and racial group in that precinct. Because this is too much information to store, we instead record the point estimate for each group's support of each candidate in addition to a simplified coarse histogram of vote counts, compressed down to just nine values, which turns out to be enough to recover the shape of the detailed histogram with remarkable fidelity, as shown in Supplementary Appendix A. During the run of the ReCom Markov chain, we can redraw samples from these coarse distributions and aggregate to the district level for each newly generated plan to determine the confidence that we have correctly identified candidates of choice.

4.3. Building raw scores of district effectiveness

We next lay out three ways to use prior election results in assigning a minority-effectiveness score

[18]Markov chains that take large steps, like ReCom, require many fewer steps to achieve approximate independence than methods that iterate very small changes.

[19]See also Sarah Cannon, Moon Duchin, Dana Randall, and Parker Rule 2020. "A Reversible Recombination Chain for Redistricting." On file with authors.

to a proposed district: an unweighted score, a score that weights elections based on statewide voting patterns, and a score that weights elections based on voting patterns restricted to the proposed district itself. We will denote these scores by s^{unw}, s^{state}, and s^{dist}, respectively. Although election-weighting schemes differ across the three effectiveness scores, each score captures the same underlying idea: the effectiveness of a district for a minority group is keyed to the district's history of voting for minority-preferred candidates running for statewide offices. Importantly, because our districts are built from whole precincts and we have prior election results matched to those precincts, no statistical inference is required to determine which candidate prevailed in each district. We simply total up the votes cast in the district for each candidate and note which candidate got the most support.

First, we need to settle on the meaning of a successful outcome for the voters of a minority group in a particular election and district. If the candidate of choice from the primary does not advance to the runoff or general, then the outcome of the general is less informative with respect to the group's preferences. Therefore, we group elections by pairing primary and general (or grouping primary–runoff–general if applicable) as Table 3 illustrates for our Texas case study. A successful election is one in which the minority-preferred candidate in the primary prevailed in both elections in the grouping (or all three, if there was a primary runoff).[20]

Our weighting scheme is keyed to the *probative* value of each statewide election in determining minority effectiveness—its value as evidence. The unweighted score treats each election equally; no election is considered more probative than any other in determining a district's effectiveness. By contrast, the statewide weighted score s^{state} and the district weighted score s^{dist} treat some statewide elections as more probative than others and weight them accordingly. These *election weighting factors* each fall on a scale from zero to one. Their product is the final weight for an election. In keeping with case law, we up-weight elections if they have certain features:

- *Recent.* More recent elections provide stronger evidence of future electoral opportunity.
- *Clear candidate of choice.* As described above in section 4.1.2, our ecological-inference outputs come with estimates of the probability that the minority-preferred candidate in the primary election has been correctly identified. Translating this to a *confidence* that EI has identified the correct candidate gives greater weight to elections in which the minority group has a clearly preferred candidate.
- *Group member preferred.* An outcome gives stronger evidence of electoral opportunity when the minority-preferred candidate is a member of the particular minority group.

The weighting factors are summarized in Table 1. We discount elections for each year of age by a multiplicative factor of $2^{-1/4} \approx .841$, so that if any one election is four years older than another, it weighs half as much. The confidence that we have correctly identified the minority-preferred candidate is the same confidence score $C(p)$ described above (see footnote 15), using draws at the state level for s^{state} and drawing from the district-level coarse histogram for s^{dist}. When gauging Latino effectiveness, we place twice as much weight on elections in which the Latino-preferred candidate is Latino; and the analogous statement holds for other minority groups. Of course, these detailed weights are choices made by the modeler. We will introduce a calibration step for our effectiveness scores in the next section that makes our outputs more robust to these parameters, and we tested this by re-running the protocol several times with slightly different choices (see footnote 31).

These weighting factors are important for the legal interpretation we intend. More recent elections are up-weighted because the predictive value of election results tends to erode over time, as older voters pass away, younger citizens reach voting age, immigrants are naturalized, people move into or out of the district, and voters change their

[20]To be precise, suppose the primary candidate of choice is Candidate X and the runoff candidate of choice is Candidate Y (who might or might not be the same person as Candidate X). Then there are three cases we count as primary success. Case one: X won the primary (in the district) and there was no runoff. Case two: X received over 50% of the vote in the primary (in the district), whether or not there was a runoff. Case three: X ranked first or second in the primary (in the district) and Y won the runoff (in the district). An election set that meets one of these primary-success conditions and in which the minority-preferred nominee wins the general election in the district is counted as a successful election in the scores below.

TABLE 1. WEIGHTING FACTORS FOR EFFECTIVENESS SCORES

<table>
<tr><th>Score/Factor</th><th>Recent</th><th>Clear candidate of choice</th><th>Group member preferred</th></tr>
<tr><td>Unweighted (s^{unw})</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>Weighted/Statewide (s^{state})</td><td rowspan="2">1 Most recent
.841 1 year prev.
.707 2 years
.595 3 years
.500 4 years
.421 5 years, etc.</td><td>Confidence from statewide EI</td><td rowspan="2">1 X belongs to group,
.5 otherwise</td></tr>
<tr><td>Weighted/District (s^{dist})</td><td>Confidence from district-level EI</td></tr>
</table>

The weighting factors for the unweighted, statewide, and district-based effectiveness scores (s^{unw}, s^{state}, and s^{dist}, respectively). All of these are computed with respect to the primary election in an election set, because the runoff and general may not contain the most-preferred candidate for the minority group. Here, Candidate X is the minority group's candidate of choice. These factors will be combined into an election-weighting term w for all elections in the dataset.

political preferences and behaviors. Confidence in correctly identifying candidates of choice is clearly pertinent, because a wrongly identified candidate of choice undermines all subsequent conclusions we will draw. Elections where the minority-preferred candidate belongs to the minority group in question are up-weighted because they are more probative: in the words of the late Judge Richard Arnold, the VRA's guarantee of equal opportunity is not met when "[c]andidates favored by [a minority group] can win, but only if the candidates are white" (*Smith* v. *Clinton* 1988, 1318).

We now have all the ingredients for the raw effectiveness score for a given district and racial group, multiplying the three factors above to get a weight $w = w(E, D)$ for each election and district. For instance, if we have 20 elections, then each w will be .05 for the s^{unw} score, no matter the election. For the statewide score s^{state}, the elections will not all count equally, so that, for example, a recent election with an in-group candidate will weigh four times as heavily as a four-year-old election with only white candidates.

Each effectiveness score is computed similarly:

$$\begin{aligned}\text{score of district } D = s(D) \;&=\; \sum_{E\in\mathscr{E}} w \cdot \delta \\ &= \text{weighted share of elections} \\ &\quad \text{won by candidate of choice,}\end{aligned}$$

where δ is 1 if the minority-preferred candidate carried the district and 0 otherwise. This expression applies to all three kinds of effectiveness scores $s = s^{\text{unw}}, s^{\text{state}}, s^{\text{dist}}$. For example, suppose there are two election groupings separated by four years, both have equal confidence weights and feature group members, and the candidate of choice is successful in one of those two election sets. Then the statewide and district raw scores of effectiveness would be 1/3 if the success was in the earlier election and 2/3 if the success was in the later election, while the unweighted score would be 1/2. The strength of using an approach that centers on electoral effectiveness rather than demographics is that we do not make evidence-free assumptions about how large a Latino population is needed to nominate and elect Latino-preferred candidates, or similarly for other minority groups. Rather, we directly and empirically answer that question by totaling up votes, district by district. Our direct, empirical approach is better keyed to actual minority electoral opportunities, and so also comports better with federal law. The VRA's plain text does not equate a minority-effective district with a majority-minority district; rather, it demands an assessment of whether minority citizens have an equal opportunity to "nominat[e]" and "elect representatives of their choice." And our empirical approach also respects the Equal Protection Clause's prohibition against relying on racial-percentage targets when drawing districts.

4.4. Calibrating effectiveness scores

The raw effectiveness scores described above combine election results in three different, reasonable ways. Each score ranges from zero (never electing minority-preferred candidates) to one (always electing them). We next convert these to calibrated scores that we will use when deciding whether to accept plans into the ensemble.

At this stage, we take a *group-control factor* into account, combining it with the raw effectiveness score because it is relevant to predicting future performance and to ensuring an emphasis on electoral success for larger numbers of minority voters. It is clear from redistricting case law that majority-minority districts are not required for VRA compliance, and indeed that setting out to draw districts with a demographic target is sometimes prohibited. At the same time, a district that has only 5% Black CVAP would not be reasonably viewed as an effective opportunity district for Black voters, on par with a district with more significant Black population. We have chosen to address this issue with a factor based on the minority group's share of district CVAP.[21] Group control of the district is relevant for two reasons. First, Section 2 of the VRA focuses on a minority group's ability to play a controlling or "decisive … role in the electoral process" and not merely one of "influence" (*LULAC* v. *Perry* 2006, 446 (plurality opinion) (citation and quotation marks omitted)). Second, because Section 2 protects the voting rights of a minority group's individual members, the effectiveness of a district should in part depend on the number of those members represented by their candidate of choice.

The goal of the calibration step is to bolster the *probabilistic* interpretation of the scores, so that, for example, a district with $s = .5$ can be described as having a 50/50 chance to perform for the minority group under consideration. To lend justification to this probabilistic interpretation, we apply a standard logistic regression to normalize the raw scores based on observed success data from actual enacted districts (specifically, all congressional, state Senate, and state House elections in the last decade).[22]

By design, the calibration step helps ensure that although the elections that are used in constructing the raw effectiveness scores are statewide contests, they still reflect election outcomes in *local* (districted) elections. We think of the logistic transformation as producing a score that best captures the observed performance of congressional, state Senate, and state House districts in the last decade. Each input (raw) score falls between zero and one; after applying the logit function we obtain an output (calibrated) effectiveness score that still falls between zero and one, but is now easier to interpret. We will reuse the same notation s^{unw}, s^{state}, s^{dist} for the outputs, taking care to refer to the scores as raw or calibrated when there is a possibility of confusion.

4.5. Counting effective districts

To assess whether a proposed plan complies with the VRA, we will need to count effective districts, and not just report scores. We elect to define a *Latino-effective* (or *Black-effective*) district as one whose calibrated effectiveness score estimates at least a certain threshold chance of both nominating and electing a Latino-preferred (or Black-preferred) candidate.

This threshold is a parameter to be set by the modeler, and it may involve considerable discretion. One consideration may be the mapmaker's level of risk aversion, since setting a lower threshold may result in a higher number of qualifying districts that can be simultaneously drawn, but some or all of those districts will be less certain to nominate and elect minority-preferred candidates. A second consideration may be how particular districts in the current enacted map have been characterized by judges and victorious litigants in prior redistricting litigation, or how they have actually performed in prior elections. A third consideration may be the number of statewide elections in the dataset: we may choose a higher effectiveness threshold if we have a smaller set of available elections, to account for the possibility that the signal from any single election is misleading.

In our Texas case study below, we have adopted the threshold condition $s > .6$—that is, to be deemed an *effective district*, we require a greater than 60% estimated chance of nominating and electing a minority-preferred candidate. We chose this figure in view of the above considerations, and because we found that districts with $s > .6$ in any one of our three scores were quite likely to have $s > .5$ in the other two versions, increasing our confidence

[21] Namely, our group-control factor for a district is $c = \min(2k, 1)$ where k is the group's share of CVAP. Alternatively, the modeler could set an election-specific group-control factor in several reasonable ways: as the minority group's estimated share of votes for the candidate of choice; the group's estimated share of the district's Democratic primary electorate; or the estimated group votes for the minority-preferred candidate divided by the total votes for all candidates, for example.

[22] We tune logit curves $f(x) = 1/(1 + \exp(-(ax + b)))$ so that $f(0) \geq 0$, $f(1) \leq 1$, and $f(c \cdot s_i) \approx \delta_i$ where s_i are the raw effectiveness scores of enacted districts, c is group control, and $\delta_i \in \{0, 1\}$ are the ground-truth outcomes (with 1 for success) for the corresponding candidates of choice. The aim is to input a raw effectiveness score s and a group-control factor c and update s to a probability of effectiveness $f(cs)$. For details and examples, see Supplementary Appendix B.

that the districts selected in this way are likely to perform more often than not.[23]

4.6. Assembling the ingredients to build a VRA-conscious ensemble

Running on a standard laptop, ReCom generates new plans at a pace of hundreds of plans per minute in the Python implementation in (MGGG Redistricting Lab 2018b), and runs about 40 times faster in the Julia implementation in (MGGG Redistricting Lab 2020b), depending on the size of the districting problem and the tightness of the constraints.[24] The VRA-conscious protocol implemented here in Python (MGGG Redistricting Lab 2020a) reassesses district effectiveness scores at each step, which slows the process somewhat, so that our runs take about 35 steps per minute for the unweighted and statewide scores and about 15 steps per minute for the district-level score on a state the size of Texas. For a smaller state like Louisiana, the speed more than doubles.

The last question to specify our protocol is how to set the numbers of effective districts that a proposed map must contain for each minority group, to be presumptively valid under the VRA and the Constitution, and thus to be included in our ensemble. Our first guide in answering this question is the state's most recent districting plan, which may have been in effect for up to a decade and either has gone unchallenged in court or has withstood legal challenges, including VRA claims.[25] The second guide, discussed above, is *rough proportionality*, within the meaning of the Supreme Court's important VRA decisions in *Gingles* and *De Grandy*: plans are frequently judged by whether the share of effective districts is similar to each group's share of statewide CVAP.

Considering these guides, we will reject proposed plans that have fewer minority-effective districts than the benchmark plan; in other words, we will treat this threshold level of effectiveness as a *validity check* in the district-generation algorithm. For instance, if we are considering a single minority group and the benchmark plan has three districts that are effective for that group, then each plan included in the ensemble must have at least three effective districts as well. On the other hand, we would reject a proposed plan if it had so many effective districts for one minority group that it would relegate another sizable demographic group to substantially sub-proportional representation.

Surveying the protocol described in this section, the key to our approach is its close reliance on detailed, precinct-level election results from both primary and general elections. We do not assume that some *a priori* demographic threshold will cleave districts that provide minority voters with realistic electoral opportunities from districts that will not. The approach is deeply empirical, focusing on whether a specific district, regardless of its precise demographic percentages, has a recent history of consistently supporting minority-preferred candidates in both primary and general elections. To quote Justice Kagan, our protocol is "evidence-based, data-based, statistics-based. Knowledge-based, one might say" (*Rucho* v. *Common Cause* 2019, 2519 (Kagan, J., dissenting)).

5. CASE STUDY: CONGRESSIONAL DISTRICTING IN TEXAS

We applied the VRA-conscious protocol described in section 4 of this article to build 36-district Texas congressional plans.

5.1. Data

We downloaded the 2018 Texas precinct shapefile and statewide election returns from the Texas Legislative Council's website (Texas Legislative Council 2020). Table 2 shows summaries of the demographic data obtained from the 2010 decennial census and the ACS rolling average for the five-year span

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[23]Case law does not dictate how certain we must be of district effectiveness. When analyzing Texas districts, we found that rejection sampling for effectiveness ran as efficiently at the $s > .7$ threshold as it did at $s > .6$, suggesting that a modeler could exercise considerable discretion in setting the effectiveness threshold.

[24]To be more precise, we conducted non-VRA trial runs on Texas, Virginia, and Pennsylvania congressional plans built out of precincts using identical machines (Intel(R) Xeon(R) CPU E5-2660 v2 @ 2.20GHz [Ivy Bridge, late 2013]), allowing districts to deviate from ideal population by only 1%. Over runs of various lengths and with various seeds, the Python implementation generated three to eight valid plans per second, while the Julia implementation generated 120 to 320 valid plans per second.

[25]Numbers derived from this benchmark may need to be adjusted if the state's political geography or demographics or the number of districts in a state's plan has changed (for example, due to reapportionment of congressional seats). Our protocol can be run using a different map as a benchmark if there is reason to believe the current plan violates the VRA or the Constitution.

Table 2. Texas Demographics

Racial group	*Share of total population*	*Share of VAP*	*Share of CVAP*
Latino	37.62%	33.61%	29.36%
Black	11.48%	11.36%	13.08%
White	45.33%	49.64%	52.28%
Other	5.57%	5.39%	5.28%
Total count	*25,145,561*	*18,279,737*	*17,858,066*

Latino, Black, White, and Other shares of Texas residents by total population, voting-age population (VAP), and citizen voting-age population (CVAP). Total population and VAP data are taken from the 2010 decennial census, while CVAP data comes from the American Community Survey (ACS) five-year rolling average ending in 2018.

ending in 2018. (We used CVAP from ACS five-year spans ending 2016, 2014, and 2012 when assessing elections from those years.) While election data could be directly joined to the shapefile, we used the maup package to disaggregate ACS data from block groups (the smallest unit for which CVAP is available) down to census blocks and then aggregated the block-level data up to precincts (MGGG Redistricting Lab 2018c). Total population and VAP were collected from the 2010 decennial census; and because these data are available at the block level, they required no proration and could be directly aggregated up to the precinct level.

We then analyzed 21 statewide Texas elections conducted from 2012 to 2018, which are recorded in Table 3. These were all the statewide elections conducted since the last round of redistricting almost a decade ago—for federal and state offices, both executive and legislative, omitting only state judicial elections.

Ultimately, we eliminated from consideration seven of those 21 elections (struck through in the table) because there was no contest in the Democratic primary, which in Texas is a critically important stage of the electoral process for determining which candidates are minority-preferred. We were left with 14 contests: nine primary/general sets and five primary/runoff/general sets, where the runoff was conducted because no candidate garnered an outright majority of the vote in the Democratic primary.

Table 3. The 14 Election Sets in the Texas Data

	2012	*2014*	*2016*	*2018*
President	P/G		P/G	
U.S. Senator	P/R/G	P/R/G		P/G
Governor		P/G		P/R/G
Lieutenant Governor		~~G~~		P/G
Attorney General		~~G~~		~~G~~
Comptroller		~~G~~		P/G
Land Commissioner		~~G~~		P/G
Ag. Commissioner		P/R/G		~~G~~
RR Commissioner	~~G~~	P/G	P/R/G	P/G

The 14 election sets in our Texas data (5 of which included a primary runoff), and the 7 general elections that we omitted because the Democratic nominee lacked any primary opposition. P means Democratic primary; R means Democratic primary runoff; and G means general election.

We also compiled district-level data for the 36 U.S. House, 31 Texas Senate, and 150 Texas House of Representatives seats, including the race and party of the winning candidates in all elections from 2012 to 2018, as well as demographic data for the districts, for use in the score calibration described in section 4.4 and carried out in section 5.3 (History, Art, and Archives, U.S. House of Representatives, Office of the Historian, 2020a, 2020b).[26]

5.2. *Racial polarization and candidates of choice*

The statewide results for general elections in Texas show a stark pattern of racial polarization. Across 14 separate contests in four election cycles, all three minority groups consistently voted Democratic, and white voters consistently voted Republican, as shown in Figure 3. In Texas, it is commonplace for more than three-quarters of white voters to vote Republican and more than three-quarters of minority voters to vote Democratic in the same election. Furthermore, this basic pattern appears to hold, to a greater or lesser degree, in every region of the state.

It therefore is not surprising that the great majority of Texas's non-white officeholders are Democrats. From 2012 through 2018, there were only two exceptions for Representatives in Congress (out of 15 Latino or Black members) and eight exceptions for Texas state Senators or Representatives (out of 83 Latino or Black state legislators).

No Democratic candidate has won a statewide general election in Texas since 1994. So none of the Latino- or Black-preferred candidates in our 14 recent contests prevailed statewide. But the vote patterns show that each of them carried a significant number of *districts* in general elections under the current Texas congressional plan and under every plan in our ensembles.

Just as the Latino-preferred and Black-preferred candidates in all 14 statewide elections were Democrats (see Figure 3), the same has held true in

[26]See also Carl Klarner. 2019. "Racial Identification of State Legislators 2001–2019." Unpublished data set. Purchased from <http://klarnerpolitics.org/>.

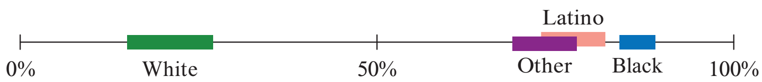

FIG. 3. The highest and lowest EI point estimates for each racial group's support of the 14 Democratic nominees in statewide general elections: White (15–27%), Other (69–78%), Latino (73–82%), and Black (84–89%).

congressional elections. The success of Latino- and Black-preferred congressional candidates in Texas therefore has hinged on their ability to win Democratic primaries (and, where applicable, primary runoffs) and then win general elections. A large majority of white voters in Texas primary elections participate in the Republican primary, while most people of color who participate in Texas primaries vote in the Democratic primary. So, for VRA purposes, we can currently forgo analysis of voting patterns in Republican primaries or Republican primary runoffs in Texas.

In Democratic primaries and primary runoffs, we found a high degree of cohesion across demographic groups. Because all 14 contests were for single-member offices (like governor), we focused on the one candidate in each Democratic primary who was preferred by each of the four demographic groups. In nine of the 14 Democratic primaries and in four of the five Democratic primary runoffs, the three minority groups (Latino, Black, Other) preferred the same candidate, as shown in Supplementary Appendix Table 7.

Given this cohesion in Democratic primaries and runoffs and especially in general elections, it might well be possible to treat Latino and Black voters, or Latino/Black/Other, as a single coalition group for VRA purposes (*Campos* v. *City of Baytown*, 1988, 1244–45). Our main analysis will treat Latino and Black voters as separate minority groups, but the same method could be adapted (and indeed simplified) for coalitional analysis.

As a final and important point relating to our EI setup, we note that we do not need to run EI on small geographies to detect regional difference.

For example, in the 2018 gubernatorial runoff, former Dallas County Sheriff Lupe Valdez and Houston's Andrew White are identified as the statewide candidates of choice for Latino voters and Black voters, respectively. But in the Dallas-Fort Worth Metroplex, Valdez carried both minority groups. As Figure 4 shows, that effect is visible in our EI outputs from a statewide run, because the hierarchical model works by computing distributions of support on each precinct. This lets us identify Valdez as the Black-preferred candidate in the Dallas-Fort Worth Metroplex while White is seen to have carried the Black vote in the Houston area.

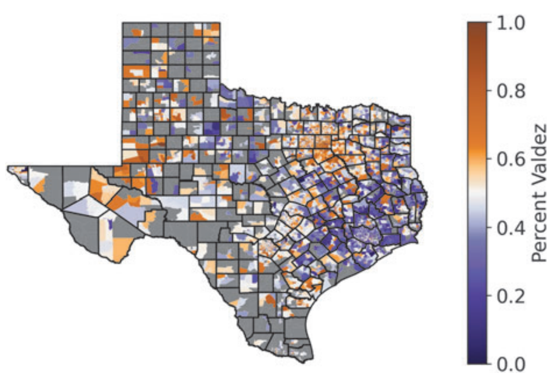

FIG. 4. The distribution of EI-estimated Black support for former Dallas County Sheriff Lupe Valdez in the 2018 gubernatorial runoff. The Dallas-Fort Worth area, in northeastern Texas, is mostly *orange* in this map, while the Houston area, in southeastern Texas, is mostly *purple.* (The map's *gray areas* contain few, if any, Black voters.) This map shows that even statewide EI can find significant regional variation in a group's voter preferences.

5.3. *Effectiveness scores and inclusion criteria*

In Texas, we have the benefit of seeing results from 33 separate contests (14 primaries, 5 primary runoffs, and 14 generals), so that 14 potential successes make up the raw effectiveness score.[27]

According to recent CVAP data (shown in Table 2 above), rough proportionality would require 10.6 districts and 4.7 districts that are effective for Latino voters and Black voters, respectively, given Texas's current congressional apportionment of 36 seats. We will round these to 11 and 5 districts, respectively. If Latino, Black, and Other voters were treated as a coalition, that coalition's proportional share would exceed 17 districts.

Using any of our three calibrated scores, Texas currently has 11 effective districts for minority groups at the 60% threshold: seven Latino-effective districts,

[27]To perform the logit calibration step described in section 4.4, we used all congressional and state-legislative winners from 2012 to 2018. This includes 145 congressional contests (36 districts), 600 state House contests (150 districts), and 77 state Senate contests (31 districts), for a total of 822 data points. This includes one special election for Congress.

TABLE 4. STATISTICS FOR EFFECTIVE DISTRICTS IN CURRENT TEXAS CONGRESSIONAL PLAN

CD	*Location*	*HCVAP %*	*Latino effective* s^{unw}	s^{state}	s^{dist}	*BCVAP %*	*Black effective* s^{unw}	s^{state}	s^{dist}	*WCVAP %*	*Representative*	*Race*
9	Houston	24.7	44	38	43	46.7	96	96	94	16.1	Al Green	Black
15	South Texas	73.7	95	97	97	2.5	8	9	7	22.1	Vicente Gonzalez	Latino
16	El Paso	76.0	99	99	97	4.2	11	12	10	17.5	Veronica Escobar	Latino
18	Houston	26.9	51	44	51	44.9	95	95	95	22.8	Sheila Jackson Lee	Black
20	San Antonio	65.0	97	97	97	5.6	12	12	12	25.8	Joaquin Castro	Latino
28	South Texas	69.2	86	93	96	5.5	10	12	8	23.2	Henry Cuellar	Latino
29	Houston	64.0	98	97	97	16.2	49	48	46	16.7	Sylvia R. Garcia	Latino
30	DFW	22.7	44	38	39	52.1	99+	99+	99	21.7	Eddie Bernice Johnson	Black
33	DFW	46.5	98	98	95	24.1	78	75	64	25.6	Marc A. Veasey	Black
34	South Texas	78.5	98	99	93	1.6	8	9	6	19.1	Filemon B. Vela	Latino
35	Austin/San Antonio	52.2	97	97	97	10.3	22	20	24	34.4	Lloyd Doggett	White

The population shares and calibrated effectiveness scores for the 11 districts in the current Texas congressional map that are labeled effective for Latino and/or Black voters. Scores over 60% have darker shading, and scores in the 50–60% range have lighter shading. Mark Veasey's District 33 is the only one that registers as effective for both Latino and Black voters, though Sheila Jackson Lee's District 18 and Sylvia Garcia's District 29 are close. All 11 Representatives are Democrats.

three Black-effective districts, and one district that is effective for both groups (see Table 4). If our protocol focused solely on the most recent elections (e.g., 2018), however, two additional districts—District 7, currently represented by Lizzie Fletcher, a white Democrat, and District 32, currently represented by Colin Allred, a Black Democrat—might meet the effectiveness thresholds for Latino voters or Black voters under some or all of our three calibrated scores. But in the early years of the decade (e.g., 2012 and 2014) both districts were still reliably voting for Republicans in statewide and congressional elections.

Since the current map has withstood judicial scrutiny under both the VRA and the Equal Protection Clause (*Abbott* v. *Perez* 2018, 2324–34), we require plans in our VRA-conscious ensemble to meet or exceed that map's level of effectiveness: so we require at least eight Latino-effective districts, at least four Black-effective districts, and a total of at least 11 districts that are effective for at least one of the groups. So, for example, a plan whose (Latino, Black, Both, Neither) effective-district count was (4, 0, 4, 28) would not qualify for the ensemble because it falls short of 11 minority-effective districts. In effect, this approach allows plans whose effective-district counts are (7, 3, 1, 25) or (8, 4, 0, 24), as well as plans that dominate one of those outcomes from the minority perspective by shifting districts from Neither to any of the other categories.[28]

5.4. Basic results

In this section we first present evidence to support the claim that our chains of districting plans have produced VRA-conscious ensembles whose statistics have stabilized after 100,000 steps. We then look at how the statistics from these ensembles compare to an ensemble built with no consideration of race and to an ensemble generated with demographic thresholds as a potential stand-in for VRA compliance. Put differently, we compare ensembles generated by our VRA-conscious protocol, which uses both racial and electoral data, with an ensemble built with racial but not electoral data and an ensemble built with neither racial nor electoral data.

We built five ReCom ensembles, by running each of the following kinds of chain until 100,000 maps are accepted.

(non-VRA) *No VRA consideration.* Only population equality is an explicit validity check, since contiguity is required and compactness is weighted into ReCom ensembles by construction, so the algorithm does not have to be manipulated to produce reasonably compact districts.

(unw) *Constrained by* s^{unw} *effectiveness.* Ensemble inclusion additionally requires at least eight districts over 60% Latino-effective, at least four districts over 60% Black-effective, and at least 11 total districts effective for one or both groups, using unweighted effectiveness scores.

(state) *Constrained by* s^{state} *effectiveness.* Same as above, but using statewide weighted scores.

[28]Although a map with fewer than 18 Neither districts could potentially give rise to a Section 2 claim by white plaintiffs and thus merit exclusion from an ensemble, our chain runs did not generate any such plan.

(dist) *Constrained by s^{dist} effectiveness.* Same as above, but using district weighted scores.

(CVAP) *Constrained by CVAP shares.* A plan must have at least eight districts over 45% HCVAP and at least four districts over 25% BCVAP to pass the validity check.[29]

5.4.1. Convergence heuristics and robustness checks. Neither ReCom nor any other MCMC method will work properly if it is not allowed to run long enough, or if designed in a way that thwarts convergence. In this article we have used ensembles built by including every plan that passes the validity checks and continuing until 500,000 maps are collected. We used two kinds of evidence to arrive at the conclusion that 500,000 plans are probably sufficient: first, we have confirmed that chains of that length have aggregate statistical properties that are approximately independent of their starting points, or "seeds," even when the seeds are quite different. This test is sometimes called the *multistart heuristic*. Second, for selected instances we have confirmed that an ensemble ten times as large has similar aggregate statistics. Passing these tests is not a rigorous proof of approximately representative sampling, but these are standard convergence heuristics used across applied statistics. If any ensemble method fails these tests, we can be sure that either the setup violates the conditions for a unique steady state, or we have not run the chain long enough to approach it.

For the multistart heuristic to have high value, we should choose plans that are initially very different and check to see that the ensembles converge to find the same summary statistics nevertheless. The first seed plan used for the multistart test for this Texas case study is the enacted congressional plan that is currently in effect, which came out of the court proceedings challenging the early-decade plan of the Republican legislature. To find two other seeds with exaggerated differences from the enacted plan, we turned to the Atlas of Redistricting project conducted by the politics team at FiveThirtyEight (Bycoffe et al. 2018). Seed 2 is their Texas plan drawn to favor Democrats, which is visibly quite different from the enacted plan and of course has very different partisan properties as well. Seed 3 is based on the plan FiveThirtyEight drew with an eye to compactness scores and county integrity.[30]

For the ensemble using the statewide effectiveness score, Figure 5 shows that a simple partisan statistic—the Clinton share of the major-party presidential vote from November 2016 across the 36 districts—gives roughly the same answers after 100,000 steps, whether the chain commences with the enacted plan or with either of the two other seed plans. Similar charts for s^{unw} and s^{dist} are found in Supplementary Appendix Figure 17. These are boxplots (or "box-and-whiskers plots") where for each plan the districts have been sorted from 1 (the district with the lowest Clinton share) to 36 (highest Clinton share). The boxes show the values at the 25th to 75th percentiles, with the median marked, and the whiskers are set at the 1st and 99th percentiles. Colored circles show the initial values for the enacted congressional plan (red) and the two additional seed plans (blue and green). The aggregate data collected from the three differently initialized runs is broadly consonant: across the districts, the three ensembles have medians, quartiles, and overall ranges within one or two percentage points of each other, even when the seeds began over 15 points apart. By contrast, Figure 6 focuses on the 18 districts with the highest Clinton share to show that our VRA-conscious ensembles, by any of the three scores, do perform differently than if a user either ignored the VRA entirely or used the CVAP demographic constraint as a VRA proxy.

We can also compare spatialized statistics such as the one shown in Figure 7, a record of the number of times that each precinct appeared in a district with $s^{\text{state}} > .6$. Just 1,000 steps from the starting point, the heatmaps are visibly different, showing that the chain has not run long enough for this statistic to converge. Much nearer visual correspondence is achieved after 10,000 steps, and the heatmaps are nearly indistinguishable after 100,000 steps.

Beyond the multistart trials, we also checked the same statistics (Clinton vote distribution and cut-edges score) after 1 million steps. We found

[29]To build a demographic-target ensemble, we searched for maps with at least eight majority-Latino districts and at least four majority-Black districts by CVAP. Initial attempts did not produce any such maps. We then lowered the thresholds to 45% for Latino CVAP and 25% for Black CVAP. While those thresholds are somewhat arbitrary, they roughly track Table 4, as well as the results of section 6 shown in Figure 9.

[30]The FiveThirtyEight compact plan did not initially meet our VRA effectiveness requirements, so we used a heuristic-optimization run as in Supplementary Appendix H to get it past the thresholds. Both FiveThirtyEight plans had to be transferred onto our precinct units with the maup package (MGGG Redistricting Lab, 2018c).

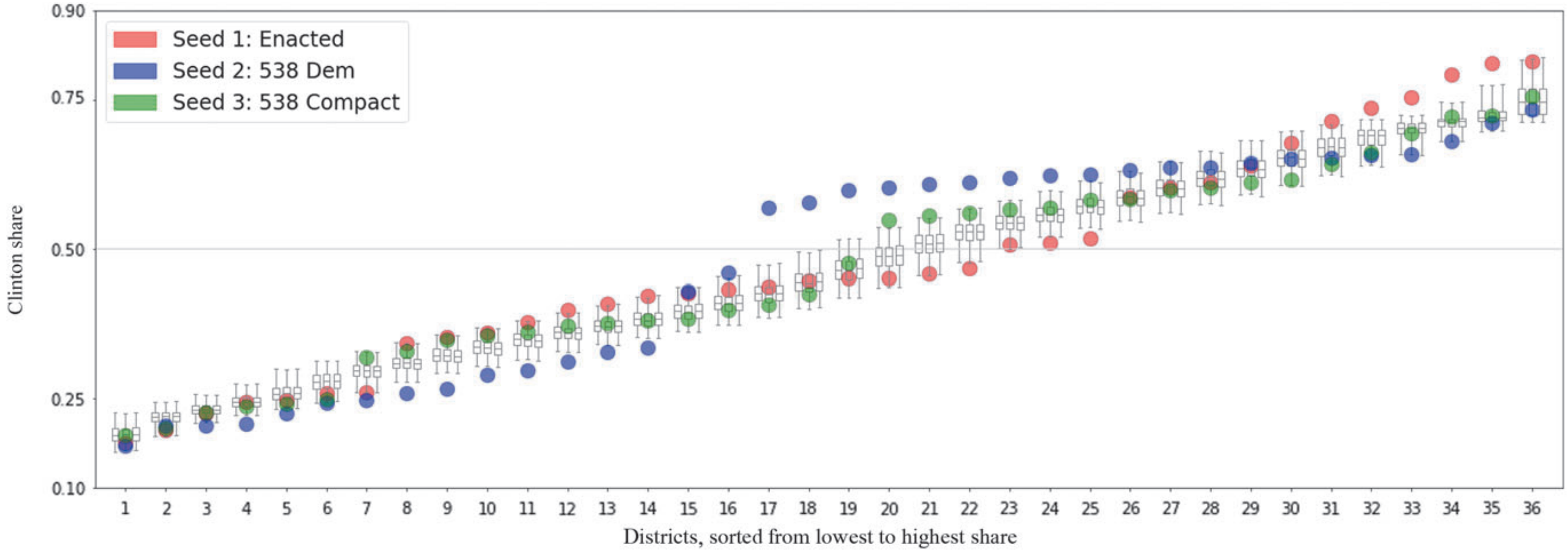

FIG. 5. In this multistart heuristic convergence test, the VRA-conscious chain for the statewide weighted effectiveness score s^{state} is run for 500,000 steps from three very different starting points. The colored dots show the Clinton share of the major-party vote from the 2016 presidential general election, district by district, in the three seed plans described in the text (with the districts sorted from lowest Clinton share to highest). The *boxes and whiskers* show Clinton share by district for each of the three ensembles—they have converged to within one or two percentage points in each district, even though the seed plans sometimes differ by 15 points or more.

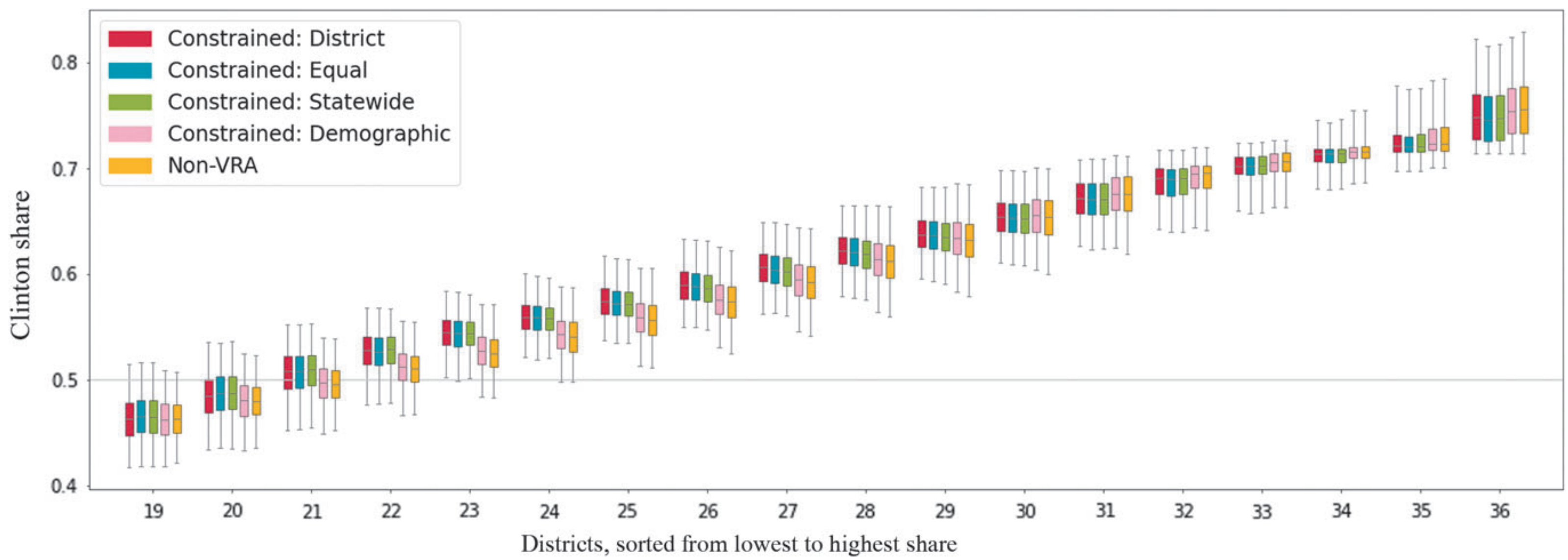

FIG. 6. Comparing the three kinds of VRA-conscious ensembles, constrained by the s^{dist}, s^{unw}, s^{state} scores, respectively, to the alternatives described in the text. Here, the Clinton share is plotted across 500,000 steps and displayed for the 18 most Democratic districts. There is a small but discernible difference that separates the partisan statistics of the VRA-conscious ensembles from those of the control ensembles, which are interestingly similar.

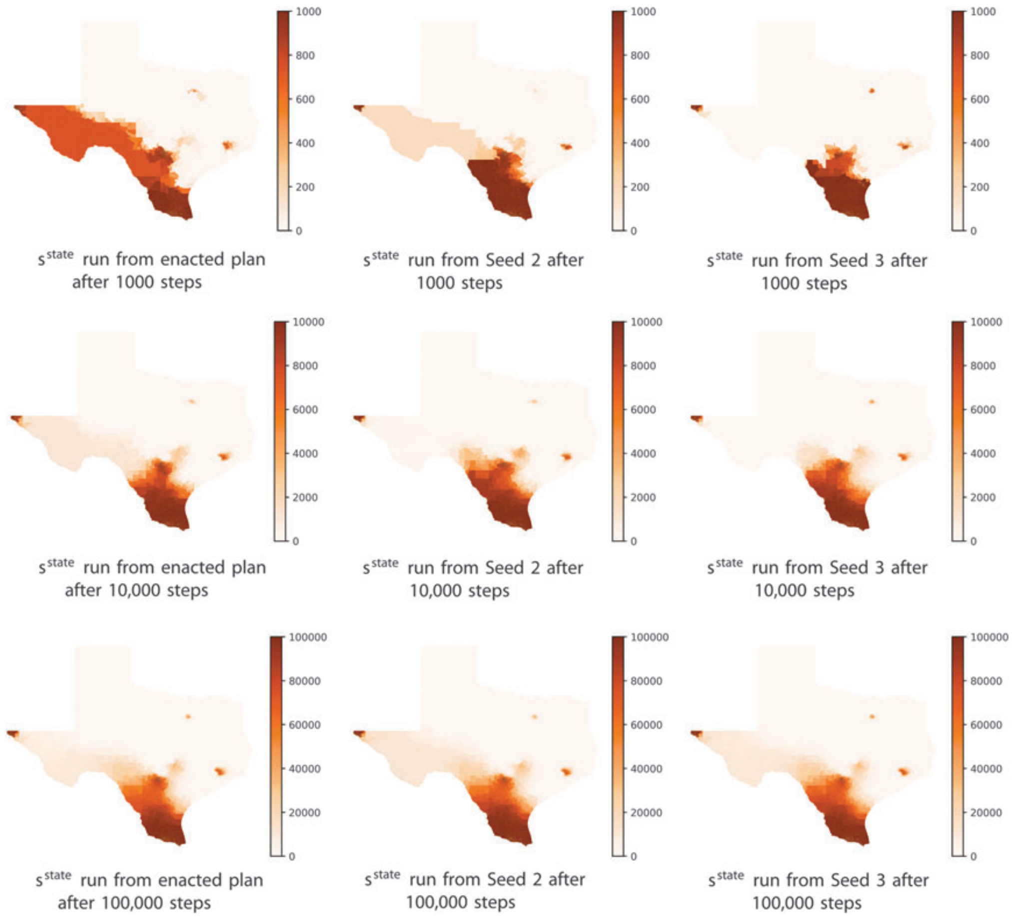

FIG. 7. The *color* of each precinct shows how many times it had appeared in a Latino-effective district after 1,000, 10,000, and 100,000 steps. These VRA-conscious ensembles are drawn with respect to the s^{state} score from the same three seed maps described in the text. There are initially significant differences across the three seeds (*top row*), but the plots converge over the course of the run (*bottom row*).

minimal difference in partisan or district-shape metrics when comparing the initial 100,000 steps, a sub-sampled 100,000-plan ensemble containing every tenth map from the set of 1 million, or the full million-plan ensemble. This raises our confidence both that the size of the sample is adequate to this level of statistical detail and that a run length in the hundreds of thousands is sufficient for convergence. Finally, we conducted slightly altered runs to confirm whether the general findings are robust to reasonable perturbations in the methodology laid out in sections 4.3, 4.4, and 4.5.[31]

[31]We conducted the following tests: using estimated share of candidate support rather than CVAP share of the district as the group-control factor *c*; replacing the confidence term for correctly identifying candidates of choice *C*(*p*) with the simpler term *p*; and dropping both the group-control factor and the calibration entirely. For the alternative group-control measure, the changes to scores on Texas congressional plans were minor for both the enacted plan and generated plans. Changes also were typically small with the simplified confidence factor, but the scores became more unstable because outcomes with high EI-based uncertainty had more weight relative to clear outcomes, producing an illusion of greater electoral success on some runs of EI. The logit calibration was valuable largely to correct for the reduction of scores by group control; we find that if we drop both of them, districts with significant shares of both Latino and Black voters are rated higher for both groups than recent electoral history warrants. Finally, we confirmed that the rate of ensemble generation is similar whether the effectiveness threshold is set at 60%, 70%, or even 75%. Taken together, these robustness runs increase our confidence that each of these parameters that requires user choice is indeed doing work in constructing a stable score that comports with electoral history, but that some of the details could be altered without breaking the protocol.

5.4.2. Comparing ensembles. In this section we compare the five ensembles defined in section 5.4 to each other, considering whether those created using our VRA-conscious protocol differ significantly from those created without electoral data or without both electoral and racial data. The answer is a definitive yes. We have already seen that the three effectiveness scores are similar to each other for the enacted plan's minority-effective districts (Table 4). Using summary statistics, we can confirm that the constrained ensembles using the three scores are similar to each other as well. But the three VRA-conscious ensembles do not resemble either the non-VRA ensemble (which uses neither electoral nor racial data) or the CVAP-shares ensemble (which uses racial, but not electoral, data as a purported stand-in for VRA compliance).

The upshot of rejecting plans with not enough effective districts is seen in Figure 8 with respect to the s^{state} score: no plan in the ensemble has fewer than eight Latino-effective or fewer than four Black-effective districts. This number of effective districts rarely happens by chance without a VRA-conscious method. Interestingly, enforcing the demographic threshold condition (bottom row) makes it somewhat more common to get at least four Black-effective districts but does not make an appreciable difference in the likelihood of creating an eighth Latino-effective district. (Supplementary Appendix F contains analogous plots for the s^{dist} and s^{unw} scores.)

Table 5 is another view of the comparison. A significant share of the plans in all the VRA-conscious ensembles pass the demographic test set forth above, but relatively few plans in the non-VRA and the

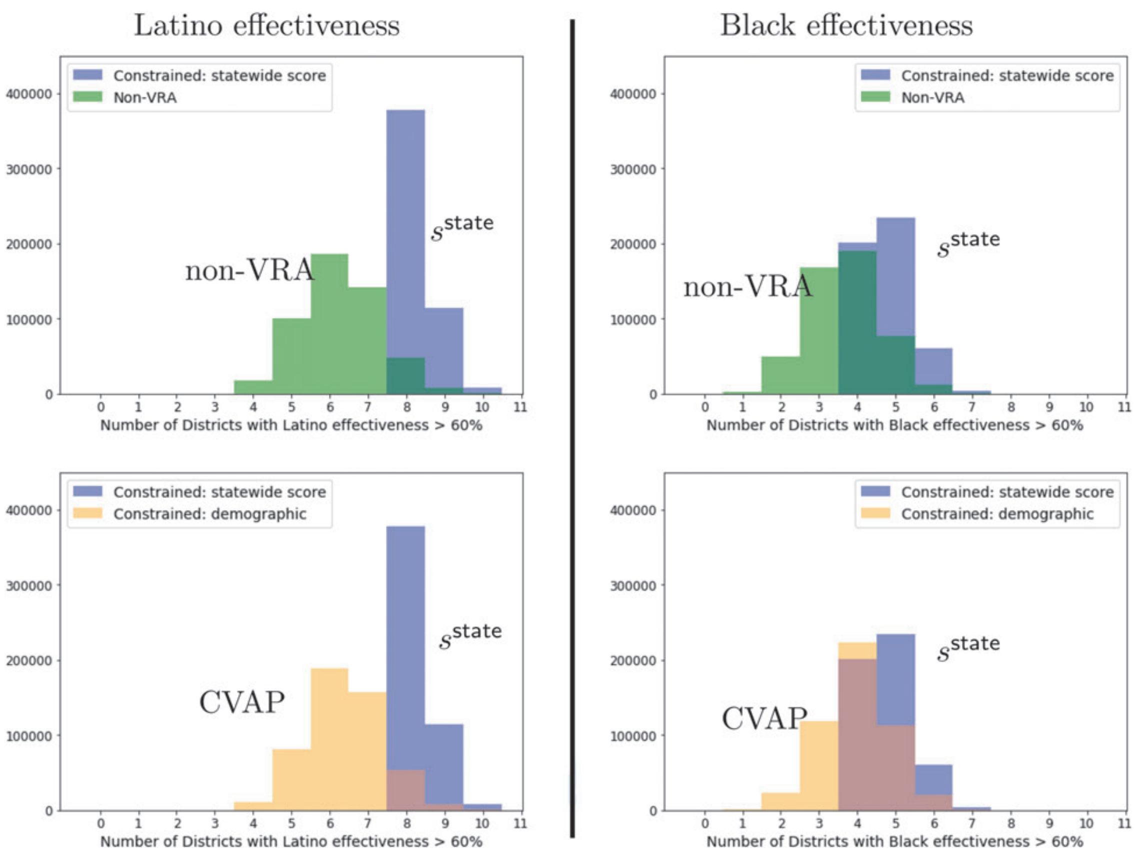

FIG. 8. The distribution of Latino- and Black-effective districts in a VRA-conscious ensemble (*purple*), compared to the non-VRA alternative (*top*, in *green*) and the CVAP-shares, demographics-based alternative (*bottom*, in *orange*). All are shown with respect to the s^{state} score. Note the very modest improvement in effectiveness for the CVAP-shares ensemble compared to the non-VRA ensemble.

TABLE 5. THE SHARE OF MAPS IN THE FIVE ENSEMBLES (COLUMNS) SATISFYING VARIOUS CRITERIA (ROWS)

		Unconstrained (non-VRA)	Constrained			Constrained (CVAP)
			(s^{unw})	(s^{state})	(s^{dist})	
Satisfies	(s^{unw})	15%	(100%)	88%	81%	20%
effectiveness	(s^{state})	20%	98%	(100%)	94%	26%
criteria	(s^{dist})	16%	72%	78%	(100%)	22%
Satisfies demographic criteria		30%	39%	46%	51%	(100%)

For the effectiveness criteria, maps must have at least eight Latino-effective districts (effectiveness over 50% for the indicated score), at least four Black-effective districts, and at least 11 distinct districts that are effective (for one or both groups) overall. Note that each VRA-conscious variant is built to satisfy effectiveness in a chosen score at the 60% level, making it likely to pass at least 11 district effectiveness tests for the other scores at the 50% level, since the scores are similar but not identical. The demographic test in the bottom row requires a map to have at least eight districts over 45% HCVAP and at least four districts over 25% BCVAP.

CVAP-shares ensembles pass our effectiveness tests.[32] This suggests that Texas ensembles built without rich electoral data—or by imposing a racial threshold—are unlikely to reflect VRA compliance and might well contain far too many maps that violate federal law. And this problem likely cannot be cured simply by changing the threshold levels for the CVAP-shares ensemble: if the CVAP thresholds are raised, it will become harder to find plans with enough qualifying districts, and many effective districts will be missed.

Comparing the three score-based ensembles against each other shows some differences but also substantial alignment in the determinations of validity. We should not be surprised that scores that typically track each other within a few percentage points can fall on the other side of a bright-line threshold: if s^{unw} is just over .6, it can certainly happen that s^{dist} is just below that level. But most districts for which one score is over .6 have the other scores over .5, making them more likely than not to be effective for the group in question. This standard is met by more than three-quarters of the s^{state} and s^{dist} ensembles. (Again, this is part of the justification to set the effectiveness threshold for ensemble inclusion at a level buffered safely above 50%.)

Considering all the evidence so far, one might ask whether any of the three calibrated effectiveness scores is to be preferred to the other two. Our determination is that all three scores can be useful. The unweighted score has the weakest claim of the three, because on its face it omits factors that are legally and factually relevant. As for the other two scores, we think it can be valuable to consider both. The district-weighted score has more regional discernment and a more sophisticated incorporation of EI outputs; the statewide-weighted score has a simpler explanation and still takes uncertainty into account. While results for different scores are not identical, the modeling methodology is robust across three reasonable ways of weighting elections to measure district effectiveness.

6. LEARNING PATTERNS IN DISTRICT EFFECTIVENESS

We have just seen that Texas congressional ensembles using demographic data but no electoral data do not resemble ensembles generated by our VRA-conscious, heavily data-driven protocol. But what about a method that uses both demographics and electoral data but in a limited way, needing only a smaller and simpler dataset? Often, scores that seem to be complicated by taking many things into account can be closely replicated using simpler inputs. In our setting, we would like to see whether our seemingly sophisticated handling of dozens of election contests could be well approximated by pared-down district metrics. To examine this question, we now model the nonlinear relationship between effectiveness scores and lower-dimensional combinations of demographic and partisan features.

In statistics and machine learning, numerous techniques have been developed to recognize patterns in data. *Classifier* models use training data to "learn" discrete labels (like yes/no effectiveness), while *regression* models "learn" continuous-

[32]That only about half the maps in the three VRA-conscious ensembles satisfy the demographic criteria implies that it is not uncommon in Texas for Latino-effective districts to have less than 45% HCVAP or for Black-effective districts to have less than 25% BCVAP. That fact in turn suggests that, at least in some parts of the state, there is significant coalitional voting between different minority groups.

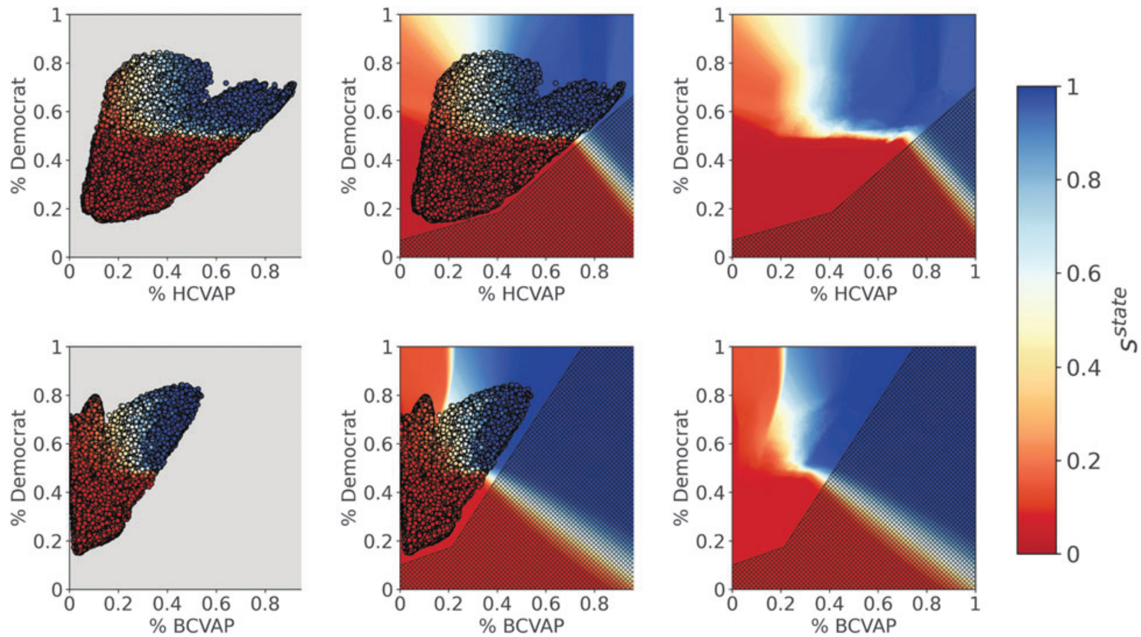

FIG. 9. The *top row* refers to effectiveness for Latino voters and to Latino CVAP; the *bottom row* to corresponding statistics for Black voters. Two-dimensional scatterplots (*left column*) show a collection of districts drawn from a non-VRA ensemble, arranged by Latino or Black CVAP share on the *x* axis and partisan lean on the *y* axis, then colored by their s^{state} score for Latino- or Black-effectiveness, respectively. The *k*-nearest-neighbors (KNN) method is "trained" on that data to infer approximate scores for all possible positions in the square (shown with the training data in the *center figures* and without it at *right*). The hatched areas in the *center and right-hand plots* contain no labeled data points, so the KNN estimates are less meaningful in those areas.

valued assignments (like effectiveness scores), on the basis of features in the data. For our examples, we are choosing to classify potential Texas congressional districts on the basis of two kinds of features:

- *Demographics*, using Latino and Black CVAP shares; and
- *Partisan lean*, obtained by averaging the Democratic shares of the 2016 and 2012 major-party presidential vote, with the more recent general election weighted twice as heavily as the older one.

We begin with a (non-VRA) ensemble of 500,000 plans, then extract the districts from each to make a large dataset, containing 997,163 districts after de-duplication. For each district, we compute its statewide weighted effectiveness score s^{state}. We randomly separate these districts into training data (80%) and data points held back for testing and validation (20%).

We attempted several kinds of models. A *k*-nearest neighbors (KNN) model assigns a value to each point based on the *k* points in the training data that are closest to its location. This can be thought of as a predicted effectiveness score for districts that may be proposed in the future. The choice of *k* is made by a validation step that attempts many different values and chooses the one that provides the highest accuracy.[33] For the regression, the learned value assigned to a point is the average value of its *k* nearest neighbors, while the yes/no classification is made by selecting the majority label among those neighbors.

The outcomes of two-dimensional KNN regression are shown in Figure 9. They show a complicated district-level relationship between effectiveness (color), Latino or Black CVAP shares (*x* axis), and partisan lean (*y* axis). If the effectiveness of districts could be captured with CVAP

[33]To be precise, we use *m*-fold cross-validation with $m=10$, then choose the *k* for KNN with the best average r^2 and mean squared error (MSE) over those ten-fold trials. Using those values of *k*, the final accuracy estimates use the full set of training data and are then corroborated against the withheld testing data.

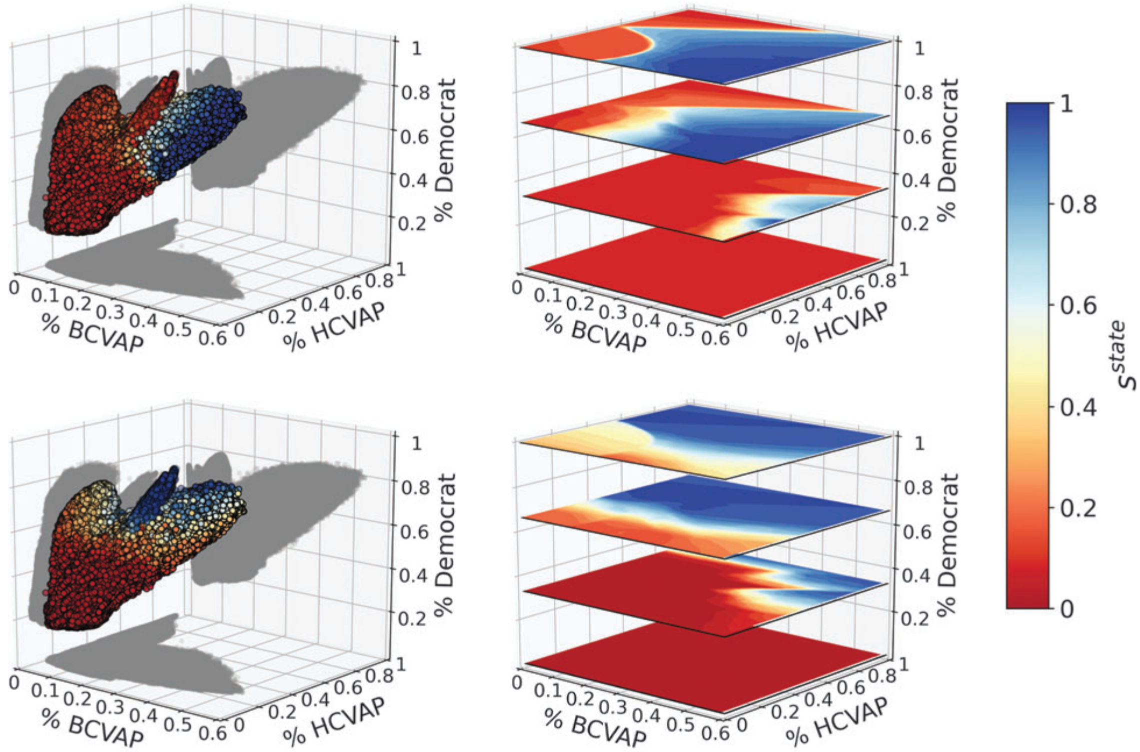

FIG. 10. KNN regression for a three-dimensional scatterplot of district effectiveness.

shares alone, we would see a vertical line dividing the effective (blue) from the ineffective (red) zones. If overall partisanship were a good predictor on its own, we might see a horizontal dividing line; this is not the case, but we note that partisanship alone is more predictive for Latino effectiveness. If effectiveness could be expressed in a simple linear relationship between partisan lean and CVAP, we would see a straight line of some slope separating the blue and red regions. Instead, we see a more complicated frontier with a large zone of ambiguity, especially in Latino effectiveness.[34]

Because Texas has two sizable minority groups, and Latino and Black voters often have overlapping electoral preferences, we might hope to do better by taking both groups' CVAP shares into account simultaneously. To this end, Figure 10 shows the same kind of regressions in three dimensions: Latino CVAP, Black CVAP, and the same measure of partisan lean. These plots still reveal complex, nonlinear frontiers and significant zones of ambiguity.

Further pattern-recognition results using various models for regression and classification are found in Supplementary Appendix G. Together, these methods indicate that scores built from our involved electoral methodology do not easily reduce to combinations of CVAP demographics and general-election partisan lean. This leads us to conclude that electoral complexity, perhaps especially the dynamics of actual primary elections, is playing an ineliminable role in our determination of district effectiveness.

7. CLOSING THE REPRESENTATION GAP

Finally, we return to where this article began: the underrepresentation of communities of color at both the federal and state level. The algorithmic techniques described in this article can be readily

[34]Grofman, Handley, and Lublin (2001) studied what amounts to effectiveness classification in a similar feature space nearly 20 years ago, positing an “elbow” or V-shaped frontier of effectiveness. For a comparison of our classification results with their framework, see Supplementary Appendix G.

reconfigured to point the way to maps that are likely to promote significant gains in minority representation.

7.1. *Searching for higher effectiveness*

Recall first that our VRA-conscious ensembles are made by imposing yes/no validity constraints rather than a probabilistic tilt or bias: the proposal of new plans is made without regard to race, and the validity criteria are given by a threshold test, with no preference for plans that exceed the threshold by a wider margin. It is therefore unsurprising that this procedure does not on its own favor the creation of plans that greatly surpass the status quo in minority electoral opportunities. But—so long as districts are population-balanced, contiguous, reasonably compact, and constructed largely or entirely from intact precincts, as is the case across all our ensembles—maps generating rough proportionality for all sizable minority groups might well be the ones that actually minimize legal exposure under both the VRA and the Equal Protection Clause.

By shifting to an algorithm that has a tilted acceptance function favoring increased minority electoral opportunities, we found it to be straightforward to create maps that fully meet (or even exceed) rough proportionality simultaneously for multiple minority groups. For example, in Texas we were able to create maps that are effective enough to typically meet rough proportionality simultaneously for both Latino and Black voters, while not sacrificing districts to double-counting—i.e., while achieving near-proportionality for people of color overall as well as for each group individually. A *heuristic optimization* algorithm can preferentially accept maps with higher minority effectiveness. We carried this out with the general "short bursts" strategy outlined in Cannon et al. 2020; for details, see Supplementary Appendix H.

To be clear: maps proposed for adoption should be developed through human deliberation based on significant community input and a broader range of criteria and values than our algorithm incorporates. No map plucked from an ensemble is likely to satisfy all human desiderata off the shelf. But just to demonstrate that a map with eight Latino-effective districts and four Black-effective districts can be replaced by one with (at least) ten and five such districts, respectively, we examine one demonstration plan found in a local search.

7.2. *A demonstration plan*

Our demonstration plan is depicted in Figure 11, and its effectiveness statistics by district are shown in Table 6.

We emphasize that this map is not intended to be an ideal map. But it does show that a carefully drawn plan could be dramatically fairer for historically underrepresented minority groups in Texas. We call it a "demonstration map" because it demonstrates that the shortfall of minority representation in the status quo map can be cured. The failure to do so can be attributed not to geography or law, but only to line-drawing.

In Table 6, we have *uncoupled* the primary and general elections, to give a more detailed view of the electoral history of these districts. In other words, this table shows the primary/runoff success independent of the general-election outcome, while our effectiveness-scoring system requires wins in both the primary (or primary and runoff) and the general, to be counted as a success. The table shows that, using any of the three scores, the demonstration plan contains at least 11, and perhaps as many as 13, effective districts for Latino voters and at least five, and perhaps as many as seven, effective districts for Black voters. Because one district in the Dallas area (District 33) and at least one in the Houston area (District 18) appear to be effective for both Black and Latino voters, the total number of minority-effective districts in the demonstration plan is 14, 15, or 16, depending on whether you rely on the unweighted, statewide, or district scores, respectively. Only one of the 16 districts is majority-white by CVAP.

Several of these 16 highlighted districts have demographics and effectiveness scores similar to those of the minority-effective districts in the current enacted plan (compare Table 4). However, in the current enacted plan, every district except Congressman Veasey's District 33 follows the rule that districts marked effective for Latino voters have HCVAP over 50% and those marked effective for Black voters have BCVAP over 40%. By contrast, the demonstration plan presented here features several effective districts with lower Latino and Black population percentages. For example, the Austin-based District 27 is a Latino-effective district with an HCVAP a shade under 40%, and the Houston-based District 9 is a Black-effective district with a BCVAP of only 28.6%. We emphasize that each

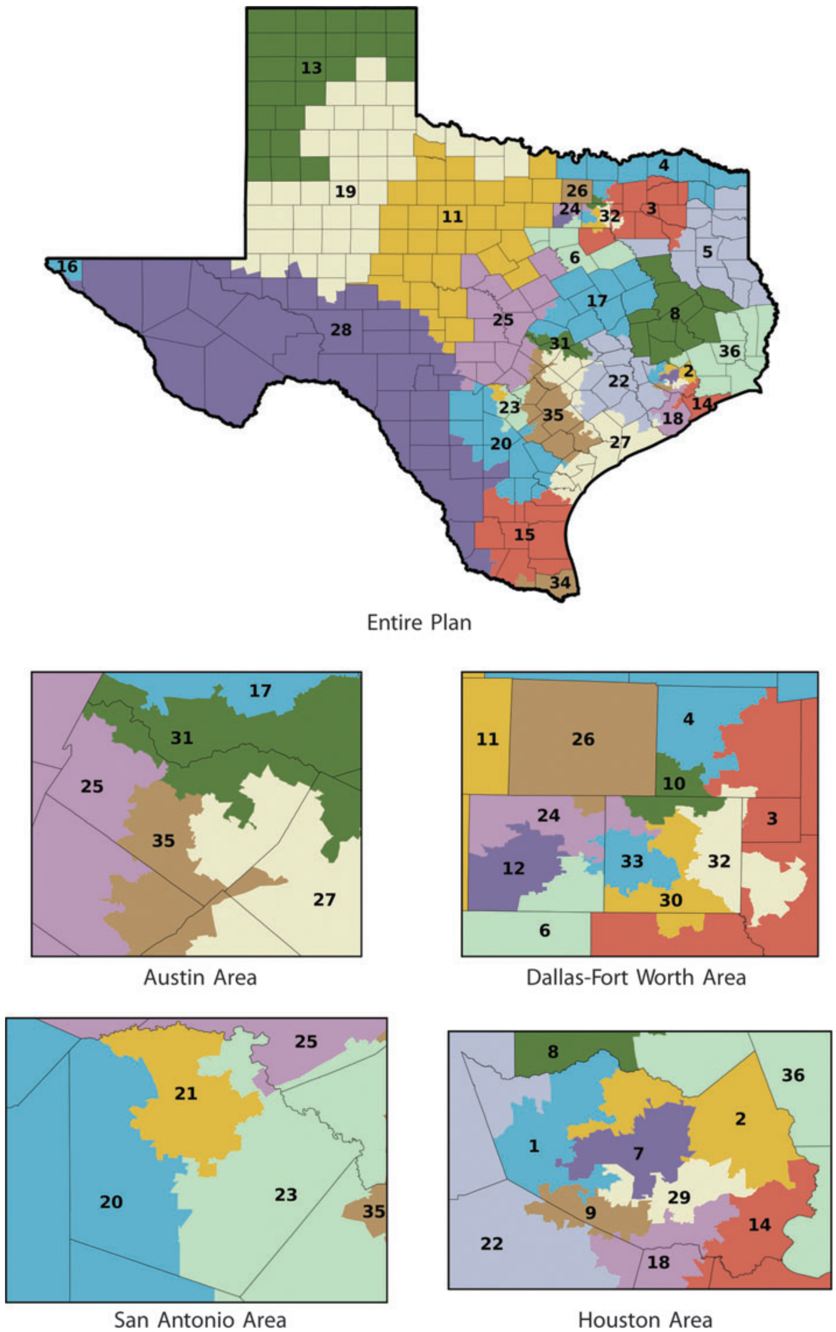

FIG. 11. An interesting demonstration plan found by heuristic optimization.

of those demonstration districts earned its effectiveness score by voting for the Latino- or Black-preferred candidates, respectively, in nearly every statewide election conducted in the last decade.

This map refutes the notion that demographics is destiny when it comes to Texas congressional districts. It contains districts that are majority-minority but not minority-effective (District 2), majority-white but Latino-effective (District 35), plurality-white but Black-effective (Districts 9, 30, and 32) or Latino-effective (Districts 27 and 29), and plurality-Latino but Black-effective

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TABLE 6. STATISTICS FOR EFFECTIVE DISTRICTS IN DEMONSTRATION TEXAS CONGRESSIONAL PLAN

Demonstration Plan

CD	Location	HCVAP %	Latino effective s^{unw}	Latino effective s^{state}	Latino effective s^{dist}	BCVAP %	Black effective s^{unw}	Black effective s^{state}	Black effective s^{dist}	WCVAP %	14 Primaries Latino	14 Primaries Black	14 Gen (Dem.)
7	Houston	36.5	77	65	77	25.5	70	58	31	31.4	9–13	9–10	14
9	Houston	23.3	40	30	33	28.6	78	66	75	31.5	10–12	10–12	14
15	South Texas	78.8	97	98	96	1.7	8	9	6	17.5	12–14	10–11	14
16	El Paso	76.1	99	99	97	4.2	11	12	10	17.4	13–14	11–14	14
18	Houston	32.0	66	59	63	30.7	76	77	69	30.4	10–13	10–12	14
20	San Antonio	60.6	77	82	76	5.5	10	11	9	30.9	12–14	12–13	9
21	San Antonio	47.5	35	74	79	5.6	8	8	8	42.9	12–14	10–14	7
23	San Antonio	51.1	77	82	79	10.7	14	15	14	34.7	12–14	10–12	9
27	Austin/Gulf Coast	39.8	84	85	85	8.8	17	16	18	47.7	12–13	10–14	13
28	South/West Texas	81.4	91	95	96	1.0	7	8	6	16.6	11–14	9–11	14
29	Houston	33.4	70	57	75	25.5	70	58	52	35.5	9–11	9–12	14
30	DFW	15.5	20	15	13	31.8	85	84	69	48.5	9–10	10–11	14
32	DFW	24.1	24	26	28	24.4	52	67	62	44.9	10–13	12–14	10
33	DFW	37.0	85	80	66	32.9	96	97	88	25.1	10–11	13	14
34	South Texas	86.7	97	98	97	0.4	6	7	5	12.3	11–14	9–11	14
35	Austin	30.7	62	62	67	4.8	10	10	9	60.6	11–13	9–10	14

District 27 (with statewide candidates of choice)

		Primary election		Primary runoff election		General election	
		Latino-pref.	Winner	Latino-pref.	Winner	Latino-pref.	Winner
President	2012	Obama	Obama ✓			Obama	Obama ✓
U.S. Senator	2012	Sadler	Sadler ✓	Sadler	Sadler ✓	Sadler	Sadler ✓
U.S. Senator	2014	Alameel	Alameel ✓	Alameel	Alameel ✓	Alameel	Cornyn ×
Governor	2014	Davis	Davis ✓			Davis	Davis ✓
Ag. Commissioner	2014	Friedman	Friedman ✓	Hogan	Hogan ✓	Hogan	Hogan ✓
RR Commissioner	2014	Brown	Brown ✓			Brown	Brown ✓
President	2016	Clinton	Clinton ✓			Clinton	Clinton ✓
RR Commissioner	2016	Yarbrough	Yarbrough ✓	Yarbrough	Yarbrough ✓	Yarbrough	Yarbrough ✓
U.S. Senator	2018	O'Rourke	O'Rourke ✓			O'Rourke	O'Rourke ✓
Governor	2018	Valdez	Valdez ✓	Valdez	Valdez ✓	Valdez	Valdez ✓
Lieutenant Governor	2018	Cooper	Collier ×			Collier	Collier ✓
Comptroller	2018	Mahoney	Chevalier ×			Chevalier	Chevalier ✓
Land Commissioner	2018	Suazo	Suazo ✓			Suazo	Suazo ✓
RR Commissioner	2018	McAllen	McAllen ✓			McAllen	McAllen ✓

The demonstration plan has up to 16 minority-effective districts, as shown in the top table, while the enacted plan has no more than 11 to 13 (compare Table 4 and accompanying text). Scores over 60% have darker shading, and scores in the 50–60% range have lighter shading. The frequency of primary and general election wins by minority-preferred candidates is shown in the last two columns. Because different candidates of choice can be identified by the statewide and district-specific method, the number of successes is given as a range. The bottom table shows that candidates preferred by Latino voters statewide prevailed in District 27 in 12 of the 14 primaries, 5 of the 5 runoffs, and 13 of the 14 general elections. (With the candidates of choice inferred from the district-specific method, there are 13 primary successes).

(the two coalition districts, 18 and 33). There are also districts that are reliably Democratic but are not effective for either Latino voters or Black voters (Districts 12 and 31).

Table 6 takes a single district and brings us back to the most basic facts about it: whether the minority-preferred candidates actually won the most votes. We use as an example the plurality-white but Latino-effective District 27, which starts in East Austin and stretches south toward the Gulf Coast. For 11 of the 14 offices, the candidate preferred by Latino voters statewide prevailed at every step in District 27: primary, runoff (when there was one), and general. In the 2014 general election, however, the Latino-preferred Democratic nominee David Alameel failed to carry District 27 against Republican incumbent U.S. Senator John Cornyn; and in the 2018 Democratic primaries for lieutenant governor and comptroller, the candidates preferred by Latino voters statewide (Michael Cooper and

Tim Mahoney, respectively) failed to carry the district. This district generated Latino-effectiveness scores of about 84 or 85%, far above our threshold for effectiveness (60%) but below the scores for the map's four most heavily Latino districts, which consistently exceeded 90%.

7.3. Aggregate effectiveness

The use of a search technique tailored to raise the number of minority-effective districts might lead us to wonder about the effect on the rest of the map. With respect to demographics alone, redistricting is a fixed-sum activity: there are only so many Latino citizens of voting age in the state, so building more districts with high HCVAP means there is less remaining HCVAP to distribute across the other districts. We might worry that we can only secure a larger number of effective districts by draining opportunities for coalitional influence from the rest of the state. But this is not the case. Because of the highly nonlinear relationship between demographics and effectiveness (see section 6), it is possible to create some plans with a greater overall effectiveness than others.

To see this, let us consider the sum of the effectiveness scores for all 36 Texas congressional districts. Because each district has a score between 0 and 1, the sum will fall between 0 and 36. To the extent that a group's effectiveness scores behave like probabilities of electoral success, the sum over the 36 districts can be regarded as the *expected value* for the group in a given election. This expected-value score takes into account the probability but not certainty of electoral success in the effective districts, and also includes contributions from other districts in which an effectiveness score could fall well below .5 yet still reflect real political influence and a chance to win.

The enacted plan has an expected-value score a bit under 12, driven by 11 highly effective districts. After a few thousand steps of a heuristic-

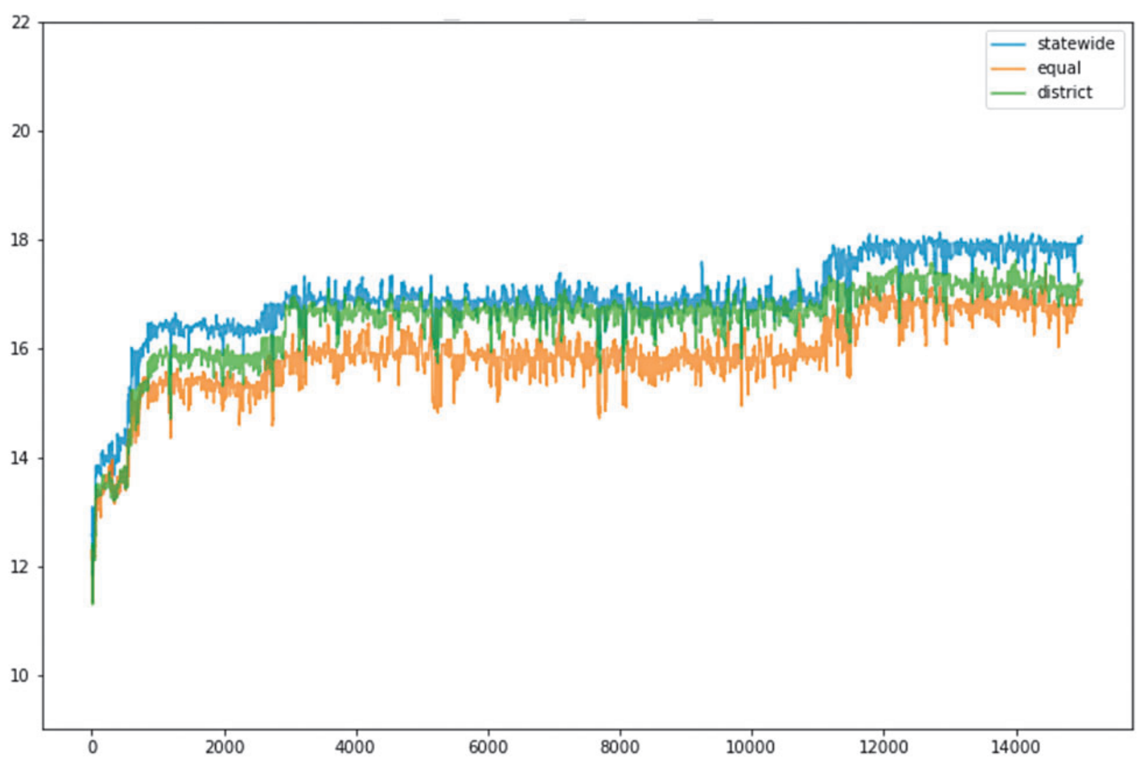

FIG. 12. This trace plot shows a kind of aggregate effectiveness for Latino and Black voters, formed by summing Latino and/or Black effectiveness scores over all 36 districts. This aggregate effectiveness trends up markedly over the course of a heuristic-optimization run that preferentially accepts plans with more districts effective for at least one minority group under the s^{state} score. This drives up the s^{state} score (in blue) most, with the other two scores following behind. (See Supplementary Appendix H for details on related optimization runs.)

optimization run (shown in Figure 12), the expected-value score is well over 15, usually over 16, and it is possible to drive the expectation up near 18 in the score being optimized. Our demonstration plan has an expectation of nearly 17, which tracks with the 16 districts highlighted in Table 6.

We find that, with respect to electoral opportunity, districting is not a fixed-sum game. We can find plans that combine Latino and Black voters with other population (including Asian American and white voters who tend to support the same candidates) in ways that lead to effective combinations. We can create safe minority districts, likely-to-elect minority districts, and some minority influence districts in a way that is especially beneficial in aggregate. This is a departure from the narrower focus on effectiveness that is directly relevant for VRA compliance, but may still point the way to a more coalitional expansion of minority opportunities beyond the demands of the law.

8. CONCLUSION

The principal goal of this project is the design and study of a protocol for building ensembles of alternative districting plans, taking closely into account the law of race and redistricting. We do this by using longitudinal electoral data, one of a choice of effectiveness scores, and a constrained district-generation algorithm.

No inclusion criterion assessed by a computer could perfectly track the conclusions of a court (not least because of variation in the judiciary itself), but ours is constructed to give us strong justification for describing it as a *representative sample* of the universe of VRA-compliant plans. We have pursued this objective in a way that also avoids overreliance on purely demographic targets that might run afoul of the Equal Protection Clause.

The structure of our protocol is described in section 4, and a detailed case study for Texas congressional districts is detailed in section 5. In section 6 we confirm that the role played by the extensive electoral data is not easily replaced by simpler proxies. And in section 7 we explore the use of similar techniques to minimize underrepresentation for minority groups—showing in particular that pushing to find plans that go the farthest to cure long-standing underrepresentation is a markedly different task from creating collections of alternatives that pass VRA muster. Studying the conditions of political and human geography that make it possible to attain near-proportionality is an interesting direction for future work.

With a detailed case study in the large, complex state of Texas, we confirm that our implementation lets us carry out the work on a time scale suitable for all stages of redistricting, from considering plans for possible adoption all the way to challenging them in litigation. We have made careful use of error estimates, performed tests of quality for ensemble generation, and confirmed robustness of the method across reasonable variations in the steps. By making our code and data public (MGGG Redistricting Lab, 2020a), we aim to make it possible for other researchers and practitioners to use this method on the ground.

This tool now makes it possible to assess proposed districting plans in racially diverse states against a baseline that takes the Voting Rights Act and the Equal Protection Clause into account. The computational tools for redistricting are continually becoming both more powerful and more refined, facilitating the creation of new maps that better meet our ideals of fairness and helping to understand maps in the context of realistic alternatives. By using novel tools in combination with renewed commitment to safeguarding minority representation, we can come closer than ever to the goal articulated by John Adams almost 250 years ago, in the midst of the American Revolution: to make our representative assemblies "in miniature an exact portrait of the people at large" (Adams, 1776, 108).

SUPPLEMENTARY MATERIAL

Supplementary Appendix

REFERENCES

Abbott v. Perez. 2018. 138 S. Ct. 2305.

Abrams v. Johnson. 1997. 521 U.S. 74.

Adams, John. 1776. "Thoughts on Government." In *The Founders' Constitution*, eds. Philip B. Kurland and Ralph Lerner. Chicago, IL: University of Chicago Press (1987).

Alabama Legislative Black Caucus v. Alabama. 2015. 575 U.S. 254.

Autrey, Eric, Daniel Carter, Gregory Herschlag, Zach Hunter, and Jonathan Mattingly. 2021. "Metropolized Forest Recombination for Monte Carlo Sampling of Graph Partitions." *arXiv.org*. arXiv:1911.01503.

Bartlett v. Strickland. 2009. 556 U.S. 1.
Becker, Amariah, and Justin Solomon. 2021. "Redistricting Algorithms." In *Political Geometry*, eds. M. Duchin and O. Walch. Basel: Birkhäuser. Under contract.
Bethune-Hill v. Virginia State Bd. of Elections. 2017. 137 S. Ct. 788.
Bialik, Kristen. 2019. "For the Fifth Time in a Row, the New Congress Is the Most Racially and Ethnically Diverse Ever." *Pew Research Center*. February 8. <https://www.pewresearch.org/fact-tank/2019/02/08/for-the-fifth-time-in-a-row-the-new-congress-is-the-most-racially-and-ethnically-diverse-ever/>.
Brown v. Thomson. 1983. 462 U.S. 835.
Bush v. Vera. 1996. 517 U.S. 952.
Bycoffe, Aaron, Ella Koeze, David Wasserman, and Julia Wolfe. 2018. "The Atlas of Redistricting." *FiveThirtyEight*.<https://projects.fivethirtyeight.com/redistricting-maps/>.
Campos v. City of Baytown. 1988. 840 F.2d 1240 (5th Cir.).
Cannon, Sarah, Ari Goldbloom-Helzner, Varun Gupta, J.N. Matthews, and Bhushan Suwal. 2020. "Voting Rights, Markov Chains, and Optimization by Short Bursts." *arXiv.org*. <https://arxiv.org/abs/2011.02288>.
Chen, Jowei. 2017. "The Impact of Political Geography on Wisconsin Redistricting: An Analysis of Wisconsin's Act 43 Assembly Districting Plan." *Election Law Journal* 16(4): 443–452.
Chen, Jowei, and Nicholas O. Stephanopoulos. 2021. "The Race-Blind Future of Voting Rights." *Yale Law Journal* 130(4): 862–946.
Common Cause v. Lewis. 2019. No. 18 CVS 014001, 2019 WL 4569584 (N.C. Super. Ct., Wake Cty. Sept. 3) (three-judge court).
Cooper v. Harris. 2017. 137 S. Ct. 1455.
Cox v. Larios. 2004. 542 U.S. 947.
DeFord, Daryl, and Moon Duchin. 2019. "Redistricting Reform in Virginia: Districting Criteria in Context." *Virginia Policy Review* 12(2): 120–146.
DeFord, Daryl and Moon Duchin. 2020. "Random Walks and the Universe of Districting Plans." In *Political Geometry*, ed. M. Duchin and O. Walch. Basel: Birkhäuser. Under contract.
DeFord, Daryl, Moon Duchin, and Justin Solomon. 2021. "Recombination: A Family of Markov Chains for Redistricting." *Harvard Data Science Review* 3(1). <https://doi.org/10.1162/99608f92.eb30390f>.
Duchin, Moon, and Bridget Tenner. 2018. "Discrete Geometry for Electoral Geography." *arXiv.org*. <https://arxiv.org/abs/1808.05860>.
Duchin, Moon and Douglas M. Spencer. 2021. "Models, Race, and the Law: A Response to 'The Race-Blind Future of Voting Rights.'" *Yale Law Journal Forum* 130:744–97.
Elmendorf, Christopher S., Kevin M. Quinn, and Marisa J. Abrajano. 2016. "Racially Polarized Voting." *University of Chicago Law Review* 83(2): 587–692.
Fifield, Benjamin, Michael Higgins, Kosuke Imai, and Alexander Tarr. 2020. "Automated Redistricting Simulation Using Markov Chain Monte Carlo." *Journal of Computational and Graphical Statistics* 29(4): 715–728. DOI: 10.1080/10618600.2020.1739532.
Grofman, Bernard, Lisa Handley, and David Lublin. 2001. "Drawing Effective Minority Districts: A Conceptual Framework and Some Empirical Evidence." *North Carolina Law Review* 79(5):1383–430.
Hebert, J. Gerald, Paul M. Smith, Martina E. Vandenberg, and Michael B. DeSanctis. 2010. *The Realist's Guide to Redistricting: Avoiding the Legal Pitfalls*, 2d edition. Chicago, IL: ABA.
Herschlag, Gregory, Han Sung Kang, Justin Luo, Christy Vaughn Graves, Sachet Bangia, Robert Ravier, and Jonathan C. Mattingly. 2020. "Quantifying Gerrymandering in North Carolina.'' *Statistics and Public Policy* 7(1): 30–38.
History, Art, and Archives, U.S. House of Representatives, Office of the Historian. 2020a. "Black-American Members by Congress, 1870–Present." *History, Art & Archives*. Available at <https://history.house.gov/Exhibitions-and-Publications/BAIC/Historical-Data/Black-American-Representatives-and-Senators-by-Congress/>.
History, Art, and Archives, U.S. House of Representatives, Office of the Historian. 2020b. "Hispanic-American Representatives, Senators, Delegates, and Resident Commissioners by Congress, 1822–Present." *History, Art & Archives*. Available at <https://history.house.gov/Exhibitions-and-Publications/HAIC/Historical-Data/Hispanic-American-Representatives,-Senators,-Delegates,-and-Resident-Commissioners-by-Congress/>.
Iowa Code § 42.4.4. N.d.
Iowa Const. art. III, § 37. N.d.
Johnson v. De Grandy. 1994. 512 U.S. 997.
Karcher v. Daggett. 1983. 462 U.S. 725.
King, Gary. 1997. *A Solution to the Ecological Inference Problem: Reconstructing Individual Behavior from Aggregate Data*. Princeton, NJ: Princeton University Press.
King, Gary, Ori Rosen, and Martin A. Tanner. 1999. "Binomial-Beta Hierarchical Models for Ecological Inference." *Sociological Methods and Research* 28(1): 61–90.
Lau, Olivia, Ryan T. Moore, and Michael Kellermann. 2020. "eiPack: Ecological Inference and Higher-Dimension Data Management (R package)." Version 0.2-1. Available at <https://CRAN.R-project.org/package=eiPack>.
League of Women Voters of Mich. v. Benson. 2019. 373 F. Supp. 3d 867 (E.D. Mich.) (three- judge court), summarily vacated and remanded sub nom. Chatfield v. League of Women Voters of Mich., 140 S. Ct. 429.
League of Women Voters v. Commonwealth. 2018. 178 A.3d 737 (Pa.).
LULAC v. Perry. 2006. 548 U.S. 399.
McCartan, Cory and Kosuke Imai. 2021. "Sequential Monte Carlo for Sampling Balanced and Compact Redistricting Plans." *arXiv.org*. <https://arxiv.org/abs/2008.06131>.
MGGG Redistricting Lab. 2018a. "Comparison of Districting Plans for the Virginia House of Delegates." White Paper. <https://mggg.org/VA-report.pdf>.
MGGG Redistricting Lab. 2018b. "GerryChain Python package." GitHub repository. <https://github.com/mggg/GerryChain>.
MGGG Redistricting Lab. 2018c. "MAUP Python package." GitHub repository. <https://github.com/mggg/MAUP>.

MGGG Redistricting Lab. 2018d. "Study of Voting Systems for Santa Clara, CA." White Paper. <https://mggg.org/santaclara>.

MGGG Redistricting Lab. 2019a. "Findings on the City of Lowell's Election Systems." White Paper. <https://mggg.org/lowell>.

MGGG Redistricting Lab. 2019b. "Study of Reform Proposals for Chicago City Council." GitHub repository. <https://mggg.org/chicago>.

MGGG Redistricting Lab. 2020a. "Computational VRA." GitHub repository. <https://github.com/mggg/VRA_ensembles>.

MGGG Redistricting Lab. 2020b. "GerryChainJulia Julia package." GitHub repository. <https://github.com/mggg/GerryChainJulia>.

Miller v. Johnson. 1995. 515 U.S. 900.

National Conference of State Legislatures. 2020. "State Legislator Demographics." *NSCL.org*. <https://www.ncsl.org/research/about-state-legislatures/state-legislator-demographics.aspx>.

Ohio A. Philip Randolph Institute v. Householder. 2019. 373 F. Supp. 3d 978 (S.D. Ohio) (three- judge court), summarily vacated and remanded, 140 S. Ct. 101.

Rucho v. Common Cause. 2019. 139 S. Ct. 2484.

Session v. Perry. 2004. 298 F. Supp. 2d 451 (E.D. Tex.) (three-judge court) (per curiam), vacated and remanded on other grounds sub nom. Jackson v. Perry, 543 U.S. 941.

Shaw v. Reno. 1993. 509 U.S. 630.

Smith v. Clinton. 1988. 687 F. Supp. 1310 (E.D. Ark.) (three-judge court), summarily aff'd, 488 U.S. 988.

Tennant v. Jefferson County Comm'n. 2012. 567 U.S. 758 (per curiam).

Texas Legislative Council. 2020. "Capitol Data Portal." Available at <https://data.capitol.texas.gov>.

Thornburg v. Gingles. 1986. 478 U.S. 30.

Unger v. Manchin. 2002. 536 U.S. 935.

U.S. Bureau of the Census. 2019a. "American Community Survey: Citizen Voting-Age Population, by Selected Characteristics." <https://data.census.gov/cedsci/table?q=S2901&tid=ACSST1Y2019.S2901>.

U.S. Bureau of the Census. 2019b. "American Community Survey: Citizen Voting-Age Population, by State." <https://www.census.gov/library/visualizations/2016/comm/citizen_voting_age_population.html>.

U.S. Const. amend. XIV. N.d.

Voinovich v. Quilter. 1993. 507 U.S. 146.

VRA. 2020. 52 U.S.C. § 10301 et seq.

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STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, et al.,

REBECCA HARPER, et al.,

Plaintiffs,

vs.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

Consolidated with
21 CVS 500085

AFFIDAVIT OF MICHAEL BARBER

Now comes affiant Michael Barber, having been first duly cautioned and sworn, deposes and states as follows:

1. I am over the age of 18 and am competent to testify regarding the matters discussed below.

2. For the purposes of this litigation, I have been asked by counsel for Legislative Defendants to analyze relevant data and provide my expert opinions.

3. To that end, I have personally prepared the report attached to this affidavit as Exhibit A, and swear to its authenticity and to the faithfulness of the opinions.

FURTHER THE AFFIANT SAYETH NAUGHT.

NCLCV v. Hall
21 CVS 15426
LDTX163

Executed on 22 December, 2021

DocuSigned by:
Michael Barber
82F8BEB03413425...

Michael Barber

Sworn or affirmed before me and subscribed in the presence the 22nd day of December, 2021, in the State of Texas and County of Harris.

NOTARY PUBLIC
STATE OF TEXAS
MARY S. LEE
Notary ID
11405696
My Commission Expires
11/4/2022

DocuSigned by:
Mary S. Lee
2FAD7787555D439...

Notary Public

Exhibit A:
Expert Report of Michael Barber, PhD

Dr. Michael Barber
Brigham Young University
724 Spencer W. Kimball Tower
Provo, UT 84604
barber@byu.edu

Exhibit #
Barber 1
12/30/2021
exhibitsticker.com

Contents

1 Introduction and Qualifications

I have been asked by counsel for the Legislative Defendants to analyze North Carolina's recently enacted redistricting plans for the General Assembly (the "Enacted Plans") and the plans submitted by the North Carolina League of Conservation Voters (the "Duchin Plans") in the context of the partisan gerrymandering claims brought against the Legislative Defendants.[1] To do this, I implement a publicly available and peer-reviewed redistricting simulation algorithm to generate 50,000 simulated district maps in each county grouping in which there are multiple districts in both the North Carolina House of Representatives and the North Carolina Senate. The redistricting algorithm generates a representative sample of districts by following neutral redistricting criteria without regard to racial or partisan data. In this way, the simulated districts establish a comparison set of plans that use purely non-partisan redistricting inputs. I then compare the simulated plans against the Enacted Plans and the Duchin Plans by reference to election results to assess whether the partisan effects of those plans are consistent with what one would expect to see in a redistricting plan composed without reference to any partisan considerations.

In the House, these simulations show that the Enacted Plans consistently score more often within the range of the non-partisan simulated maps than the Duchin Plans. In addition, the simulations show that the Enacted Plans contain one county grouping, the Guilford County grouping in the House of Representative, that is a partisan outlier. However, this grouping largely follows the boundaries of a 2019 court-approved district plan. In contrast, the Duchin Plans generate partisan outliers in four county groupings.

In the Senate analysis both the Enacted and Duchin plans generate partisan outliers when compared to the simulated district maps in two clusters each. Furthermore, neutral redistricting criteria such as following municipal lines support the decisions by the map drawers in the Enacted Plan in more districts, while in these same districts the Duchin Plan divides Democratic-leaning municipalities into more pieces in order to combine Democratic-

[1] These plans were attached to the NCLCV complaint, filed on November 16, 2021.

leaning voters in cities with Republican voters in suburban and rural parts of North Carolina to create additional competitive or Democratic-leaning districts. Given these results, as well as the otherwise high degree of agreement between the Enacted and Duchin maps, it is my opinion that the Enacted Maps are not "extreme partisan gerrymanders" as plaintiffs allege.

I am an associate professor of political science at Brigham Young University and faculty fellow at the Center for the Study of Elections and Democracy in Provo, Utah. I received my PhD in political science from Princeton University in 2014 with emphases in American politics and quantitative methods/statistical analyses. My dissertation was awarded the 2014 Carl Albert Award for best dissertation in the area of American Politics by the American Political Science Association.

I teach a number of undergraduate courses in American politics and quantitative research methods.[2] These include classes about political representation, Congressional elections, statistical methods, and research design.

I have worked as an expert witness in a number of cases in which I have been asked to analyze and evaluate various political and elections-related data and statistical methods. Cases in which I have testified at trial or by deposition are listed in my CV, which is attached to the end of this report. I have previously provided expert reports in a number of cases related to voting, redistricting, and election-related issues: *Nancy Carola Jacobson, et al., Plaintiffs, vs. Laurel M. Lee, et al., Defendants. Case No. 4:18-cv-00262 MW-CAS (U.S. District Court for the Northern District of Florida); Common Cause, et al., Plaintiffs, vs. Lewis, et al., Defendants. Case No. 18-CVS-14001 (Wake County, North Carolina); Kelvin Jones, et al., Plaintiffs, v. Ron DeSantis, et al., Defendants, Consolidated Case No. 4:19-cv-300 (U.S. District Court for the Northern District of Florida); Community Success Initiative, et al., Plaintiffs, v. Timothy K. Moore, et al., Defendants, Case No. 19-cv-15941 (Wake County, North Carolina); Richard Rose et al., Plaintiffs, v. Brad Raffensperger, Defendant, Civil Action No. 1:20-cv-02921-SDG (U.S. District Court for the Northern Dis-*

[2]The political science department at Brigham Young University does not offer any graduate degrees.

trict of Georgia); Georgia Coalition for the People's Agenda, Inc., et. al., Plaintiffs, v. Brad Raffensberger, Defendant. Civil Action No. 1:18-cv-04727-ELR (U.S. District Court for the Northern District of Georgia); Alabama, et al., Plaintiffs, v. United States Department of Commerce; Gina Raimondo, et al., Defendants. Case No. CASE NO. 3:21-cv-00211-RAH-ECM-KCN (U.S. District Court for the Middle District of Alabama Eastern Division); League of Women Voters of Ohio, et al., Relators, v. Ohio Redistricting Commission, et al., Respondents. Case No. 2021-1193 (Supreme Court of Ohio).

In my position as a professor of political science, I have conducted research on a variety of election- and voting-related topics in American politics and public opinion. Much of my research uses advanced statistical methods for the analysis of quantitative data. I have worked on a number of research projects that use "big data" that include millions of observations, including a number of state voter files, campaign contribution lists, and data from the US Census. I have also used geographic information systems and other mapping techniques in my work with political data.

Much of this research has been published in peer-reviewed journals. I have published nearly 20 peer-reviewed articles, including in our discipline's flagship journal, *The American Political Science Review* as well as the inter-disciplinary journal,*Science Advances*. My CV, which details my complete publication record, is attached to this report as Appendix A.

The analysis and opinions I provide in this report are consistent with my education, training in statistical analysis, and knowledge of the relevant academic literature. These skills are well-suited for this type of analysis in political science and quantitative analysis more generally. My conclusions stated herein are based upon my review of the information available to me at this time. I reserve the right to alter, amend, or supplement these conclusions based upon further study or based upon the availability of additional information. I am being compensated for my time in preparing this report at an hourly rate of $400/hour. My compensation is in no way contingent on the conclusions reached as a result of my analysis. The opinions in this report are my own, and do note represent the view of Brigham Young

University.

2 Summary of Conclusions

Based on the evidence and analysis presented below, my opinions regarding the 2021 enacted redistricting plans in the North Carolina General Assembly can be summarized as follows:

- The contemporary political geography of North Carolina is such that Democratic majorities are often geographically clustered in the largest cities of the state while Republican voters often dominate the suburban and rural portions of the state.
- This is not the case in the rural northeastern region of the state, where there are also significant Democratic majorities.
- This geographic clustering in cities an in the rural northeast puts the Democratic Party at a natural disadvantage when single-member districts are drawn.
- This is further amplified by the 'county grouping' process that is unique to North Carolina's redistricting process where districts are constrained to remain within county groups.
- This disadvantage partially arises from the difficulty, and in many cases impossibility, of drawing Democratic-leaning districts in many of the county groupings that comply with constitutional requirements, even though Democratic voters make up roughly 40% of voters in these parts of the state.
- Based on a comparison between the Enacted Plan, the Duchin Plan, and a set of 50,000 simulated maps, the Enacted Plan is less of a partisan outlier than the Duchin Plan in the State House. In 39 of the 40 clusters the Enacted Plan is not a partisan outlier in

comparison to the simulation results. In 36 of the 40 clusters the Duchin Plan is not a partisan outlier in comparison to the simulation results.

- In the Senate analysis both the Enacted and Duchin plans generate partisan outliers when compared to the simulated district maps in two clusters each.

- Areas of disagreement between proposed plans often arise because the Duchin plan divides Democratic leaning municipalities into more pieces in order to combine Democratic-leaning voters with Republican voters in suburban and rural parts of the state to create additional competitive or Democratic leaning districts.

- Given these results, as well as the otherwise high degree of agreement between the Enacted and Duchin maps, it is my opinion that the Enacted Maps are not "extreme partisan gerrymanders" as plaintiffs allege.

3 Political Geography of North Carolina

For the last several decades, North Carolina has been relatively competitive in statewide elections. Democratic and Republican candidates have won the state at the presidential, gubernatorial, congressional, and state level. Figure 1 below shows the results of the average of all statewide elections in North Carolina from 2000 through 2020. These races include: president, US Senate, governor, lieutenant governor, attorney general, secretary of state, state auditor, treasurer, superintendent, commissioner of agriculture, commissioner of labor, insurance commissioner, and partisan judicial elections in 2018.[3] While not all races are up for election in each year, I create the index by averaging the two-party vote share of those races that occurred in each two-year cycle. State-level races in North Carolina occur in presidential election years while US senate races occur every six years. There were no statewide partisan races in 2006. As can be seen in the figure, the statewide Democratic margin in North Carolina peaked in 2008 at 55% of the two-party vote and reached its nadir in 2010 with 44% of the vote.

The relative stability of the statewide results over the last 10 years masks a dramatic variation in the spatial location of Democratic and Republican voters within the state. The following section details this and shows in a variety of different ways that Democratic voters are more likely to be spatially clustered in the state while Republican voters tend to live in more politically diverse areas.

Scholarship in political science has noted that the spatial distribution of voters throughout a state can have an impact on the partisan outcomes of elections when a state is, by necessity, divided into a number of legislative districts. This is largely the case because Democratic-leaning voters tend to cluster in dense, urban areas while Republican-leaning voters tend to be more equally distributed across the remainder of the state.[4] One prominent

[3]To create the index I sum by party all votes cast for each candidate in each race by year. I then take the fraction of votes cast for candidates of the two major parties that were cast for Democratic candidates in that year. There are other possible measures and methods one could use, such as considering candidate percentages before averaging or including third party voters.

[4]See for example Stephanopoulos, N. O. and McGhee, E. M., Partisan Gerrymandering and the Efficiency

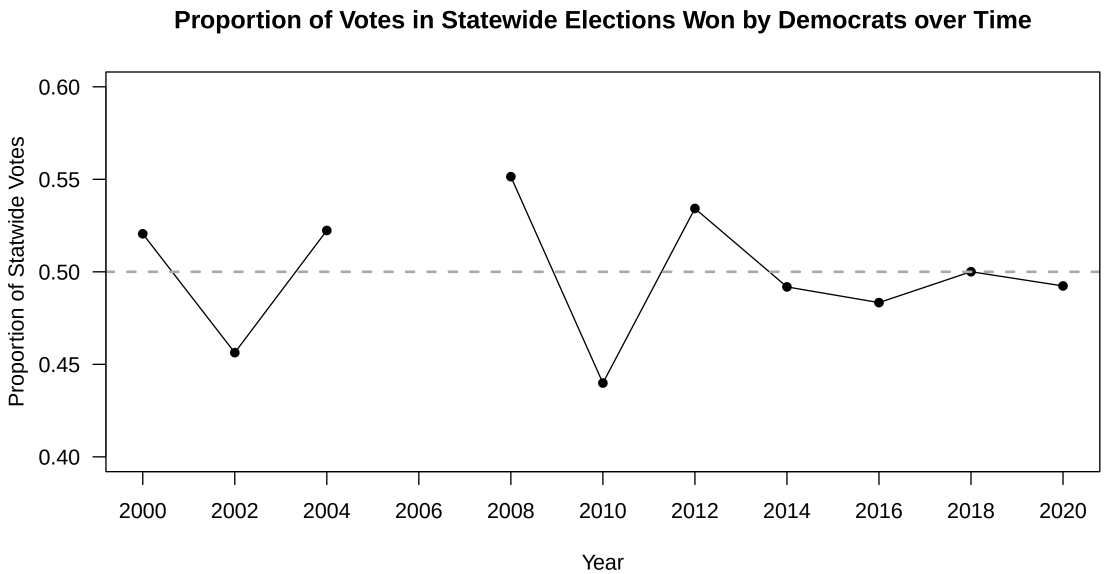


Figure 1: **Democratic Proportion of Statewide Election Contests, 2000-2020**

study of the topic (Chen and Rodden, 2013) finds that "Democrats are highly clustered in dense central city areas, while Republicans are scattered more evenly through the suburban, exurban, and rural periphery...Precincts in which Democrats typically form majorities tend to be more homogenous and extreme than Republican-leaning precincts. When these Democratic precincts are combined with neighboring precincts to form legislative districts, the nearest neighbors of extremely Democratic precincts are more likely to be similarly extreme than is true for Republican precincts. As a result, when districting plans are completed, Democrats tend to be inefficiently packed into homogenous districts."[5]

The upshot of this pattern is that political parties stand at a disadvantage when their voters are not "efficiently" distributed across the state. To understand what I mean

Gap, *The University of Chicago Law Review* 82: 831-900, (2015); Chen, J. and Rodden, J., Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures, *Quarterly Journal of Political Science* 8: 239-269, (2013); Nall, C., The Political Consequences of Spatial Policies: How Interstate Highways Facilitated Geographic Polarization, *Journal of Politics*, 77(2): 394-406, (2015); Gimple, J. and Hui, I., . Seeking politically compatible neighbors? The role of neighborhood partisan composition in residential sorting, *Political Geography* 48: 130-142 (2015); Bishop, B., *The Big Sort: Why the Clustering of Like-Minded America is Tearing Us Apart*, Houghton Mifflin Press (2008); and Jacobson, G. C., and Carson, J. L., *The Politics of Congressional Elections*, 9th ed. Lanham, MD: Rowman and Littlefield (2016).

[5]Chen, J. and Rodden, J., Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures, *Quarterly Journal of Political Science* 8: 239-269, (2013)

by efficient, imagine two different scenarios. First, imagine a party with a slim majority of voters statewide in which every precinct's vote share perfectly reflected the overall state. In other words, the party has a slight majority in every precinct that adds up to a slight majority statewide. In this case, this party's voters are extremely efficiently distributed in such a way that the party will win every single district despite only a slim majority statewide. Now imagine a different arrangement, a party who still holds a slim majority statewide, but whose voters are heavily concentrated in a few areas and sparsely populated throughout the rest of the state. In this case, despite holding a majority of votes statewide, the party will only win a few seats where their voters are heavily concentrated. The political geography of North Carolina more closely resembles the second scenario.

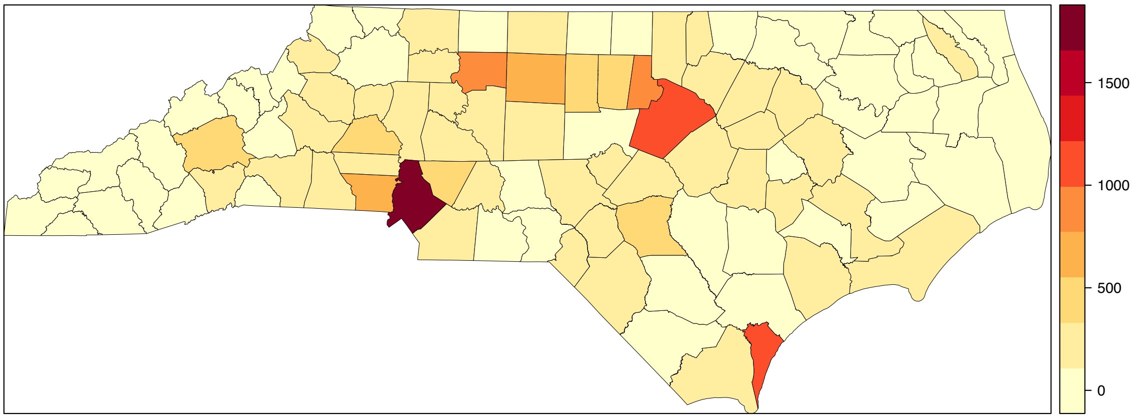
Figure 2 shows two maps of North Carolina. The top map shows the population density across counties. The bottom map shows the distribution of partisan preference across the state. Comparing the two shows that the most dense and urban counties (Wake, Mecklenburg, Durham, Guilford, Forsyth, New Hanover) in the state tend to also be where we see clusters of Blue on the bottom map.

North Carolina adds an additional wrinkle to this trend that also works to create heavily Democratic state legislative districts. Figure 2 shows that the rural counties of north eastern North Carolina are strongly Democratic.[6] This further works to facilitate the creation of strongly Democratic state legislative districts because each of these rural counties, and sometimes in combination with other adjacent rural counties, can form a legislative district. This is because the state constitution again emphasizes that counties be kept together when drawing district boundaries, and when grouping counties to collect a sufficient number of people, the minimum grouping of contiguous counties should be used. Because these rural counties all share the common feature of being strongly Democratic, any grouping of these counties together will further generate legislative districts with large majorities in support of Democratic candidates.

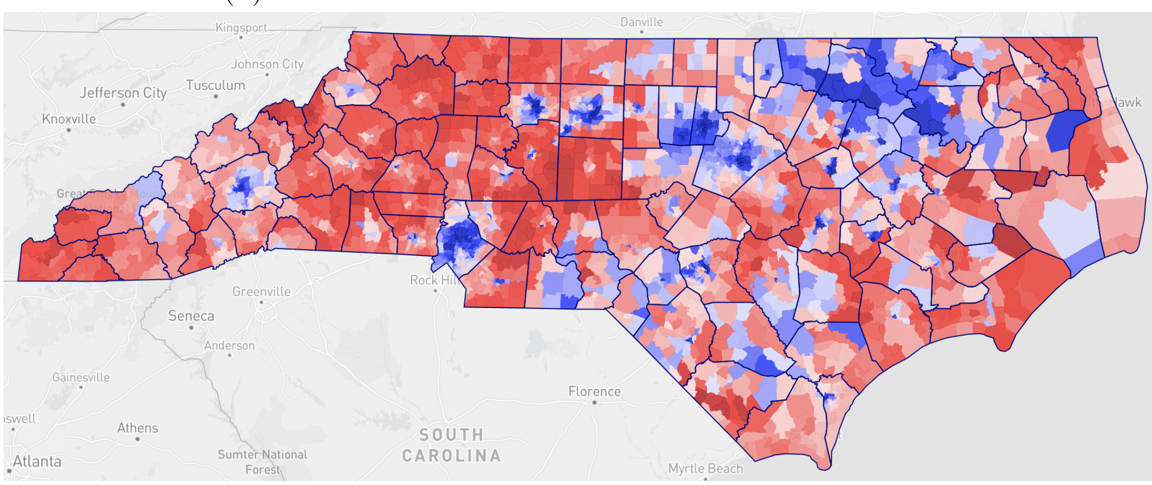
[6]This would include Vance, Warren, Halifax, Northampton, Hertford, Bertie, and Edgecomb counties.

Figure 2: **Distribution of People and Partisan Preferences in North Carolina.**

(a) Population Density of North Carolina Counties



(b) Partisan Preferences in North Carolina Counties



Note: Blue = Democratic, Red = Republican

Thus, the geographic concentration of a party's voters tends to harm that party when single-member districts are drawn by creating districts that favor that party by very large majorities, thus 'wasting' many votes in running up large majorities far beyond 50%+1.[7] This occurs in North Carolina in the urban counties of the state as well as the northeastern counties of the state where there are also sizeable Democratic majorities. Importantly, the discussion is not about *where* Democratic voters are heavily clustered together, but simply that they are. It is less important if this clustering takes place in large urban cities or in

[7]McGhee, E. (2017). Measuring Efficiency in Redistricting. Election Law Journal: Rules, Politics, and Policy, 16(4), 417–442. doi:10.1089/elj.2017.0453

rural portions of the state. The overwhelming margins for the party are what drives 'wasted votes,' which, in turn translate to fewer seats than the statewide proportion of the vote would suggest.

Another way to consider this is to look at a lower level of geography, the Voter Tabulation District (VTD), which is similar to a precinct. Figure 3 shows the distribution of partisan preferences for 11 statewide partisan elections for all VTDs in North Carolina.[8] The left panel notes VTDs where there are strong majorities for either party and labels them as "inefficient" VTDs. They are inefficient based on the discussion above that a party wastes votes if it builds majorities far beyond the needed 50%+1. Note that the distribution is not symmetric and that there are more VTDs with very large democratic majorities than there are VTDs with equally large Democratic majorities. The right panel shows the same distribution by labels "efficient" VTDs — those where a party has a majority, but not an overwhelming majority. Note here that there are many more VTDs with efficient Republican majorities than there are VTDs with efficient Democratic majorities.

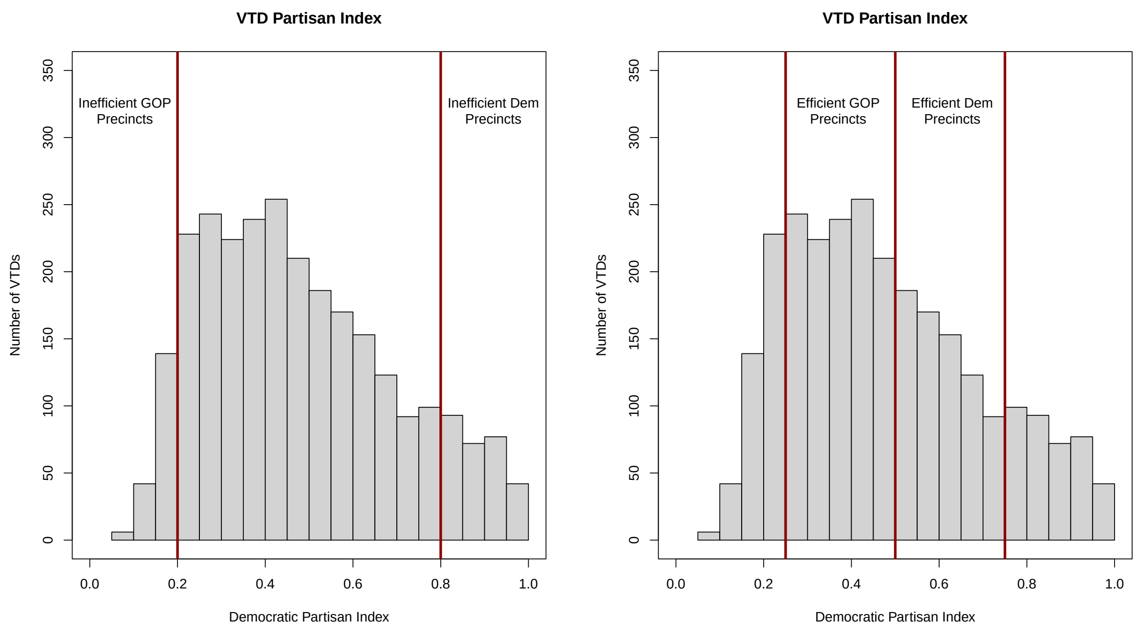
This inefficient distribution of votes would not be a problem for Democrats if districts were able to amble about the state so as to create districts that had less overwhelming Democratic support. Rodden (2019) notes this by saying: "Democrats would need a redistricting process that intentionally carved up large cities like pizza slices or spokes of a wheel, so as to combine some very Democratic urban neighborhoods with some republican exurbs in an effort to spread Democrats more efficiently across districts (pg. 155).[9]" Alternatively, as districts get larger in size (i.e. congressional districts) "Democratic communities can easily string together and overwhelm the surrounding rural Republicans (pg. 149)." However, the laws governing redistricting in North Carolina run counter to either of these strategies.

[8] I use these elections because they were the most comprehensive set of statewide elections I could obtain, given the tight time constraints, that were aggregated and matched to the level of the VTD. The elections are 2020: President, Senate, Governor, Lieutenant Governor, Attorney General; 2016: President, Senate, Governor, Lieutenant Governor, Attorney General; 2014: Senate.

[9] Rodden, Jonathan A. Why cities lose: The deep roots of the urban-rural political divide. Hachette UK, 2019.. While Rodden is specifically discussing Pennsylvania in this quote, the statement is true of any location with Democrats clustered in urban areas.

Figure 3: **Distribution of Votes Across VTDs in North Carolina.**

(a) Inefficient VTDs (b) Efficient VTDs



Note: Partisan Index based on the average of 11 statewide partisan races between 2014-2020.

North Carolina's strict rules that require districts to remain within pre-determined county clusters prohibit the type of meandering districts that Rodden describes above. Furthermore, additional restrictions requiring geographic compactness and minimizing the splitting of municipalities further eliminates the possibility of taking the strategy described above. In the end, this means that Republicans begin the redistricting process with a natural advantage due to the combination of laws requiring where and how districts are drawn combined with the particular spatial distribution of their voters. Thus, as I will show below, the advantage we observe between the expected Republican seat share in the state legislature compared to the statewide Republican vote share in the recent past is more due to geography than partisan activity by Republican map drawers.[10]

[10]Rodden (2019) notes regarding North Carolina, "Due to the presence of a sprawling knowledge-economy corridor, a series of smaller automobile cities with relatively low partisan gradients, and the distribution of rural African Americans, Democrats are relatively efficiently distributed in North Carolina at the scale of congressional districts (pg. 173)." It is important to note that this statement is not true for state legislative districts, which contain much smaller populations than congressional districts (and thus often cannot span

To measure the expected seat share in the state House and Senate, I compute a partisan index of statewide elections for 11 statewide partisan elections between 2014-2020.[11]

Figure 4 shows this for the 120 House seats. Districts are ordered from least Democratic at the bottom to most Democratic at the top. Districts with a partisan index less than 0.50 (i.e. Republican leaning) are shown as squares and districts with a partisan index greater than 0.50 (i.e. Democratic leaning) are displayed as triangles. In the House there are 71 districts with an index less than 0.50 (shown as squares) and 49 districts with an index greater than 0.50 (shown as triangles). A vertical dashed line is placed at 0.50 in each panel for reference. The grey lines around each point show the range of election outcomes for all of the 11 statewide elections used to generate the index. Districts in which the Republican candidate for statewide elections won the majority of the two-party vote share in all 11 races are colored red while districts where the Democratic candidate for statewide elections won the majority of the two-party vote share in all 11 races are colored blue. Districts where both parties have won a majority of the two-party vote share in these 11 races are colored green. Looking at the range across the index, there are 60 districts colored red (reliably Republican) in the House figure, 40 blue districts (reliable Democratic), and 20 green districts (competitive) in the House map. Using an alternative definition of competitiveness based on the closeness of the index to 0.50, there are 57 districts with an index less than 0.45, 24 districts between 0.45 and 0.55 (a commonly used range to define competitive seats), and 39 districts with an index of greater than 0.55.

Using the same method for the Senate, there are 30 squares (i.e. Republican leaning districts) and 20 triangles in the figure (i.e. Democratic leaning districts). Using the color scheme described above, there are 26 red districts (reliably Republican), 17 blue districts (reliable Democratic), and 7 green districts in the Senate map (competitive). Using an alternative definition of competitiveness based on the closeness of the index to 0.50, there

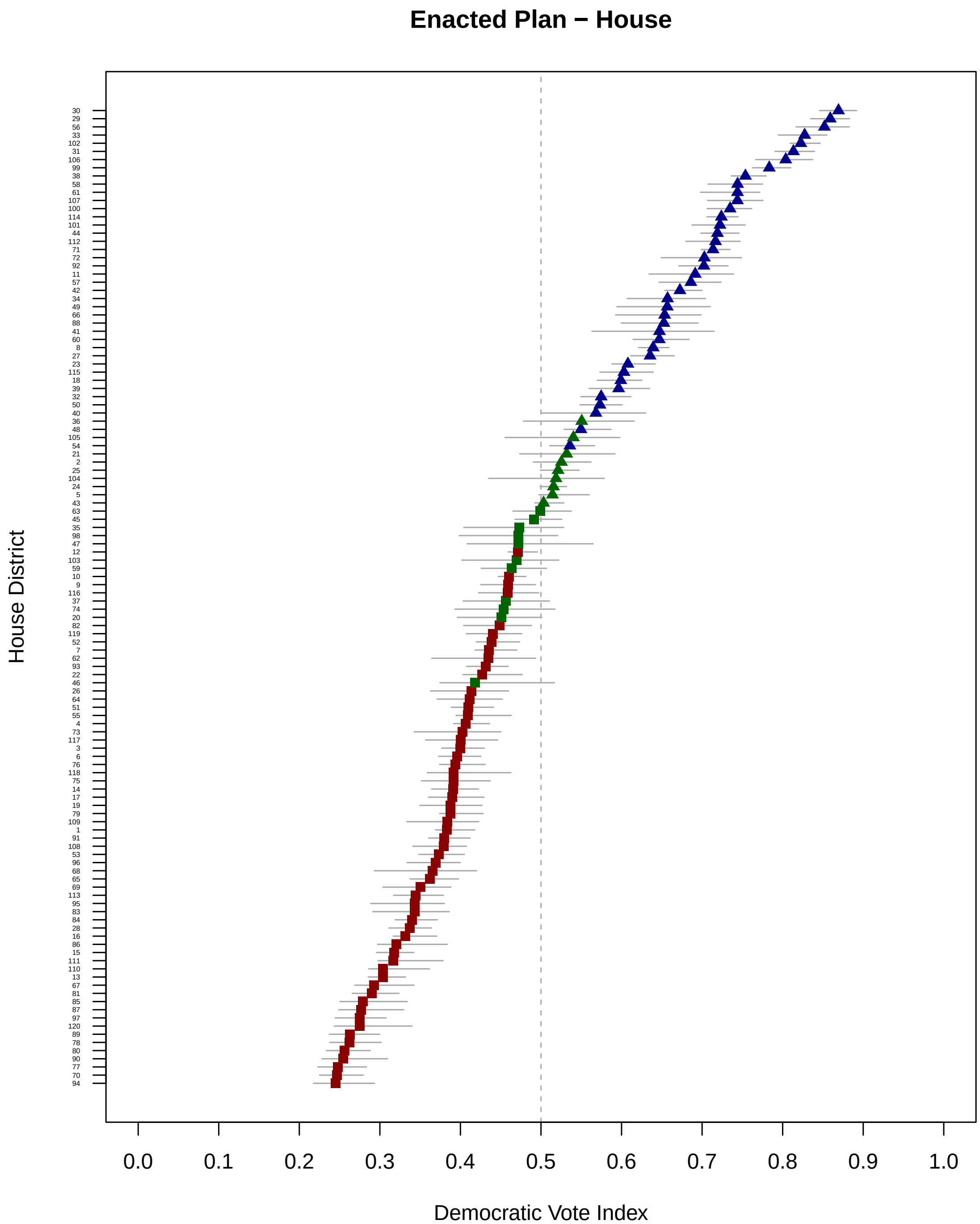
across multiple cities) and are much more constrained to remain within the county clusters, unlike the congressional district maps.

[11] The elections are 2020: President, Senate, Governor, Lieutenant Governor, Attorney General; 2016: President, Senate, Governor, Lieutenant Governor, Attorney General; 2014: Senate

are 24 districts with an index less than 0.45, 17 districts between 0.45 and 0.55, and 9 districts with an index of greater than 0.55. Figure 5 shows this for the 50 Senate seats.

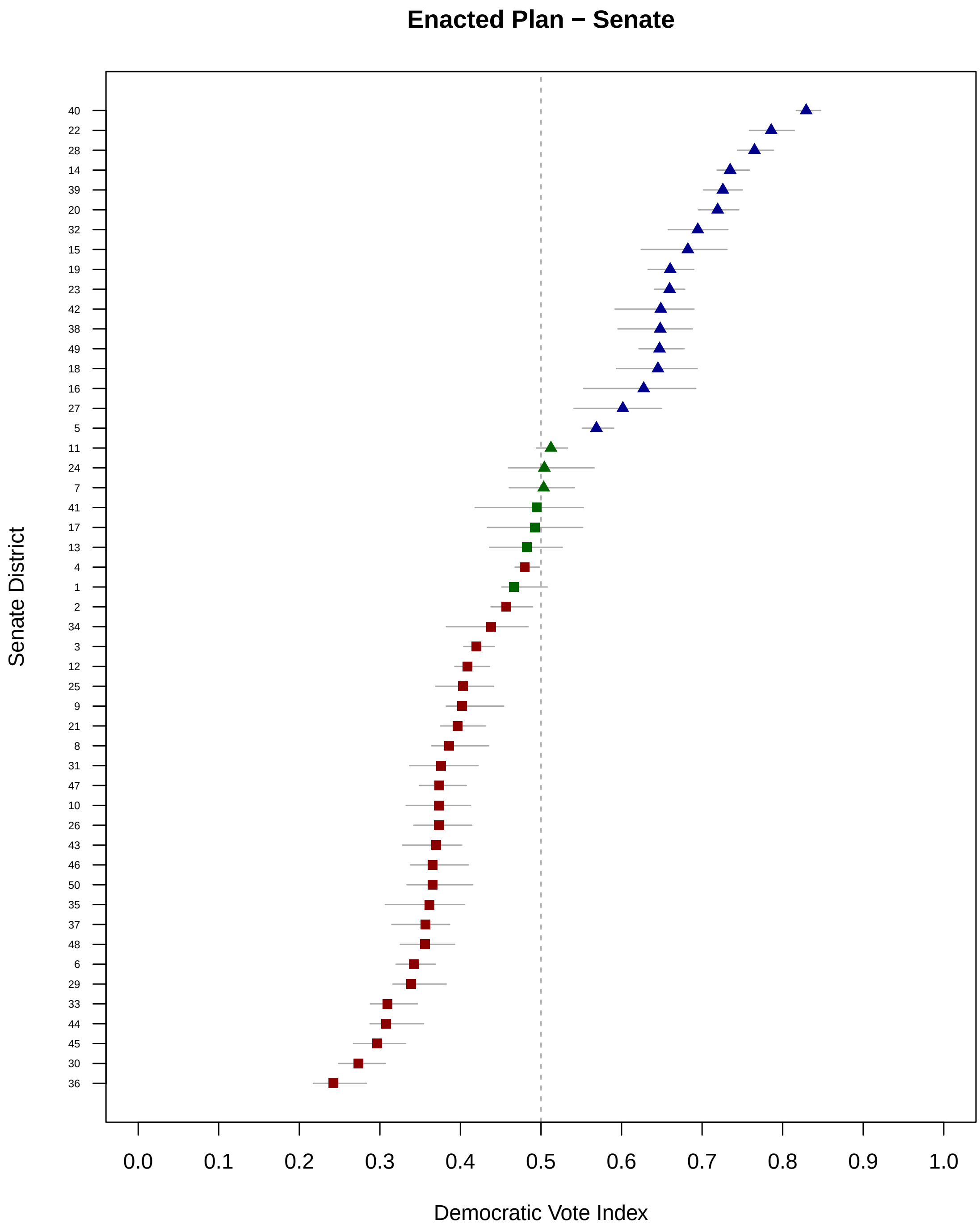
When looking at these figures, we cannot make any immediate determinations about why this distribution of seats, which has more Republican leaning districts than Democratic leaning districts, does not exactly reflect the statewide of average of votes in the state, which is much closer to parity between the parties. The reason for this is that, as discussed above, the distribution of voters who favor one party or the other is not even across the state. Furthermore, districts in North Carolina are restricted to remain within the pre-determined county clusters, further complicating the connection between district boundaries and statewide vote shares. This unique feature of North Carolina's redistricting process significantly constrains any map maker and can furthermore exacerbate the geographic disparities that exist across the state.

Figure 4: **Partisan Index of Senate Districts in 2021 Enacted Plan**



Note: Partisan Index based on the average of 11 statewide partisan races between 2014-2020. Districts with a partisan index less than .50 (i.e. Republican leaning) are shown as squares and districts with a partisan index greater than .50 (i.e. Democratic leaning) are displayed as triangles. A vertical dashed line is placed at .50 in each panel for reference. The grey lines around each point show the range of election outcomes for all of the 11 statewide elections used to generate the index. Districts in which the Republican candidate for statewide elections won the majority of the two-party vote share in all 11 races are colored red while districts where the Democratic candidate for statewide elections won the majority of the two-party vote share in all 11 races are colored blue. Districts where both parties have won a majority of the two-party vote share in these 11 races are colored green.

Figure 5: **Partisan Index of Senate Districts in 2021 Enacted Plan**



Note: Partisan Index based on the average of 11 statewide partisan races between 2014-2020. Districts with a partisan index less than .50 (i.e. Republican leaning) are shown as squares and districts with a partisan index greater than .50 (i.e. Democratic leaning) are displayed as triangles. A vertical dashed line is placed at .50 in each panel for reference. The grey lines around each point show the range of election outcomes for all of the 11 statewide elections used to generate the index. Districts in which the Republican candidate for statewide elections won the majority of the two-party vote share in all 11 races are colored red while districts where the Democratic candidate for statewide elections won the majority of the two-party vote share in all 11 races are colored blue. Districts where both parties have won a majority of the two-party vote share in these 11 races are colored green.

4 Introduction to Simulations Analysis

To gauge the range of partisan outcomes in the North Carolina General Assembly, I conduct simulated districting analyses to allow me to produce a large number of districting plans that follow traditional districting criteria using small geographic units as building blocks for hypothetical legislative districts (voting tabulation districts, or VTDs). This simulation process ignores all partisan and racial considerations when drawing districts. Instead, the computer simulations are programmed to create districting plans that follow traditional districting goals without paying attention to partisanship, race, or the location of incumbent legislators.

The process of simulating districting plans has been recognized and used in a variety of redistricting cases, including in North Carolina.[12] While different people employ slightly different methods, the overall process is much the same. For my simulations, I use a program developed by Fifield et al. (2020).[13]

A significant advantage of the simulation-based approach in general is the ability to compare a proposed map to a set of maps that are drawn without consideration of criteria such as partisanship or race. If the proposed map is similar to the set of simulated maps, it is reasonable to assume that the proposed map was not drawn primarily with partisan intent. If the map differs from the simulations, it is important to recognize that a variety of factors could have played into the deviation, but the underlying idea is that a deviation from the simulations reflects a choice by the map-maker to prioritize some factor that was not

[12]See League of Women Voters of Ohio v. Ohio Redistricting Commission (2021); Harper v. Hall (2021); Common Cause v. Lewis (2019); Harper v. Lewis (2019); League of Women Voters of Pennsylvania v. Commonwealth of Pennsylvania (2018).

[13]Fifield, Benjamin, , Michael Higgins, Kosuke Imai, and Alexander Tarr. "Automated redistricting simulation using Markov chain Monte Carlo." Journal of Computational and Graphical Statistics 29, no. 4 (2020): 715-728.

Fifield, Benjamin, Kosuke Imai, Jun Kawahara, and Christopher T Kenny. 2020. "The essential role of empirical validation in legislative redistricting simulation." Statistics and Public Policy 7 (1): 52–68.

Kenny, Christopher T., Cory McCartan, Benjamin Fifield, and Kosuke Imai. 2020. redist: Computational Algorithms for Redistricting Simulation. https://CRAN.R-project.org/package= redist.

McCartan, Cory, and Kosuke Imai. 2020. "Sequential Monte Carlo for sampling balanced and compact redistricting plans." arXiv preprint arXiv:2008.06131.

made a priority in the simulations. This could include partisanship, but could also include incumbency protection, preservation of media markets, keeping particular counties, cities, or neighborhoods together that have historically been joined in districts, or some other factor that is important to a map maker or legislator involved in the process.

A major factor in the validity of the simulated maps is whether or not they constitute a representative sample of the trillions of possible maps that could be drawn.[14] If the sample produced by the simulations is not representative, then we may be comparing a proposed map to a biased selection of alternative maps, which renders the value of the comparison meaningless.

A specific benefit of the particular algorithm I use here is that the authors show mathematically and in a small-scale validation study that their method produces a representative sample of maps. With regards to this issue, the authors state:

> Yet, until recently, surprisingly few simulation algorithms have existed in the published scholarship. In fact, most of these existing studies use essentially the same Monte Carlo simulation algorithm where a geographical unit is randomly selected as a "seed" for each district and then neighboring units are added to contiguously grow this district until it reaches the pre-specified population threshold (e.g., Cirincione, Darling, and O'Rourke 2000; Chen and Rodden 2013). Unfortunately, no theoretical justification is given for these simulation algorithms, and hence they are unlikely to yield a representative sample of redistricting plans for a target population....Unlike the aforementioned standard simulation algorithms, the proposed algorithms are designed to yield a representative sample of redistricting plans under contiguity and equal population constraints.[15]

[14]Tam Cho, Wendy K., and Yan Y. Liu. "Toward a talismanic redistricting tool: A computational method for identifying extreme redistricting plans." Election Law Journal 15, no. 4 (2016): 351-366. Cho, Wendy K. Tam, and Bruce E. Cain. "Human-centered redistricting automation in the age of AI." Science 369, no. 6508 (2020): 1179-1181. McCartan, Cory, and Kosuke Imai. "Sequential Monte Carlo for sampling balanced and compact redistricting plans." arXiv preprint arXiv:2008.06131 (2020).

[15]Cirincione, C., Darling, T. A., and O'Rourke, T. G. (2000), "Assessing South Carolina's 1990s Congressional Districting," Political Geography, 19, 189–211. DOI: 10.1016/S0962-6298(99)00047-5. Chen, J., and

With a representative set of maps in hand, we can then analyze the difference between the proposed map and the simulated maps on a variety of metrics. As discussed above, it is well established that the party whose voters are more geographically compact stands at a natural disadvantage when single member districts are drawn. "The party that's more spread out has a geographic advantage," says applied mathematician Jonathan Mattingly of Duke University. "That's our system.[16]" The comparison between the simulated districts and the proposed map overcomes this hurdle and allows for an apples-to-apples comparison that accounts for the unique political geography of a state, such as the spatial distribution of voters or the location and number of administrative boundaries, such a counties. Simulation methods can also incorporate a state's other unique redistricting rules. The simulation-based approach therefore permits us to compare a particular plan to a large number of representative districting plans in the North Carolina House and Senate using criteria specific to North Carolina. In the simulations I run, I instruct the model to generate plans that adhere to the restrictions included in the North Carolina Constitution as well as the *Stephenson* criteria of roughly equal population, adherence to county cluster boundaries, minimization of county traversals within clusters, and geographic compactness.

Specifically, the model is constrained to conduct 50,000 simulations separately in each county cluster by assembling VTDs into districts that meet the redistricting criteria of equal population, contiguity, compactness, and minimal county and municipal divisions.[17] Within each cluster the model generates 50,000 maps with the number of districts equal to the number of districts allocated to that cluster that are of roughly equal population ($< 5\%$ deviation above or below the target population of 86,995 in the House and 208,788 in the Senate). The model is also instructed to generate districts that cross county boundaries as few times as possible. Of course, county populations do not always add up to round units

Rodden, J. (2013), "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures," Quarterly Journal of Political Science, 8, 239–269. DOI: 10.1561/100.00012033.

[16]https://www.sciencenews.org/article/gerrymandering-elections-next-gen-computer-generated-maps

[17]The simulations are not allowed to split VTDs as this is the lowest level of geography for which I have election results.

of districts, and so of necessity some county boundaries will be split. The model is further instructed that when a county boundary needs to be crossed, it should avoid splitting the county more times than necessary. After the model is run, I discard any simulations that include more county traversals than the Enacted Plan.

I also instruct the model to generate districts that are geographically compact. After the model is run, I compute the average geographic compactness of the simulated districts in the county cluster and compare that to the average geographic compactness of the Enacted Plan. I use the Polsby-Popper measure of compactness, which is a common measure of geographic compactness.[18] After the model is run, I also discard any simulations that are less compact, on average, than the Enacted Plan.

The final constraint is an instruction to avoid splitting municipal boundaries. This constraint is second order to the constraint to avoid county boundaries. In other words, the model prioritizes avoiding county splits over municipal splits. Once the county split constraint is accounted for, then the model places priority on avoidance of municipal splits. Because municipalities and VTDs do not perfectly overlap, it is difficult to calculate the exact number of municipal splits from the model. I make a simplifying assumption and assign each VTD to a municipality if any part of the VTD intersects that municipality. Furthermore, if a VTD overlaps multiple municipalities, I assign the VTD to the municipality in which the most area of the VTD is contained. In a few cases a city spans multiple counties. Here I consider each portion of the city as a separate municipality.

Once the simulated district plans are complete, I then compute the partisan lean of each district in each plan. For the partisan composition of each district I rely on the two-party election results from statewide elections disaggregated to the level of the VTD. I then reassemble these election results at the district level to compute the proportion of votes

[18]The Polsby-Popper measure is computed by taking is the ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district. A district's Polsby-Popper score falls with the range of [0,1] and a score closer to 1 indicates a more compact district. Polsby, Daniel D., and Robert D. Popper. 1991. "The Third Criterion: Compactness as a procedural safeguard against partisan gerrymandering." Yale Law & Policy Review 9 (2): 301–353.

in each statewide election that were won by the Democratic and Republican candidates in those districts. I compute the index of district partisanship using the two-party vote share in eleven elections from the past ten years.[19] The index is an average of all eleven of these statewide races in North Carolina from 2012-2020. Averages of multiple elections have the benefit of "washing out" the impact of any particular election, since individual elections can vary due to particular candidate features and other idiosyncrasies and particular years can vary due to national electoral waves (i.e. 2020 was a good electoral year for Democrats while 2016 was a good year for Republicans nationwide). As such, my preferred metric is the partisan index. However, I also compute the two-party vote share for each of the 11 statewide elections individually and report these as well for completeness. Occasionally, seeing how a plan or set of simulations varies across individual elections can shed light on the variation and shifts in political preferences in a locality.

5 NC House Analysis

A unique feature of the redistricting process in North Carolina is the use of "county grouping (or clusters)" wherein redistricting takes place entirely inside of each cluster. In essence, this means that the process of redistricting the state House (or Senate) in North Carolina is not a single problem in which a map maker draws 120 (or 50 for the Senate) districts throughout the state. Instead, the map maker faces many distinct redistricting problems that are all self contained. Cooper et al. (2021, "The Duke Study"), have addressed this issue using the 2020 census data and reported on the optimal set of clusters in both the House and Senate. They state, "Determining the county clusters for the NC House and for the NC Senate is the first step in the redistricting process for the NC General Assembly. The county clusters are largely algorithmically determined through an optimization procedure

[19]The particular races are 2020: President, US Senate, Governor, Lieutenant Governor, and Attorney General; 2016: President, US Senate, Governor, Lieutenant Governor, and Attorney General; 2014: US Senate. There are other partisan statewide races in these years, but I was unable to locate election results disaggregated to the VTD level.

outlined by the NC Supreme Court in *Stephenson v. Bartlett*.[20]" While there are a few choices that a map maker can make in choosing between different sets of clusters, the county cluster design significantly constrains any map maker as he or she is forced to work only within the counties contained in a given cluster. Because of this, any analysis of the Enacted Plan must consider each cluster separately, as they are independent of one another.

In the state House, there are 40 county clusters. 33 clusters containing 107 of the 120 districts are fixed based on the county cluster arrangement determined by Cooper et al. (2021, "Duke Study"). The remaining 7 clusters were selected by the General Assembly from three sets of choices between clusters.

5.1 House Groupings with only 1 District

Of the 40 county clusters, there are 13 of them composed of 31 counties in which the cluster contains only 1 House district. In these clusters there is no discretion for any map maker. The district is simply the boundaries of the county cluster. These counties collectively have a population of 1,128,328, or approximately 11% of the state's total population and account for 13 of the 120 seats in the state House.

Figure 6 shows a map of the counties that constitute these single-district clusters. Table 1 below shows each cluster, the counties included in the cluster, and the corresponding districts in the House Enacted Plan. The final two columns of the table show the partisan lean of the cluster using the 11 statewide partisan elections index discussed above and whether or not, based on that index, the cluster leans Democratic (or Republican). I classify a district (in the Enacted Plan and in the simulations as well) as being Democratic leaning if the partisan index for that district is greater than 0.50. In other words, if more than fifty percent of the ballots cast for the two major parties were for Democratic candidates, that district is classified as a Democratic leaning district. Obviously, districts with index values much larger than (smaller than) 0.50 will be more likely to elect a Democrat (Republican)

[20] https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf

than districts that are very close to 0.50.

The bottom row of the Table 1 shows the results for all 13 clusters together. Collectively these counties have a partisan index of 0.43, meaning roughly four in ten voters in these counties cast ballots for Democratic candidates in the 11 statewide races I consider here. However, the location of voters for the different parties is not uniformly distributed across these counties. Given this spatial distribution of voters across the counties, 4 of the 13 clusters lean Democratic, or roughly 30 percent. In this case, the proportion of Democratic leaning districts is lower that the proportion of voters in these counties who favor Democratic candidates. However, this is not due to any district boundaries. It is purely a function of the political geography of the state since all of these districts are entire county units and are, as such, fixed.

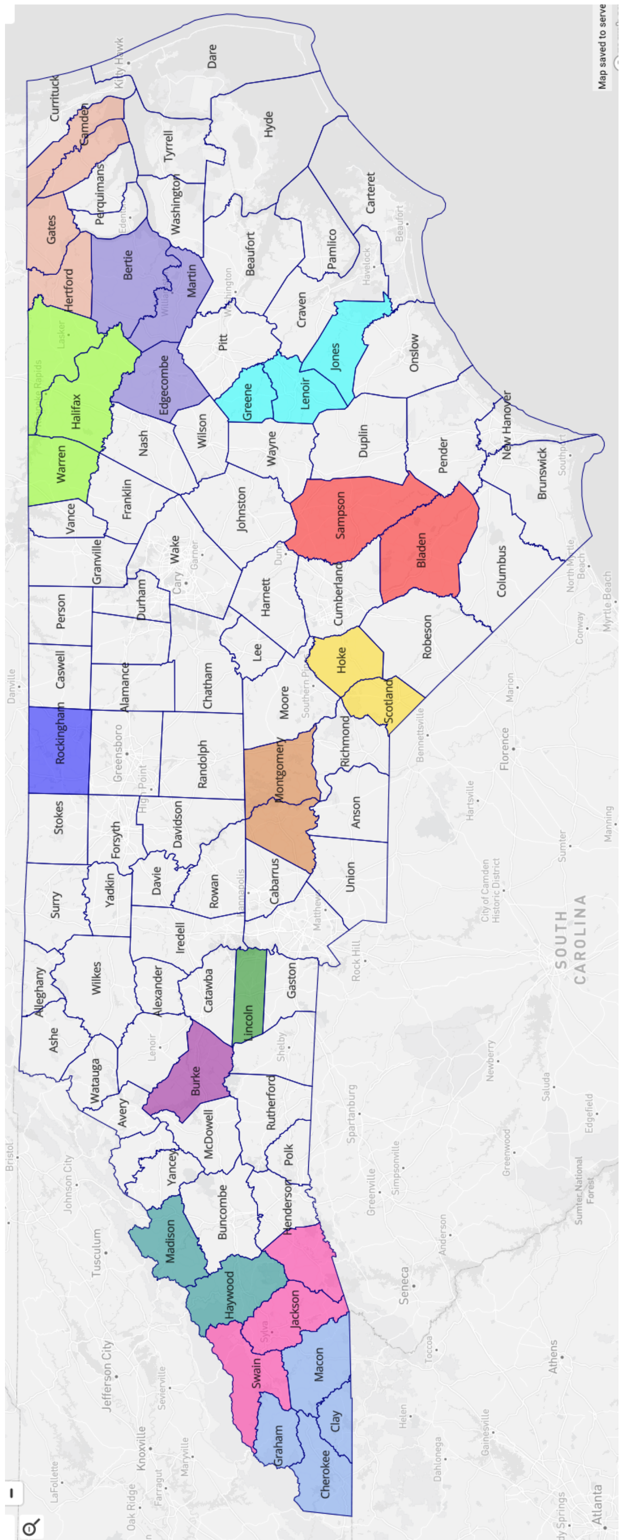


Figure 6: Map of Counties and County Clusters with only 1 House District

Table 1: County Grouping Containing 1 House District

County Cluster	# Counties	# Districts	District #	County Cluster Democratic Partisan Index	# of districts that are Democratic leaning
Rockingham	1	1	65	0.36	0
Lincoln	1	1	97	0.28	0
Burke	1	1	86	0.32	0
Bladen-Sampson	2	1	22	0.43	0
Hoke-Scotland	2	1	48	0.55	1
Haywood-Madison	2	1	118	0.40	0
Montgomery-Stanly	2	1	67	0.30	0
Bertie-Edgecomb-Martin	3	1	23	0.61	1
Greene-Jones-Lenoir	3	1	12	0.47	0
Jackson-Swain-Transylvania	3	1	119	0.44	0
Halifax-Northampton-Warren	3	1	27	0.64	1
Cherokee-Clay-Graham-Macon	4	1	120	0.28	0
Camden-Gates-Hertford-Pasquotank	4	1	5	0.52	1
Total:	31	13		0.43	4

6 House Groupings with More than 1 District:

There are 27 county clusters that contain multiple districts where a map drawer has some discretion to draw district boundaries. I consider each cluster separately in the simulations analysis because the districts are constrained to remain within each county cluster.

These clusters collectively account for 107 of the 120 districts in the North Carolina House of Representatives. In addition to calculating the number of Democratic leaning districts for the Enacted Plan, I also compute the same partisan index for the plaintiffs proposed map (hereafter, 'Duchin Map') and compare how the Enacted Map and the Duchin Map perform on this same metric.[21] An overview of the results are as follows. In these 107 districts, the Enacted Plan creates 62 districts that lean Republican and 45 districts that lean Democratic according to the statewide partisan elections index. The Duchin Plan creates 52 districts that lean Republican and 52 districts that lean Democratic according to the statewide partisan elections index.

I then place both maps in relation to the distribution of partisan outcomes from the simulated districts. In each cluster I consider the number of Democratic districts generated by each plan in comparison to the distribution of results from the simulations. I consider a plan to be a partisan outlier if the number of Democratic districts generated by the plan falls outside the middle 50% of simulation results. This is a conservative definition of an outlier. In the social sciences, medicine, and other disciplines it is traditional to consider something an outlier if it falls outside the middle 95% or 90% of the comparison distribution.

In 26 of the 27 clusters, the Enacted Map produces a number of Democratic districts that falls within the middle 50% of simulation results and are not partisan outliers. This leaves 1 cluster in which the Enacted Plan is a partisan outlier in comparison to the simulation results.[22] The Enacted Map also produces the same number of Democratic leaning districts as the modal (most common) number of Democratic leaning districts in the simulations in

[21] Plaintiffs refer to this as an "optimized map." It is unclear what this means as optimization is a choice made by the researcher as to which factors to prioritize at the expense of others.

[22] This occurs in Guilford County.

22 of the 27 clusters.

In 23 of the 27 clusters, the Duchin Map produces a number of Democratic districts that fall within the middle 50% of simulation results and are not partisan outliers. This leaves 4 clusters in which the Duchan Plan is a partisan outlier in comparison to the simulation results.[23] This is three more clusters that are partisan outliers than the Enacted Map. The Duchin Map also produces the same number of Democratic leaning districts as the modal (most common) number of Democratic leaning districts in the simulations in 20 of the 27 clusters.

By these metrics the Duchin Map is less in alignment with the results of the non-partisan simulations than the Enacted Map and is a greater partisan outlier.

In 20 of the 27 clusters the Enacted Map and the Duchin map are in agreement on the number of Democratic leaning districts.[24] This means there is disagreement in 7 of the 40 total clusters. Figure 7 shows a map of the locations in which the Enacted Plan and the Duchin Plan are in agreement on the number of Democratic leaning districts. Figure 8 shows a map of the locations in which the Enacted Plan and the Duchin Plan disagreement on the number of Democratic leaning districts.

Table 2 summarizes the results of the simulation analysis for these 27 House clusters with multiple districts. Thereafter, I present the results cluster-by-cluster.

[23]These are Brunswick-New Hanover, Cumberland, Duplin-Wayne, and Pitt

[24]These county groupings are: Davidson, Columbus-Robeson, Carteret-Craven, Nash-Wilson, Caswell-Orange, Alexander et al., Franklin et al., Alleghany et al., Beaufort et al., Anson-Union, Onslow-Pender, Harnett-Johnston, Catawba-Iredell, Durham-Person, Forsyth-Stokes, Cabarrus et al., Chatham et al., Avery et al., Mecklenburg, and Wake.

Table 2: House County Grouping Analysis Summary

County Cluster	Cluster Democratic Partisan Index	# Districts	# of Districts that are Democratic Leaning: Enacted Map	Duchin Map	Simulations
Davidson	0.27	2	0	0	0
Pitt	0.54	2	1	**2**	1
Alamance	0.45	2	0	1	0-1
Columbus-Robeson	0.45	2	0	0	0
Carteret-Craven	0.35	2	0	0	XXX
Duplin-Wayne	0.43	2	0	**1**	0
Nash-Wilson	0.52	2	2	2	2
Caswell-Orange	0.71	2	2	2	2
Alexander-Surry-Wilkes	0.25	2	0	0	0
Franklin-Granville-Vance	0.51	2	1	1	1
Alleghany-Ashe-Caldwell-Watauga	0.36	2	0	0	0
Beaufort-Chowan-Currituck Dare-Hyde-Pamlico Perquimans-Tyrrell-Washington	0.39	2	0	0	0
Buncombe	0.60	3	2	3	2-3
Anson-Union	0.37	3	0	0	0
Onslow-Pender	0.35	3	0	0	0
Cumberland	0.59	4	3	**4**	3
Harnett-Johnston	0.38	4	0	0	0
Catawba-Iredell	0.33	4	0	0	0
Durham-Person	0.76	4	4	4	4
Brunswick-New Hanover	0.45	4	1	**2**	1
Forsyth-Stokes	0.52	5	2	2	2-3
Cabarrus-Davie-Rowan-Yadkin	0.36	5	0	0	0
Chatham-Lee-Moore-Randolph-Richmond	0.38	5	1	1	1
Guilford	0.61	6	**4**	5	5
Avery-Cleveland-Gaston-Henderson-McDowell-Mitchell-Polk-Rutherford-Yancey	0.35	7	0	0	0
Mecklenburg	0.65	13	11	11	11-12
Wake	0.61	13	11	11	11-12
Total:		107	45	52	46-51

Note: Number of Democratic leaning districts is measured using the average two-party vote share in each district from the 11 statewide races noted earlier. Simulations range represents the middle 50% of outcomes from the simulations results. There are no simulations results conducted in Carteret-Craven cluster, see later section for explanation. Groupings where a plan falls outside the middle 50% range of the simulations are bolded.

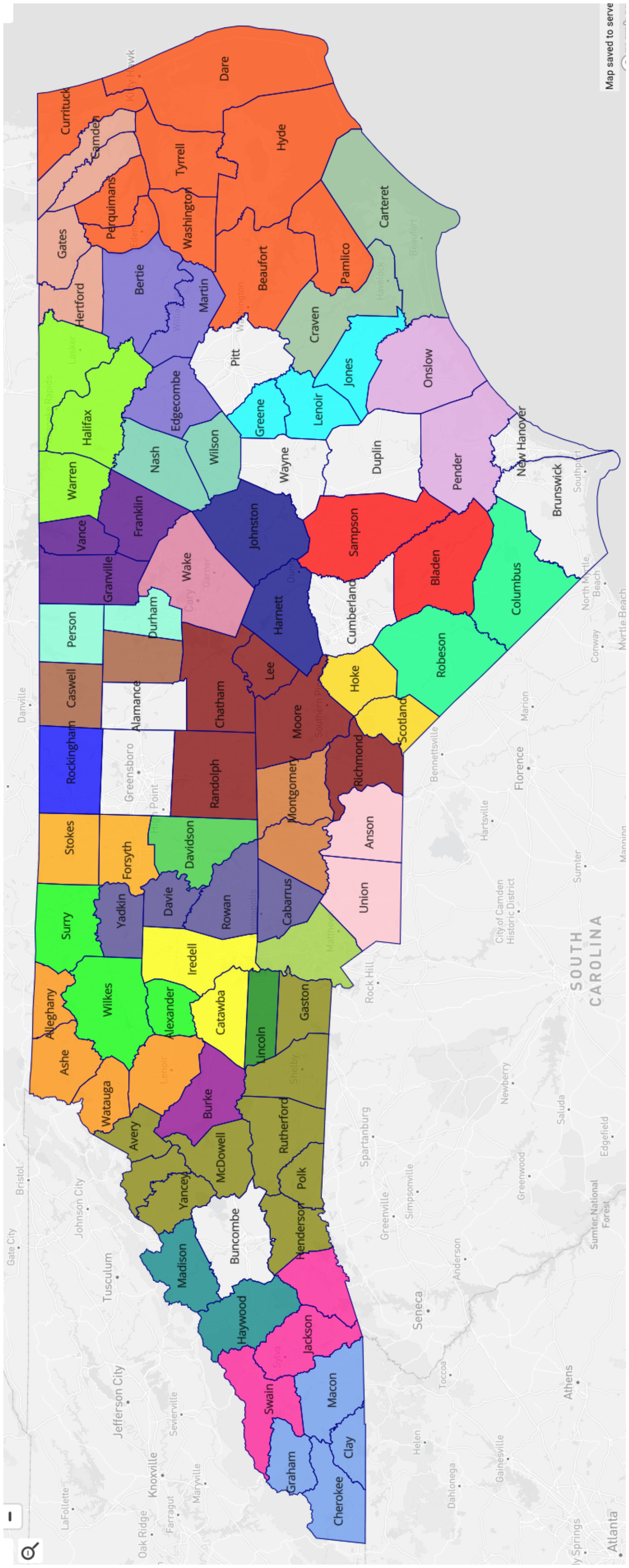
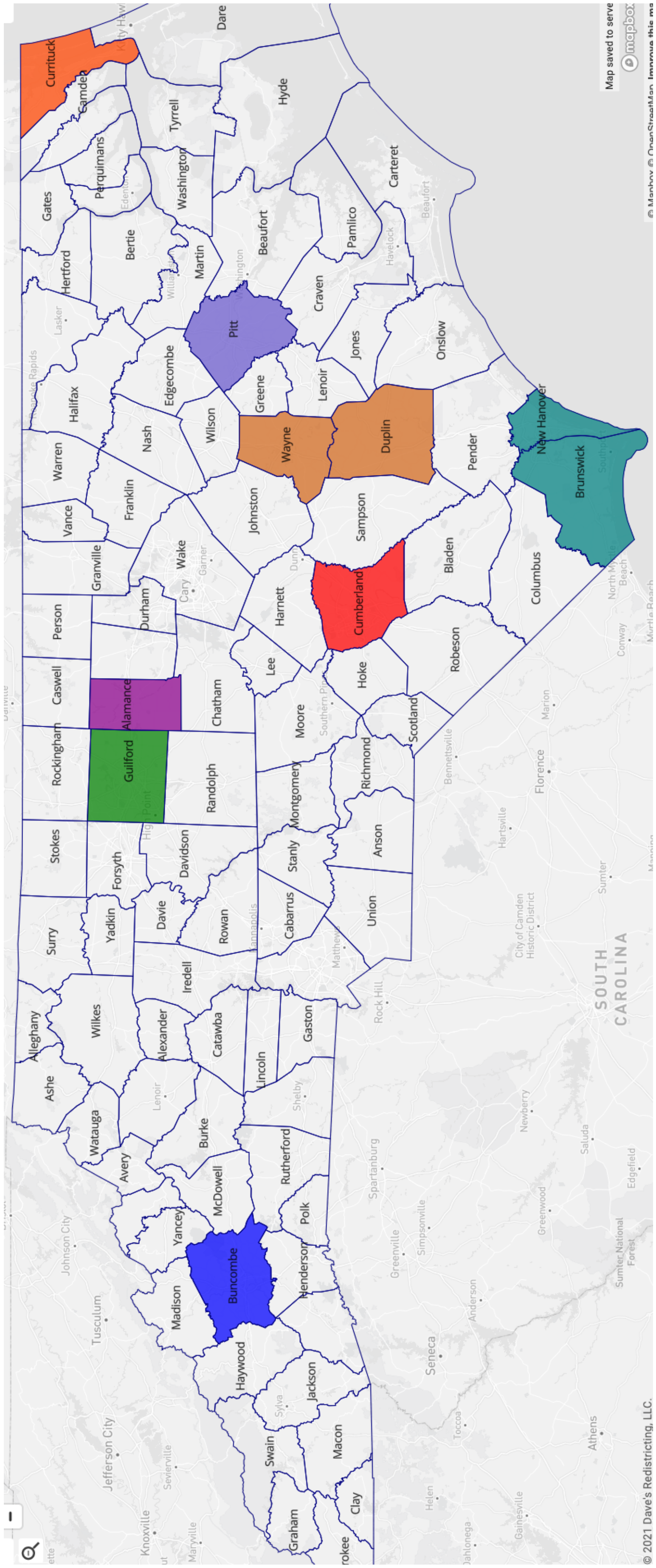


Figure 7: Map of House County Clusters Where Enacted and Duchin Plans Agree on Partisan Lean of Districts

Figure 8: Map of House County Clusters Where Enacted and Duchin Plans Disagree on Partisan Lean of Districts



6.1 Davidson House County Grouping

Davidson County contains 2 districts. In the Enacted Map these are Districts 80 and 81. The county cluster has an overall partisan index of 0.27, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, in this case the county cluster is only one county (Davidson) and so the simulations are constrained to keep both districts entirely within the county, and thus, by definition there will be no county traversals in all 50,000 simulations as well as in the Enacted Map. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 37,252 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 9. A map of the Enacted Plan's districts within this cluster is shown in Figure 10.

The distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 11. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In this cluster the simulations, the Enacted Map, and the Duchin Map are in agreement, and all generate 0 Democratic leaning districts.

Table 3 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In this case there is unanimous agreement across all 11 elections.

Figure 9: **Map of Davidson House County Cluster**

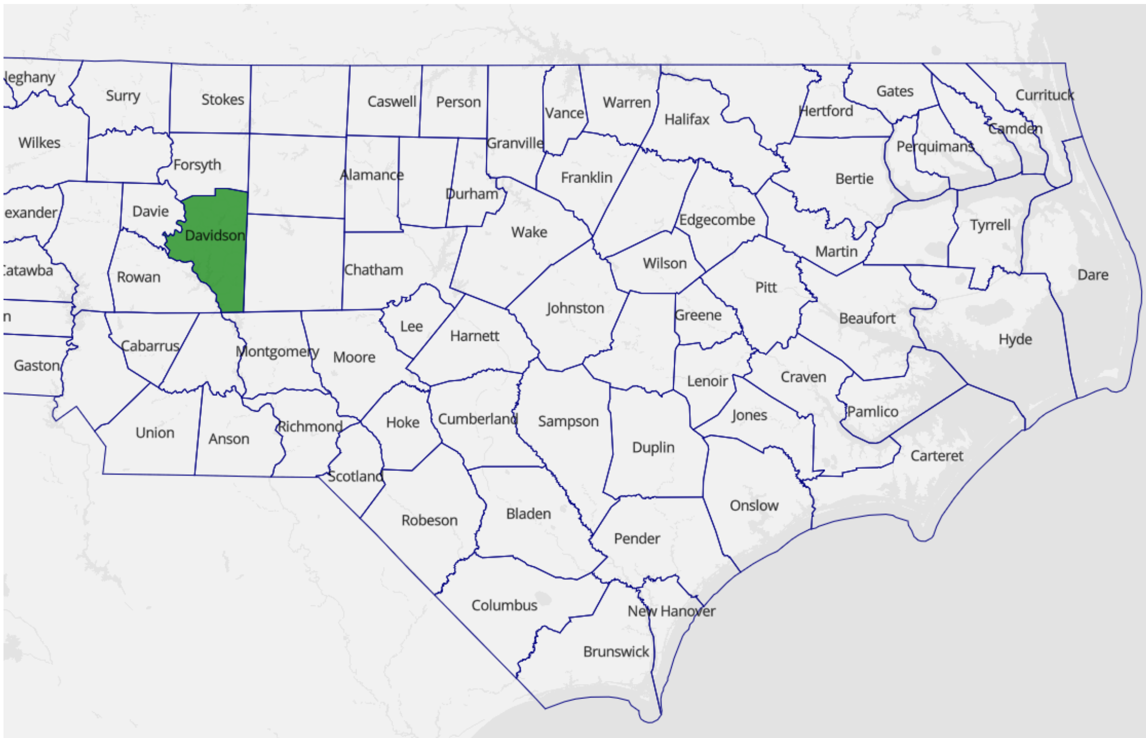
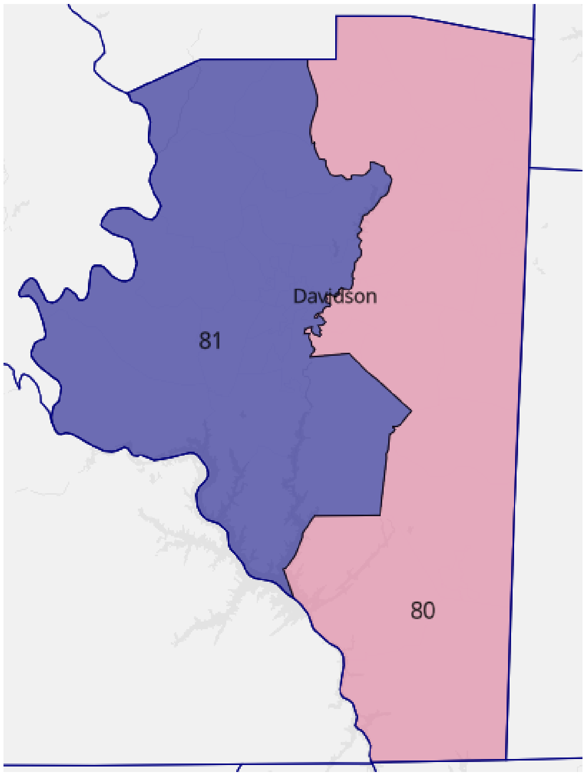
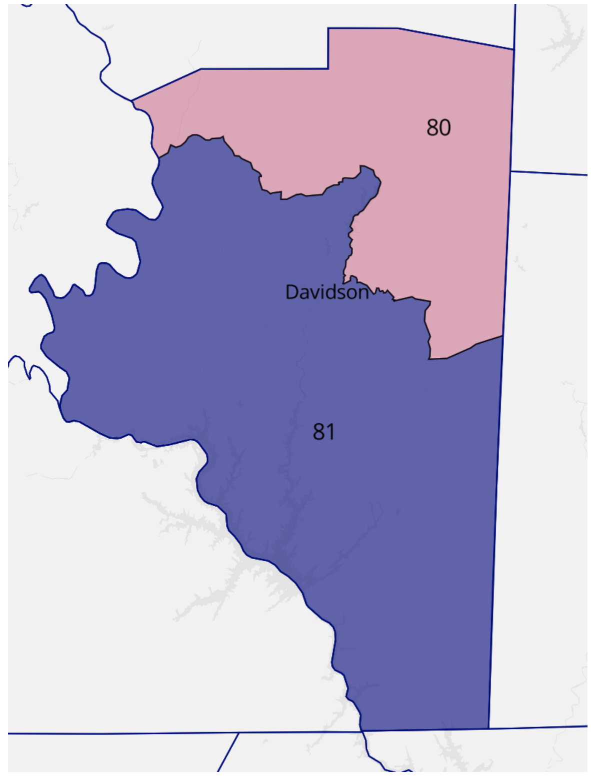


Figure 10: **Map of House Enacted Plan in Davidson County Cluster**

(a) Enacted Map



(b) Duchin Map



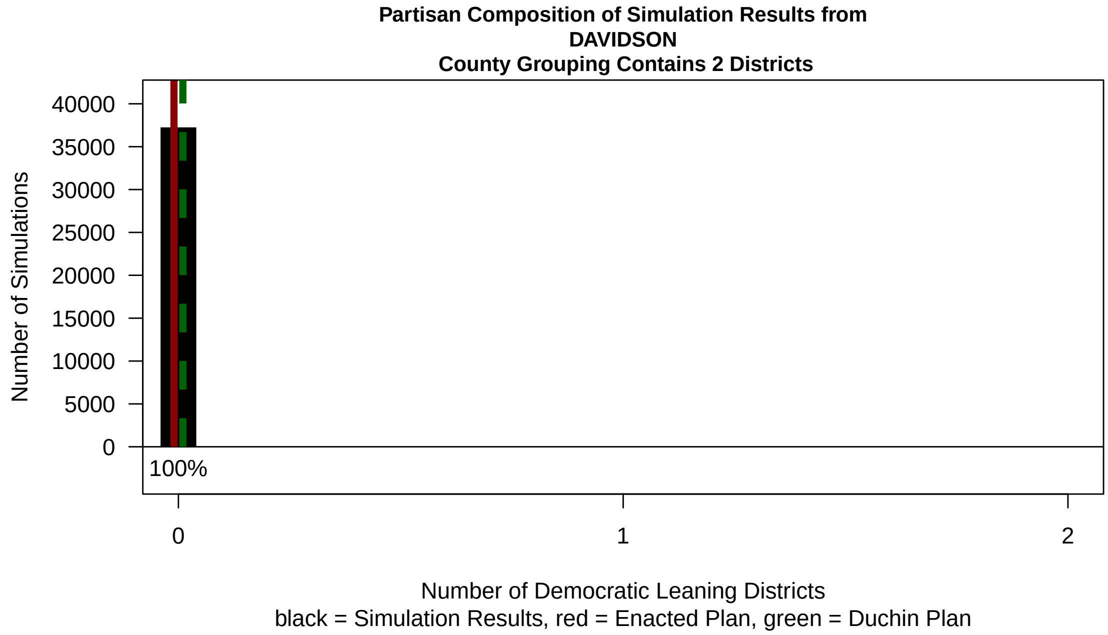
Note: The left map shows the district lines for the Enacted Map and the right map shows the district lines for the Duchin Map.

Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
80	0.26	0.28
81	0.29	0.27

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 11: **Distribution of Partisan Districts from Simulations in Davidson House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 3: Simulation Results by Individual Elections

Davidson House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.2 Pitt House County Grouping

Pitt County contains 2 districts. In the Enacted Map these are Districts 8 and 9. The county cluster has an overall partisan index of 0.54, which is slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, in this case the county cluster is only one county and so the simulations are constrained to keep both districts entirely within the county, and thus, by definition there will be no county traversals in all 50,000 simulations as well as in the Enacted Map. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 5,189 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 12. A map of the Enacted Maps' districts and the Duchin Map's district boundaries within this cluster are shown in Figure 13.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 14. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 91% of the simulations there is 1 Democratic leaning district and in the remaining 9% of the simulations there are two Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by creating one Democratic district. The Duchin Map generates two Democratic districts.

Table 4 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Demo-

cratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 12: **Map of Pitt House County Cluster**

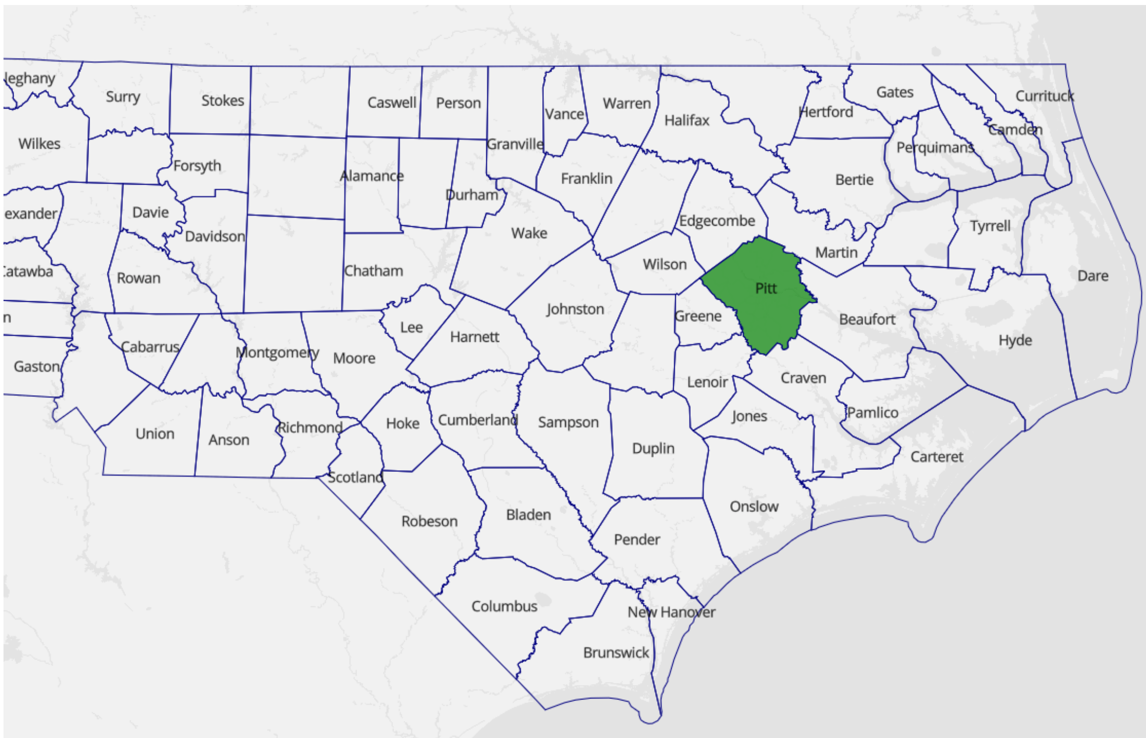
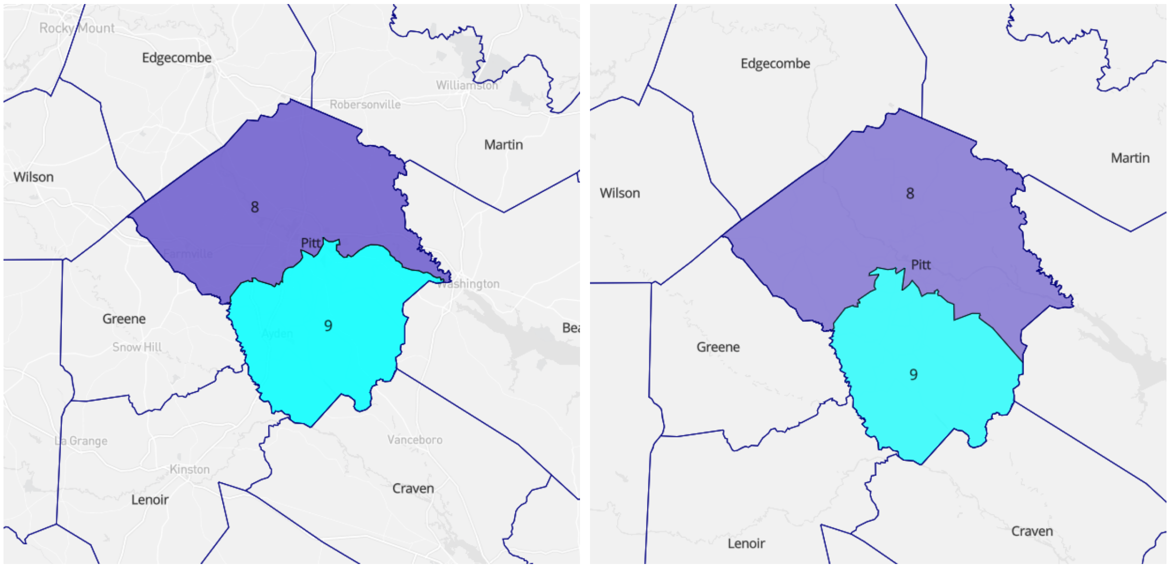


Figure 13: **Enacted Map and Duchin Map in Pitt House County Cluster**

(a) Enacted Map (b) Duchin Map

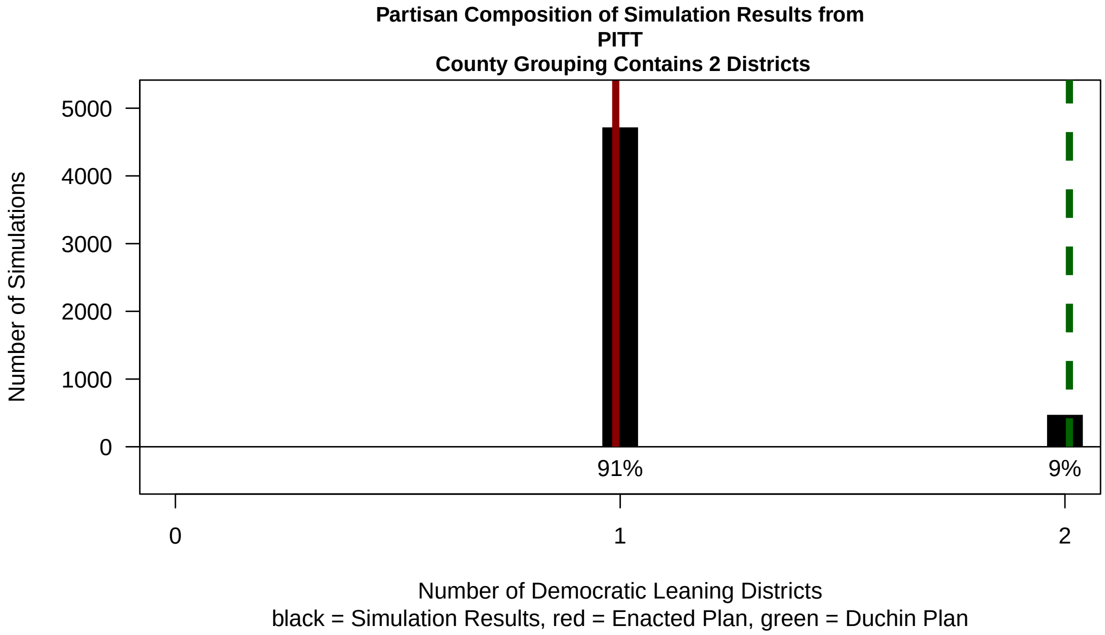


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
8	0.64	0.55
9	0.46	0.53

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 14: **Distribution of Partisan Districts from Simulations in Pitt House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 4: Simulation Results by Individual Elections

Pitt House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	**89%**	11%
2020 Senate	0%	**91%**	9%
2020 Governor	0%	**44%**	56%
2020 Lt. Governor	0%	**94%**	6%
2020 Attorney General	0%	**71%**	29%
2016 President	0%	**97%**	3%
2016 Senate	0%	**100%**	0%
2016 Governor	0%	**97%**	3%
2016 Lt. Governor	0%	**100%**	0%
2016 Attorney General	0%	**83%**	17%
2014 Senate	0%	**100%**	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 89% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

6.3 Alamance House County Grouping

Alamance County contains 2 districts. In the Enacted Map these are Districts 63 and 64. The county cluster has an overall partisan index of 0.45, which is slightly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, in this case the county cluster is only one county and so the simulations are constrained to keep both districts entirely within the county, and thus, by definition there will be no county traversals in all 50,000 simulations as well as in the Enacted Map. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 47,482 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 15. A map of the Enacted Maps' districts and the Duchin Map's district boundaries within this cluster are shown in Figure 16. I also include the map of districts in this county from the 2020 plan for comparison here.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 17. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 44% of the simulations there are 0 Democratic leaning districts and in the remaining 56% of the simulations there is 1 Democratic leaning district. The Enacted Map is within the middle 50% if the simulation results, but is not in alignment with the modal outcome of the simulations. The Duchin Map generates 1 Democratic district.

Table 5 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 10 of the 11 elections considered the Enacted Plan agrees with the modal outcome of the simulations. The only case in which it does not agree with the modal result is in the 2020 Lt. Governor's race. However, in this race the simulations were nearly equally split between generating 0 and 1 Democratic district.

The Enacted Plan is also extremely similar to the maps used in Alamance County in the 2020 elections. These districts were approved by a court in 2019. The Enacted Plan is different by only two and one half precincts - South Burlington precinct is now placed in District 64 (it was in District 63 in the 2020 map) and North Thompson and the part of Melville 3 precinct that was split into District 64 is now placed into District 63, making it whole and keeping the municipality of Swepsonville entirely in District 63.

Another consideration is that while the Enacted Plan does not generate a Democratic leaning district using the partisan index, there is one district that is effectively a 50/50 split between Republicans and Democrats. The partisan index of District 63 is 0.4994, which is about as close to a perfect split between Republican and Democratic votes as a district could get. It is very likely that both parties will win this district a number of times over the next several years.

Figure 15: **Map of Alamance House County Cluster**

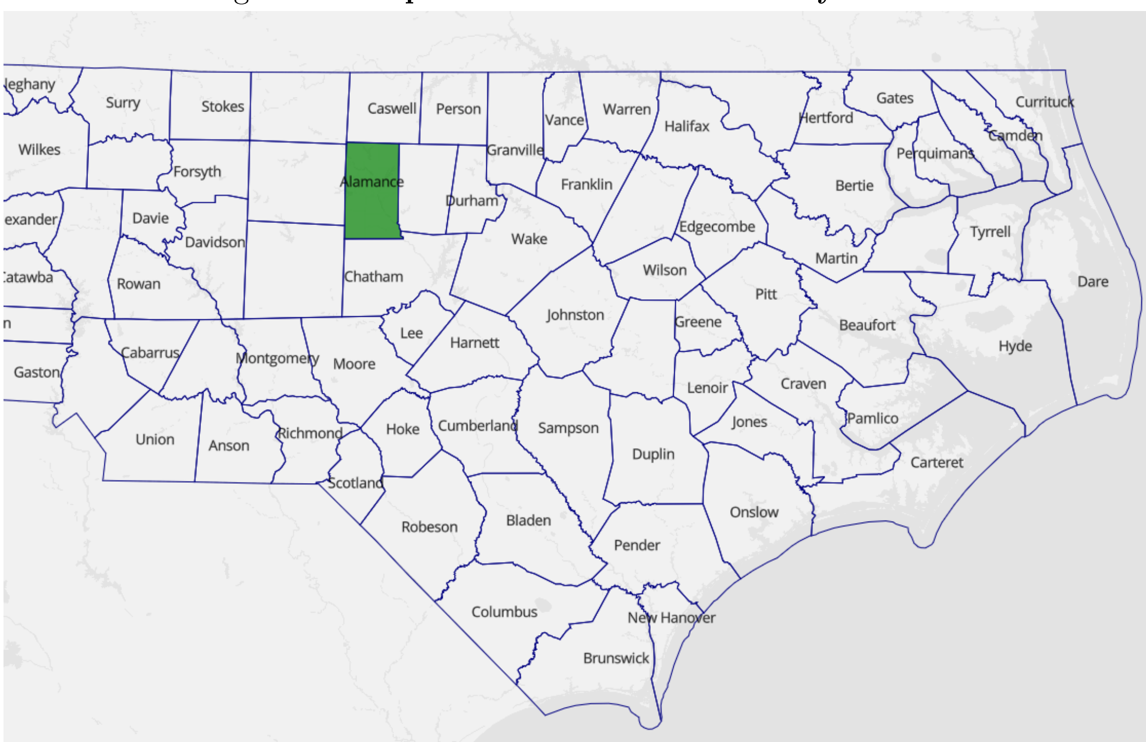
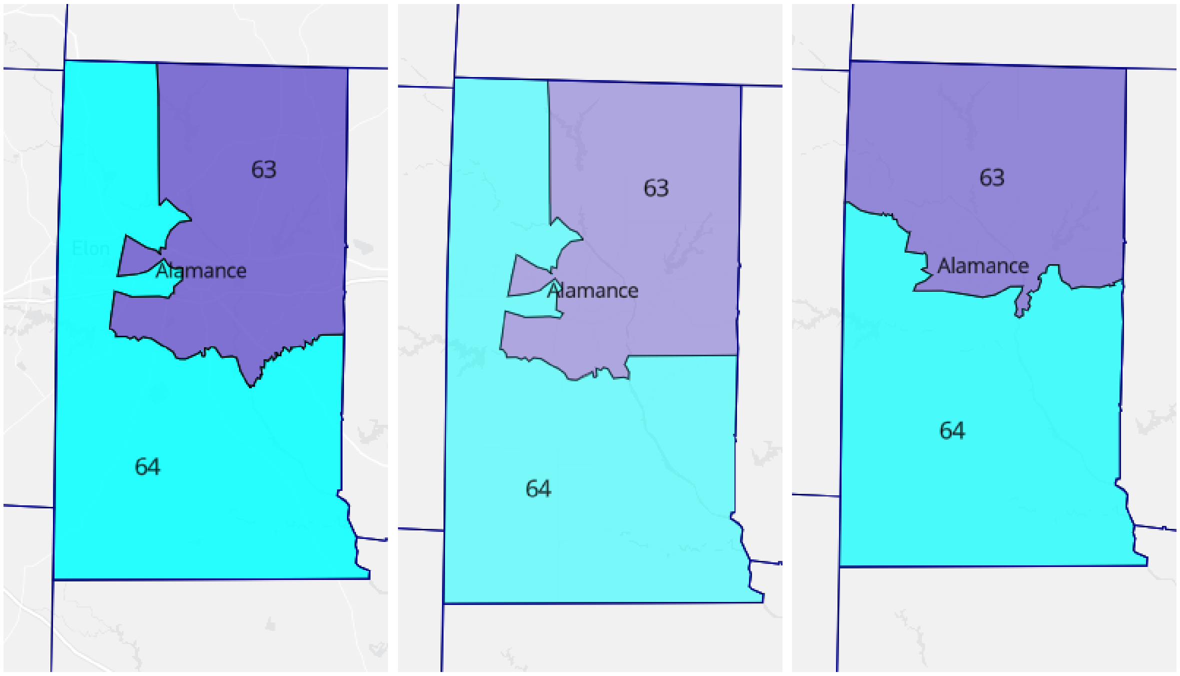


Figure 16: **Enacted Map, 2020 Map, and Duchin Map in Pitt House County Cluster**

(a) Enacted Map (b) 2020 Map (c) Duchin Map

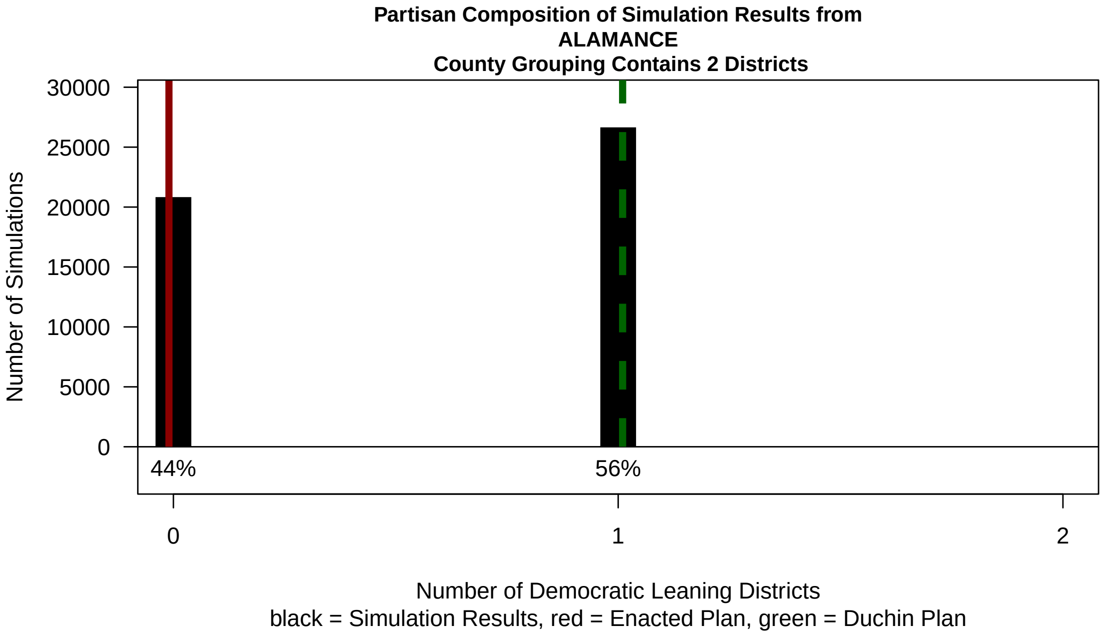


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
63	0.50	0.54
64	0.41	0.38

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 17: **Distribution of Partisan Districts from Simulations in Alamance House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 5: Simulation Results by Individual Elections

Alamance House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	40%	**60%**	0%
2020 Senate	38%	**62%**	0%
2020 Governor	3%	**97%**	0%
2020 Lt. Governor	**47%**	53%	0%
2020 Attorney General	13%	**87%**	0%
2016 President	**77%**	23%	0%
2016 Senate	**98%**	2%	0%
2016 Governor	39%	**61%**	0%
2016 Lt. Governor	**99%**	1%	0%
2016 Attorney General	42%	**58%**	0%
2014 Senate	**97%**	3%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 60% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

6.4 Columbus and Robeson House County Grouping

The Columbus-Robeson House county grouping contains 2 districts. In the Enacted Map these are Districts 46 and 47. The county cluster has an overall partisan index of 0.45, which is slightly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 46,076 remaining simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 2,664 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 18. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 19.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 20. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by creating 0 Democratic districts. The Duchin Map also generates 0 Democratic district.

Table 6 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 18: **Map of Columbus and Robeson House County Cluster**

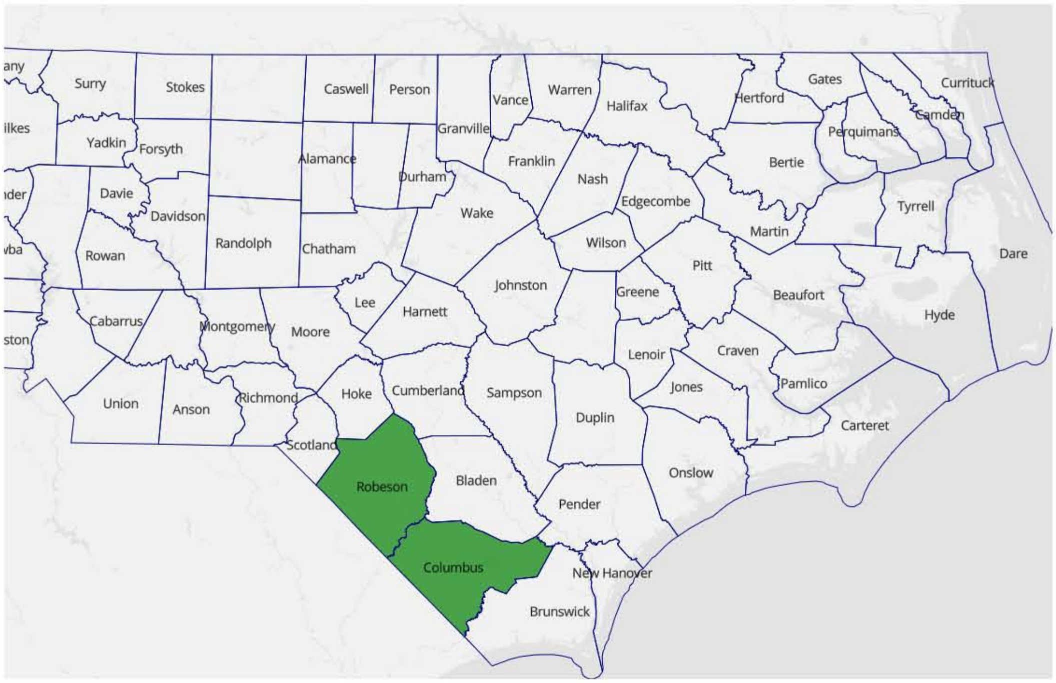
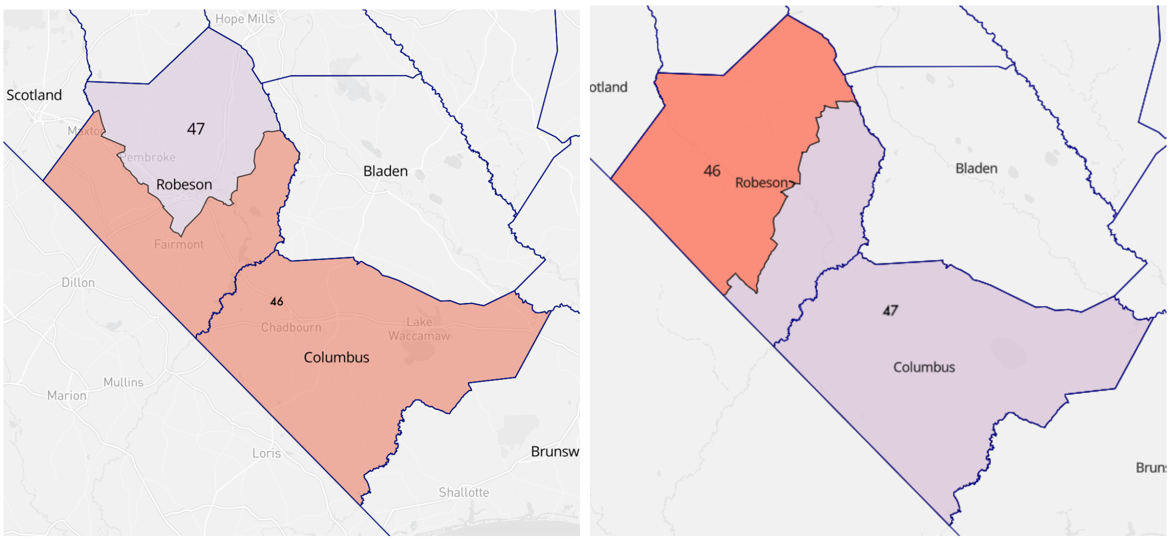


Figure 19: **Map of House Enacted Plan and Duchin Plan in Columbus and Robeson County Cluster**

(a) Enacted Map (b) Duchin Map

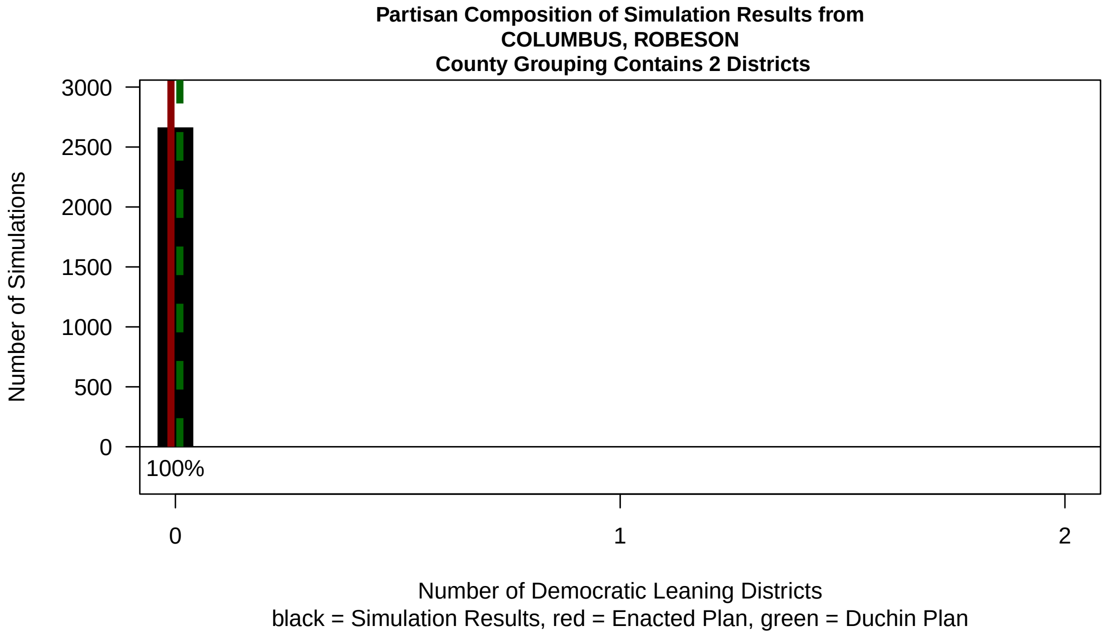


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
46	0.42	0.49
47	0.48	0.42

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 20: **Distribution of Partisan Districts from Simulations in Columbus and Robeson House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 6: Simulation Results by Individual Elections

Columbus and Robeson House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	0%	**100%**	0%
2016 Attorney General	0%	**53%**	47%
2014 Senate	0	0%	**100%**

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.5 Carteret and Craven House County Grouping

The Carteret-Craven House county grouping contains 2 districts. In the Enacted Map these are Districts 3 and 13. The county cluster has an overall partisan index of 0.35, which is strongly Republican. I do not conduct simulations in this cluster because there is no possible way to assemble VTDs in this county grouping and produce two districts that meet the equal population criteria. To do so requires splitting a VTD, something both the Enacted Plan and Duchin Plans do, but the simulations are not capable of. However, there is agreement between the Enacted Plan and the Duchin Plan, as both plans create two Republican leaning districts that are nearly identical in shape. Furthermore, given the strong Republican lean of the county grouping and relatively even distribution of partisan preferences in the county, it would be impossible to assemble any district that leans Democratic.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 21. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 22.

Figure 21: **Map of Carteret and Craven County Cluster**

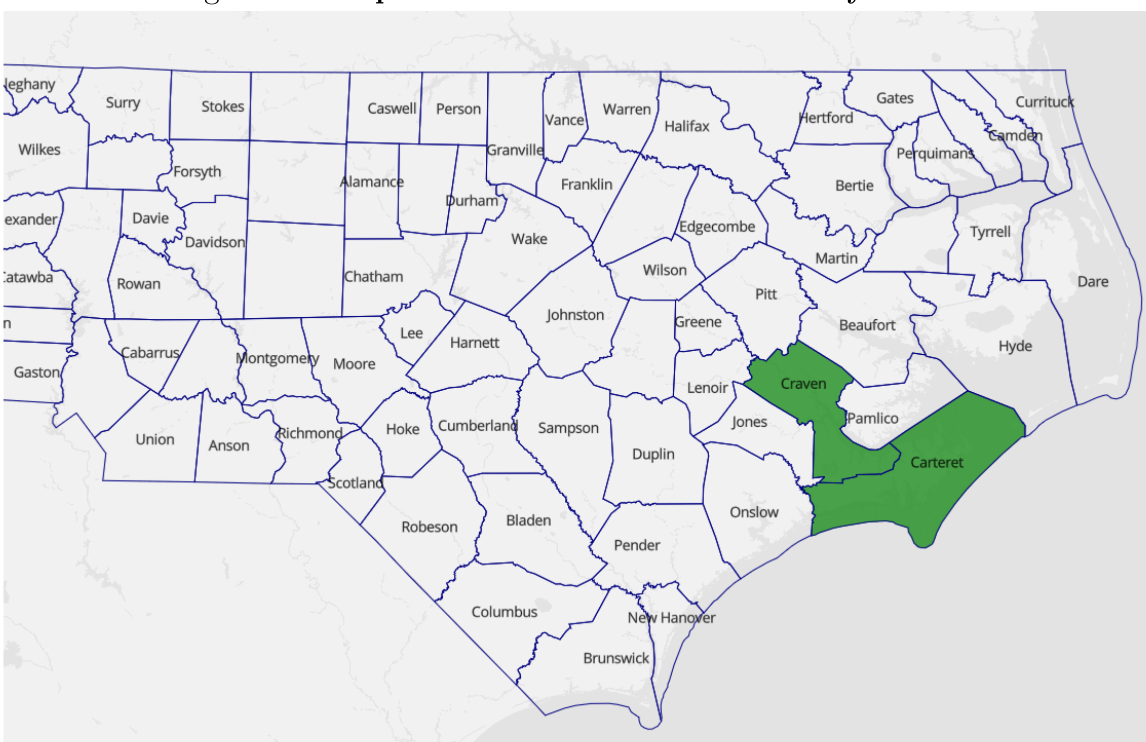
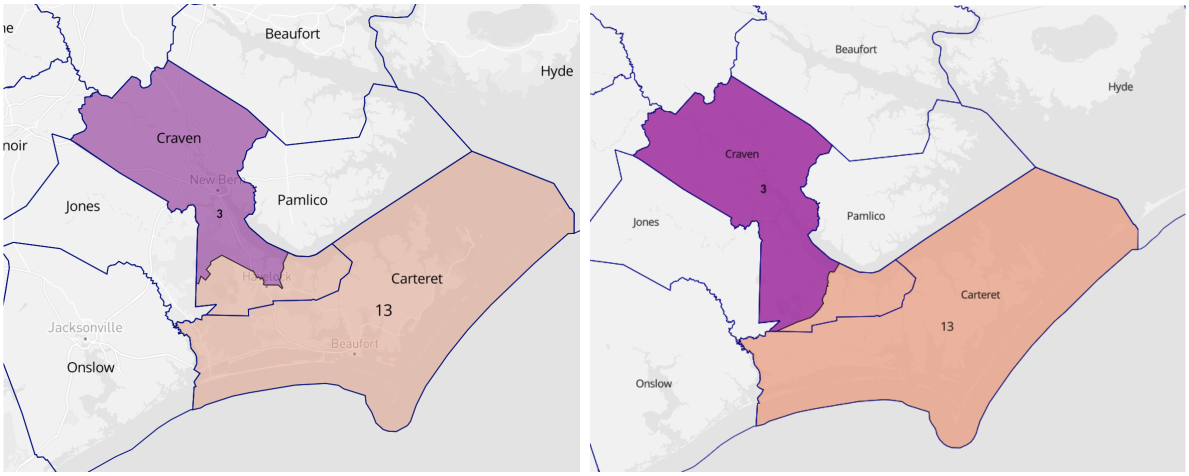


Figure 22: **Map of House Enacted Plan in Carteret and Craven County Cluster**

(a) Enacted Map (b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
3	0.40	0.40
13	0.31	0.31

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

6.6 Duplin and Wayne House County Grouping

The Duplin-Wayne House county grouping contains 2 districts. In the Enacted Map these are Districts 4 and 10. The county cluster has an overall partisan index of 0.43, which is moderately Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any maps that contain more county traversals than the Enacted Plan, leaving 23,399 maps. Next, I would normally discard any simulations in which the average compactness score of the districts in the simulations that are not as large or larger than the compactness score of the Enacted Map. However, this leaves 0 simulated maps, as the Enacted Plan is more compact than any of the simulations (an average Polsby-Popper score of .50, which is very high). To have some simulations to compare to the Enacted Plan and the Duchin plan, I retained the 10% of the simulated maps that have the highest compactness score (2,704 maps).

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 23. A map of the Enacted Maps' districts and the Duchin Map's district boundaries within this cluster are shown in Figure 24.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 25. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in agreement with the simulation results and generates 0 Democratic leaning districts. The Duchin Map creates one Democratic leaning district (District 21) surrounding the town of Goldsboro. However to avoid Republican leaning VTDs in the north and western portions of Wayne County, District 4 in the Duchin Plan joins these VTDs with Duplin County to the south. This creates a district that has a northern "hook," which is much less compact than the districts in the Enacted Plan. The average Polsby-Popper score for Districts 21 and 4 in the Duchin plan is 0.32. What reason could there be for the shape of District 4? One possibility is that the district is attempting to keep Goldsboro, the largest city in Wayne County whole. However, both the Enacted and Duchin plans keep Goldsboro whole.[25] Given this, it is hard to imagine another explanation for the unusual shape of District 4 aside from an attempt to avoid Republican precincts so as to create a Democratic leaning seat in District 21.

Table 7 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the elections considered the Enacted Plan agrees with the modal (most common) outcome of the simulations.

[25]The Enacted Plan places 5 residents from Goldsboro and the Goldsboro wastewater treatment plant in District 4. The remaining 99.99% of Goldsboro is in District 10.

Figure 23: **Map of Duplin and Wayne House County Cluster**

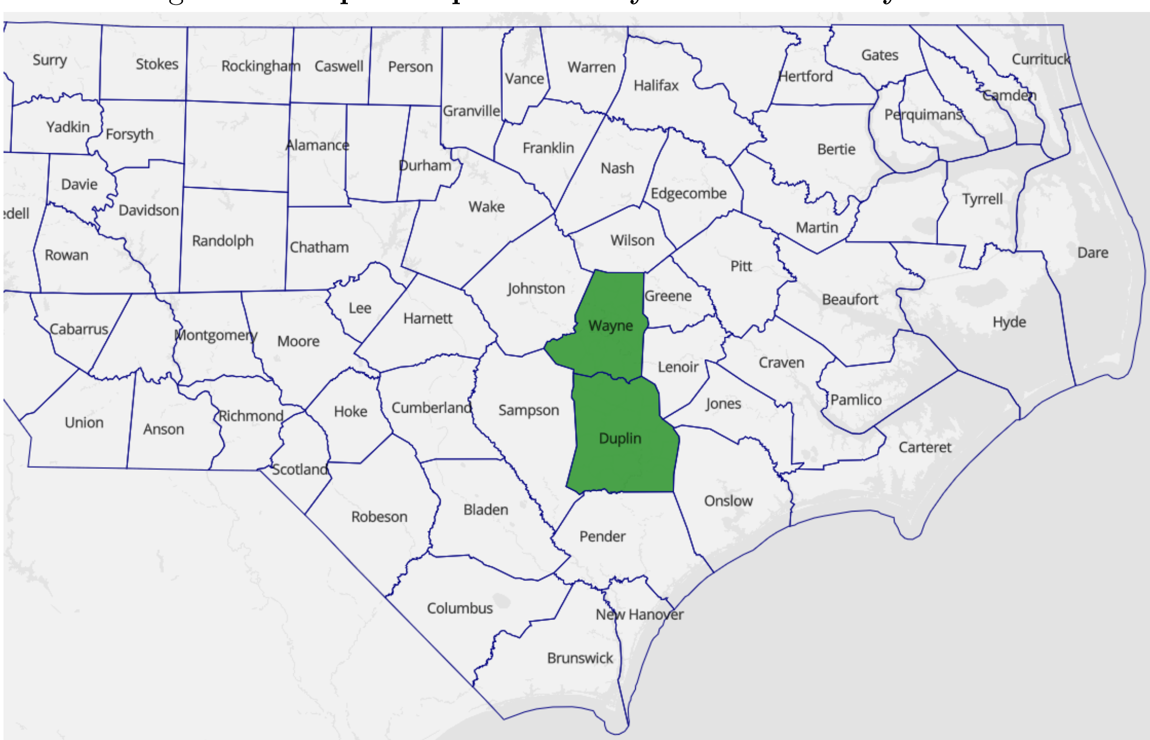
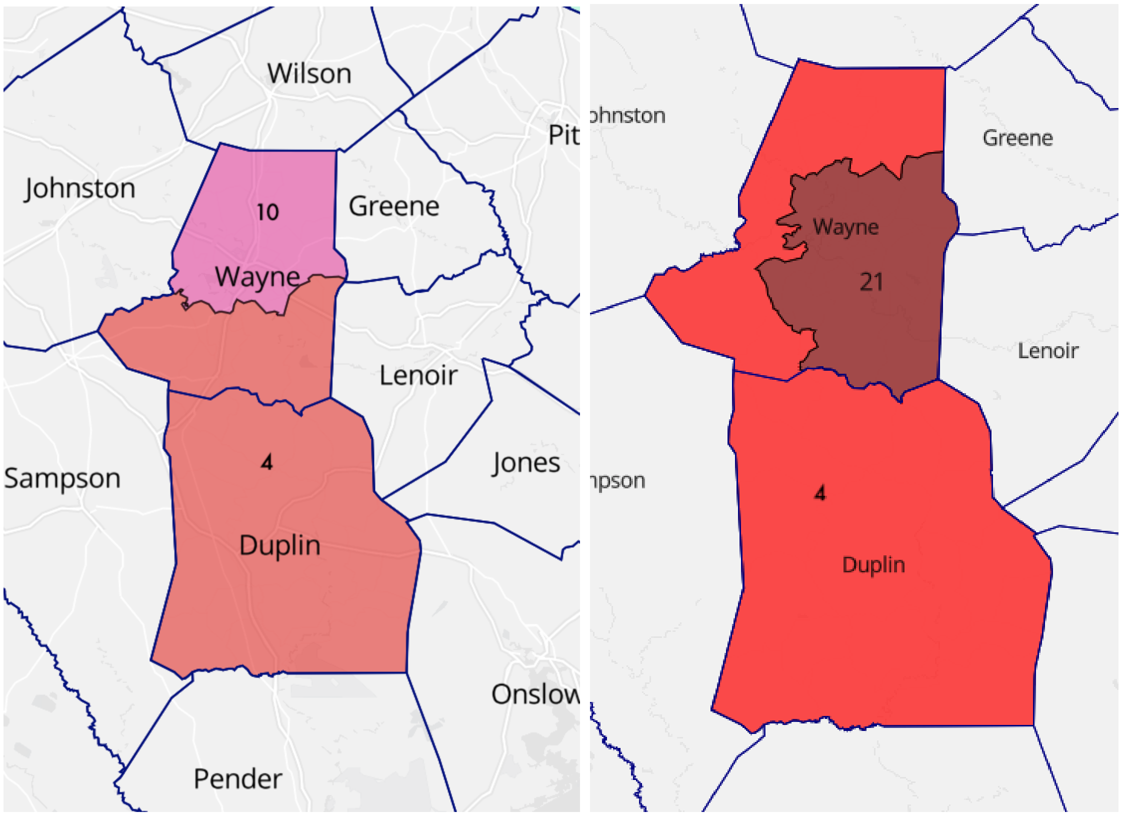


Figure 24: **Map of House Enacted Plan in Duplin and Wayne County Cluster**

(a) Enacted Map (b) Duchin Map

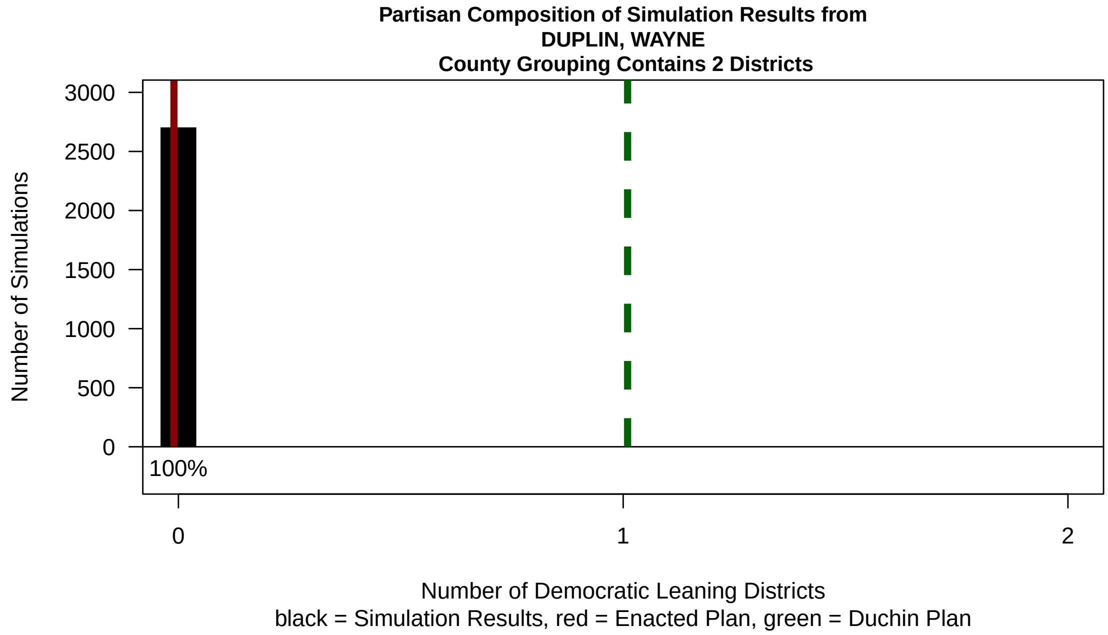


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
4	0.41	0.36
10 (21 in Duchin)	0.46	0.51

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 25: **Distribution of Partisan Districts from Simulations in Duplin and Wayne House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 7: Simulation Results by Individual Elections

Duplin and Wayne House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**95%**	5%	0%
2014 Senate	**95%**	5%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.7 Nash and Wilson House County Grouping

The Nash-Wilson House county grouping contains 2 districts. In the Enacted Map these are Districts 24 and 25. The county cluster has an overall partisan index of 0.52, which is slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 41,476 remaining simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 14,569 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 26. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 27.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 28. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 2 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 2 Democratic districts. The Duchin Map also generates 2 Democratic districts.

Table 8 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 26: **Map of Nash and Wilson House County Cluster**

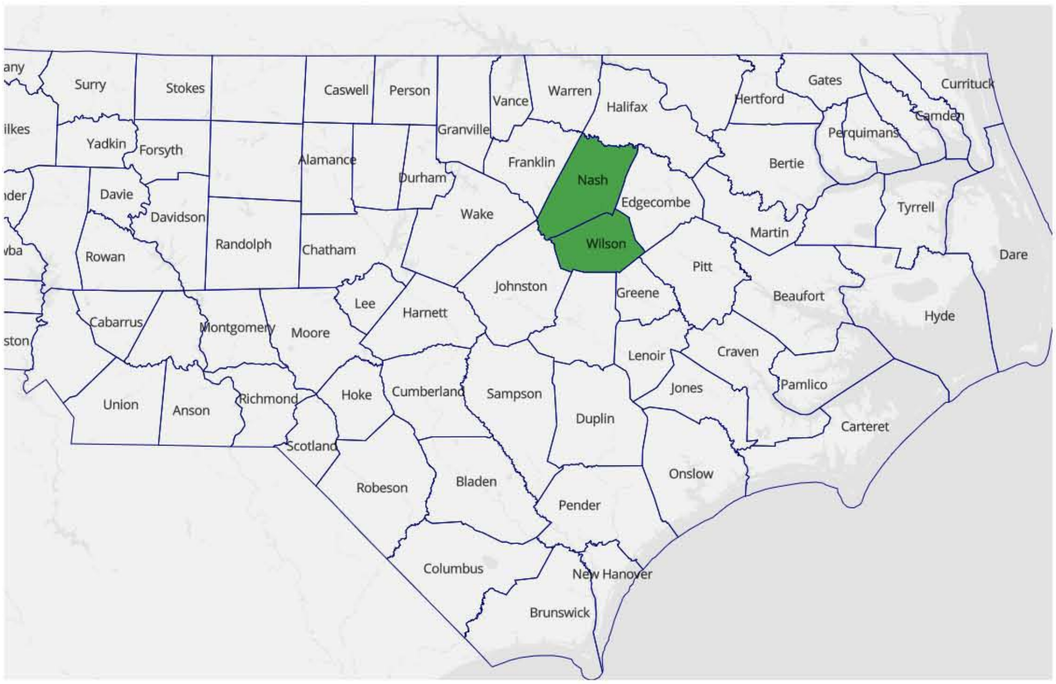
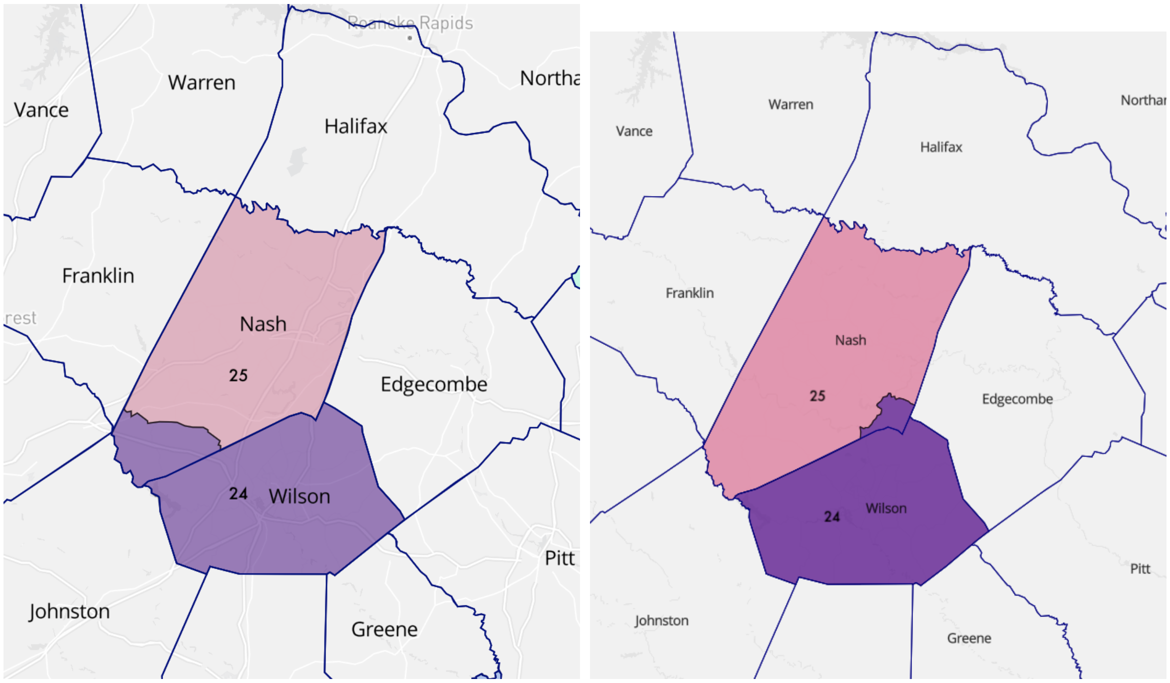


Figure 27: **Map of House Enacted Plan in Nash and Wilson County Cluster**

(a) Enacted Map (b) Duchin Map

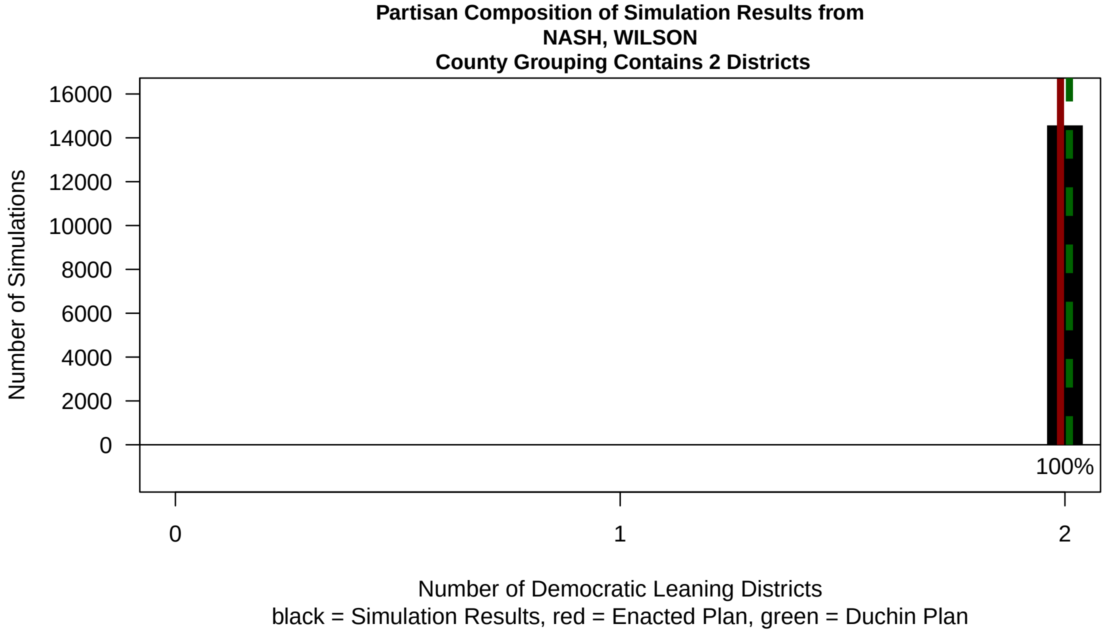


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
24	0.52	0.52
25	0.52	0.52

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 28: **Distribution of Partisan Districts from Simulations in Nash and Wilson House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 8: Simulation Results by Individual Elections

Nash and Wilson House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	**88%**	12%
2020 Senate	0%	0%	**100%**
2020 Governor	0%	0%	**100%**
2020 Lt. Governor	0%	**88%**	12%
2020 Attorney General	0%	0%	**100%**
2016 President	0%	0%	**100%**
2016 Senate	0%	0%	**100%**
2016 Governor	0%	0%	**100%**
2016 Lt. Governor	0%	0%	**100%**
2016 Attorney General	0%	0%	**100%**
2014 Senate	0%	**88%**	12%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 88% of the simulations produce 1 Democratic leaning districts. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

6.8 Caswell and Orange House County Grouping

The Caswell-Orange House county grouping contains 2 districts. In the Enacted Map these are Districts 50 and 56. The county cluster has an overall partisan index of 0.71, which is strongly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 50,000 simulated maps since in this case all of the simulation results only include one county traversal, as does the Enacted Map. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 40,012 simulated maps, each containing two districts.

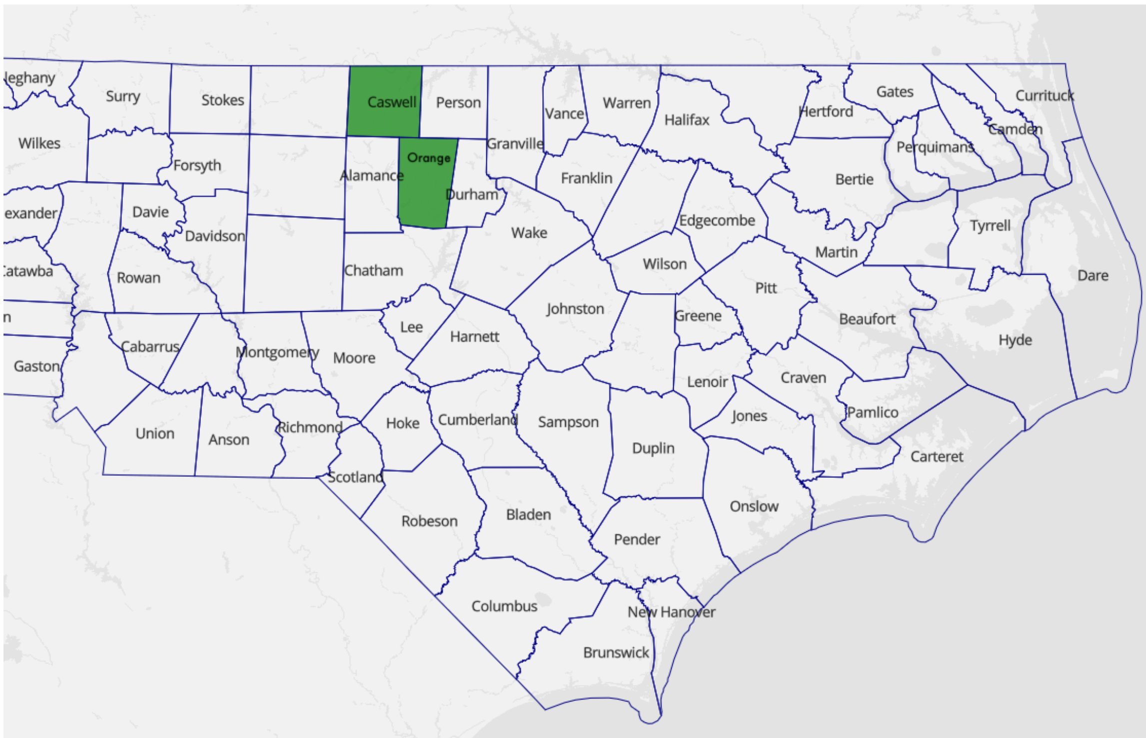
A map of the location of this county cluster in relation to the rest of the state is shown in Figure 29. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 30.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 31. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 2 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 2 Democratic districts. The Duchin Map also generates 2 Democratic districts.

Table 9 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 29: **Map of Caswell and Orange House County Cluster**



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
50	0.57	0.56
56	0.85	0.85

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 30: **Map of House Enacted Plan in Caswell and Orange County Cluster**

(a) Enacted Map (b) Duchin Map

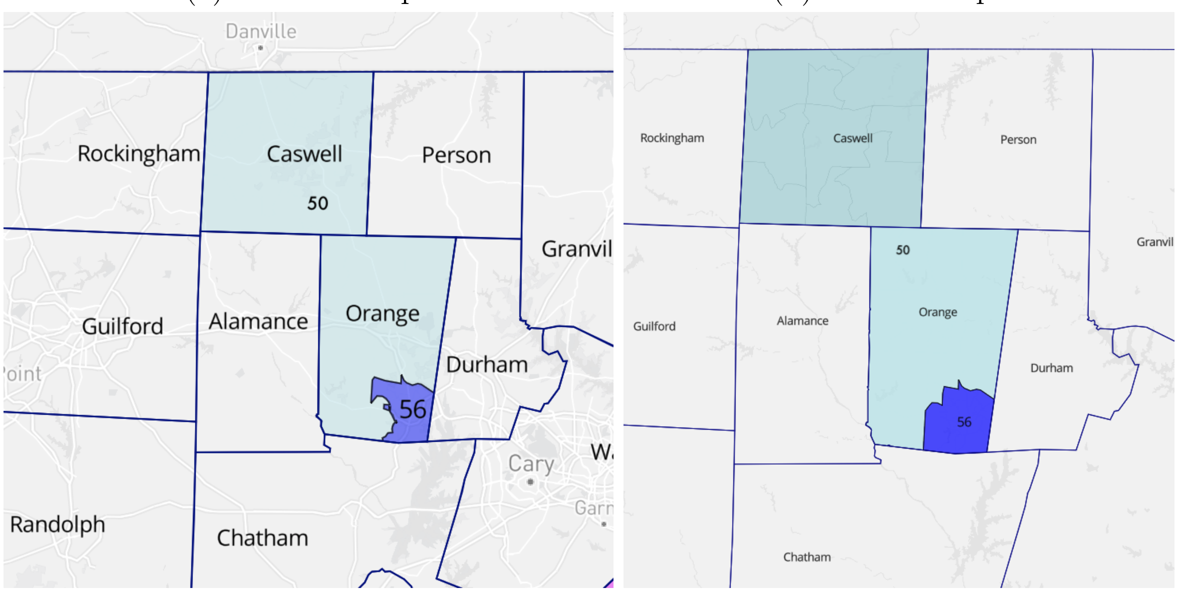
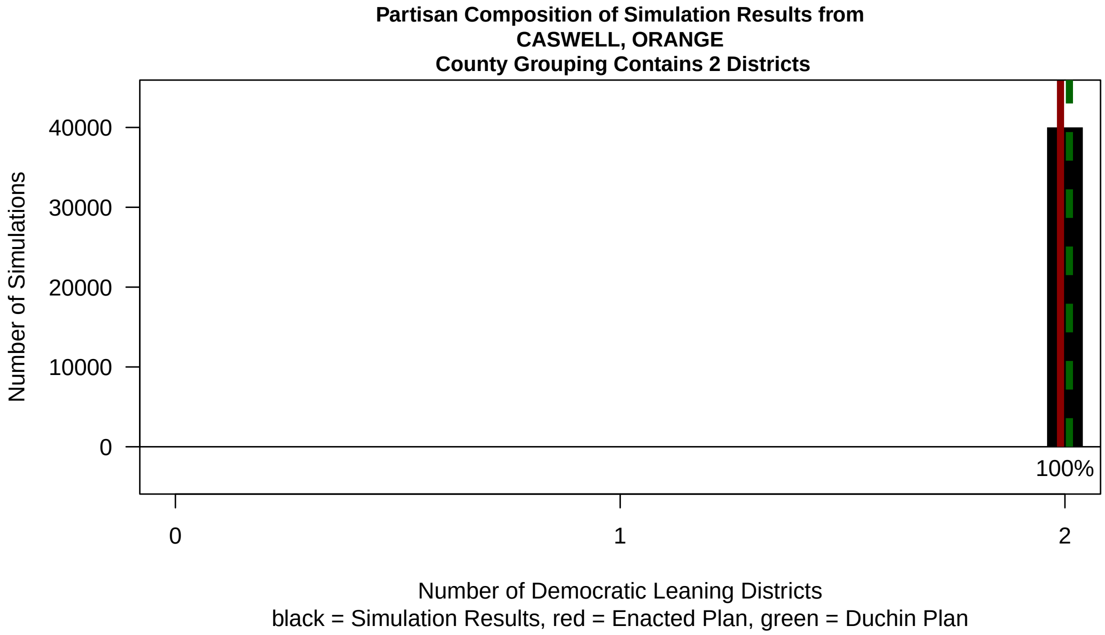


Figure 31: **Distribution of Partisan Districts from Simulations in Caswell and Orange House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 9: Simulation Results by Individual Elections

Caswell and Orange House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	0%	**100%**
2020 Senate	0%	0%	**100%**
2020 Governor	0%	0%	**100%**
2020 Lt. Governor	0%	0%	**100%**
2020 Attorney General	0%	0%	**100%**
2016 President	0%	0%	**100%**
2016 Senate	0%	0%	**100%**
2016 Governor	0%	0%	**100%**
2016 Lt. Governor	0%	0%	**100%**
2016 Attorney General	0%	0%	**100%**
2014 Senate	0%	0%	**100%**

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 2 Democratic leaning districts. The Enacted Plan does as well, as the '2 District' cell is bolded in that row.

6.9 Alexander, Surry, and Wilkes House County Grouping

The Alexander-Surry-Wilkes House county grouping contains 2 districts. In the Enacted Map these are Districts 90 and 94. The county cluster has an overall partisan index of 0.25, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 49,931 simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 20,124 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 32. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 33.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 34. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic districts. The Duchin Map also generates 0 Democratic districts.

Table 10 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 32: **Map of Alexander, Surry, and Wilkes County House County Cluster**

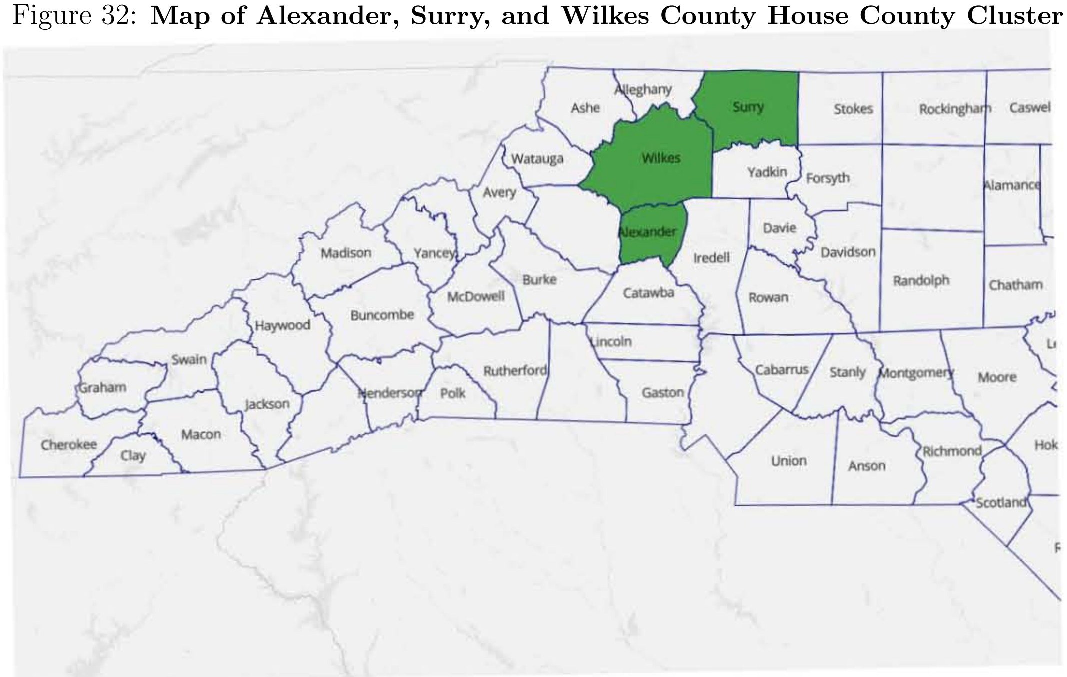
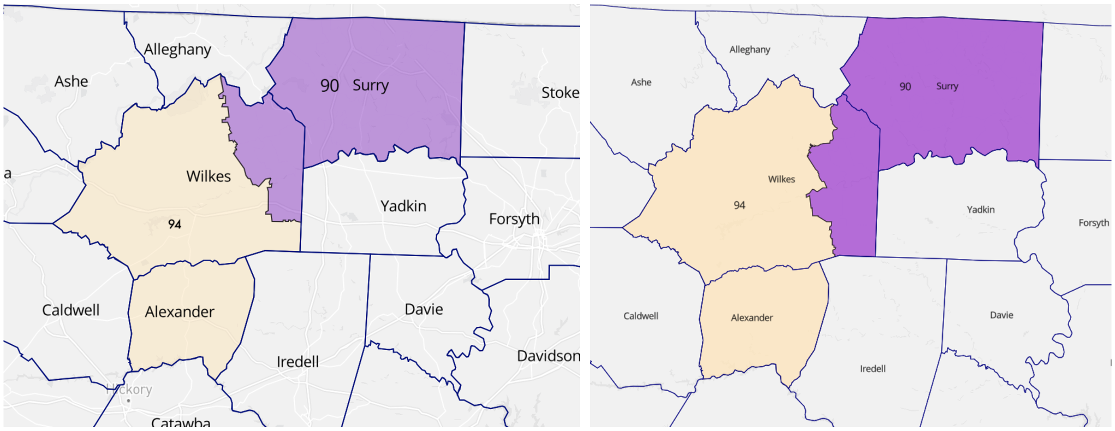


Figure 33: **Map of House Enacted Plan in Alexander, Surry, and Wilkes County Cluster**

(a) Enacted Map (b) Duchin Map

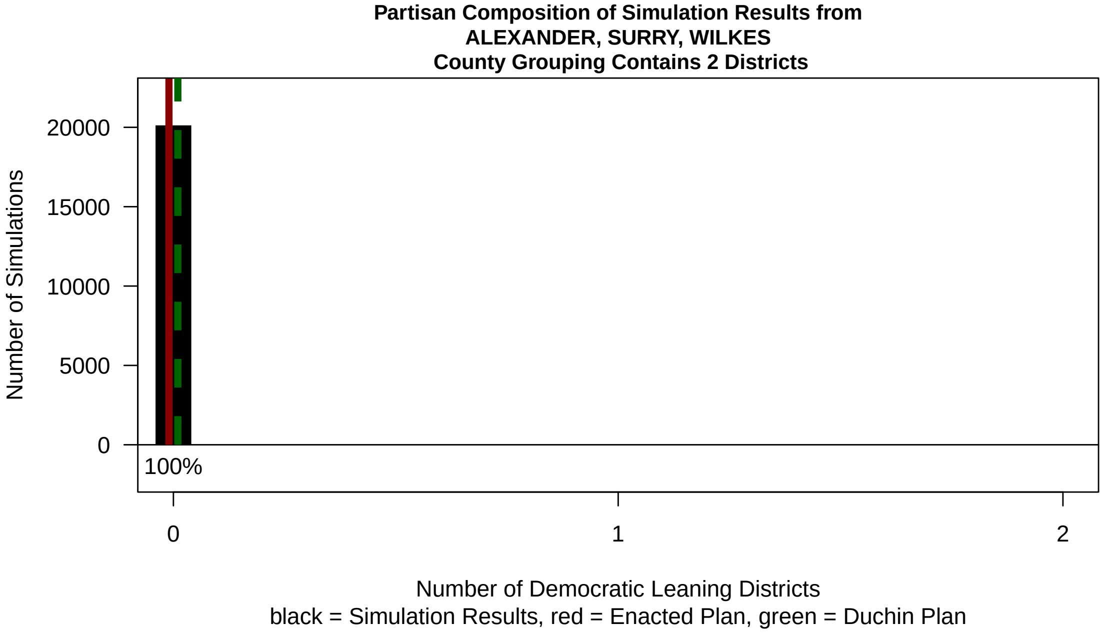


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
90	0.26	0.26
94	0.25	0.25

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 34: **Distribution of Partisan Districts from House Simulations in Alexander, Surry, and Wilkes CountyCluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 10: Simulation Results by Individual Elections

Alexander, Surry, and Wilkes House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.10 Franklin, Granville, and Vance House County Grouping

The Franklin-Granville-Vance House county grouping contains 2 districts. In the Enacted Map these are Districts 32 and 7. The county cluster has an overall partisan index of 0.51, which is very slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 17,823 simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 7,682 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 35. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 36.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 37. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there is 1 Democratic leaning district. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 1 Democratic district. The Duchin Map also generates 1 Democratic district.

Table 11 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 35: **Map of Franklin, Granville, and Vance House County Cluster**

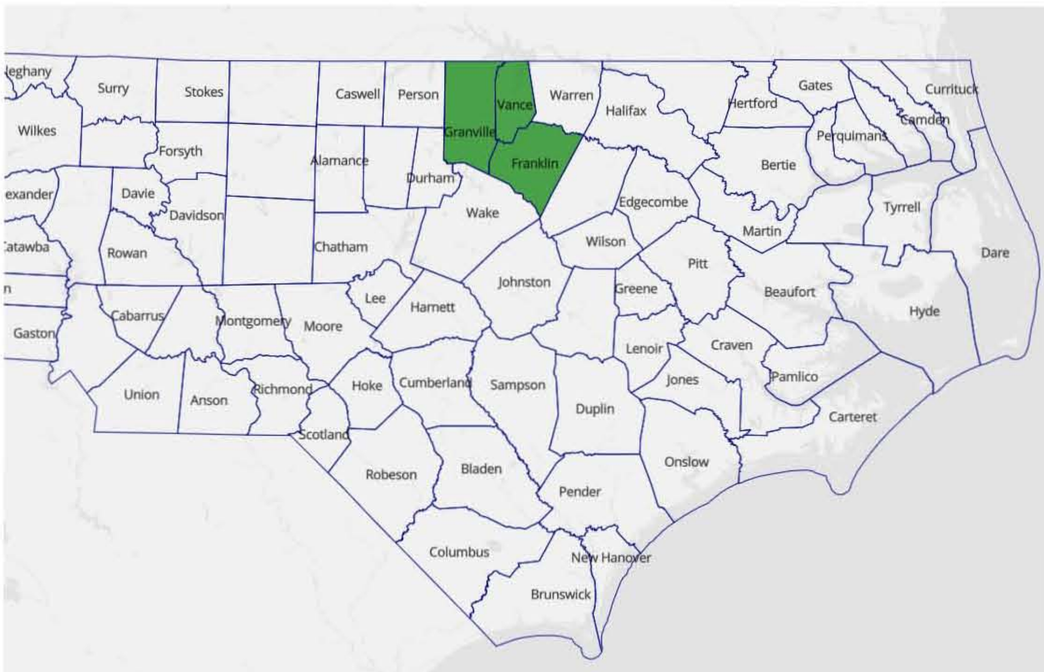
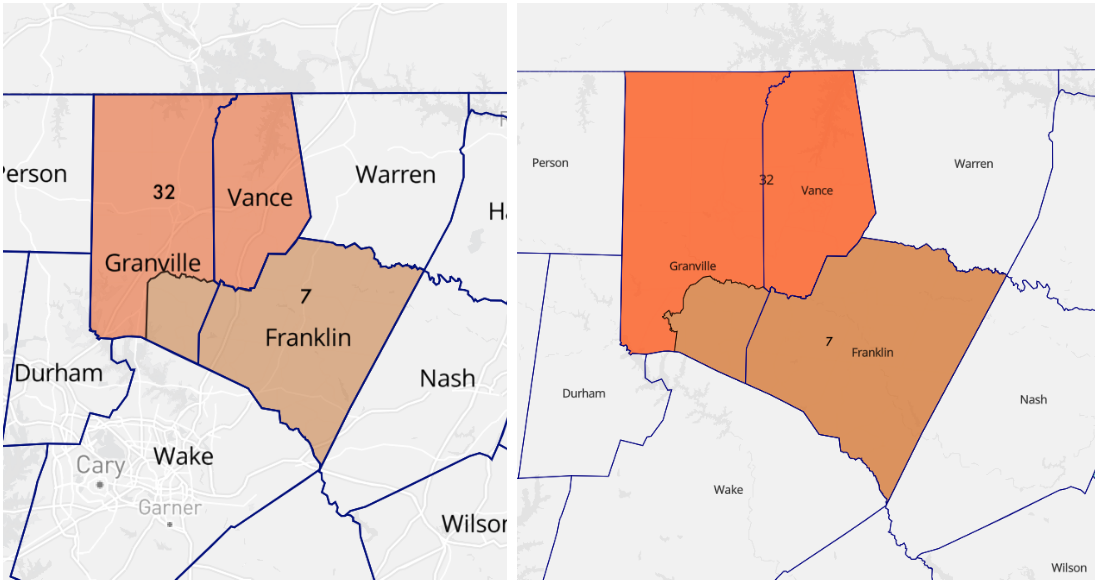


Figure 36: **Map of House Enacted Plan in Franklin, Granville, and Vance County Cluster**

(a) Enacted Map (b) Duchin Map

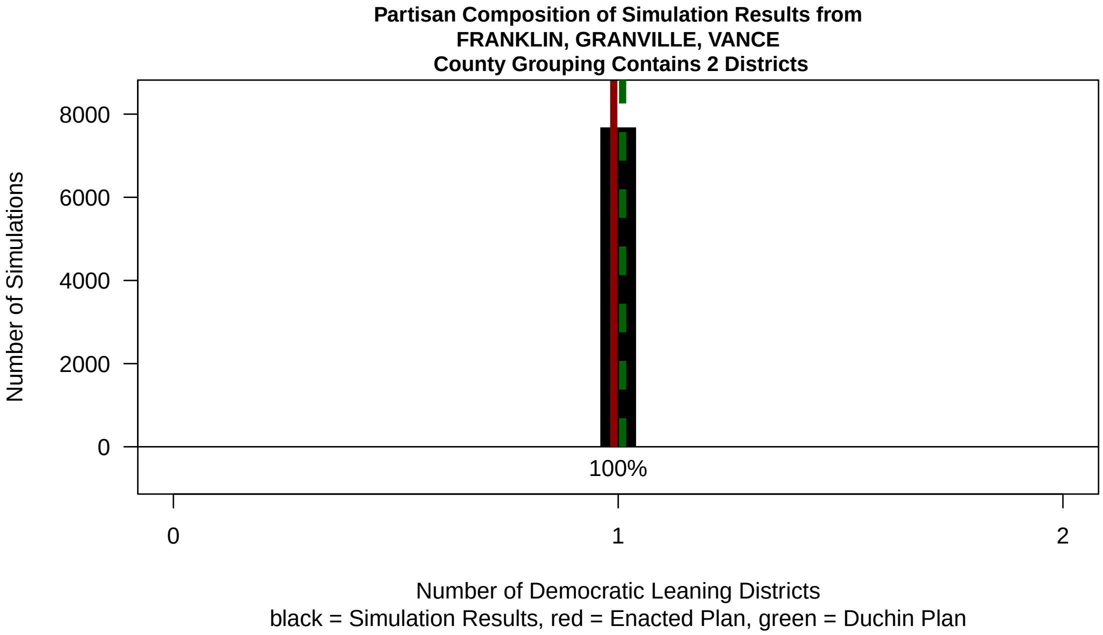


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
7	0.44	0.44
32	0.58	0.58

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 37: **Distribution of Partisan Districts from Simulations in Franklin, Granville, and Vance House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 11: Simulation Results by Individual Elections

Franklin, Granville, and Vance House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	**100%**	0%
2020 Senate	0%	**100%**	0%
2020 Governor	0%	**100%**	0%
2020 Lt. Governor	0%	**100%**	0%
2020 Attorney General	0%	**100%**	0%
2016 President	0%	**100%**	0%
2016 Senate	0%	**100%**	0%
2016 Governor	0%	**100%**	0%
2016 Lt. Governor	0%	**100%**	0%
2016 Attorney General	0%	**100%**	0%
2014 Senate	0%	**100%**	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

6.11 Alleghany, Ashe, Caldwell, and Watauga House County Grouping

The Alleghany-Ashe-Caldwell-Watauga House county grouping contains 2 districts. In the Enacted Map these are Districts 93 and 87. The county cluster has an overall partisan index of 0.36, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 47,843 simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves only six unique maps that are as compact as the Enacted Plan.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 38. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 39.

Because there are only six maps that fit the criteria I use of equal population, county traversals, and compactness equal to or better than the Enacted Map, I do not present the distribution of district partisanship for the simulations here. It is sufficient to say that in the Enacted Map, the Duchin map, and the six remaining simulations, all create 2 Republican districts and 0 Democratic leaning districts, regardless of the index or election used. Table 12 shows this below.

Figure 38: **Map of Alleghany, Ashe, Caldwell, and Watauga House County Cluster**

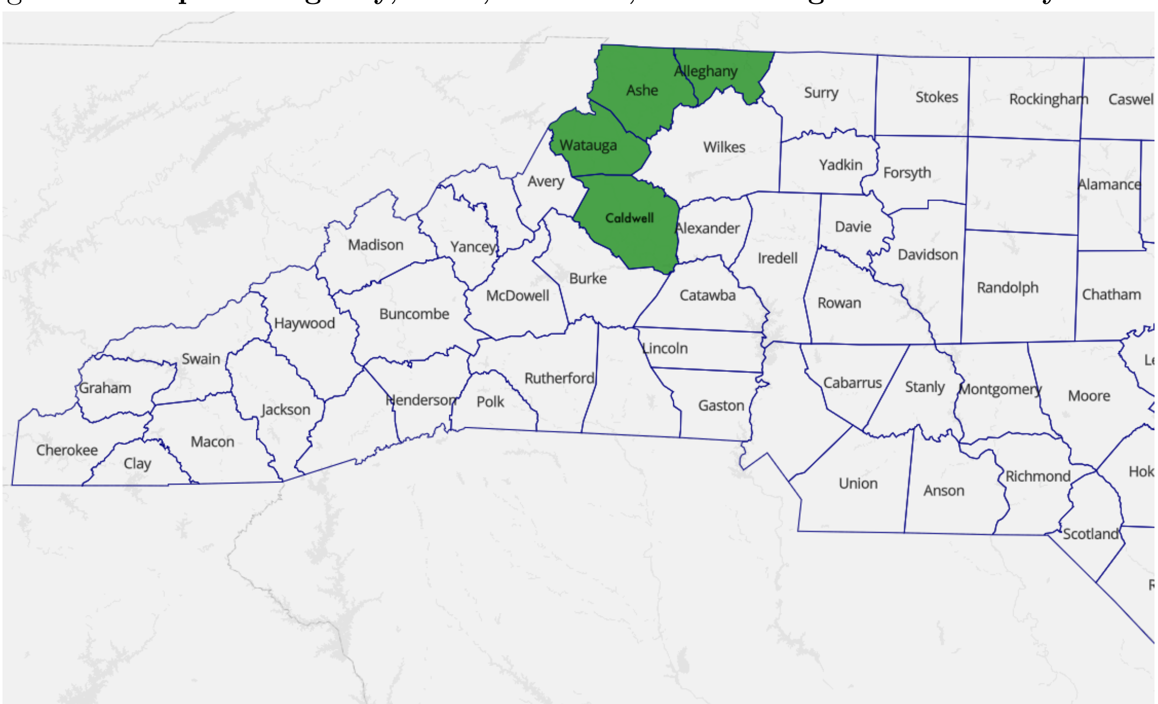
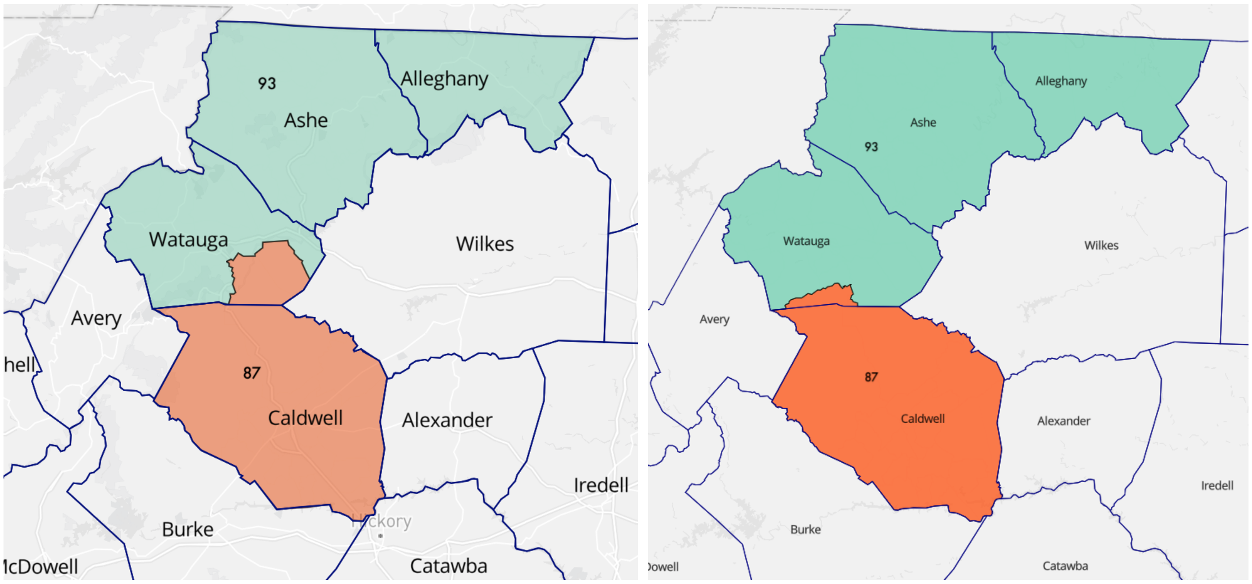


Figure 39: **Map of House Enacted Plan inAlleghany, Ashe, Caldwell, and Watauga County Cluster**

(a) Enacted Map (b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
87	0.28	0.27
93	0.43	0.43

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Table 12: Simulation Results by Individual Elections

Alleghany, Ashe, Caldwell, and Watauga House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Election Indices:	Percentage of Simulations		
All Elections Index	**100%**	0%	0%
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.12 Beaufort, Chowan, Currituck, Dare, Hyde, Pamlico, Perquimans, Tyrrell, and Washington House County Grouping

The Beaufort-Chowan-Currituck-Dare-Hyde-Pamlico-Perquimans-Tyrrell-Washington House county grouping contains 2 districts. In the Enacted Map these are Districts 1 and 79. The county cluster has an overall partisan index of 0.39, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 379 simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves only two unique maps that are as compact as the Enacted Plan.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 40. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 41.

Because there are only two maps that fit the criteria I use of equal population, county traversals, and compactness equal to or better than the Enacted Map, I do not present the distribution of district partisanship for the simulations here. It is sufficient to say that in the Enacted Map, the Duchin map, and the two remaining simulations, all create 2 Republican districts and 0 Democratic leaning districts, regardless of the index or election used. Table 13 shows this below.

Figure 40: **Map of Beaufort, Chowan, Currituck, Dare, Hyde, Pamlico, Perquimans, Tyrrell, and Washington House County Cluster**

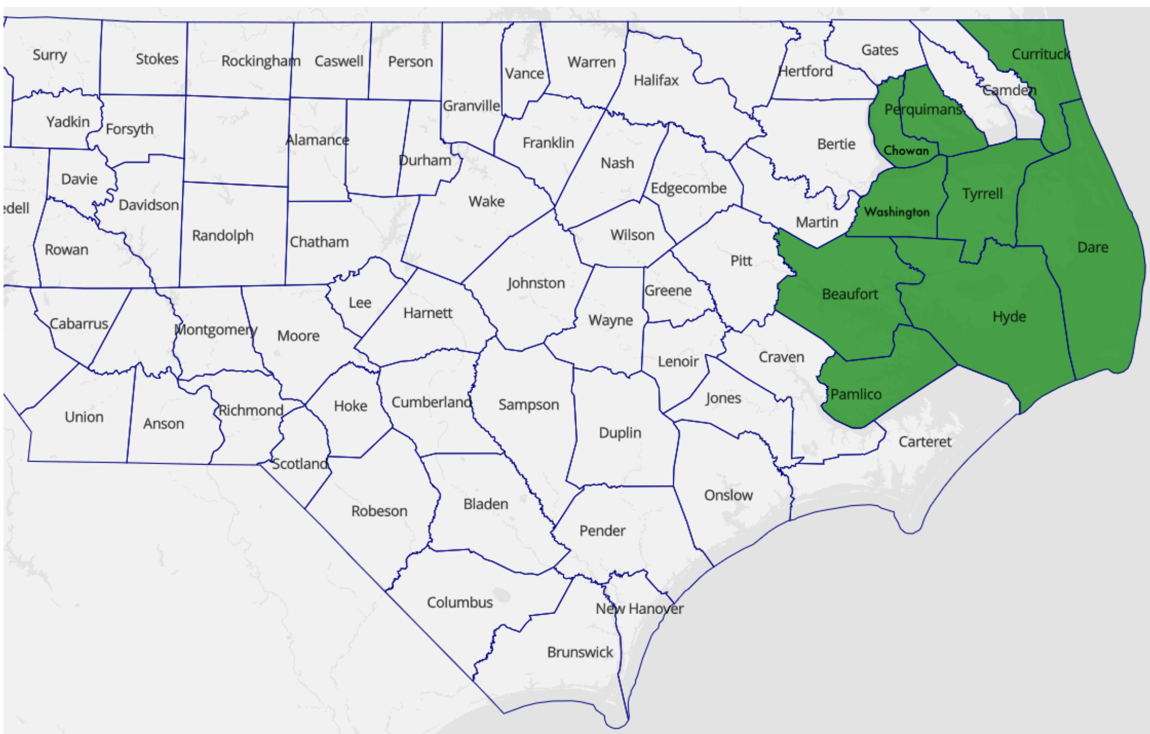
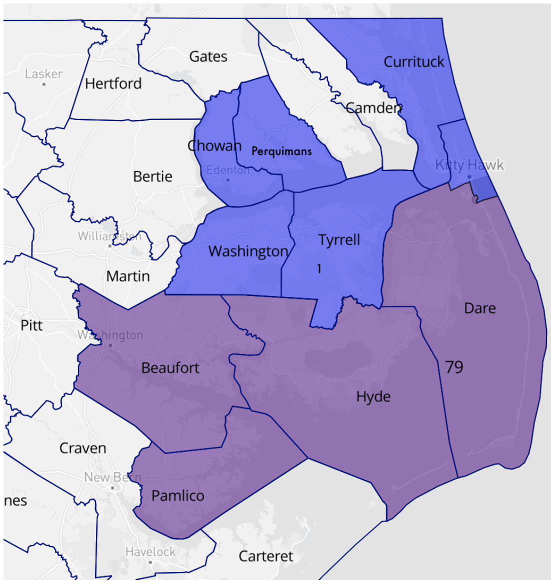
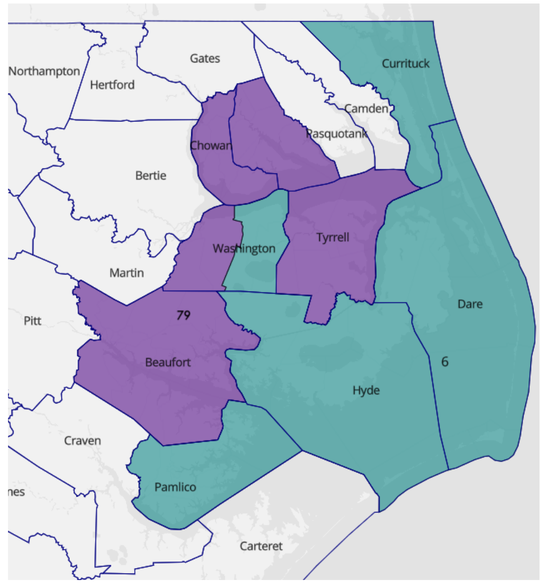


Figure 41: **Map of House Enacted Plan in Beaufort, Chowan, Currituck, Dare, Hyde, Pamlico, Perquimans, Tyrrell, and Washington County Cluster**

(a) Enacted Map



(b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
1 (6 in Duchin)	0.39	0.36
79	0.39	0.41

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Table 13: Simulation Results by Individual Elections

Beaufort, Chowan, Currituck, Dare, Hyde, Pamlico, Perquimans, Tyrrell, and Washington House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Election Indices:	Percentage of Simulations		
All Elections Index	**100%**	0%	0%
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.13 Buncombe House County Grouping

The Buncombe House county grouping contains 3 districts. In the Enacted Map these are Districts 114, 115, and 116. The county cluster has an overall partisan index of 0.60, which is moderately Democratic. After conducting 50,000 initial simulations to create three districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, this grouping contains only one county, so all of the simulations will contain the same number of traversals as the Enacted Map. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 38,664 simulated maps, each containing three districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 42. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 43.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 45. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 28% of the simulations there are 2 Democratic leaning districts. in 72% oft he simulations there are three Democratic leaning districts. The Enacted Map is in alignment with the minority outcome of the simulations by also creating 2 Democratic districts. The Duchin Map generates 3 Democratic districts.

Table 15 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In this case the Enacted Plan creates 2 Democratic leaning districts, regardless of the election considered. However, the frequency with which the simulations produce 2 Democratic districts varies from a low of 2% in the 2020 Governor race to a 51% majority in the 2016 Presidential race.

One consideration for why the Enacted Plan diverges from the Duchin Plan and the modal outcome of the simulations is because it keeps a larger portion of the town of Asheville, the county seat and largest city in Buncombe County, in fewer districts. Figure 44 shows a map of the city and how the two different plans divide the city. The Duchin Plan splits Asheville nearly equally across all three districts in a pie shape while the Enacted Plan keeps much more of Asheville within two districts. There is a small portion of the southern most part of the city in District 116. The tactic of dividing Democratic cities in a 'pinwheel' or 'pizza' shape and grouping those 'slices' with more Republican suburban and exurban areas is a classic tactic to generate more Democratic districts and overcome the geographic clustering that is common among Democratic voters. The Enacted Plan keeps much more of Asheville within two districts. Table 14 shows the percent of Asheville voters in each district in each plan. It is clear that the Duchin plan splits Ashville into three roughly equal parts while the Enacted Plan places a much larger majority of Asheville into only two districts.

Table 14: Division of Asheville in Enacted Plan and Duchin Plan

	Percent of Asheville in district	
District:	Enacted Plan	Duchin Plan
114	55.6	27.7
115	30.9	39.9
116	13.5	32.5
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: `https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf` Population numbers for city by district for Duchin Plan from Dave's Redistricting online. `https://davesredistricting.org/`

Figure 42: **Map of Buncombe House County Cluster**

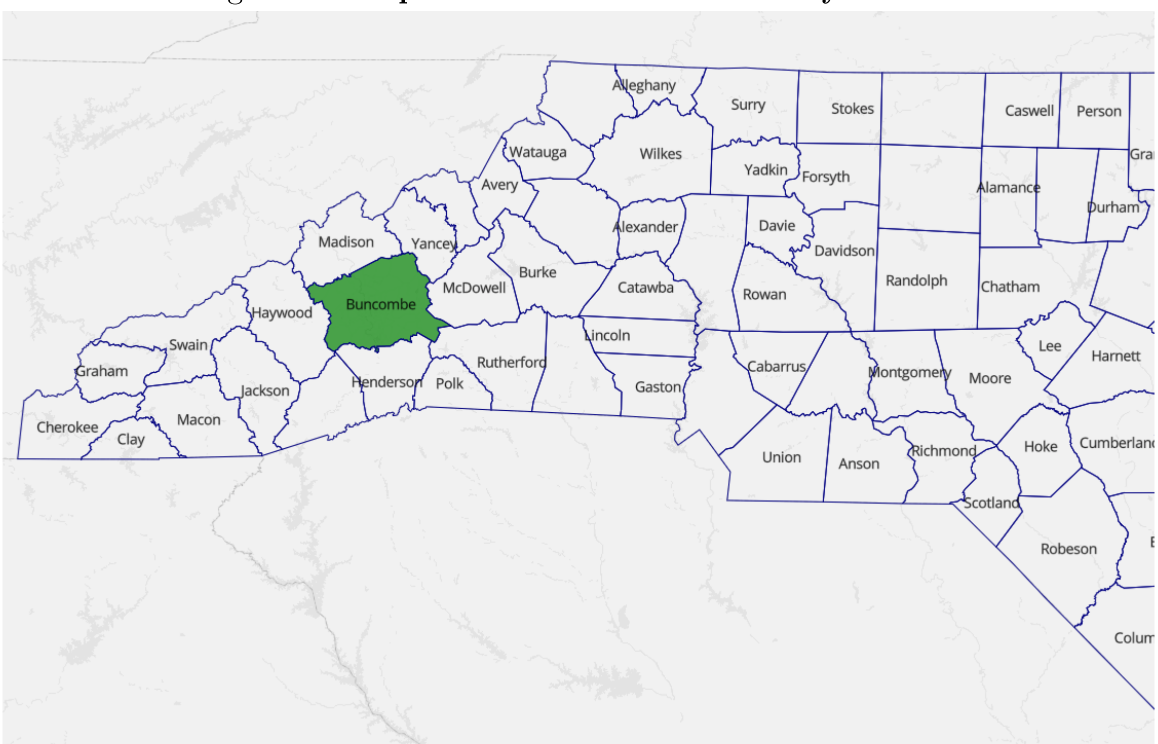
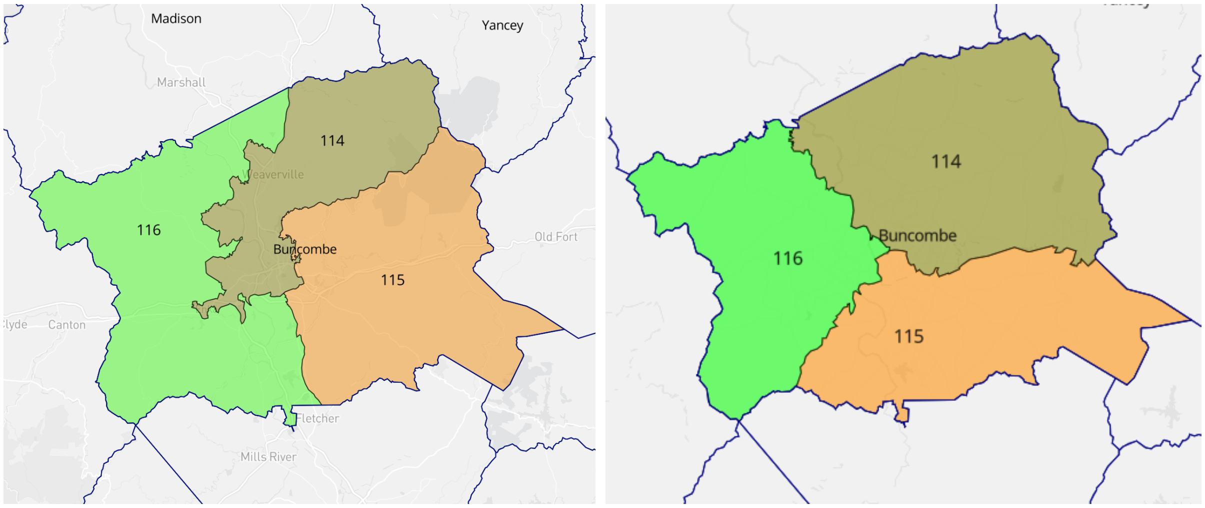


Figure 43: **Map of House Enacted Plan and Duchin Plan in Buncombe County Cluster**

(a) Enacted Map (b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
114	0.72	0.62
115	0.60	0.60
116	0.46	0.57

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 44: **Map of Asheville Divisions in Buncombe County Cluster**

(a) Enacted Map

(b) Duchin Map

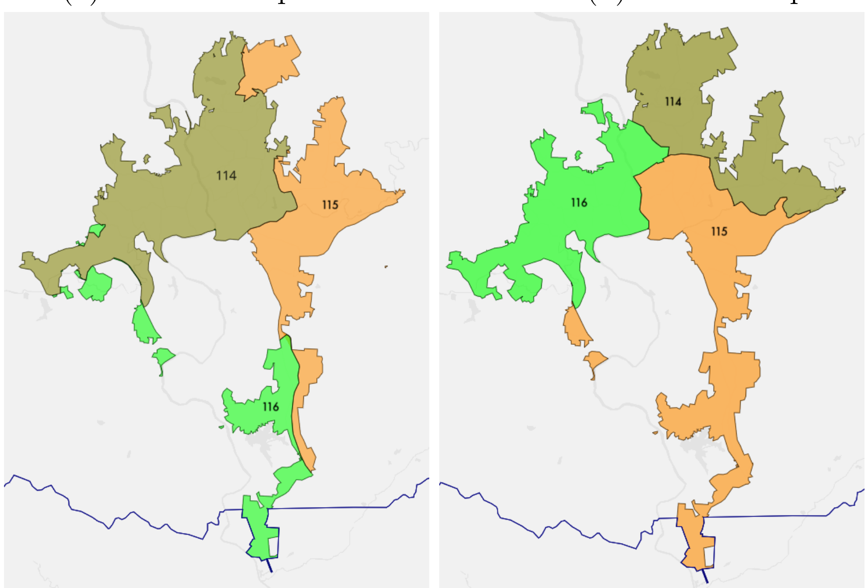
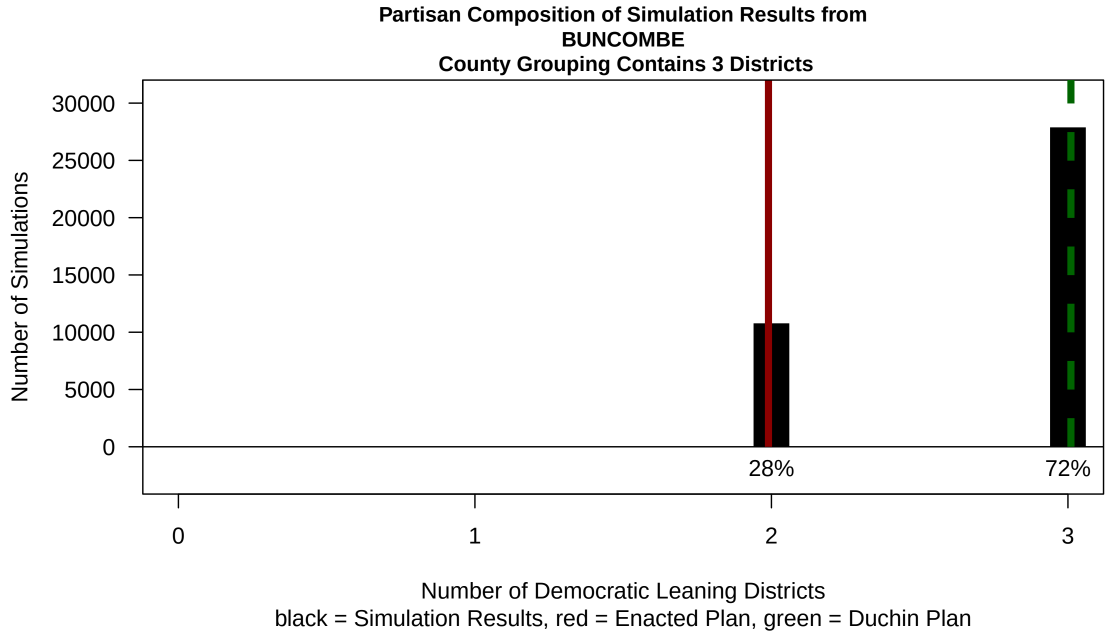


Figure 45: **Distribution of Partisan Districts from Simulations in Buncombe House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 15: Simulation Results by Individual Elections

Buncombe House County Cluster

Number of Democratic Leaning Districts:				
	0	1	2	3
Individual Elections:				
2020 President	0%	0%	**26%**	74%
2020 Senate	0%	0%	**23%**	77%
2020 Governor	0%	0%	**2%**	98%
2020 Lt. Governor	0%	0%	**31%**	69%
2020 Attorney General	0%	0%	**16%**	84%
2016 President	0%	1%	**51%**	48%
2016 Senate	0%	1%	**46%**	53%
2016 Governor	0%	0%	**12%**	88%
2016 Lt. Governor	0%	1%	**43%**	56%
2016 Attorney General	0%	0%	**20%**	80%
2014 Senate	0%	0%	**24%**	76%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 26% of the simulations produce 2 Democratic leaning districts. The Enacted Plan does as well, as the '2 Districts' cell is bolded in that row.

6.14 Anson and Union House County Grouping

The Anson-Union House county grouping contains 3 districts. In the Enacted Map these are Districts 55, 68 and 69. The county cluster has an overall partisan index of .37, which is strongly Republican. After conducting 50,000 initial simulations to create three districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 43,555 simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 20,759 simulated maps, each containing three districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 46. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 47.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 48. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic districts. The Duchin Map also generates 0 Democratic districts.

Table 16 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 46: **Map of Anson and Union House County Cluster**

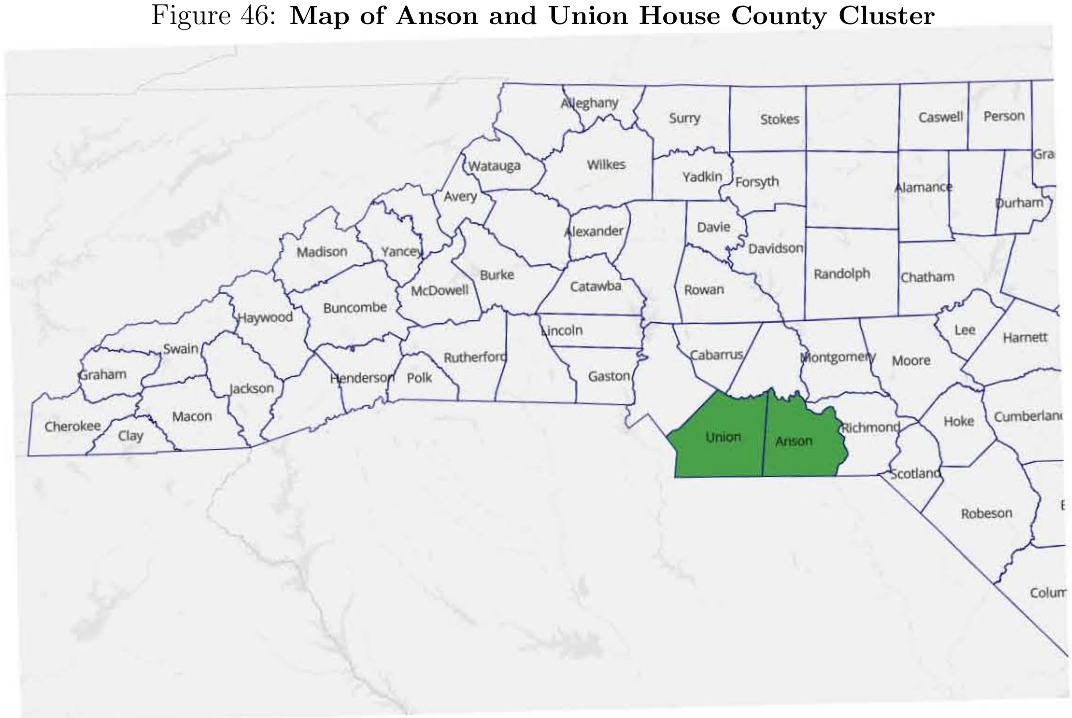
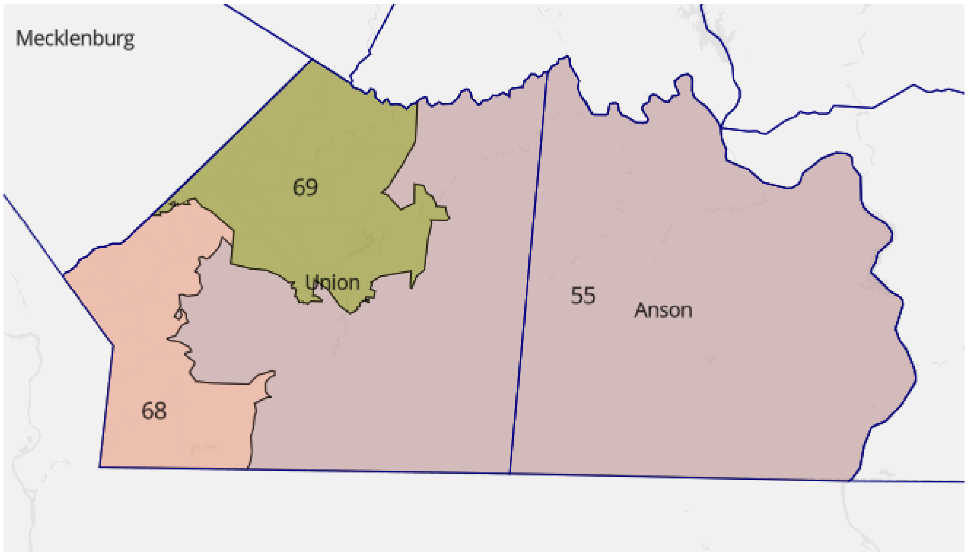
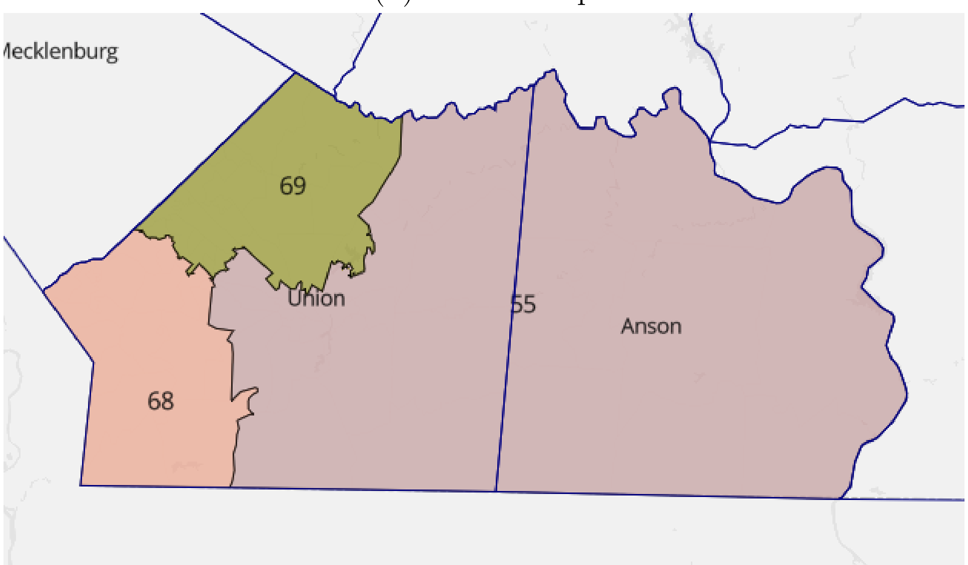


Figure 47: **Map of House Enacted Plan in Anson and Union House County Cluster**

(a) Enacted Map



(b) Duchin Map

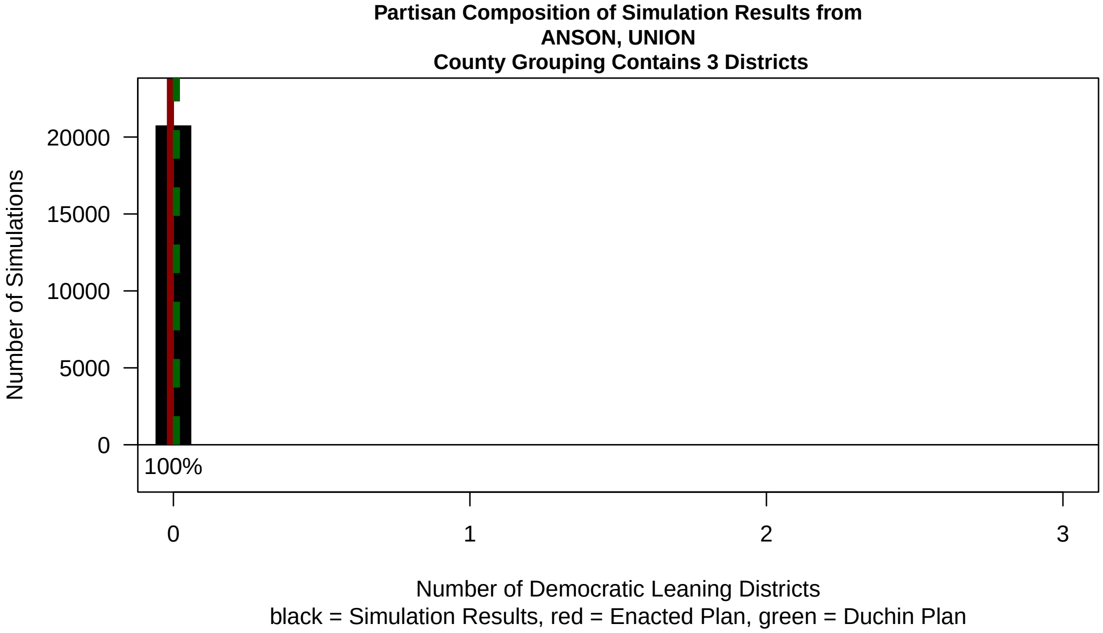


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
55	0.41	0.44
68	0.36	0.35
69	0.35	0.34

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 48: **Distribution of Partisan Districts from Simulations in Anson and Union House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 16: Simulation Results by Individual Elections

Anson and Union House County Cluster

Number of Democratic Leaning Districts:				
	0	1	2	3
Individual Elections:				
2020 President	**100%**	0%	0%	0%
2020 Senate	**100%**	0%	0%	0%
2020 Governor	**100%**	0%	0%	0%
2020 Lt. Governor	**100%**	0%	0%	0%
2020 Attorney General	**100%**	0%	0%	0%
2016 President	**100%**	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%
2014 Senate	**73%**	27%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.15 Onslow and Pender House County Grouping

The Onslow-Pender House county grouping contains 3 districts. In the Enacted Map these are Districts 14, 15, and 16. The county cluster has an overall partisan index of .35, which is heavily Republican. After conducting 50,000 initial simulations to create three districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 48,928 simulated maps. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 35,873 simulated maps, each containing three districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 49. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 50.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 51. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic districts. The Duchin Map also generates 0 Democratic districts.

Table 17 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In this case there is unanimous agreement between the modal outcome in the simulations and the Enacted Map across all 11 elections.

Figure 49: **Map of Onslow and Pender House County Cluster**

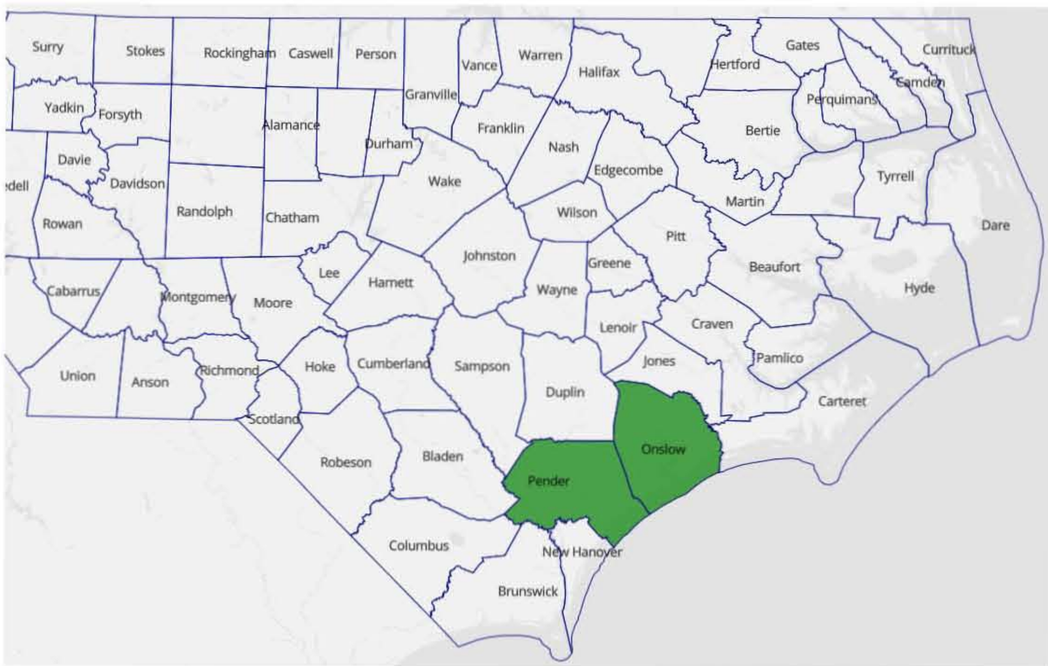
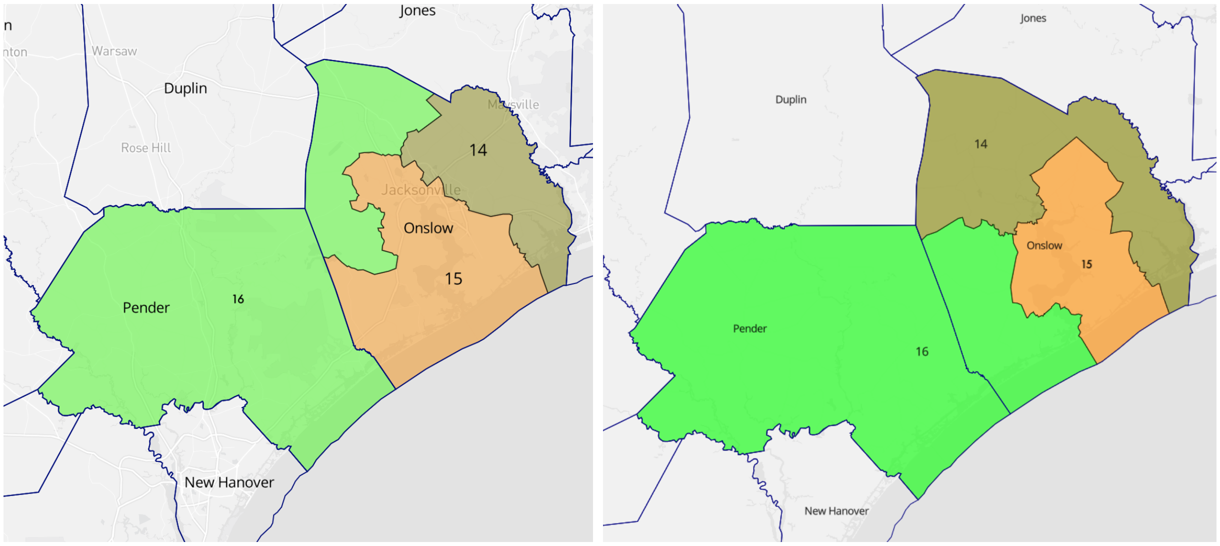


Figure 50: **Map of House Enacted Plan in Onslow and Pender County Cluster**

(a) Enacted Map (b) Duchin Map

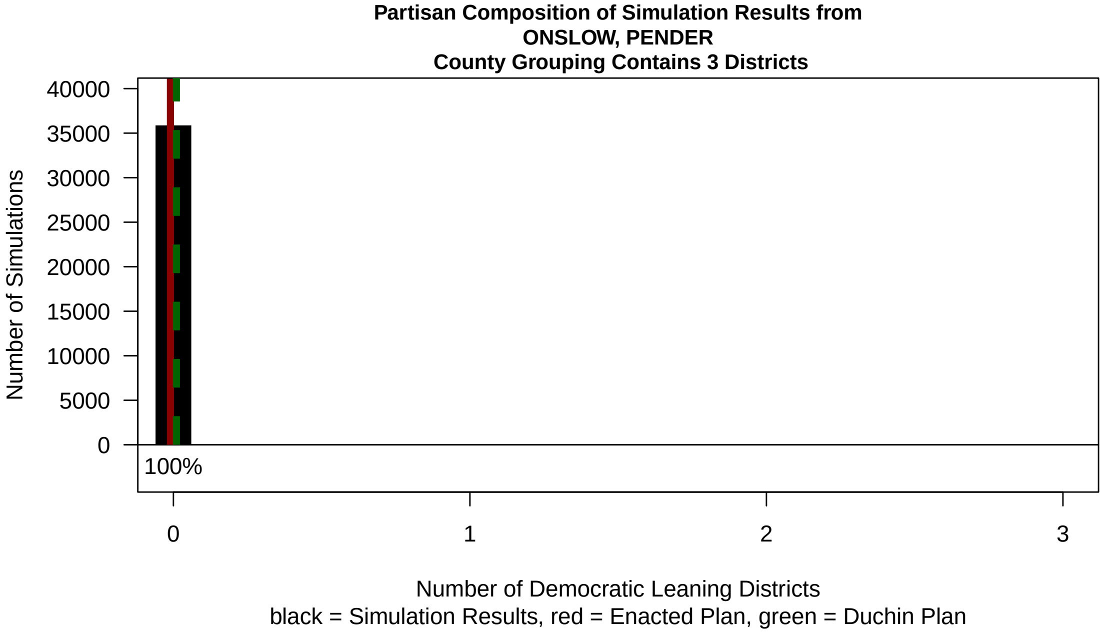


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
14	0.39	0.29
15	0.32	0.49
16	0.33	0.33

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 51: **Distribution of Partisan Districts from Simulations in Onslow and Pender House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 17: Simulation Results by Individual Elections

Onslow and Pender House County Cluster

Number of Democratic Leaning Districts:				
	0	1	2	3
Individual Elections:				
2020 President	**100%**	0%	0%	0%
2020 Senate	**100%**	0%	0%	0%
2020 Governor	**100%**	0%	0%	0%
2020 Lt. Governor	**100%**	0%	0%	0%
2020 Attorney General	**100%**	0%	0%	0%
2016 President	**100%**	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%
2014 Senate	**100%**	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.16 Cumberland House County Grouping

The Cumberland House county group contains 4 districts. In the Enacted Map these are Districts 42, 43, 44, and 45. The county cluster has an overall partisan index of .59, which is moderately Democratic. After conducting 50,000 initial simulations to create four districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, Cumberland is a single county group, and so all of the simulations have the same number of traversals as the Enacted Map. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 10,521 simulated maps, each containing four districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 52. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 53.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 55. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 82% of the simulations there are 3 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 3 Democratic districts. In 18% of the simulations there are 4 Democratic leaning districts. The Duchin Map generates 4 Democratic districts. This falls outside of the 50% range of simulation results and is thus classified as a partisan outlier result.

Table 19 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 5 of the 11 elections there is agreement between the modal outcome in the simulations and the Enacted Map. In 6 of the 11 elections the Enacted Plan results fall outside the middle 50% range of the simulations and would be classified as outliers.

One consideration for why the Enacted Plan diverges from the Duchin Plan is because it keeps a larger portion of the town of Fayetteville, the county seat and largest city in Cumberland County, in fewer districts. Figure 54 shows a map of the city and how the two different plans divide the city. The Duchin Plan splits Fayetteville nearly equally across all four districts in a pie shape. The tactic of dividing Democratic cities in a 'pinwheel' or 'pizza' shape and grouping those 'slices' with more Republican suburban and exurban areas is a classic tactic to generate more Democratic districts and overcome the geographic clustering that is common among Democratic voters. The Enacted Plan keeps much more of Fayetteville within three districts. A small portion of the southern most part of the city is located in District 45. Table 18 shows the percent of Fayetteville voters in each district in each plan. It is clear that the Duchin plan splits Fayetteville into 4 roughly equal parts while the Enacted Plan places a much larger majority of Fayetteville into only three districts.

Table 18: Division of Fayetteville in Enacted Plan and Duchin Plan

	Percent of Feyetville in district	
District:	Enacted Plan	Duchin Plan
42	31.4	33.4
43	21.4	21.5
44	39.9	26.8
45	7.3	18.3
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: `https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf` Population numbers for city by district for Duchin Plan from Dave's Redistricting online. `https://davesredistricting.org/`

Figure 52: **Map of Cumberland House County Cluster**

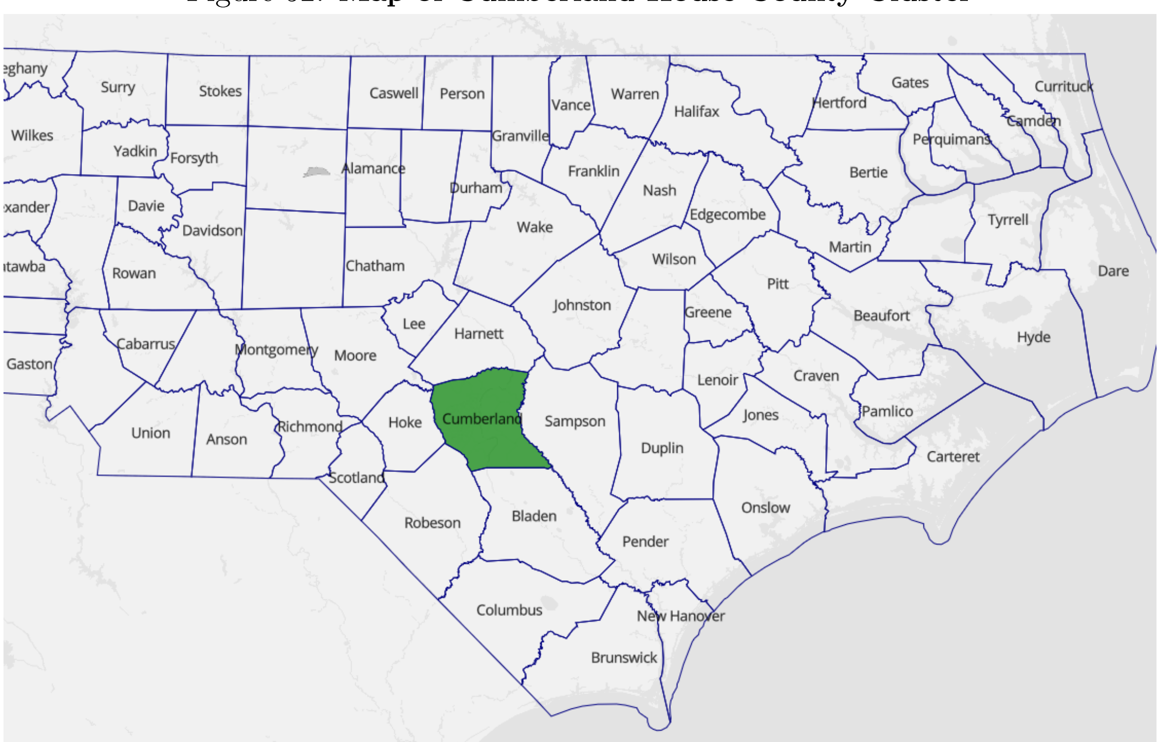
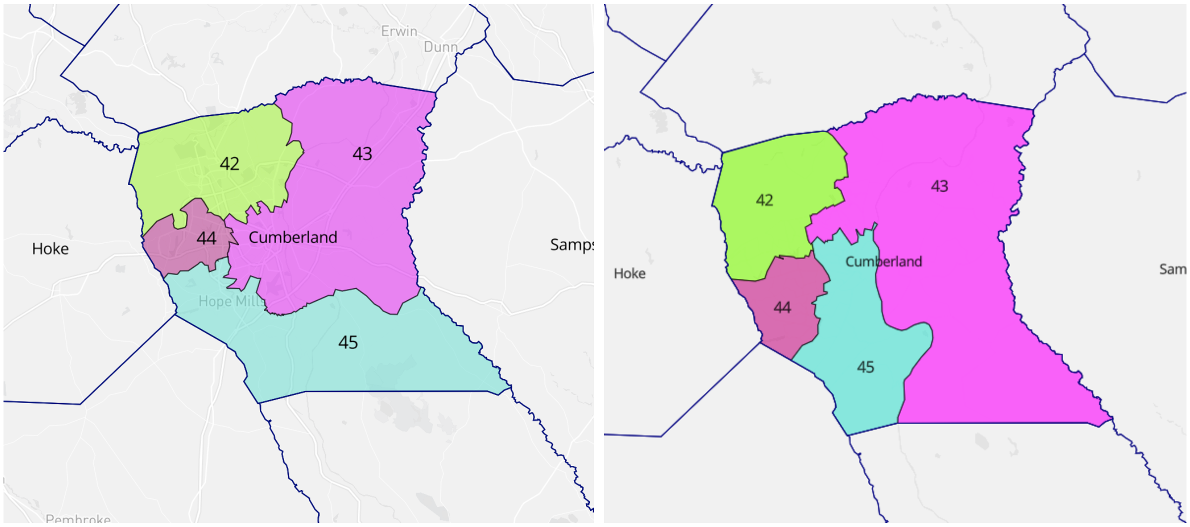


Figure 53: **Map of House Enacted Plan in Cumberland County Cluster**

(a) Enacted Map (b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
42	0.67	0.72
43	0.50	0.55
44	0.72	0.60
45	0.49	0.53

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 54: **Map of Fayetteville Divisions in Cumberland County Cluster**

(a) Enacted Map (b) Duchin Map

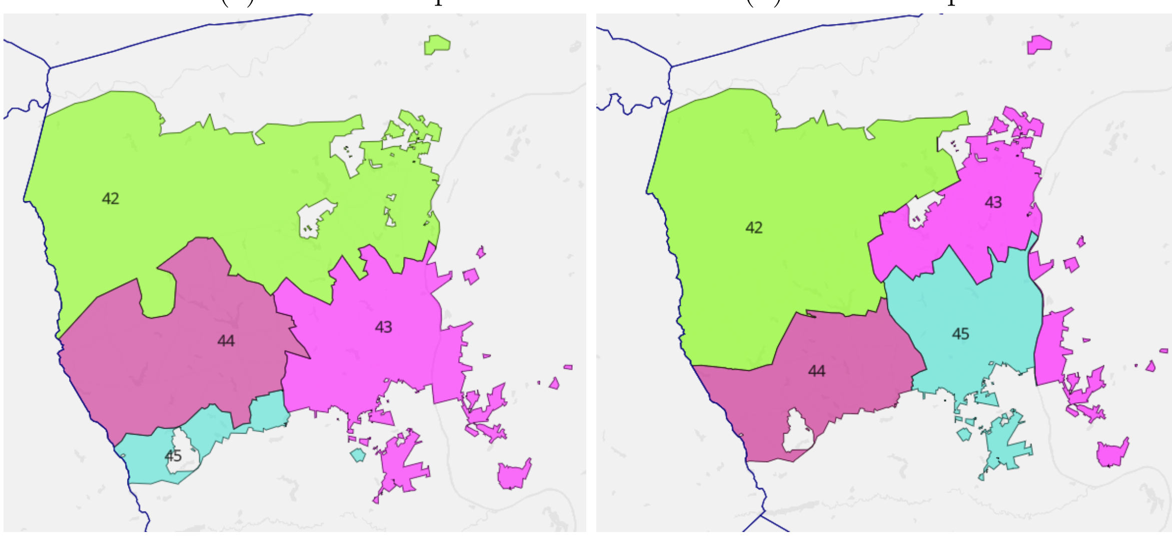
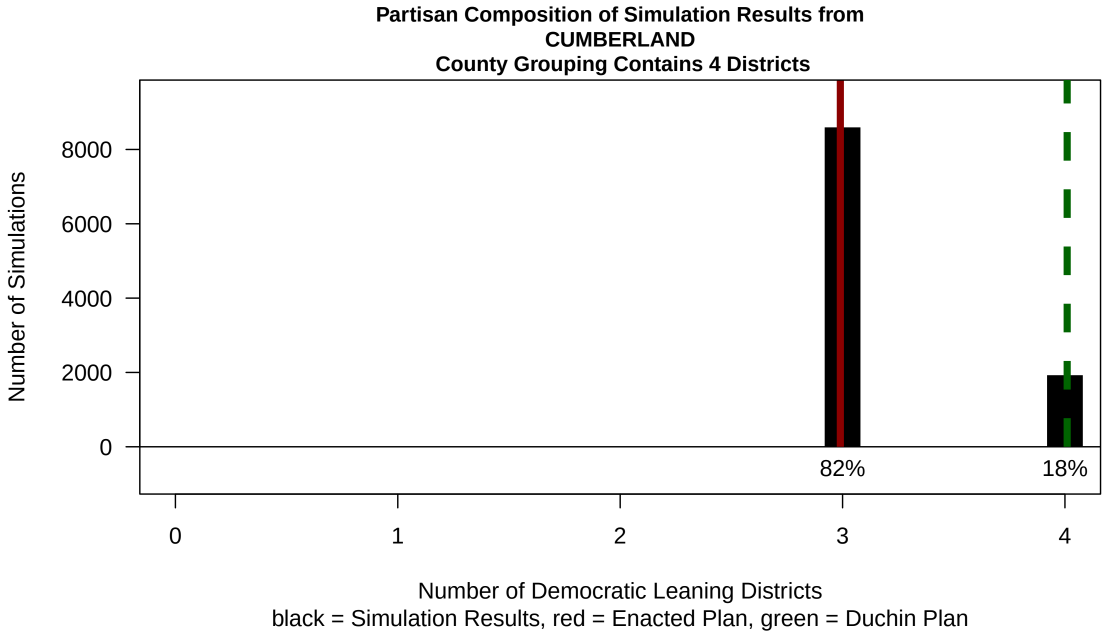


Figure 55: **Distribution of Partisan Districts from Simulations in Cumberland House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 19: Simulation Results by Individual Elections

Cumberland House County Cluster

	Number of Democratic Leaning Districts:				
	0	1	2	3	4
Individual Elections:					
2020 President	0%	0%	**0%**	91%	9%
2020 Senate	0%	0%	**0%**	88%	12%
2020 Governor	0%	0%	0%	23%	**77%**
2020 Lt. Governor	0%	0%	**0%**	90%	10%
2020 Attorney General	0%	0%	0%	49%	**51%**
2016 President	0%	0%	0%	**90%**	10%
2016 Senate	0%	0%	**0%**	94%	6%
2016 Governor	0%	0%	**0%**	94%	6%
2016 Lt. Governor	0%	0%	**0%**	94%	6%
2016 Attorney General	0%	0%	0%	**48%**	52%
2014 Senate	0%	0%	0%	**89%**	11%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 0% of the simulations produce 2 Democratic leaning districts. The Enacted Plan does as well, as the '3 Districts' cell is bolded in that row.

One thing to note regarding the instances in which the Enacted Plan does not align with the simulation results in individual elections. In all six cases the Enacted Plan creates one district (and occasionally two districts) that is extremely competitive and is effectively tied (less than 1% from 50/50), but is just below 0.50 and is thus not classified as a Democratic district. For example, in the 2020 Presidential race the Enacted Plan districts have a partisan lean of 0.719, 0.672, 0.495, and 0.492. Thus, two of the districts, while not classified as Democratic leaning will be heavily contested and both parties will likely win these districts at different times in the coming years.

6.17 Harnett and Johnston House County Grouping

The Harnett-Johnston House county group contains 4 districts. In the Enacted Map these are Districts 6, 26, 28, and 53. The county cluster has an overall partisan index of .38, which is moderately Republican. After conducting 50,000 initial simulations to create four districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 34,976 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 593 simulated maps, each containing four districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 56. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 57.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 58. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic districts. The Duchin Map also generates 0 Democratic districts.

Table 20 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In all 11 of the individual elections there is agreement between the modal outcome in the simulations and the Enacted Map.

Figure 56: **Map of Harnett and Johnston House County Cluster**

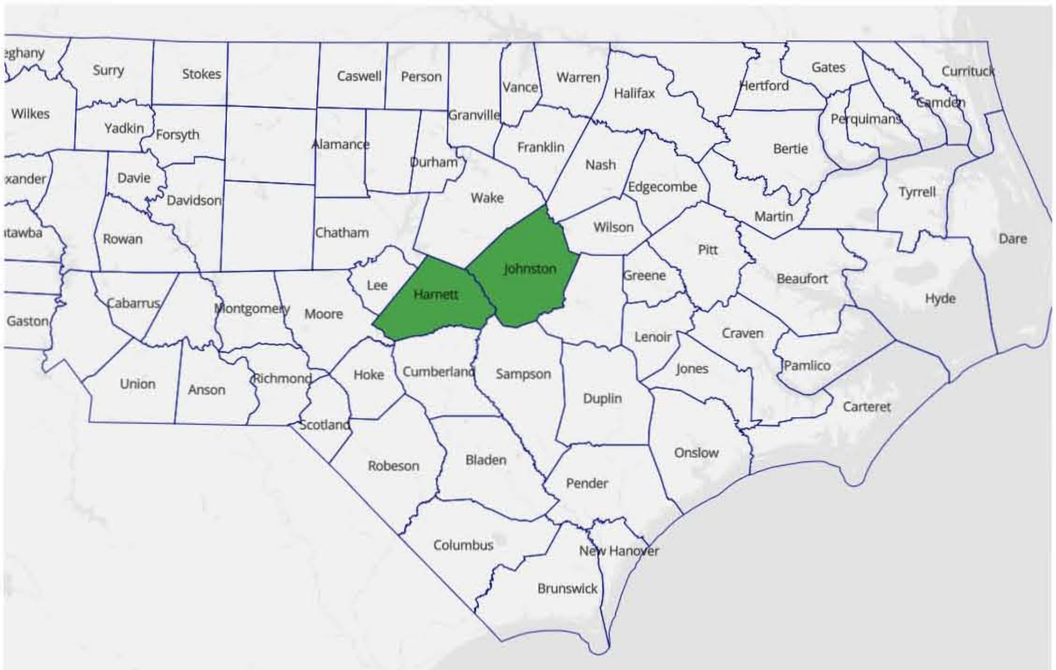
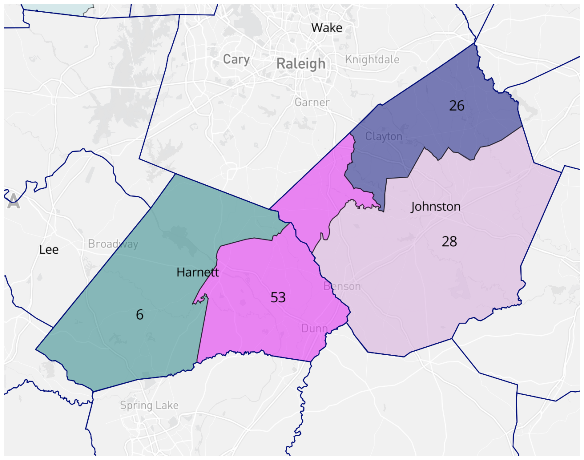
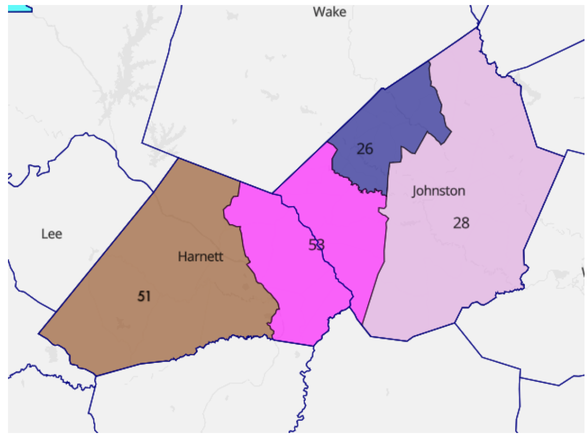


Figure 57: **Map of House Enacted Plan in Harnett and Johnston County Cluster**

(a) Enacted Map



(b) Duchin Map

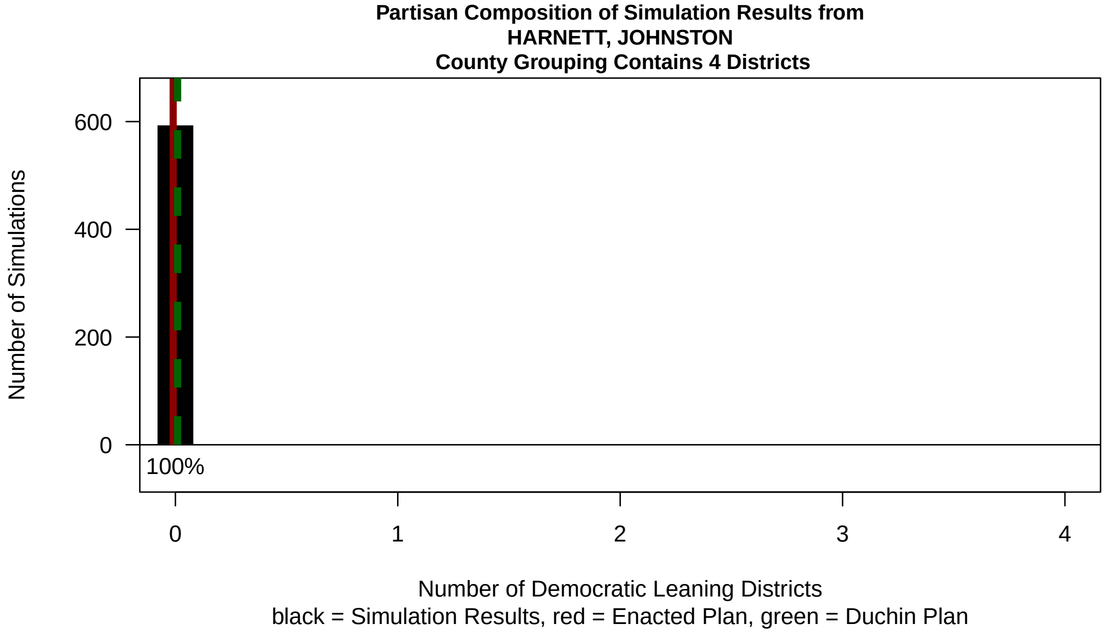


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
6 (51 in Duchin)	0.40	0.42
26	0.41	0.43
28	0.34	0.35
53	0.37	0.33

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 58: **Distribution of Partisan Districts from Simulations in Harnett and Johnston House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 20: Simulation Results by Individual Elections

Harnett and Johnston House County Cluster

	Number of Democratic Leaning Districts:				
	0	1	2	3	4
Individual Elections:					
2020 President	**100%**	0%	0%	0%	0%
2020 Senate	**100%**	0%	0%	0%	0%
2020 Governor	**100%**	0%	0%	0%	0%
2020 Lt. Governor	**100%**	0%	0%	0%	0%
2020 Attorney General	**100%**	0%	0%	0%	0%
2016 President	**100%**	0%	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%	0%
2014 Senate	**100%**	0%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.18 Catawba and Iredell House County Grouping

The Catawba-Iredell House county group contains 4 districts. In the Enacted Map these are Districts 84, 89, 95, and 96. The county cluster has an overall partisan index of .33, which is strongly Republican. After conducting 50,000 initial simulations to create four districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 14,955 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 2,944 simulated maps, each containing four districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 59. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 60.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 61. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic districts. The Duchin Map also generates 0 Democratic districts.

Table 21 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In all 11 of the individual elections there is agreement between the modal outcome in the simulations and the Enacted Map.

Figure 59: **Map of Catawba and Iredell House County Cluster**

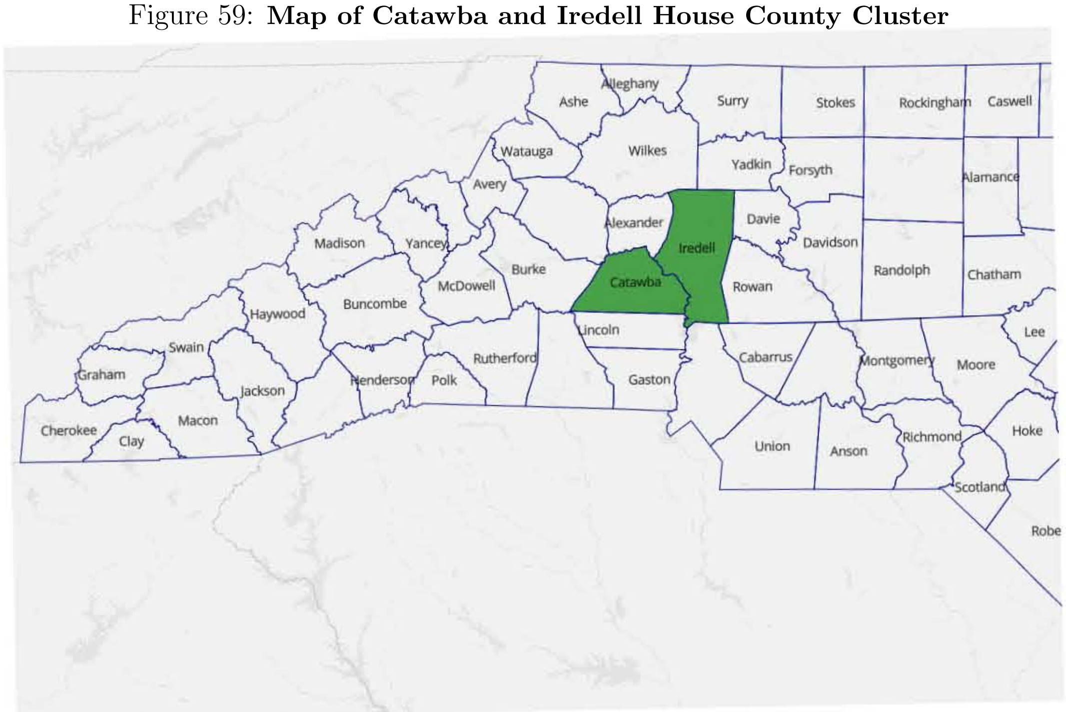
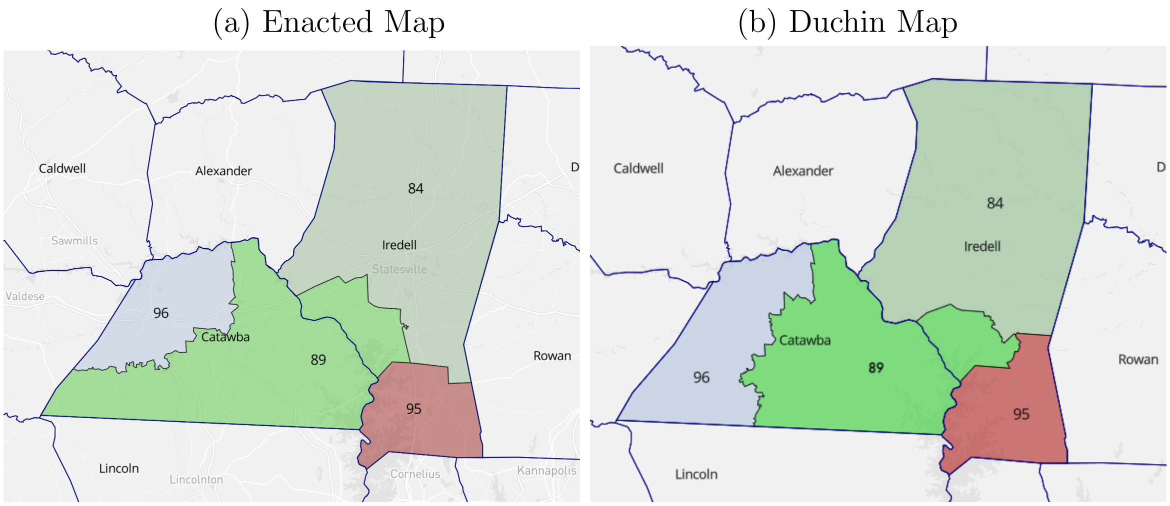


Figure 60: **Map of House Enacted Plan in Catawba and Iredell County Cluster**

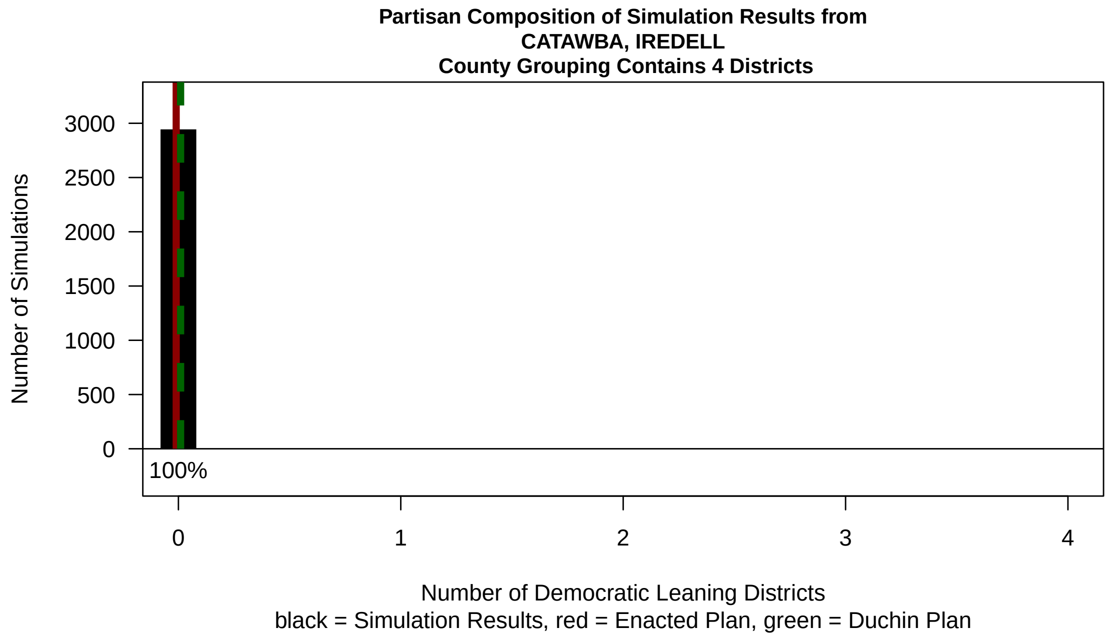


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
84	0.34	0.34
89	0.26	0.28
95	0.34	0.34
96	0.37	0.36

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 61: **Distribution of Partisan Districts from Simulations in Catawba and Iredell House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 21: Simulation Results by Individual Elections

Catawba and Iredell House County Cluster

	Number of Democratic Leaning Districts:				
	0	1	2	3	4
Individual Elections:					
2020 President	**100%**	0%	0%	0%	0%
2020 Senate	**100%**	0%	0%	0%	0%
2020 Governor	**100%**	0%	0%	0%	0%
2020 Lt. Governor	**100%**	0%	0%	0%	0%
2020 Attorney General	**100%**	0%	0%	0%	0%
2016 President	**100%**	0%	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%	0%
2014 Senate	**100%**	0%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the ‘0 District’ cell is bolded in that row.

6.19 Durham and Person House County Grouping

The Durham-Person House county group contains 4 districts. In the Enacted Map these are Districts 2, 29, 30, and 31. The county cluster has an overall partisan index of .76, which is strongly Democratic. After conducting 50,000 initial simulations to create four districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 49,896 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 37,800 simulated maps, each containing four districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 62. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 63.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 64. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 4 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 4 Democratic districts. The Duchin Map also generates 4 Democratic districts.

Table 22 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In all 11 of the individual elections there is agreement between the modal outcome in the simulations and the Enacted Map.

Figure 62: **Map of Durham and Person House County Cluster**

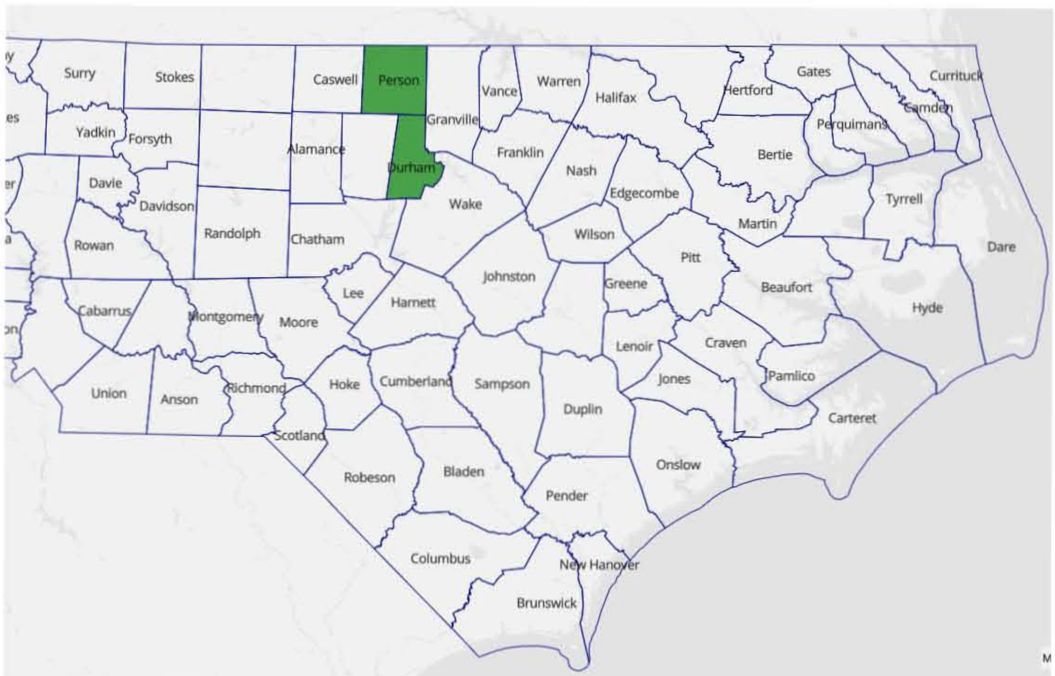
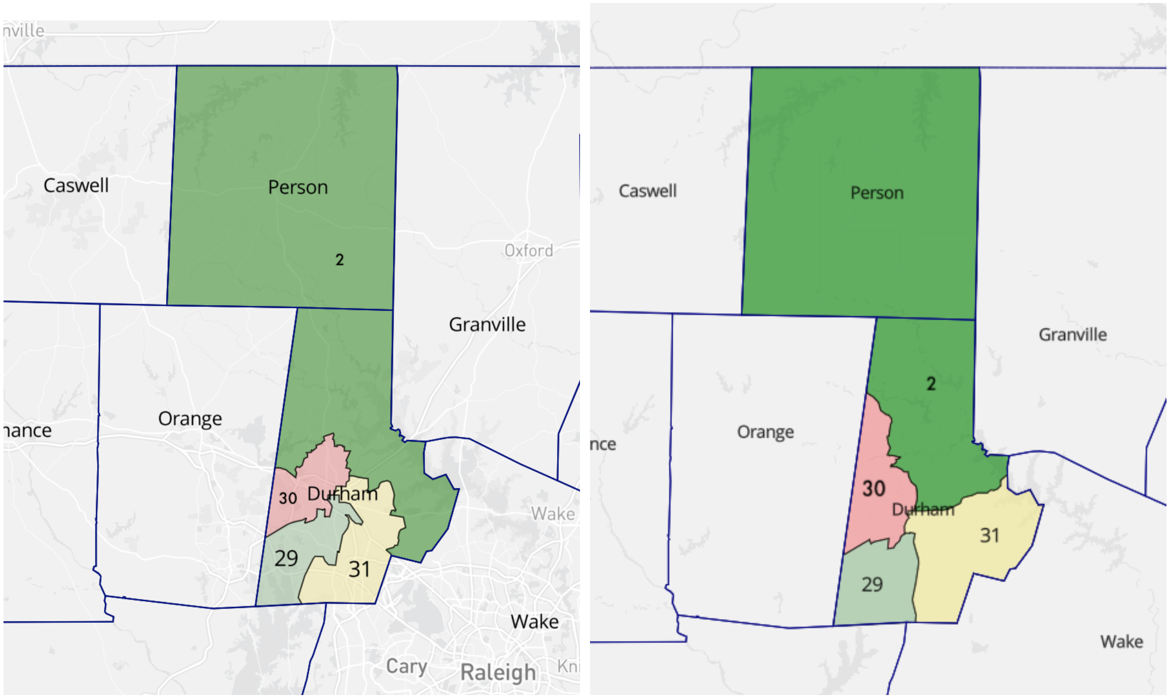


Figure 63: **Map of House Enacted Plan in Durham and Person House County Cluster**

(a) Enacted Map (b) Duchin Map

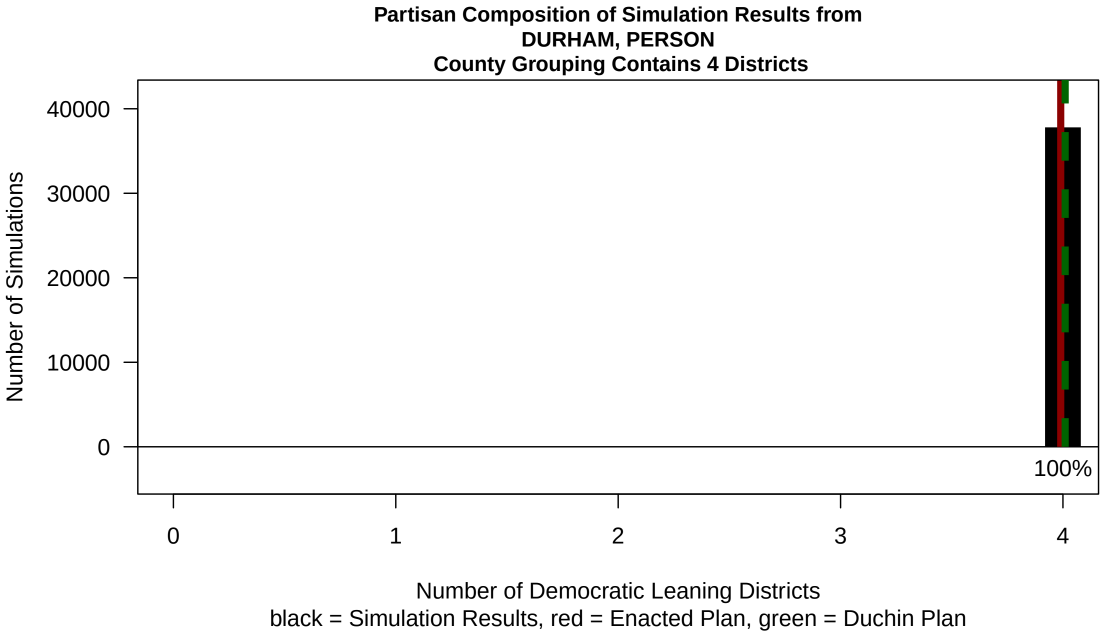


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
2	0.52	0.58
29	0.86	0.83
30	0.87	0.81
31	0.81	0.81

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 64: **Distribution of Partisan Districts from Simulations in Durham and Person House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 22: Simulation Results by Individual Elections

Durham and Person House County Cluster

	Number of Democratic Leaning Districts:				
	0	1	2	3	4
Individual Elections:					
2020 President	0%	0%	0%	0%	**100%**
2020 Senate	0%	0%	0%	0%	**100%**
2020 Governor	0%	0%	0%	0%	**100%**
2020 Lt. Governor	0%	0%	0%	0%	**100%**
2020 Attorney General	0%	0%	0%	0%	**100%**
2016 President	0%	0%	0%	0%	**100%**
2016 Senate	0%	0%	0%	**0%**	100%
2016 Governor	0%	0%	0%	0%	**100%**
2016 Lt. Governor	0%	0%	0%	**0%**	100%
2016 Attorney General	0%	0%	0%	0%	**100%**
2014 Senate	0%	0%	0%	0%	**100%**

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 4 Democratic leaning districts. The Enacted Plan does as well, as the '4 District' cell is bolded in that row.

6.20 Brunswick and New Hanover House County Grouping

The Brunswick-New Hanover House county group contains 4 districts. In the Enacted Map these are Districts 17, 18, 19, and 20. The county cluster has an overall partisan index of .45, which is Republican leaning. After conducting 50,000 initial simulations to create four districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 12,087 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 562 simulated maps, each containing four districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 65. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 66.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 67. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there is 1 Democratic leaning district. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 1 Democratic district. The Duchin Map generates 2 Democratic districts. The Duchin Map does not align with any of the simulations because it is less compact (average Polsby-Popper score of 0.35) than the Enacted Map (average Polsby-Popper score of 0.36) and the simulated maps, which are constrained to be at least as compact, on average, as the Enacted Map. This is evident by looking at the maps of the districts in the Duchin Plan. District 20 is a long and narrow district that begins south of Wilmington (the largest city in the cluster), takes in the eastern side of Wilmington, which is more Republican, and then loops around to the north west. In doing this, the Duchin map then splits the more Democratic portion of Wilmington between districts 18 and 19 in order to create two Democratic leaning districts. As a result, the town of Wilmington is a part of districts 18, 19, and 20. This is also true of the Enacted Map, however, the Enacted map does this while creating more compact districts.

Table 23 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 10 of the 11 individual elections there is agreement between the modal outcome in the simulations and the Enacted Map. In the 1 scenario in which they do not agree (2020 Governor race), the Enacted Map generates one more Democratic district than the simulations do.

Figure 65: **Map of Brunswick and New Hanover House County Cluster**

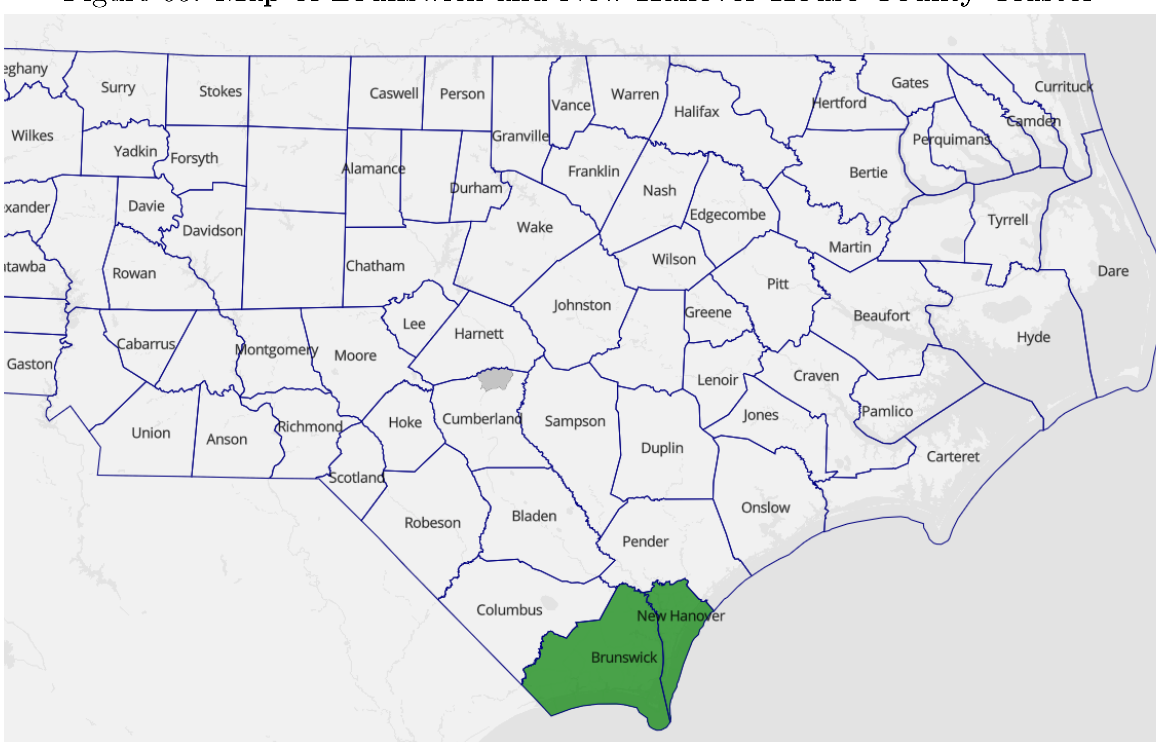
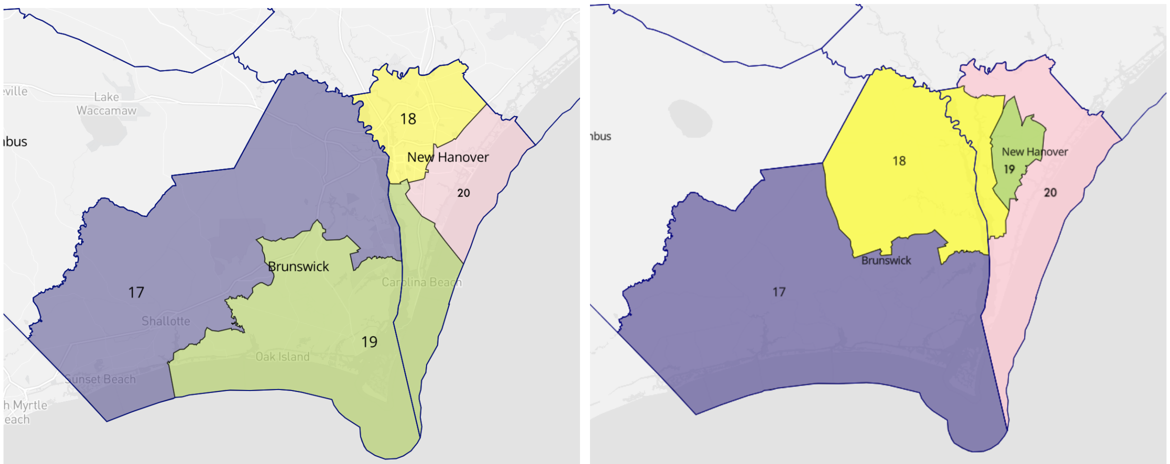


Figure 66: **Map of House Enacted Plan in Brunswick and New Hanover County Cluster**

(a) Enacted Map (b) Duchin Map

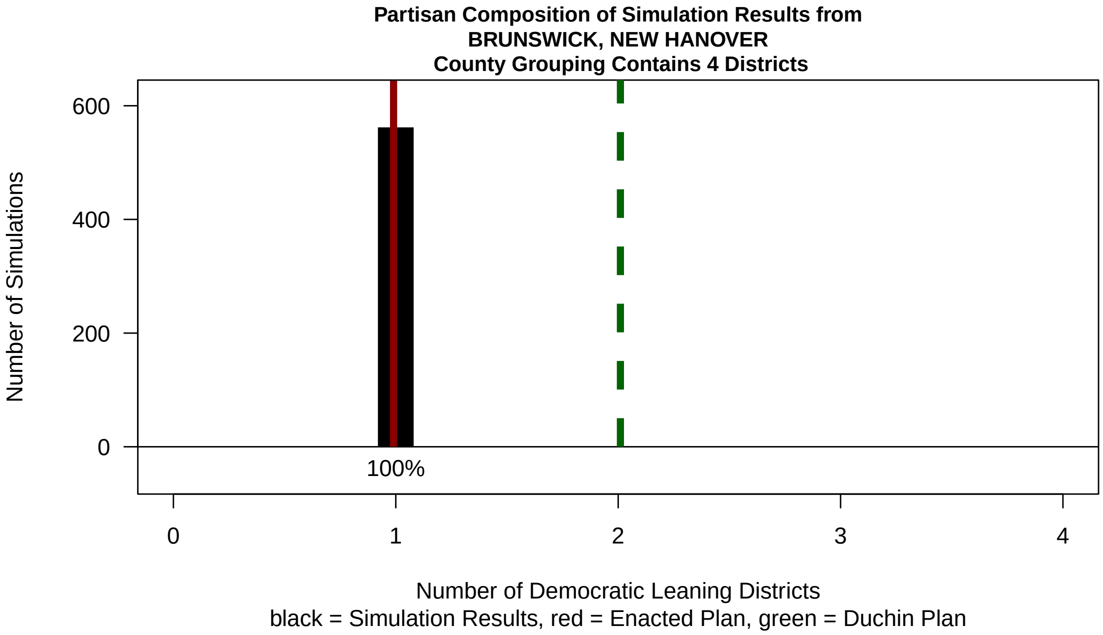


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
17	0.39	0.35
18	0.60	0.53
19	0.39	0.55
20	0.45	0.41

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 67: **Distribution of Partisan Districts from Simulations in Brunswick and New Hanover House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 23: Simulation Results by Individual Elections

Brunswick and New Hanover House County Cluster

	Number of Democratic Leaning Districts:				
	0	1	2	3	4
Individual Elections:					
2020 President	0%	**100%**	0%	0%	0%
2020 Senate	0%	**100%**	0%	0%	0%
2020 Governor	0%	100%	**0%**	0%	0%
2020 Lt. Governor	0%	**100%**	0%	0%	0%
2020 Attorney General	0%	**100%**	0%	0%	0%
2016 President	0%	**100%**	0%	0%	0%
2016 Senate	0%	**100%**	0%	0%	0%
2016 Governor	0%	**100%**	0%	0%	0%
2016 Lt. Governor	0%	**100%**	0%	0%	0%
2016 Attorney General	0%	**100%**	0%	0%	0%
2014 Senate	0%	**100%**	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

6.21 Forsyth and Stokes House County Grouping

The Forsyth-Stokes House county group contains 5 districts. In the Enacted Map these are Districts 71, 72, 74, 75, and 91. The county cluster has an overall partisan index of .52, which is slightly Democratic leaning. After conducting 50,000 initial simulations to create five districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 17,147 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 3,726 simulated maps, each containing five districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 68. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 69. I also include the 2020 map's boundaries for comparison.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 70. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 33% of the simulations there are 2 Democratic leaning districts. In 50% of the simulations there are 3 Democratic leaning districts, and in 17% of the simulations there are 4 Democratic leaning districts. The Enacted Map creates 2 Democratic districts. The Duchin Map also generates 2 Democratic districts.

Table 24 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 10 of the 11 individual elections the Enacted Map generates 2 Democratic districts. In 1 scenario (2020 Governor race), the Enacted Map generates 3 Democratic districts.

The Enacted Plan is also extremely similar to the maps used in Forsyth County in the 2020 elections. These districts were approved by a court in 2019. The county grouping was different, and Forsyth was combined with Yadkin County in 2020, however, in both plans the less populous county is kept whole and combined with a portion of Forsyth County. Within the more populated Forsyth County, the boundaries are extremely similar. The Enacted Plan is different by only 5 precincts total, and no district differs from the 2020 maps by more than a 3 precinct shift.

Figure 68: **Map of Forsyth and Stokes House County Cluster**

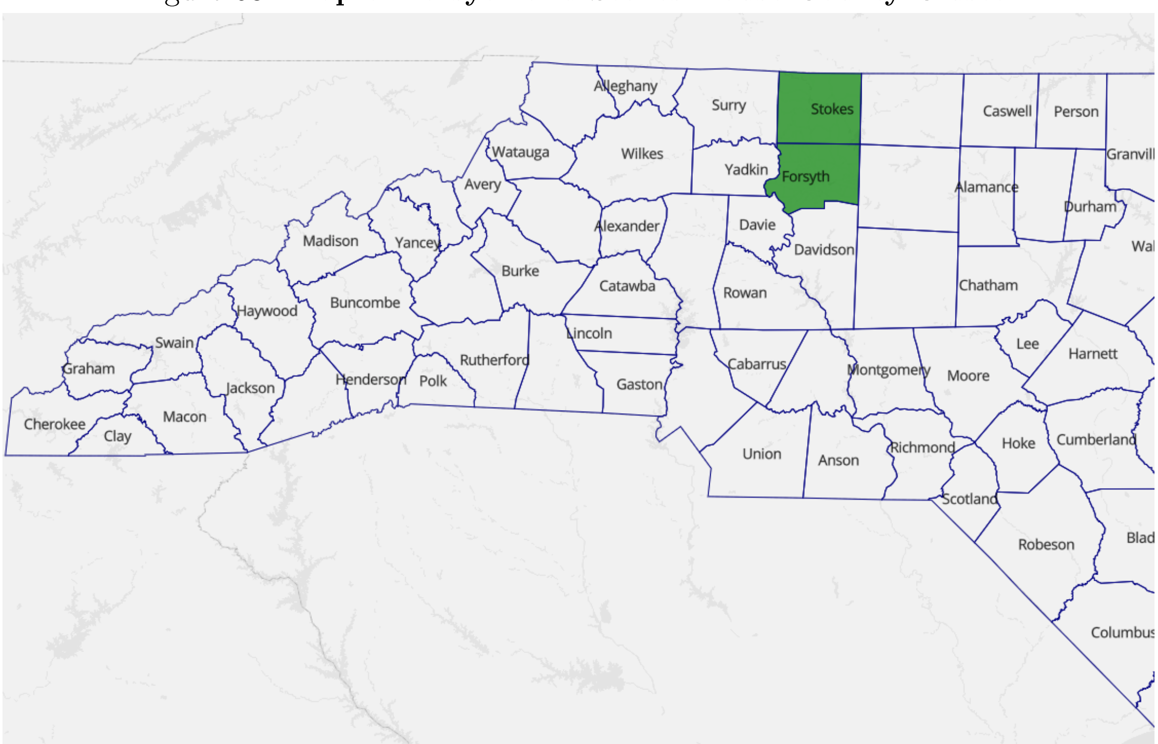
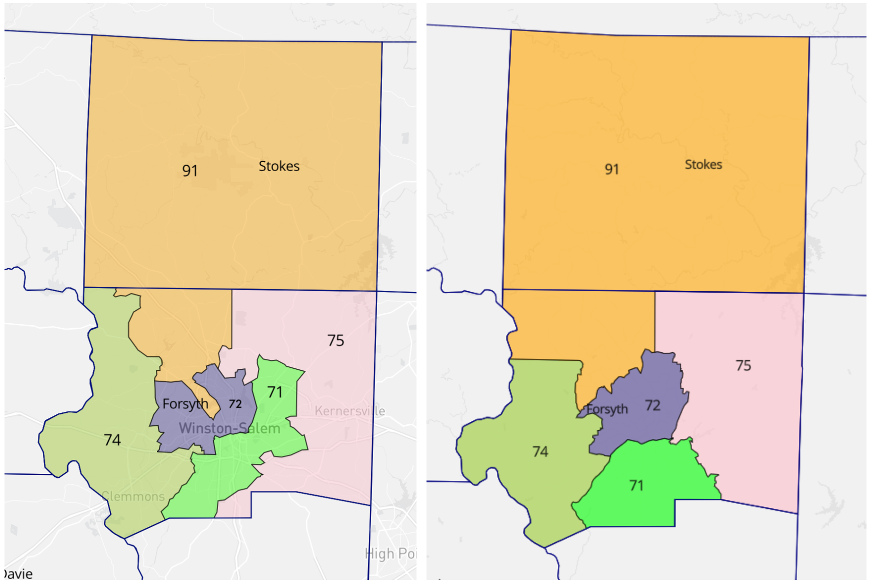
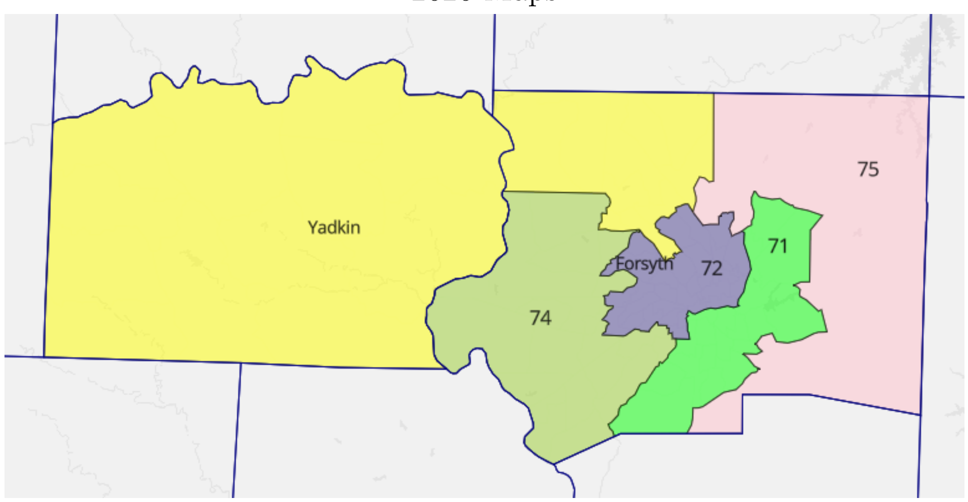


Figure 69: **Map of House Enacted Plan in Forsyth and Stokes County Cluster**

(a) Enacted Map (b) Duchin Map



2020 Maps

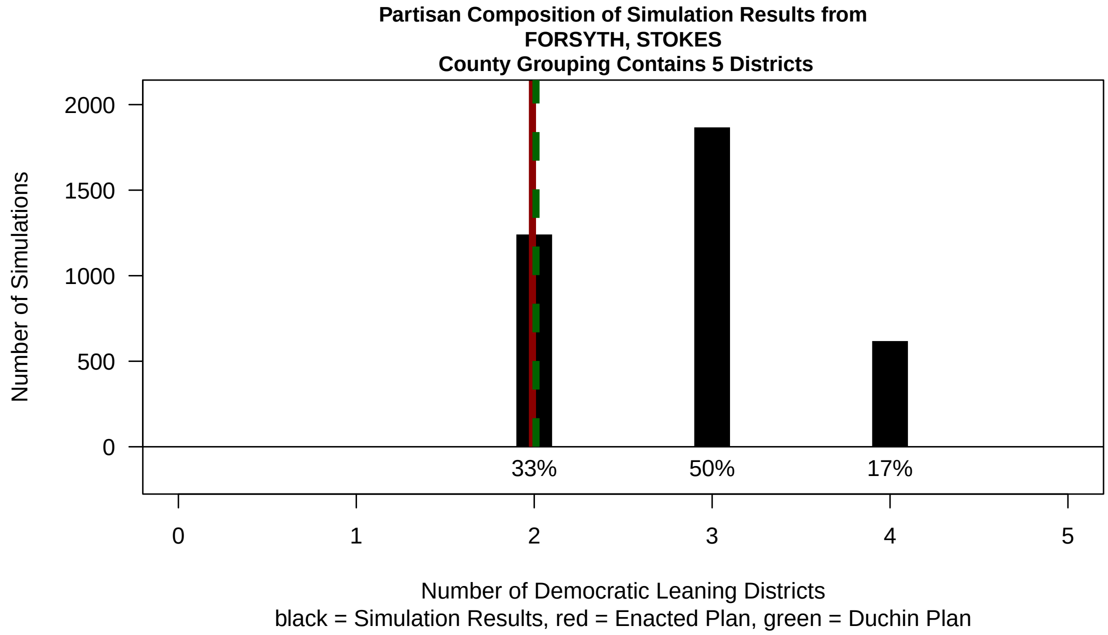


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
71	0.71	0.69
72	0.70	0.74
74	0.45	0.46
75	0.39	0.42
91	0.38	0.35

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 70: **Distribution of Partisan Districts from Simulations in Forsyth and Stokes House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 24: Simulation Results by Individual Elections

Forsyth and Stokes House County Cluster

	Number of Democratic Leaning Districts:					
	0	1	2	3	4	5
Individual Elections:						
2020 President	0%	0%	**14%**	50%	35%	0%
2020 Senate	0%	0%	**29%**	52%	19%	0%
2020 Governor	0%	0%	0%	**21%**	79%	0%
2020 Lt. Governor	0%	0%	**44%**	44%	13%	0%
2020 Attorney General	0%	0%	**30%**	52%	18%	0%
2016 President	0%	0%	**45%**	45%	11%	0%
2016 Senate	0%	5%	**67%**	28%	0%	0%
2016 Governor	0%	0%	**21%**	55%	24%	0%
2016 Lt. Governor	0%	4%	**66%**	30%	0%	0%
2016 Attorney General	0%	0%	**25%**	56%	19%	0%
2014 Senate	0%	3%	**58%**	38%	1%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 14% of the simulations produce 2 Democratic leaning districts. The Enacted Plan does as well, as the '2 District' cell is bolded in that row.

6.22 Cabarrus, Davie, Rowan, and Yadkin House County Grouping

The Cabarrus-Davie-Rowan-Yadkin House county group contains 5 districts. In the Enacted Map these are Districts 73, 76, 77, 82, and 83. The county cluster has an overall partisan index of .36, which is strongly Republican. After conducting 50,000 initial simulations to create five districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 6,649 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 283 simulated maps, each containing five districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 71. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 72.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 73. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 99% of the simulations there are 0 Democratic leaning districts. The Enacted Map creates 0 Democratic districts. The Duchin Map also generates 0 Democratic districts.

Table 25 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In all of the 11 individual elections the Enacted Map generates 0 Democratic districts and is in agreement with the majority of the simulations results in 8 of the 11 individual elections considered.

Figure 71: **Map of Cabarrus, Davie, Rowan, and Yadkin House County Cluster**

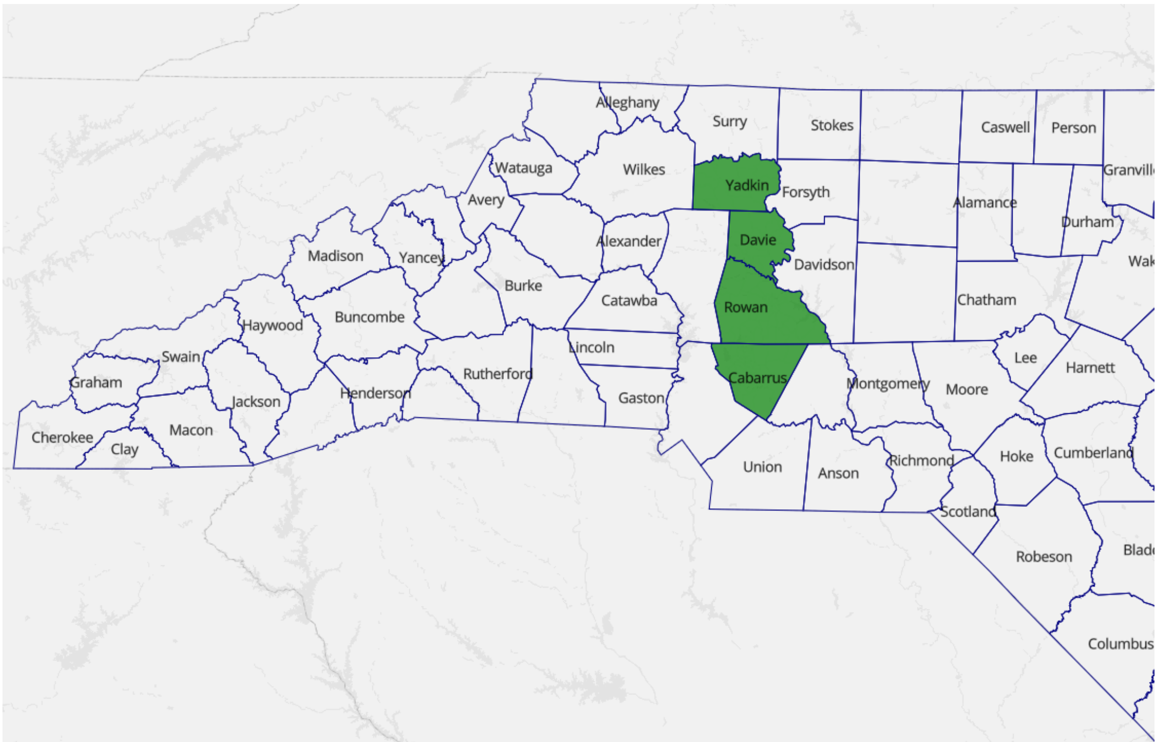
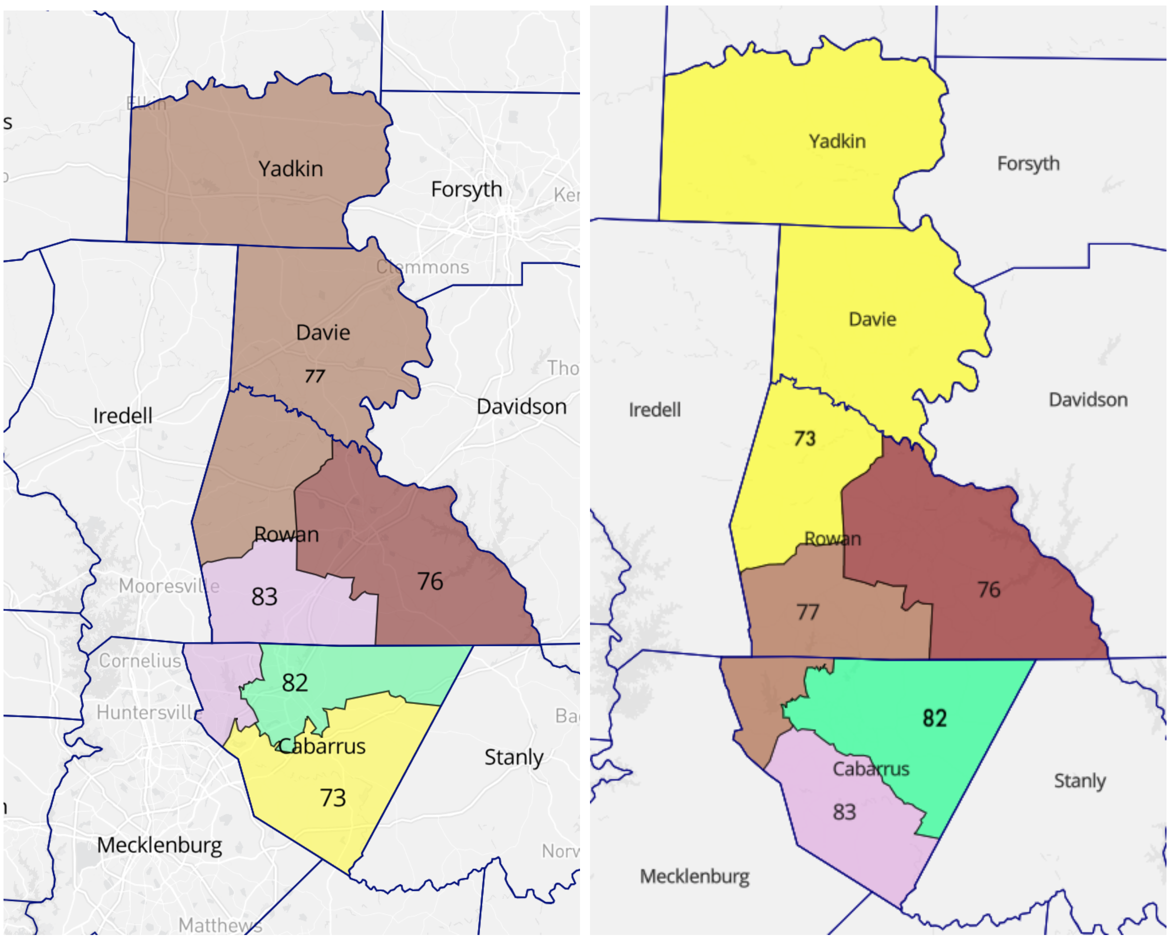


Figure 72: **Map of House Enacted Plan in Cabarrus, Davie, Rowan, and Yadkin County Cluster**

(a) Enacted Map (b) Duchin Map

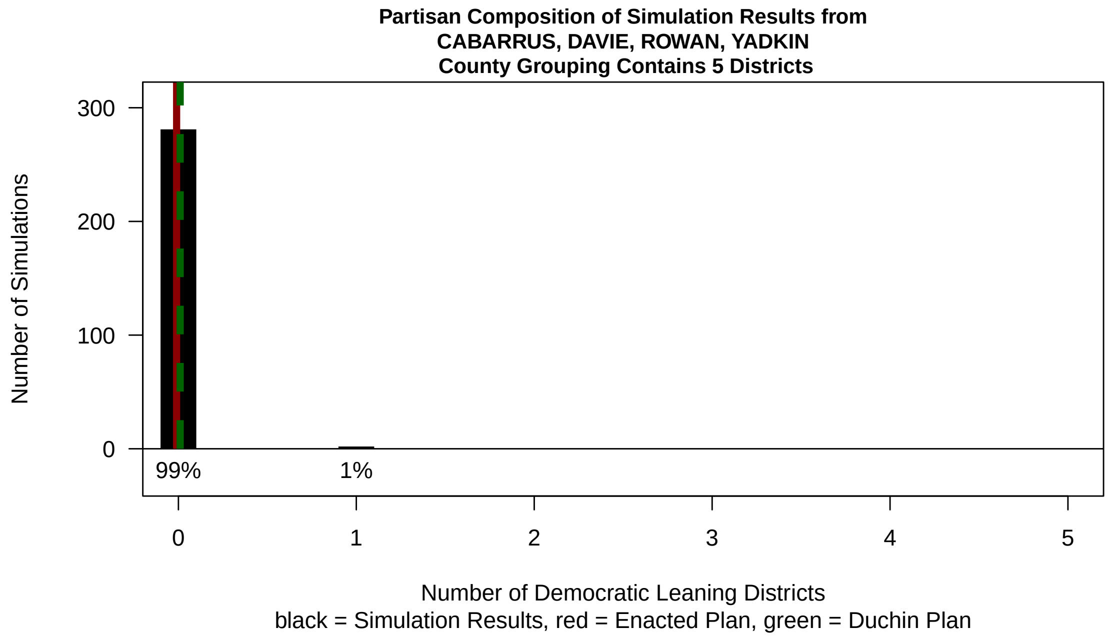


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
73	0.40	0.25
76	0.40	0.40
77	0.25	0.35
82	0.45	0.41
83	0.34	0.43

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 73: **Distribution of Partisan Districts from Simulations in Cabarrus, Davie, Rowan, and Yadkin House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 25: Simulation Results by Individual Elections

Cabarrus, Davie, Rowan, and Yadkin House County Cluster

	Number of Democratic Leaning Districts:					
	0	1	2	3	4	5
Individual Elections:						
2020 President	**10%**	90%	0%	0%	0%	0%
2020 Senate	**85%**	15%	0%	0%	0%	0%
2020 Governor	**2%**	98%	0%	0%	0%	0%
2020 Lt. Governor	**87%**	13%	0%	0%	0%	0%
2020 Attorney General	**9%**	91%	0%	0%	0%	0%
2016 President	**100%**	0%	0%	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%	0%	0%
2014 Senate	**100%**	0%	0%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 10% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.23 Chatham, Lee, Moore, Randolph, and Richmond House County Grouping

The Chatham-Lee-Moore-Randolph-Richmond House county group contains 5 districts. In the Enacted Map these are Districts 51, 52, 54, 70, and 78. The county cluster has an overall partisan index of .38, which is strongly Republican. After conducting 50,000 initial simulations to create five districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 1,868 simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 939 simulated maps, each containing five districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 74. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 75.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 76. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 18% of the simulations there are 0 Democratic leaning districts. In 82% of the simulations there is 1 Democratic leaning district. The Enacted Map creates 1 Democratic district. The Duchin Map also generates 1 Democratic district.

Table 26 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all of the 11 individual elections the Enacted Map generates 1 Democratic district and is in agreement with the majority of the simulations results in all 11 individual elections considered.

Figure 74: **Map of Chatham, Lee, Moore, Randolph, and Richmond House County Cluster**

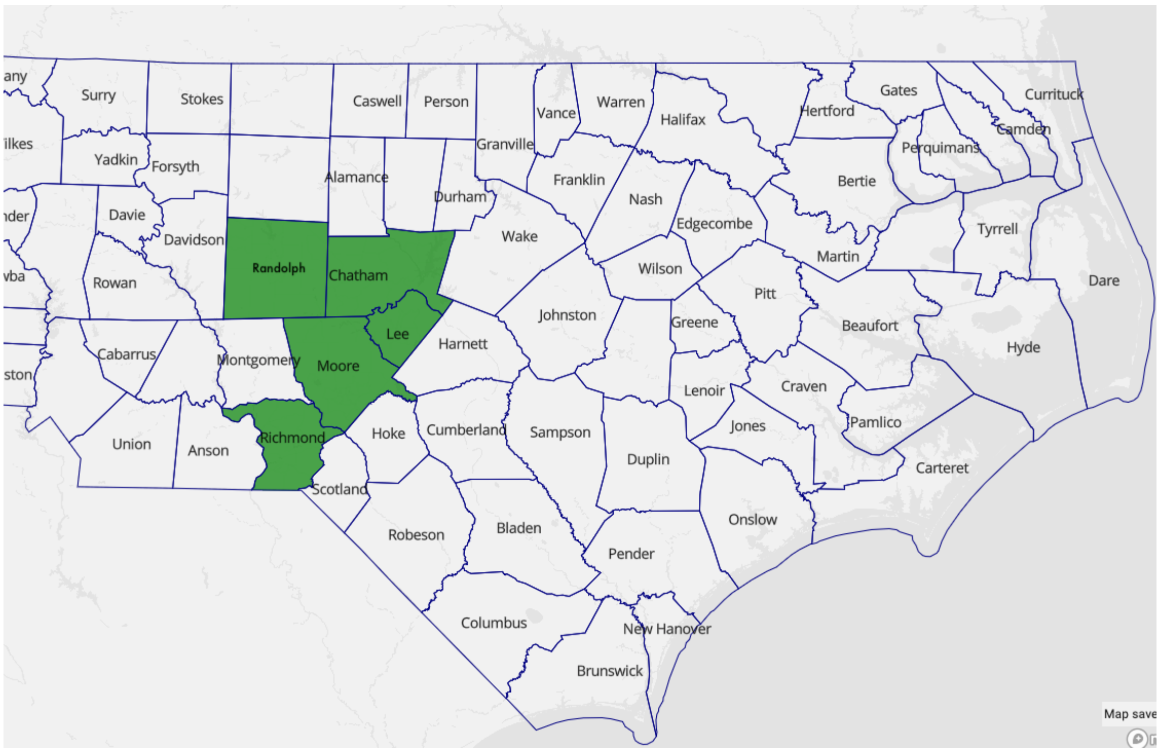
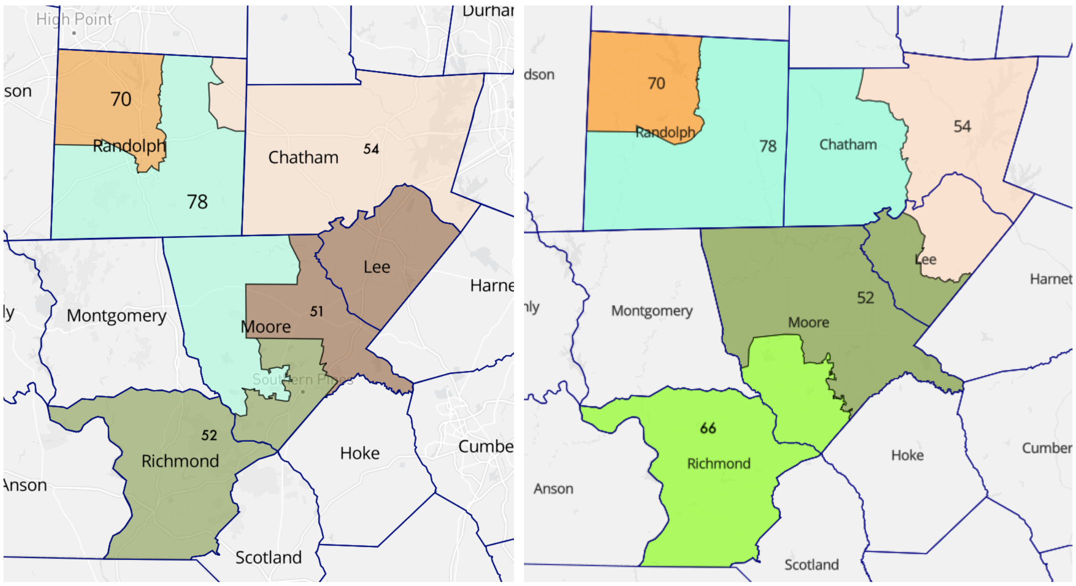


Figure 75: **Map of House Enacted Plan in Chatham, Lee, Moore, Randolph, and Richmond County Cluster**

(a) Enacted Map (b) Duchin Map

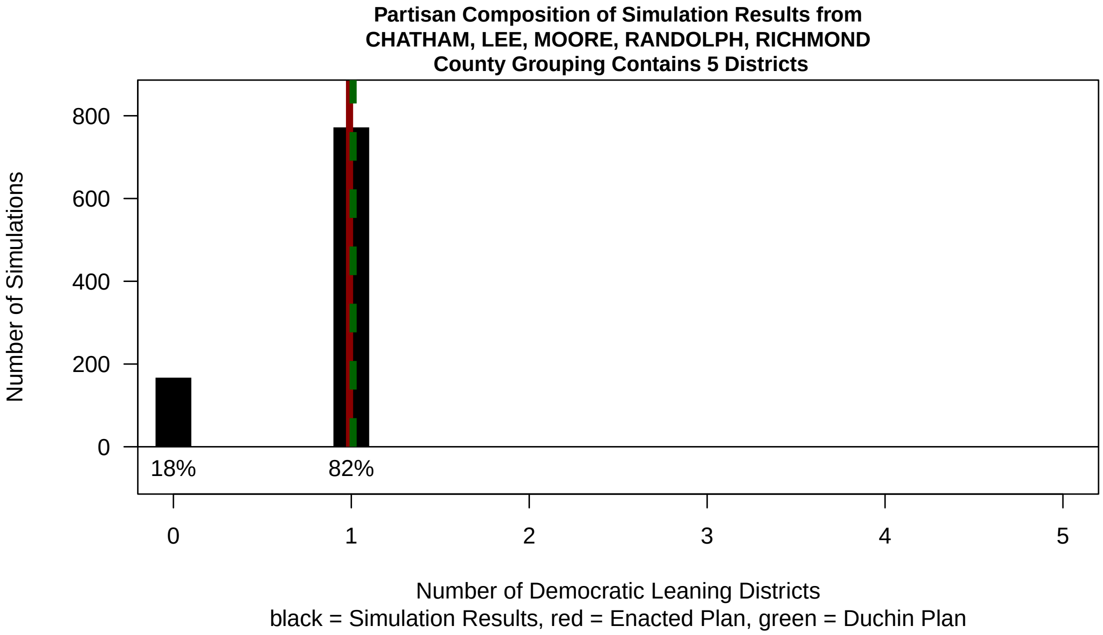


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
51(66 in Duchin)	0.41	0.42
52	0.44	0.35
54	0.54	0.58
70	0.25	0.24
78	0.26	0.27

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 76: **Distribution of Partisan Districts from Simulations in Chatham, Lee, Moore, Randolph, and Richmond House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 26: Simulation Results by Individual Elections

Chatham, Lee, Moore, Randolph, and Richmond House County Cluster

	Number of Democratic Leaning Districts:					
	0	1	2	3	4	5
Individual Elections:						
2020 President	17%	**83%**	0%	0%	0%	0%
2020 Senate	18%	**82%**	0%	0%	0%	0%
2020 Governor	0%	**100%**	0%	0%	0%	0%
2020 Lt. Governor	18%	**82%**	0%	0%	0%	0%
2020 Attorney General	15%	**85%**	0%	0%	0%	0%
2016 President	18%	**82%**	0%	0%	0%	0%
2016 Senate	19%	**81%**	0%	0%	0%	0%
2016 Governor	15%	**85%**	0%	0%	0%	0%
2016 Lt. Governor	29%	**71%**	0%	0%	0%	0%
2016 Attorney General	14%	**86%**	0%	0%	0%	0%
2014 Senate	15%	**85%**	0%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 83% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

6.24 Guilford House County Grouping

The Guilford House county group contains 6 districts. In the Enacted Map these are Districts 57, 58, 59, 60, 61, and 62. The county cluster has an overall partisan index of .61, which is strongly Democratic. After conducting 50,000 initial simulations to create six districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, this grouping contains only one county, and thus the Enacted Plan will contain as many traversals as all of the simulations. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 15,489 simulated maps, each containing six districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 77. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 78. I also include the map of districts in this county from the 2020 plan for comparison here.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 79. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 1% of the simulations there are 4 Democratic leaning districts. In 79% of the simulations there is 5 Democratic leaning district. in 21% of the simulations there are 6 Democratic districts. The Enacted Map creates 4 Democratic districts. The Duchin Map generates 5 Democratic districts.

Table 27 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Demo-

cratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 10 of the 11 individual elections the Enacted Map generates 4 Democratic districts and in 1 election (2020 Governor) the map contains 5 Democratic leaning districts.

An important point to consider when looking at the Enacted Map is that it closely adheres to the map used in Guilford County the 2020 election, which was approved by a court in 2019. The Enacted Plan is different by only four precincts. District 57 is identical across the two plans. Districts 59, 61, and 62 differ from the 2020 map by only 1 precinct each. District 60 differs from the 2020 map by 2 precincts and District 58 differs by only 3 precincts.

Figure 77: **Map of Guilford House County Cluster**

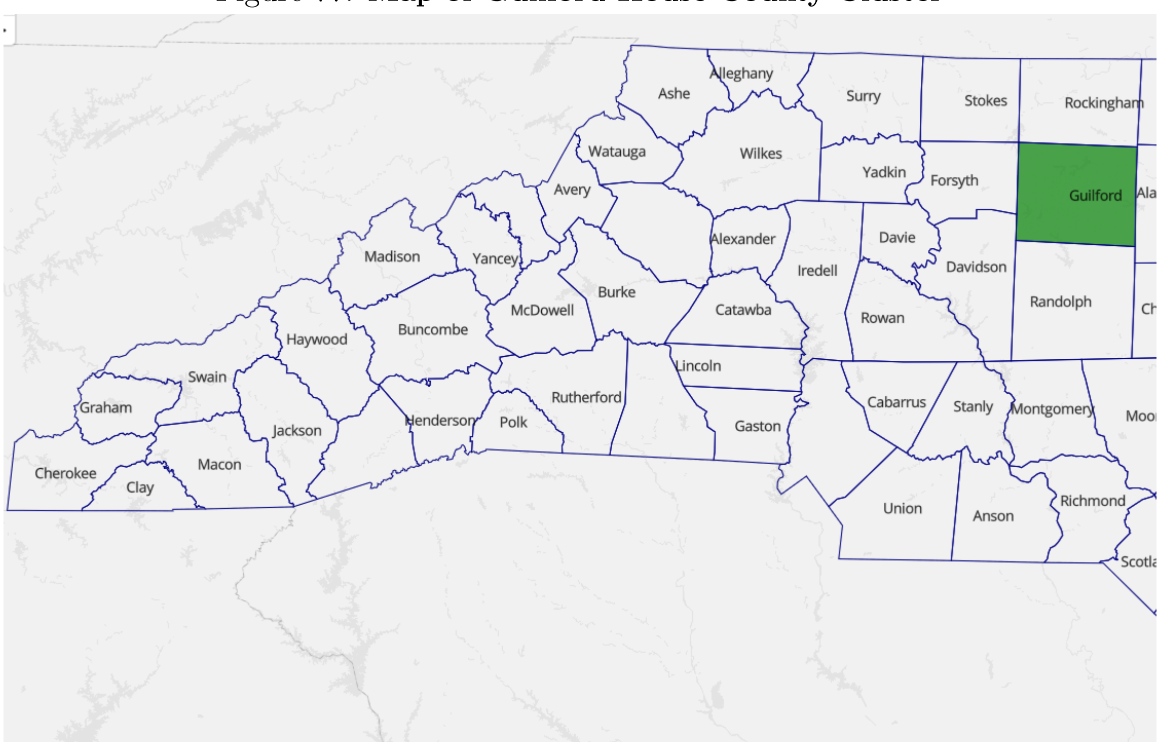
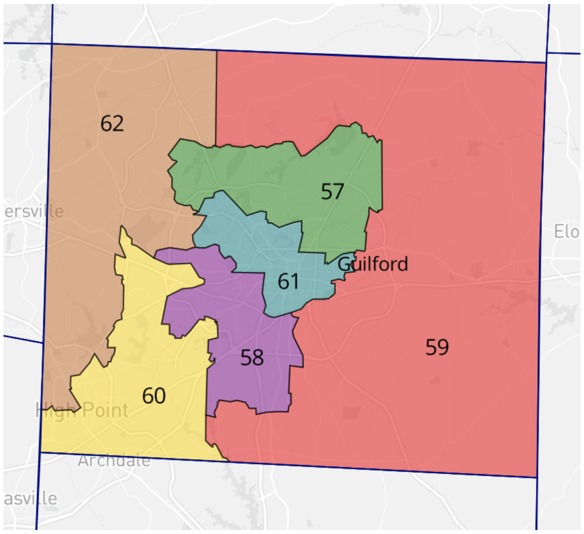
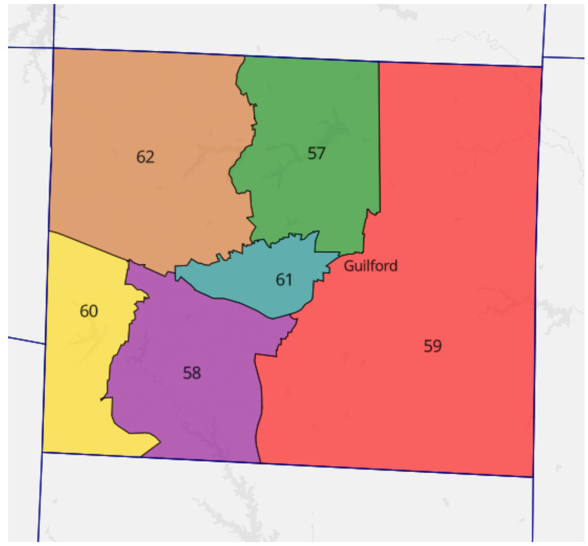


Figure 78: **Map of House Enacted Plan in Guilford County Cluster**

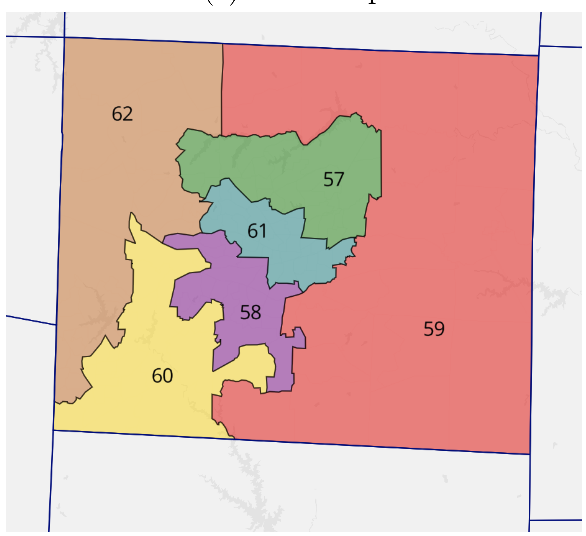
(a) Enacted Map



(b) Duchin Map



(c) 2020 Map

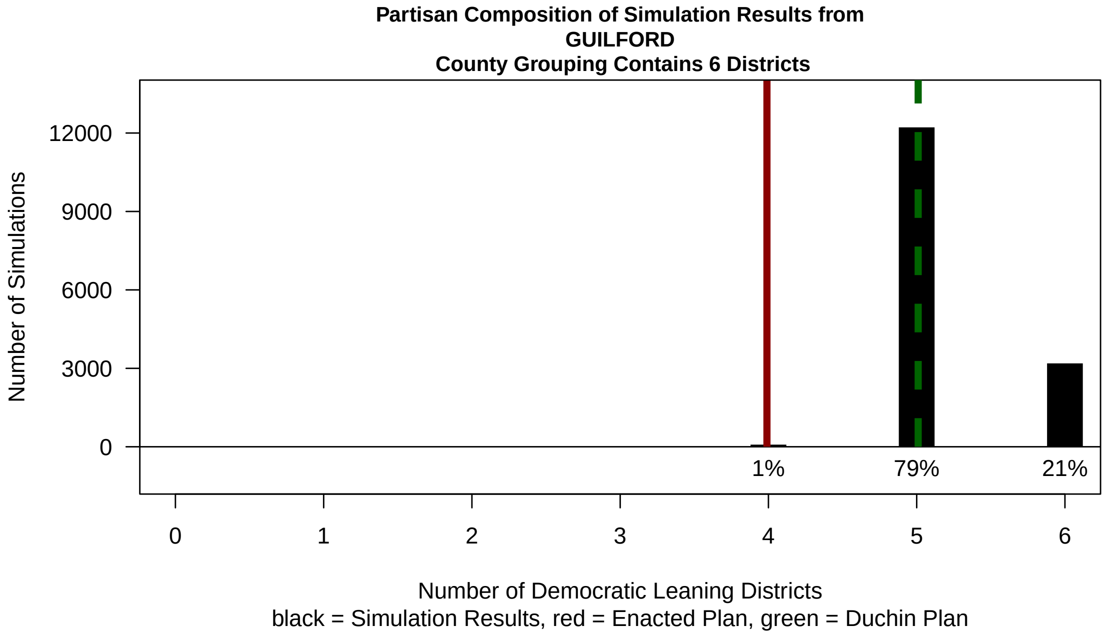


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
57	0.68	0.65
58	0.74	0.65
59	0.46	0.54
60	0.64	0.57
61	0.74	0.80
62	0.43	0.48

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 79: **Distribution of Partisan Districts from Simulations in Guilford House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 27: Simulation Results by Individual Elections

Guilford HouseCounty Cluster

	Number of Democratic Leaning Districts:						
	0	1	2	3	4	5	6
Individual Elections:							
2020 President	0%	0%	0%	0%	**0%**	41%	59%
2020 Senate	0%	0%	0%	0%	**0%**	73%	27%
2020 Governor	0%	0%	0%	0%	0%	**1%**	99%
2020 Lt. Governor	0%	0%	0%	0%	**1%**	80%	19%
2020 Attorney General	0%	0%	0%	0%	**0%**	53%	47%
2016 President	0%	0%	0%	0%	**2%**	84%	13%
2016 Senate	0%	0%	0%	0%	**7%**	90%	3%
2016 Governor	0%	0%	0%	0%	**0%**	44%	56%
2016 Lt. Governor	0%	0%	0%	0%	**8%**	90%	3%
2016 Attorney General	0%	0%	0%	0%	**1%**	82%	17%
2014 Senate	0%	0%	0%	0%	**21%**	78%	1%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 0% of the simulations produce 4 Democratic leaning districts. The Enacted Plan does, as the '1 District' cell is bolded in that row.

6.25 Avery, Cleveland, Gaston, Henderson, McDowell, Mitchell, Polk, Rutherford, and Yancey House County Grouping

The Avery-Cleveland-Gaston-Henderson-McDowell-Mitchell-Polk-Rutherford-Yancey House county group contains 7 districts. In the Enacted Map these are Districts 85, 108, 109, 110, 111, 113, and 117. The county cluster has an overall partisan index of .35, which is strongly Republican. After conducting 50,000 initial simulations to create seven districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 14,667 simulated plans. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 11,815 simulated maps, each containing seven districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 80. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 81.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 82. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map creates 0 Democratic leaning districts. The Duchin Map generates 0 Democratic leaning districts.

Table 28 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the individual elections the Enacted Map generates 0 Democratic districts and is in agreement with all of the simulated results across all 11 elections.

Figure 80: **Map of Avery, Cleveland, Gaston, Henderson, McDowell, Mitchell, Polk, Rutherford, and Yancey House County Cluster**

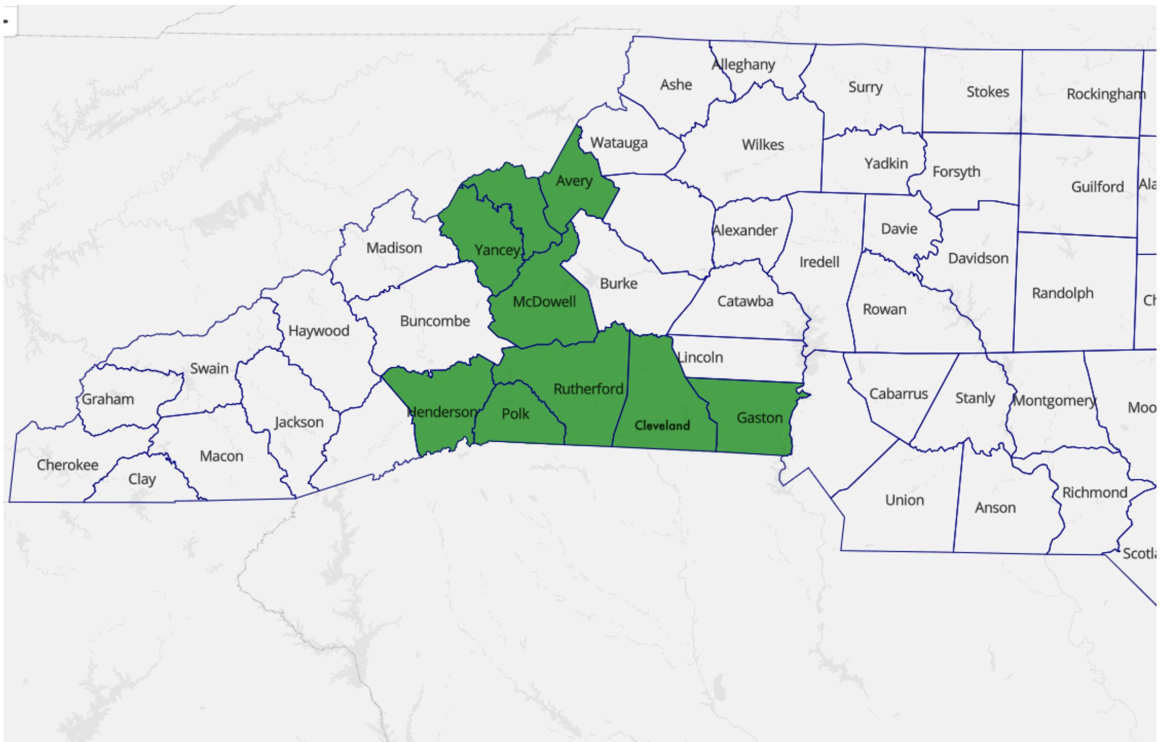
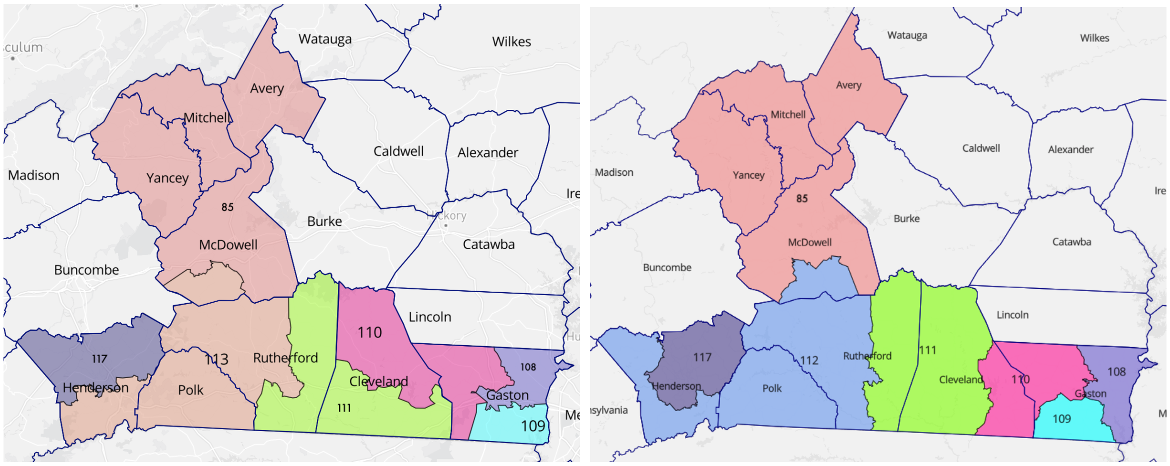


Figure 81: **Map of House Enacted Plan in Avery, Cleveland, Gaston, Henderson, McDowell, Mitchell, Polk, Rutherford, and Yancey County Cluster**

(a) Enacted Map (b) Duchin Map

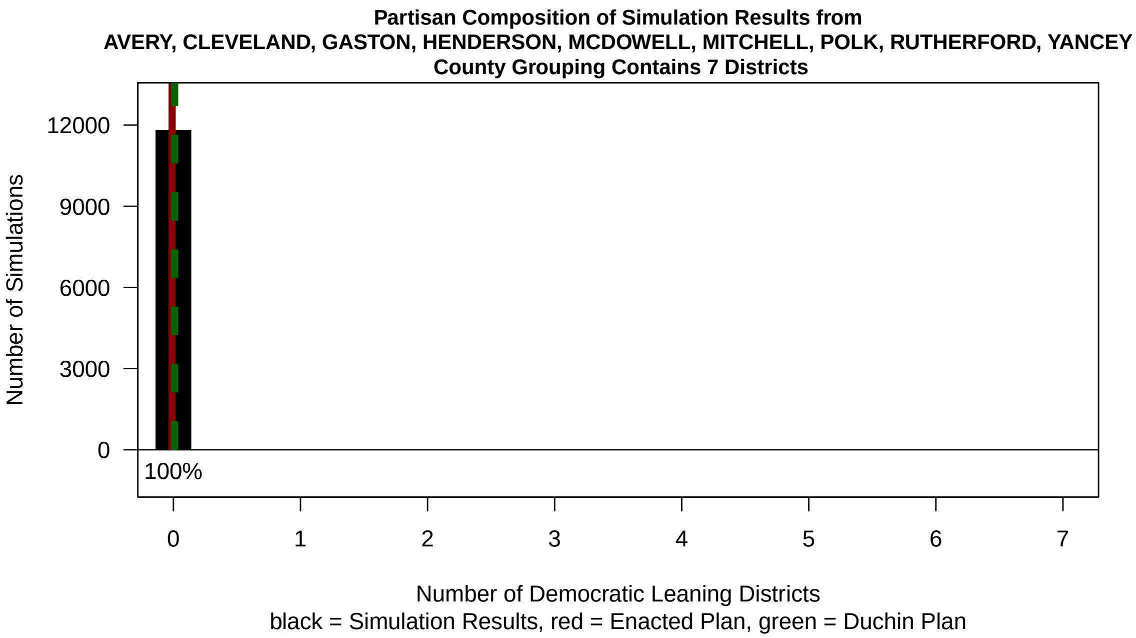


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
85	0.28	0.28
108	0.38	0.32
109	0.38	0.43
110	0.31	0.32
111	0.32	0.34
113	0.35	0.33
117	0.40	0.40

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 82: **Distribution of Partisan Districts from Simulations in Avery, Cleveland, Gaston, Henderson, McDowell, Mitchell, Polk, Rutherford, and Yancey House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 28: Simulation Results by Individual Elections

Avery, Cleveland, Gaston, Henderson, McDowell, Mitchell, Polk, Rutherford, and Yancey House County Cluster

Number of Democratic Leaning Districts:			
	0	1	2-7
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**99%**	1%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

6.26 Mecklenburg House County Grouping

The Mecklenburg House county group contains 13 districts. In the Enacted Map these are Districts 88, 92, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, and 112. The county cluster has an overall partisan index of .65, which is strongly Democratic. After conducting 50,000 initial simulations to create 13 districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, this cluster is a single county, and thus, there are no traversals. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 3,161 simulated maps, each containing 13 districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 83. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 84.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 85. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 1% of the simulations there are 10 Democratic leaning districts. In 56% of the simulations there are 11 Democratic leaning districts, and in 44% of the simulations there are 12 Democratic leaning districts. The Enacted Map aligns with the majority of simulations and creates 11 Democratic leaning districts. The Duchin Map generates 11 Democratic leaning districts as well.

Table 29 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Demo-

cratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. Across the 11 individual elections the Enacted Map generates between 9-13 Democratic districts and is in agreement with the majority of the simulated results in 7 of the 11 elections. In 10 of the 11 elections the Enacted Plan is within the middle 50% of the simulation results.

Figure 83: **Map of Mecklenburg House County Cluster**

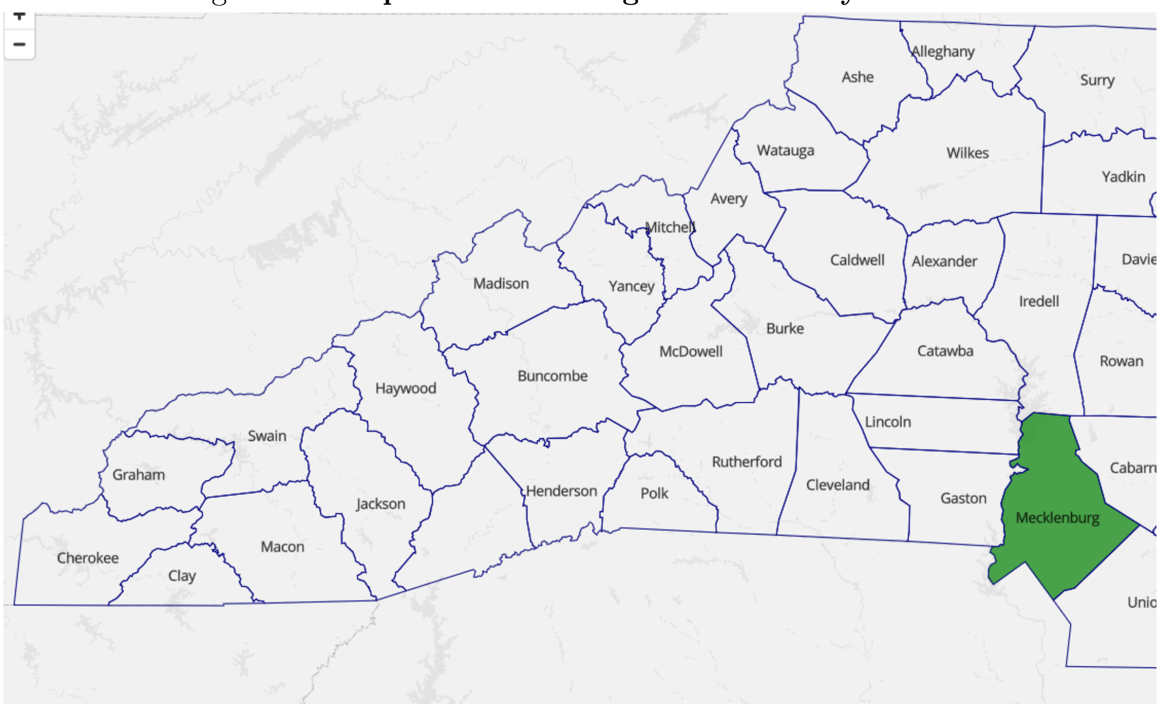
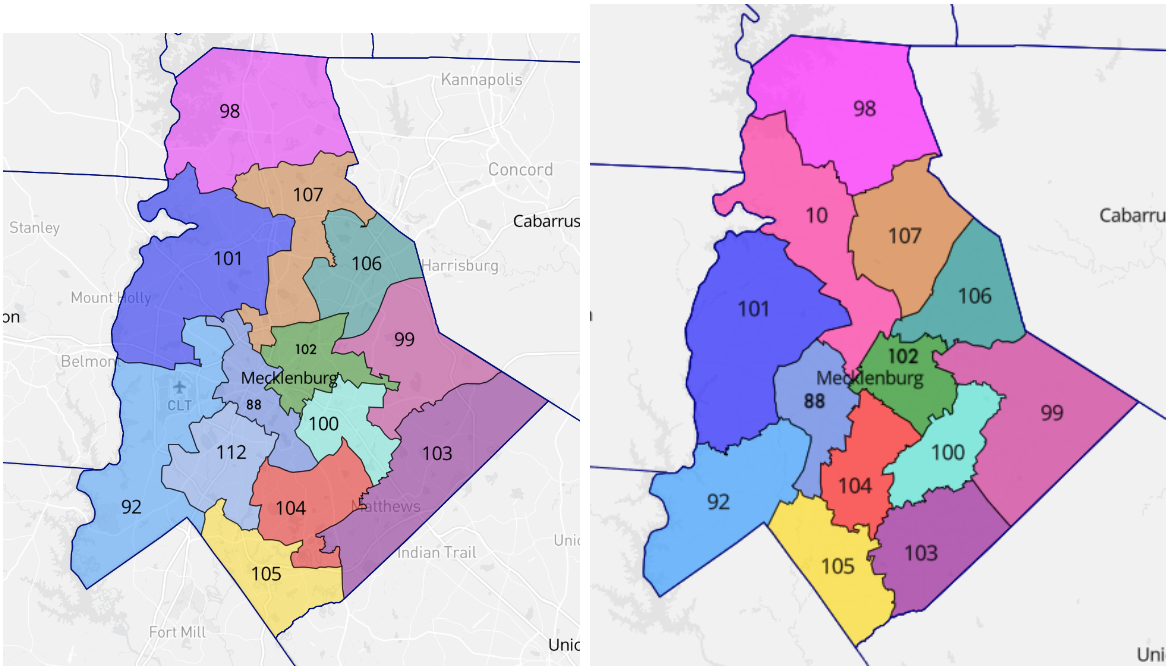


Figure 84: **Map of House Enacted Plan in Mecklenburg County Cluster**

(a) Enacted Map (b) Duchin Map

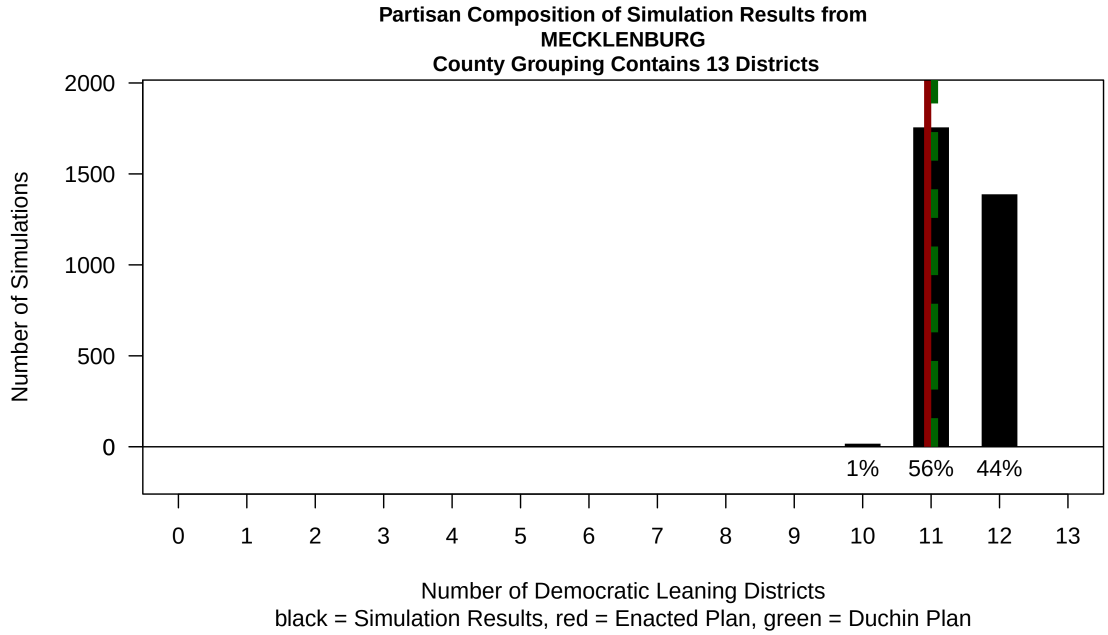


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
88	0.65	0.75
92	0.70	0.69
98	0.47	0.47
99	0.78	0.59
100	0.73	0.68
101	0.72	0.74
102	0.82	0.80
103	0.47	0.49
104	0.51	0.55
105	0.54	0.55
106	0.80	0.82
107	0.74	0.75
112 (10 in Duchin)	0.72	0.75

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 85: **Distribution of Partisan Districts from Simulations in Mecklenburg House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 29: Simulation Results by Individual Elections

Mecklenburg House County Cluster

	Number of Democratic Leaning Districts:						
	0-7	8	9	10	11	12	13
Individual Elections:							
2020 President	0%	0%	0%	0%	0%	0%	**100%**
2020 Senate	0%	0%	0%	0%	**39%**	61%	0%
2020 Governor	0%	0%	0%	0%	0%	0%	**100%**
2020 Lt. Governor	0%	0%	0%	0%	**36%**	64%	0%
2020 Attorney General	0%	0%	0%	0%	**9%**	91%	0%
2016 President	0%	0%	0%	3%	**69%**	28%	0%
2016 Senate	0%	3%	**50%**	45%	2%	0%	0%
2016 Governor	0%	0%	0%	0%	11%	**76%**	13%
2016 Lt. Governor	0%	4%	**58%**	38%	0%	0%	0%
2016 Attorney General	0%	0%	5%	**34%**	57%	4%	0%
2014 Senate	0%	4%	**60%**	35%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 13 Democratic leaning districts. The Enacted Plan does as well, as the '13 District' cell is bolded in that row.

6.27 Wake House County Grouping

The Wake House county group contains 13 districts. In the Enacted Map these are Districts 11, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 49, and 66. The county cluster has an overall partisan index of .61, which is strongly Democratic. After conducting 50,000 initial simulations to create 13 districts in this cluster, I would normally discard any simulations that contain more county traversals than the Enacted Plan. However, this cluster is a single county, and thus, there are no traversals. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 14,305 simulated maps, each containing 13 districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 86. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 87.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 88. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 2% of the simulations there are 10 Democratic leaning districts. In 32% of the simulations there are 11 Democratic leaning districts, and in 66% of the simulations there are 12 Democratic leaning districts. The Enacted Map creates 11 Democratic leaning districts. The Duchin Map generates 11 Democratic leaning districts as well.

Table 30 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Demo-

cratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. Across the 11 individual elections the Enacted Map generates between 9-13 Democratic districts and is in agreement with the majority of the simulated results in 7 of the 11 elections.

Figure 86: **Map of Wake House County Cluster**

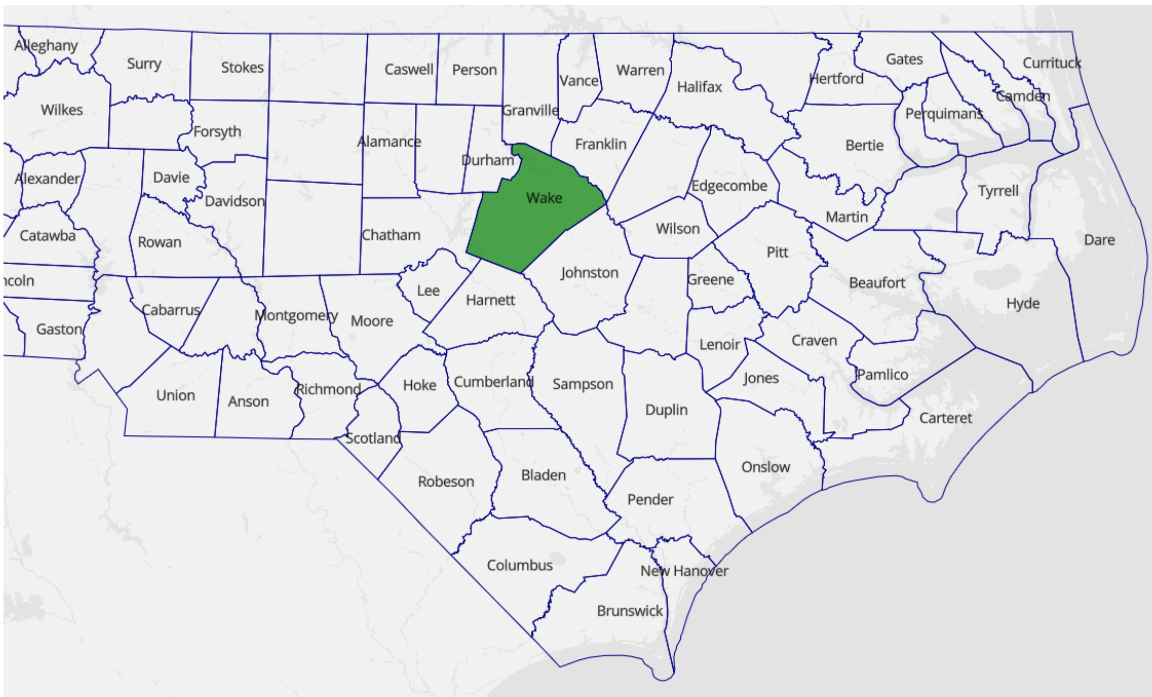
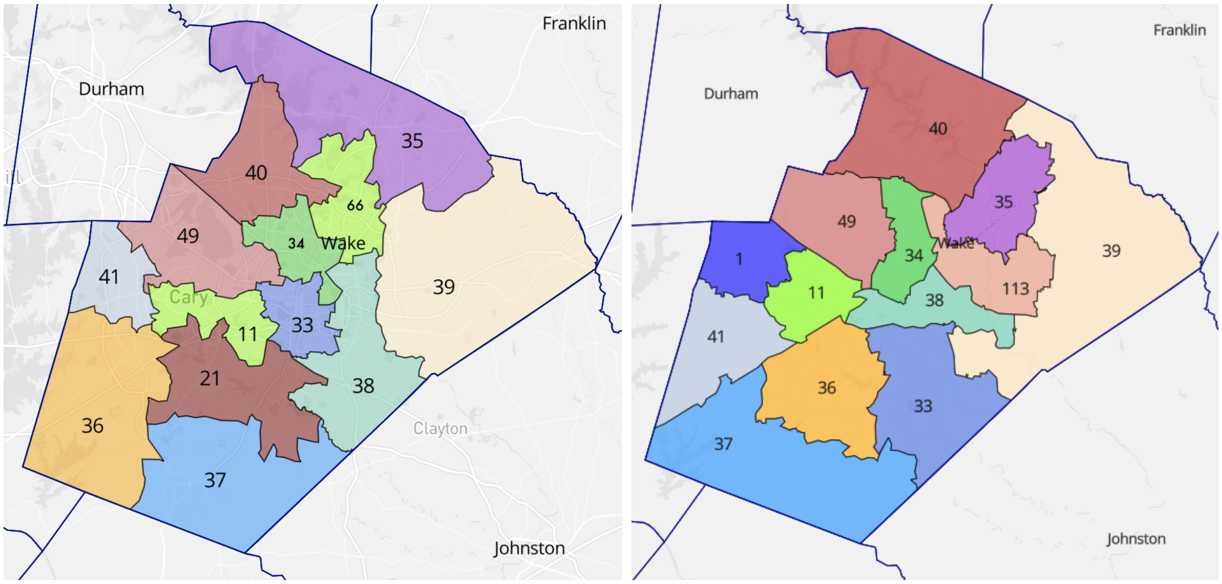


Figure 87: **Map of House Enacted Plan in Wake County Cluster**

(a) Enacted Map (b) Duchin Map

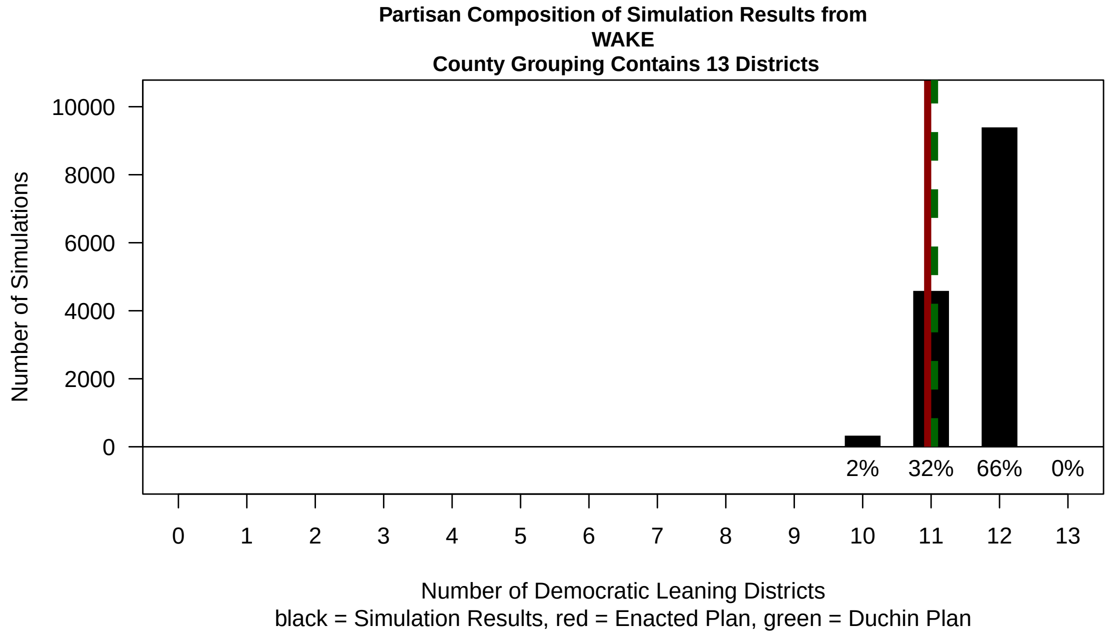


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
11	0.69	0.65
21 (1 in Duchin)	0.53	0.65
33	0.83	0.65
34	0.65	0.62
35	0.47	0.63
36	0.55	0.53
37	0.45	0.46
38	0.75	0.84
39	0.59	0.59
40	0.56	0.49
41	0.64	0.58
49	0.65	0.64
66 (113 in Duchin)	0.65	0.69

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 88: **Distribution of Partisan Districts from Simulations in Wake House County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 30: Simulation Results by Individual Elections

Wake House County Cluster

	Number of Democratic Leaning Districts:						
	0-7	8	9	10	11	12	13
Individual Elections:							
2020 President	0%	0%	0%	0%	**2%**	81%	17%
2020 Senate	0%	0%	0%	0%	**9%**	88%	2%
2020 Governor	0%	0%	0%	0%	0%	0%	**100%**
2020 Lt. Governor	0%	0%	0%	0%	**14%**	85%	0%
2020 Attorney General	0%	0%	0%	0%	**2%**	78%	20%
2016 President	0%	0%	2%	21%	**58%**	19%	0%
2016 Senate	0%	21%	**57%**	21%	1%	0%	0%
2016 Governor	0%	0%	0%	6%	**60%**	34%	0%
2016 Lt. Governor	0%	33%	**57%**	9%	0%	0%	0%
2016 Attorney General	0%	0%	2%	19%	**62%**	18%	0%
2014 Senate	0%	28%	**61%**	12%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 2% of the simulations produce 11 Democratic leaning districts. The Enacted Plan does as well, as the '11 District' cell is bolded in that row.

7 NC Senate Analysis

7.1 Senate Groupings with only 1 District

In the state Senate, there are 26 county clusters. 17 clusters containing 36 of the 50 districts are fixed based on the optimal county clusters determined by Cooper et al. (2021, 'Duke Study'). The remaining 9 clusters were selected by the General Assembly from four sets of choices between clusters as presented by the Duke Study.

In the Enacted Plan there are 14 county clusters composed of 48 counties in which the cluster contains only 1 Senate district. In these clusters there is no discretion for any map maker. The district is simply the boundaries of the county group. These counties collectively have a population of 2,906,456, or approximately 28% of the state's total population and account for 14 of the 50 seats in the state senate.

Figure 89 shows a map of the counties that constitute these single-district clusters in the Enacted Plan. Figure 90 shows a map of the countie that constitute these single-district clusters chosen in the Duchin Plan. Table 31 below shows each cluster, the counties included in the cluster, and the corresponding districts in the Senate Enacted Plan. The final two columns of the table show the partisan lean of the cluster using the 11 statewide partisan elections index discussed above and whether or not, based on that index, the cluster leans Democratic (or Republican). I classify a district (in the Enacted Plan and in the simulations as well) as being Democratic leaning if the partisan index for that district is greater than 0.50. In other words, if more than fifty percent of the ballots cast for the two major parties were for Democratic candidates, that district is classified as a Democratic leaning district. Obviously, districts with numbers much larger than (smaller than) 0.50 will be more likely to elect a Democrat (Republican) than districts that are very close to 0.50.

The bottom row of Table 31 shows the results for all 14 clusters together. Collectively these counties have a partisan index of 0.43, meaning roughly four in ten voters in these counties cast ballots for Democratic candidates in the 11 statewide races I consider here.

However, the location of voters for the different parties is not uniformly distributed across these counties. Given this spatial distribution of voters across the counties, 4 of the 14 clusters lean Democratic, or roughly 30 percent. In this case, the proportion of Democratic leaning districts is lower than the proportion of voters in these counties who favor Democratic candidates. However, this is not due to any district boundaries. It is again purely a function of the political geography of the state since all of these districts are entire county units and are, as such, fixed.

In some cases the Enacted Plan and the Duchin Plan use different county groupings from one another. This occurs in 4 cases and is shown in Table 31 below. This results in a net change of 3 counties included in single district groupings.[26]

In the Duchin Plan 5 of the 14 clusters lean Democratic, or approximately 36% of the districts. As in the Enacted Plan, the proportion of Democratic leaning districts is lower that the proportion of voters in these counties who favor Democratic candidates. However, this is not due to any district boundaries. It is again purely a function of the political geography of the state since all of these districts are entire county units and are, as such, fixed.

[26] Stokes replaces Yadkin, Henderson and Polk are replaced by McDowell and Cleveland.

Table 31: County Clusters Containing 1 Senate District

County Cluster	# Counties	District #	County Cluster Democratic Partisan Index	Democratic District
Clusters Used by Both Enacted and Duchin Plans				
Johnston	1	10	0.37	0
Onslow	1	6	0.34	0
Rowan-Stanly	2	33	0.31	0
Edgecombe-Pitt	2	5	0.57	1
Davidson-Davie	2	30	0.27	0
Caswell-Orange-Person	3	23	0.66	1
Franklin-Nash-Vance	3	11	0.51	1
Beaufort-Craven-Lenoir	3	3	0.42	0
Hoke-Robeson-Scotland	3	24	0.51	1
Greene-Wayne-Wilson	3	4	0.48	0
Clusters Used by Enacted Plan				
Henderson-Polk-Rutherford	3	48	0.36	0
Alexander-Surry-Wilkes-Yadkin	4	36	0.24	0
Carteret-Chowan-Halifax-Hyde-Martin-Pamlico-Warren-Washington	8	2	0.46	0
Bertie-Camden-Currituck-Dare-Gates-Hertford-Northampton-Pasquotank-Perquimans-Tyrrell	10	1	0.47	0
Alternative Clusters Used by Duchin Plan				
Cleveland-McDowell-Rutherford	3	47	0.32	0
Alexander-Stokes-Surry-Wilkes	4	45	0.25	0
Carteret-Chowan-Dare-Hyde-Pamlico-Pasquotank-Perquimans-Washington	8	2	0.39	0
Bertie-Camden-Currituck-Gates-Halifax-Hertford-Martin- Northampton-Tyrrell-Warren	10	1	0.54	1
Total Enacted:	48		0.43	4

Figure 89: **Map of Counties and County Clusters with only 1 Senate District in Enacted Plan**

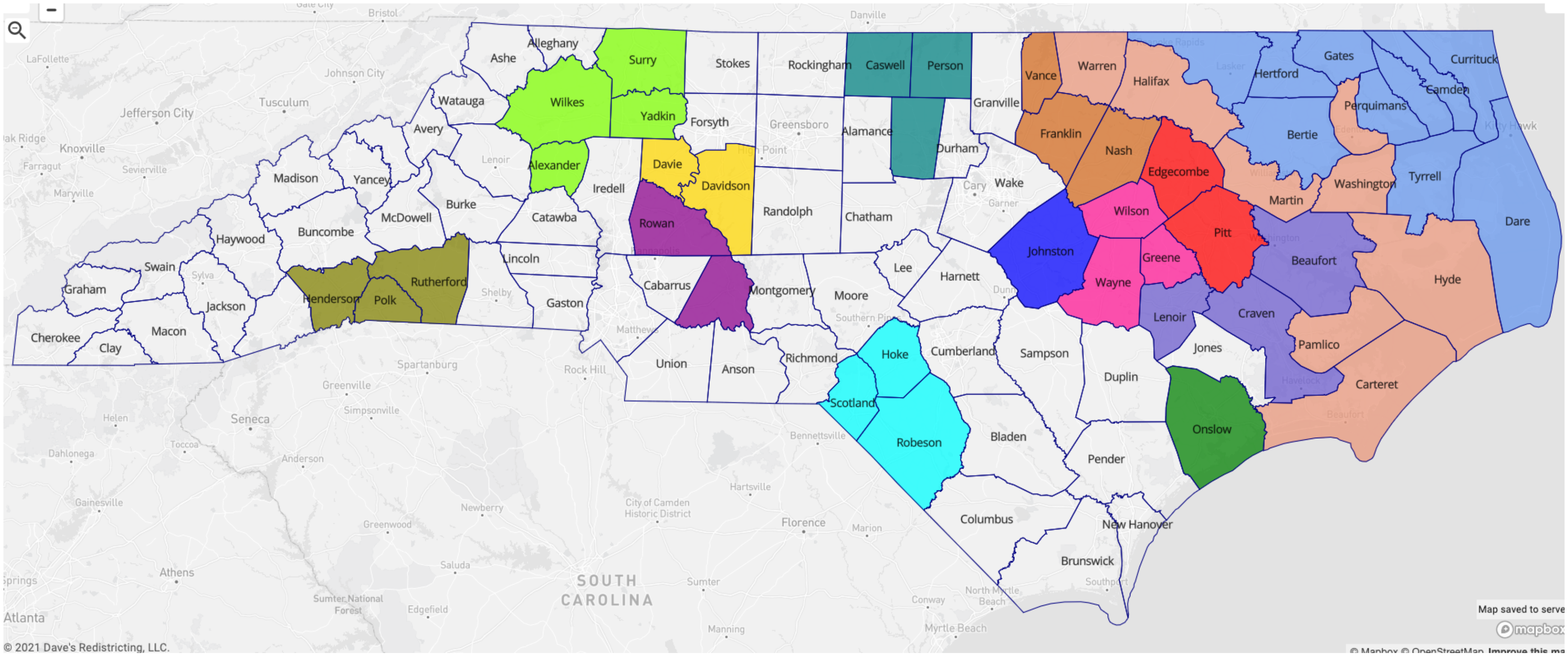
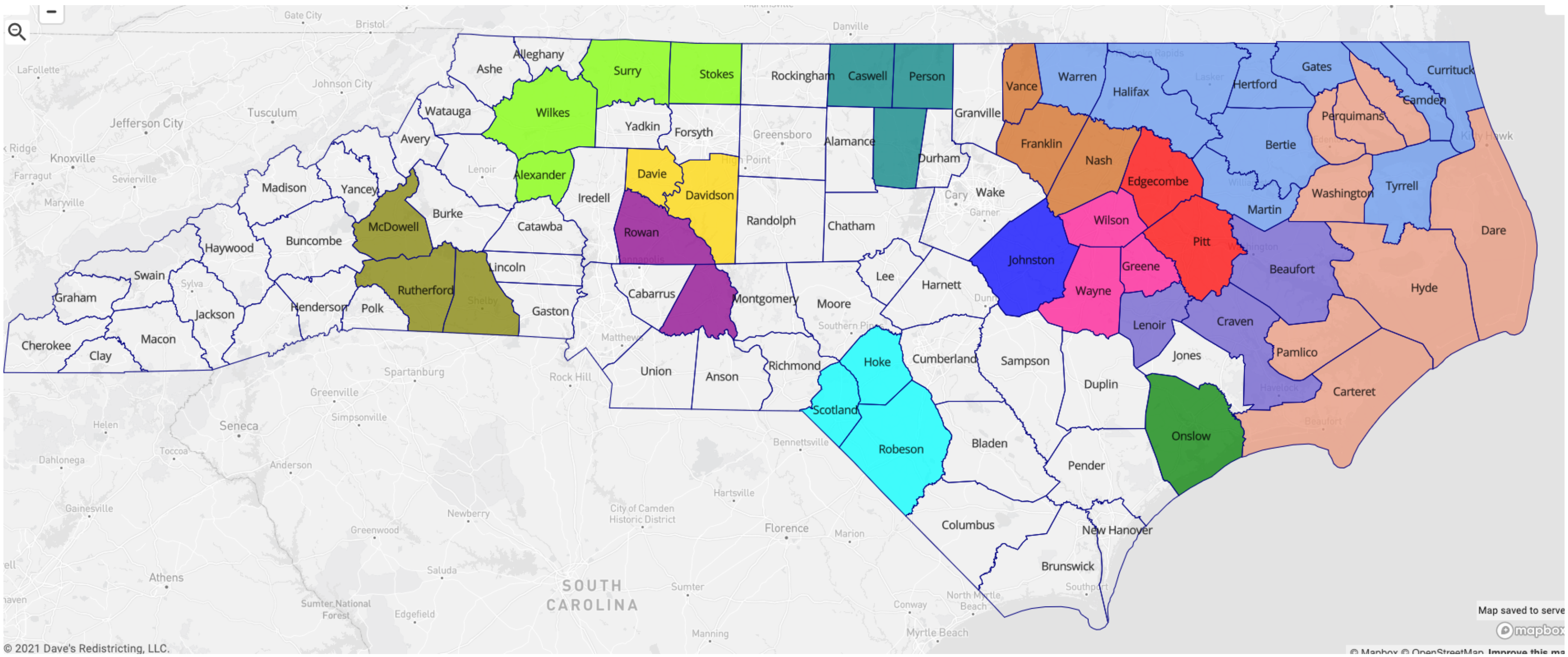


Figure 90: **Map of Counties and County Clusters with only 1 Senate District in Duchin Plan**



8 Senate Groupings with More than 1 District:

There are 12 county groups with more than 1 district where a map drawer has some discretion to draw districts. I consider each cluster separately because the districts are constrained to remain within the county cluster as the redistricting process is North Carolina is a series of discrete redistricting problems within each county cluster.

I conduct simulations in the 12 clusters that contain more than one Senate district. These clusters collectively account for 36 of the 50 districts in the North Carolina Senate. In the Enacted Plan, 20 of these districts lean Republican and 16 lean Democratic according to the statewide partisan elections index. In addition to calculating the number of Democratic leaning districts for the Enacted Plan, I also compute the same partisan index for the plaintiffs' Duchin Plan and compare how the Enacted Plan and the Duchin Plan perform on this same metric. The Duchin Plan creates 17 districts that lean Republican and 19 districts that lean Democratic according to the statewide partisan elections index in these districts.

I then place both maps in relation to the distribution of partisan outcomes from the simulated districts. In each cluster I consider the number of Democratic districts generated by each plan in comparison to the distribution of results from the simulations. I consider a plan to be a partisan outlier if the number of Democratic districts generated by the plan falls outside the middle 50% of simulation results. This is a conservative definition of an outlier. In the social sciences, medicine, and other disciplines it is traditional to consider something an outlier if it falls outside the middle 95% or 90% of the comparison distribution.

In the Senate, the Duchin Map chooses a different set of county clusters from those that have an alternative option presented in the Cooper et al. (2021, 'Duke Study') report. This occurs in three different county groupings. As a result, in these three different clusters the Duchin Senate Map and the Enacted Senate Map are not comparable because they use different groupings of counties. I compare the remaining nine clusters that are common between the two proposals. An overview of the results are as follows.

In 10 of the 12 clusters, the Enacted Map produces a number of Democratic districts that falls within the middle 50% of simulation results and are not partisan outliers. Furthermore, the Enacted Map produces the same number of Democratic leaning districts as the modal (most common) number of Democratic leaning districts in the simulations in 10 of the 12 clusters.

In 10 of the 12 clusters, the Duchin Map produces a number of Democratic districts that fall within the middle 50% of simulation results and are not partisan outliers. Furthermore, the Duchin Map produces the same number of Democratic leaning districts as the modal (most common) number of Democratic leaning districts in the simulations in 10 of the 12 clusters.

In 6 of the 9 clusters that are common between the Enacted Map and the Duchin Map there is agreement between the two plans on the number of Democratic leaning districts.[27] This means there is disagreement in 4 of the 26 total clusters. Table 32 summarizes the results of the simulation analysis for the 12 Senate clusters with multiple districts. Figure 91 shows a map of the counties where the Enacted Plan and the Duchin Plan are in agreement on the number of Democratic leaning seats. Figure 92 shows a map of the counties where the Enacted Plan and the Duchin Plan disagree on the number of Democratic leaning seats.

Thereafter, I present the results cluster-by-cluster.

[27] These groupings are: Cumberland-Moore, Chatham-Durham, Alleghany et al., Brunswick-Columbus-New Hanover, Bladen et al., Alamance et al., and the combination of Buncombe, Burke, McDowell, Cleveland, Gaston, Lincoln, Henderson, Polk, Forsyth, Stokes, and Yadkin into four different groupings.

Table 32: Senate County Grouping Analysis Summary

			# of Districts that are Democratic Leaning		
County Cluster	**Cluster Democratic Partisan Index**	**# Districts**	**Enacted Map**	**Duchin Map**	**Simulations**
Clusters Used by both Enacted and Duchin Plans					
Cumberland-Moore	0.52	2	1	1	1
Chatham-Durham	0.75	2	2	2	2
Alleghany-Ashe-Avery-Caldwell-Catawba-Cherokee-Clay-Graham-Haywood-Jackson-Macon-Madison-Mitchell-Swain-Transylvania-Watauga-Yancy	0.36	2	0	0	0
Brunswick-Columbus-New Hanover	0.45	2	1	1	1
Bladen-Duplin-Harnett-Jones-Lee-Pender-Sampson	0.41	2	0	0	0
Guilford-Rockingham	0.57	3	2	**3**	2
Alamance-Anson-Cabarrus-Montgomery-Randolph-Richmond-Union	0.38	4	0	0	0
Granville-Wake	0.61	6	**4**	**5**	6
Iredell-Mecklenburg	0.60	6	**4**	5	5
Clusters Used by Enacted Plan					
Buncombe-Burke-McDowell	0.51	2	1		1
Cleveland-Gaston-Lincoln	0.34	2	0		0
Forsyth-Stokes	0.52	2	1		1
Alternative Clusters Used by Duchin Plan					
Buncombe-Henderson-Polk	0.54	2		1	1
Burke-Gaston-Lincoln	0.34	2		0	0
Forsyth-Yadkin	0.54	2		1	1
Total:		35	16	19	19

Note: Number of Democratic leaning districts is measured using the average two-party vote share in each district from the 11 statewide races noted earlier. Simulations range represents the middle 50% of outcomes from the simulations results. Clusters that fall outside of the simulation range are bolded.

Figure 91: **Map of Senate Counties Where Enacted and Duchin Plans Agree on Partisan Lean of Districts**

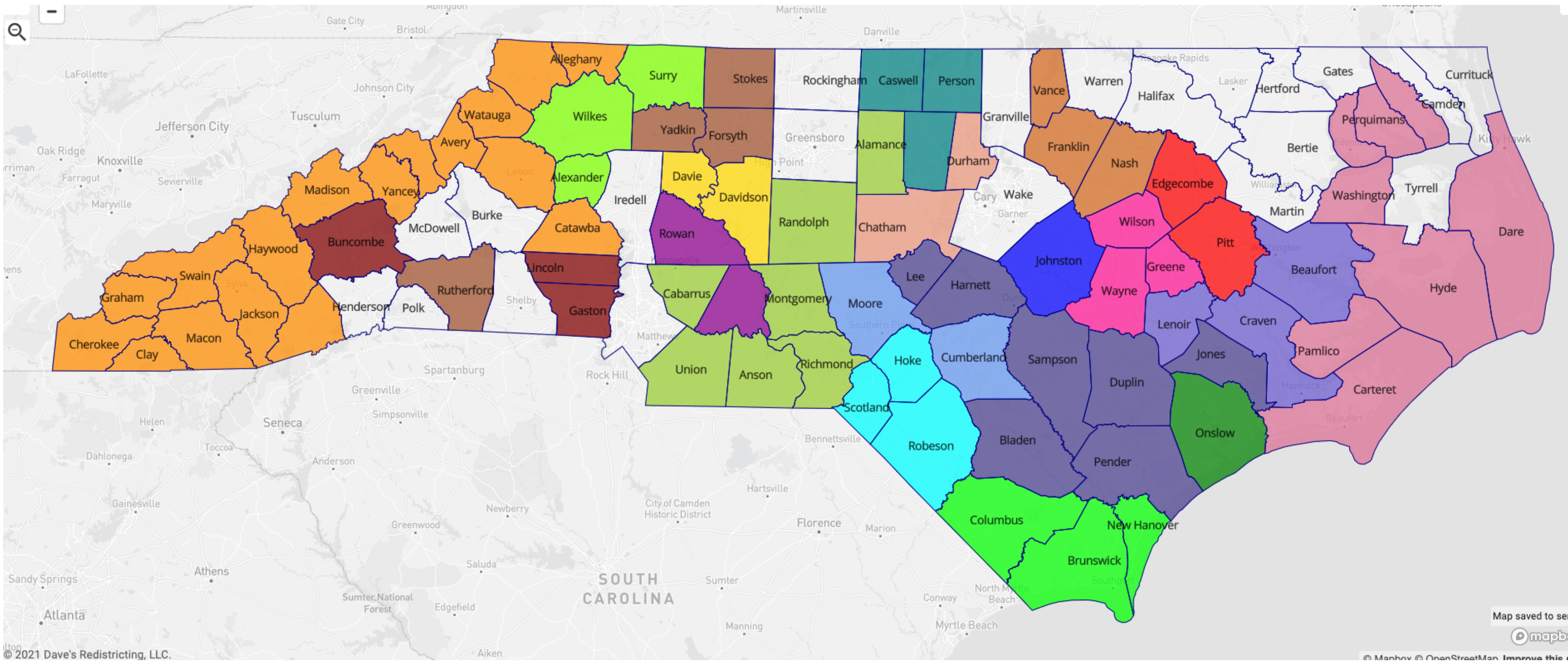
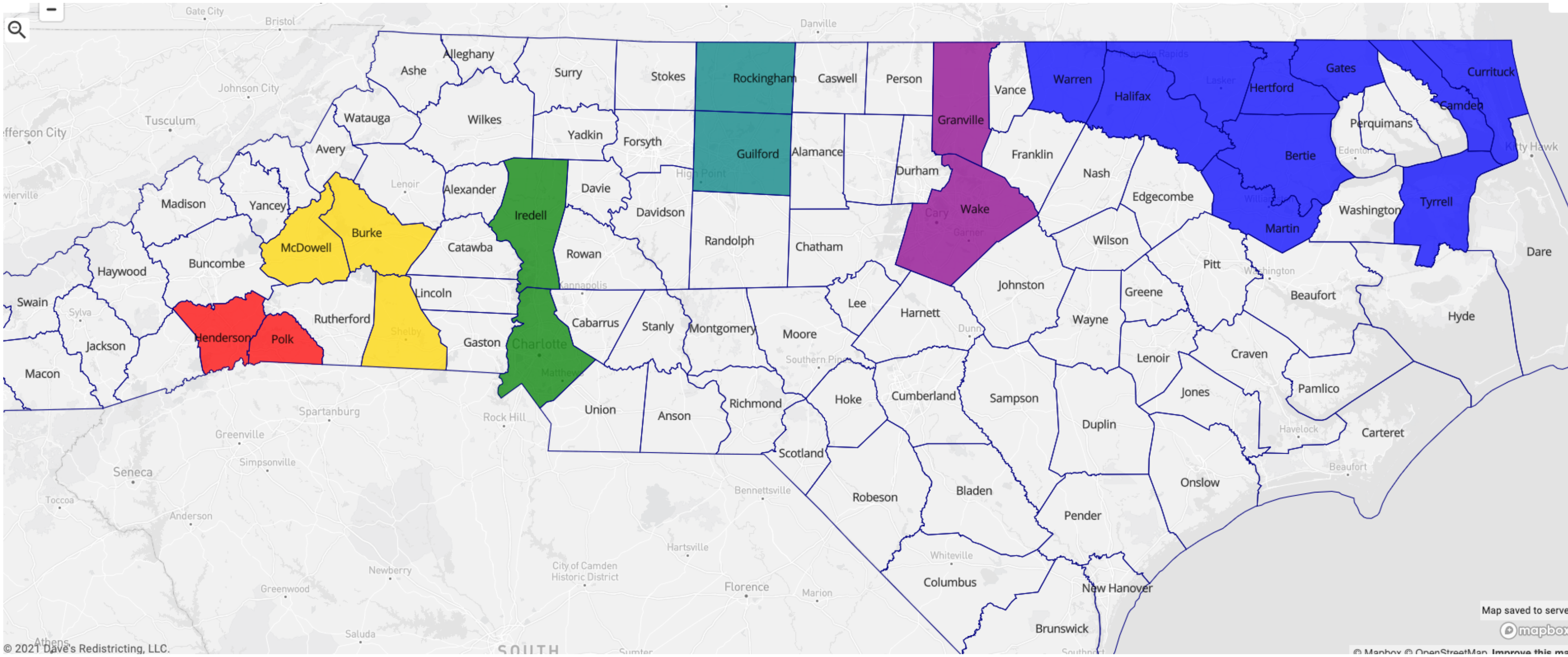


Figure 92: **Map of Senate Counties Where Enacted and Duchin Plans Disagree on Partisan Lean of Districts**



8.1 Cumberland and Moore Senate County Grouping

The Cumberland-Moore Senate county group contains 2 districts. In the Enacted Map these are Districts 19 and 21. The county cluster has an overall partisan index of .52, which is slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. All 50,000 simulations meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 42,625 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 93. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 94.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 95. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 77% of the simulations there is 1 Democratic leaning district. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 1 Democratic district. The Duchin Map also generates 1 Democratic district.

Table 33 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 10 of the 11 individual elections there is agreement between the modal outcome in the simulations and the Enacted Map.

Figure 93: **Map of Cumberland and Moore Senate County Cluster**

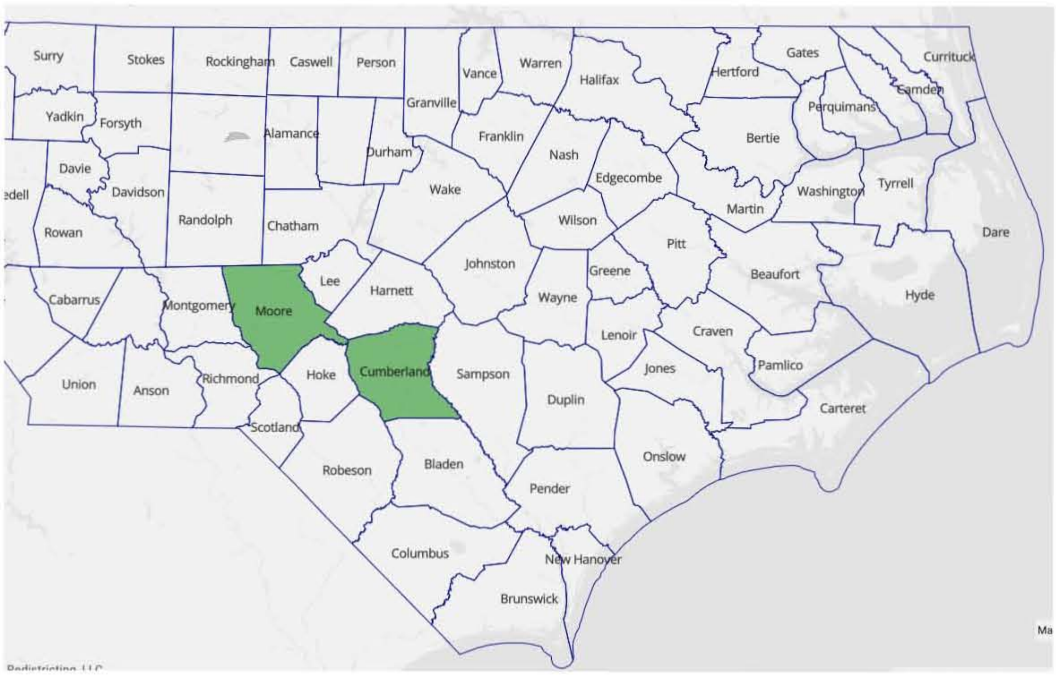
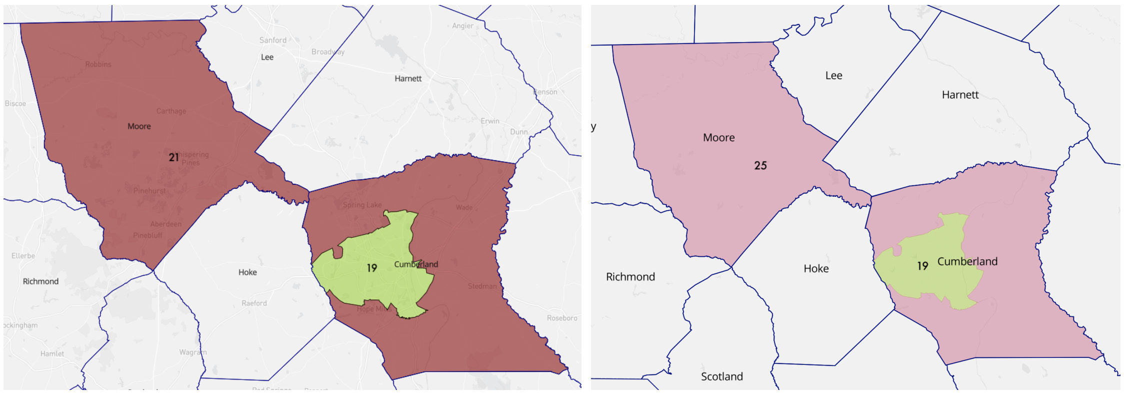


Figure 94: **Map of Enacted Plan in Cumberland and Moore Senate County Cluster**

(a) Enacted Map (b) Duchin Map

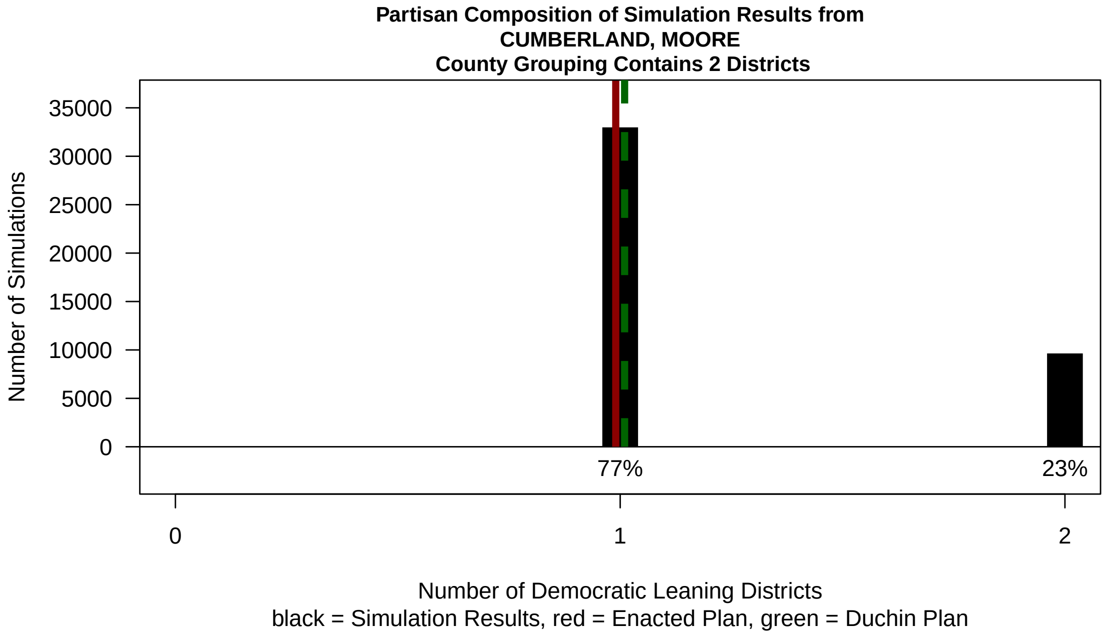


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
19	0.66	0.66
25 (21 in Duchin)	0.40	0.40

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 95: **Distribution of Partisan Districts from Simulations in Cumberland and Moore Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 33: Simulation Results by Individual Elections

Cumberland and Moore Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	**82%**	18%
2020 Senate	0%	**91%**	9%
2020 Governor	0%	**7%**	93%
2020 Lt. Governor	0%	**94%**	6%
2020 Attorney General	0%	**58%**	42%
2016 President	0%	**84%**	16%
2016 Senate	0%	**97%**	3%
2016 Governor	0%	**71%**	29%
2016 Lt. Governor	0%	**99%**	1%
2016 Attorney General	0%	**57%**	43%
2014 Senate	0%	**96%**	4%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 82% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

8.2 Chatham and Durham Senate County Grouping

The Chatham-Durham Senate county group contains 2 districts. In the Enacted Map these are Districts 20 and 22. The county cluster has an overall partisan index of .75, which is strongly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 49,721 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 1,750 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 96. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 97.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 98. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 2 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 2 Democratic leaning districts. The Duchin Map also generates 2 Democratic leaning districts.

Table 34 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In all 11 of the 11 individual elections there is agreement between the modal outcome in the simulations and the Enacted Map.

Figure 96: **Map of Chatham and Durham Senate County Cluster**

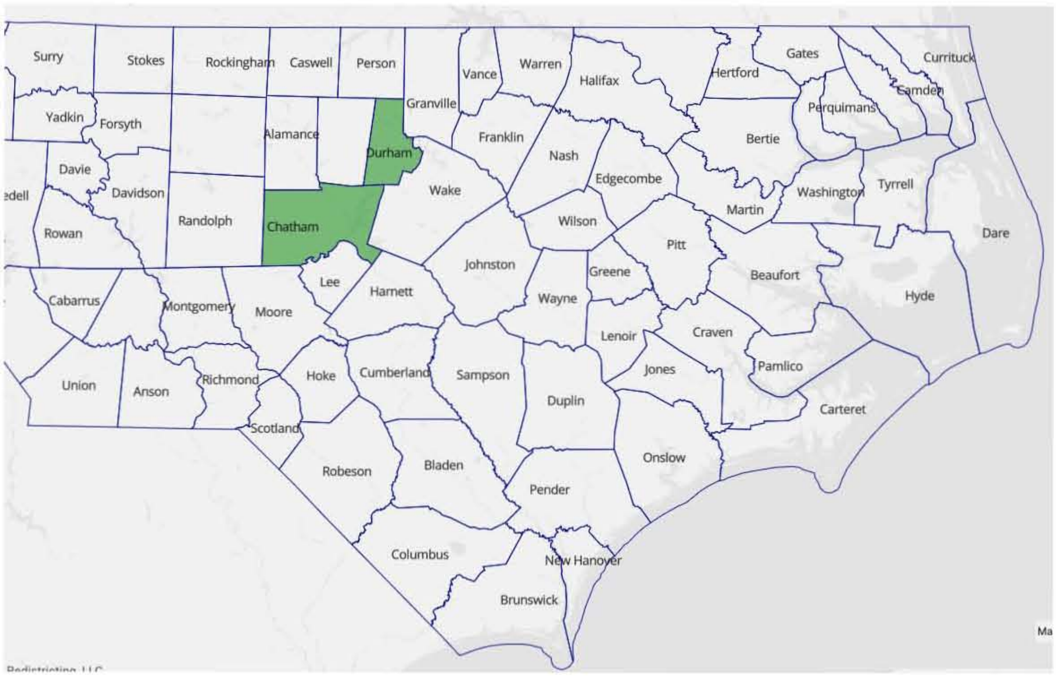
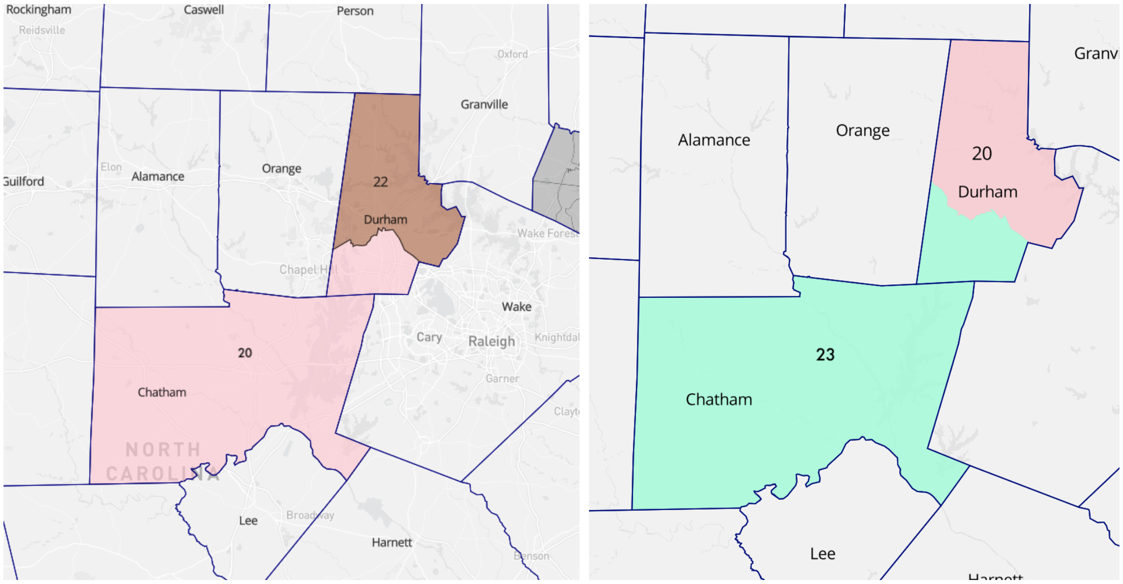


Figure 97: **Map of Enacted Plan in Chatham and Durham Senate County Cluster**

(a) Enacted Map (b) Duchin Map

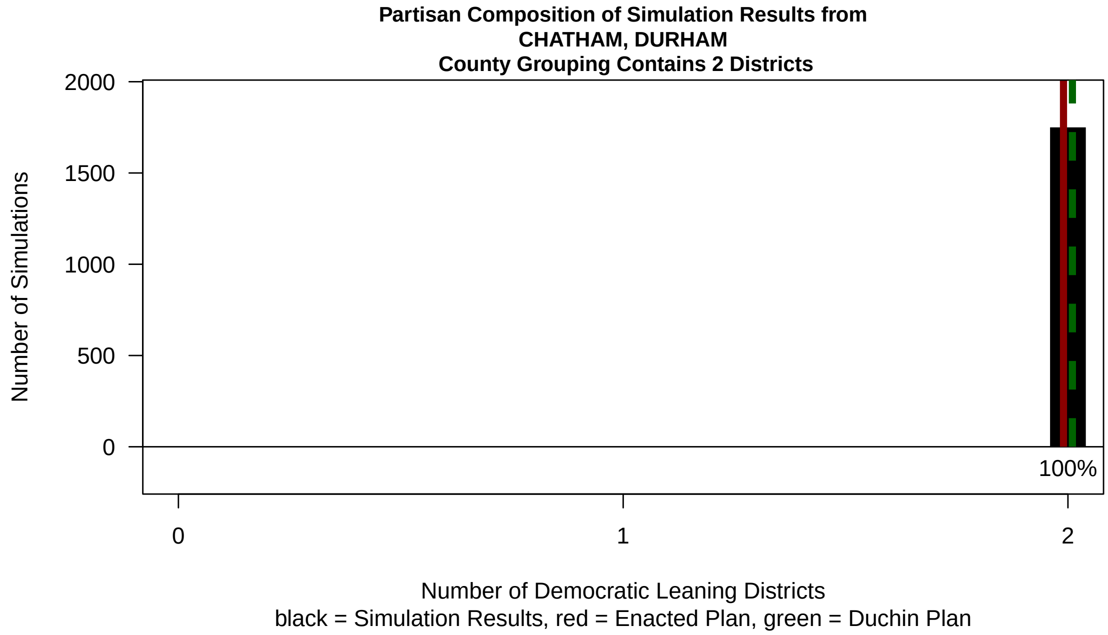


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
20 (23 in Duchin)	0.72	0.71
22 (20 in Duchin)	0.79	0.79

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 98: **Distribution of Partisan Districts from Simulations in Chatham and Durham Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 34: Simulation Results by Individual Elections

Chatham and Durham Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	0%	**100%**
2020 Senate	0%	0%	**100%**
2020 Governor	0%	0%	**100%**
2020 Lt. Governor	0%	0%	**100%**
2020 Attorney General	0%	0%	**100%**
2016 President	0%	0%	**100%**
2016 Senate	0%	0%	**100%**
2016 Governor	0%	0%	**100%**
2016 Lt. Governor	0%	0%	**100%**
2016 Attorney General	0%	0%	**100%**
2014 Senate	0%	0%	**100%**

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 2 Democratic leaning districts. The Enacted Plan does as well, as the '2 Districts' cell is bolded in that row.

8.3 Bladen, Duplin, Harnett, Jones, Lee, Pender, and Sampson Senate County Grouping

The Bladen-Duplin-Harnett-Jones-Lee-Pender-Sampson Senate county grouping contains 2 districts. In the Enacted Map these are Districts 9 and 12. The county cluster has an overall partisan index of 0.41, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. All 50,000 simulated maps meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves only one unique map that is as compact as the Enacted Plan.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 99. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 100.

Because there is only 1 map that fits the criteria I use of equal population, county traversals, and compactness equal to or better than the Enacted Map, I do not present the distribution of district partisanship for the simulations here. It is sufficient to say that in the Enacted Map, the Duchin map, and the remaining simulated map all create 2 Republican districts and 0 Democratic leaning districts, regardless of the index or election used. Table 35 shows this below.

Figure 99: **Map of Bladen, Duplin, Harnett, Jones, Lee, Pender, and Sampson Senate County Cluster**

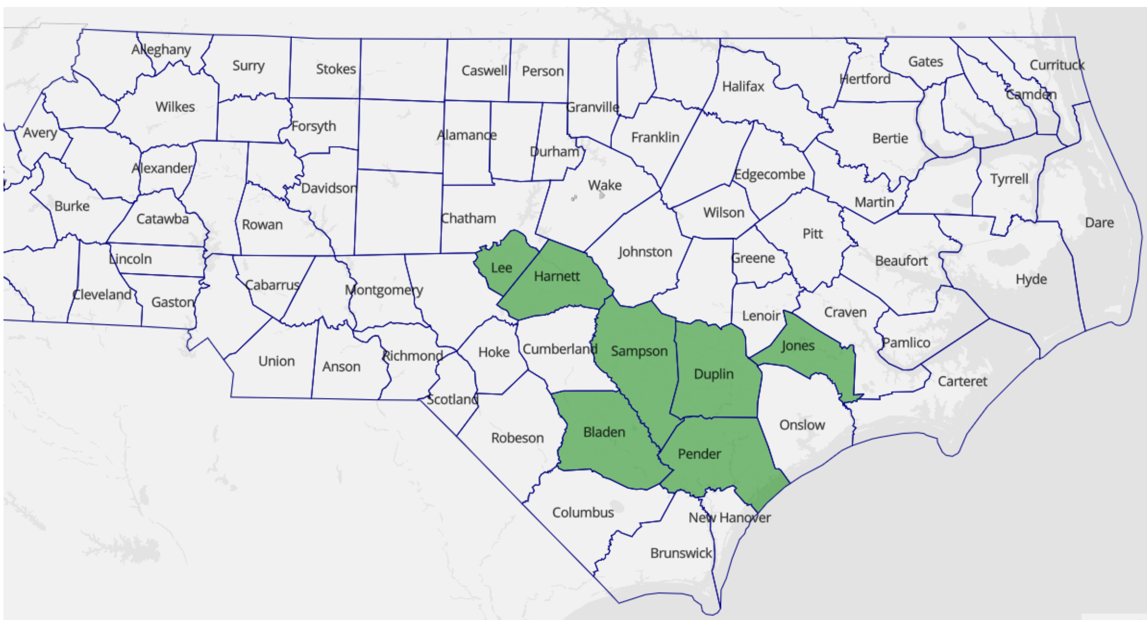
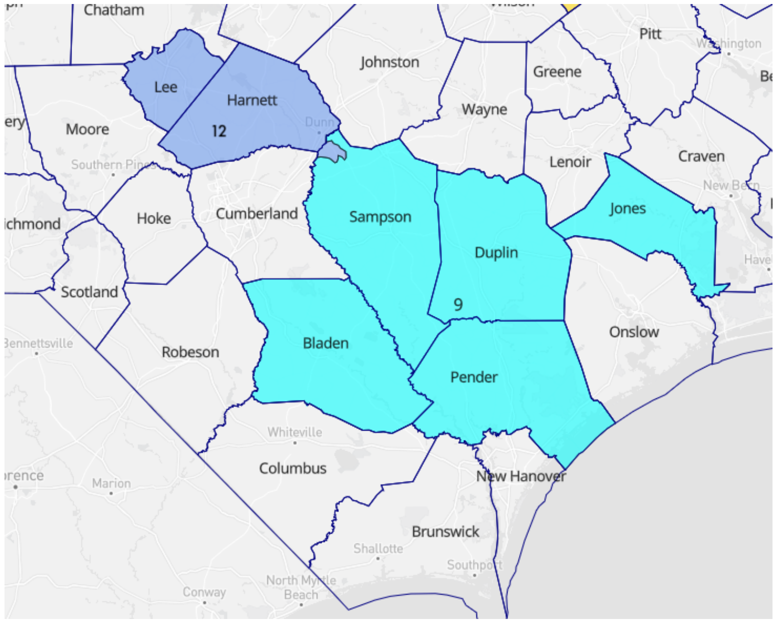
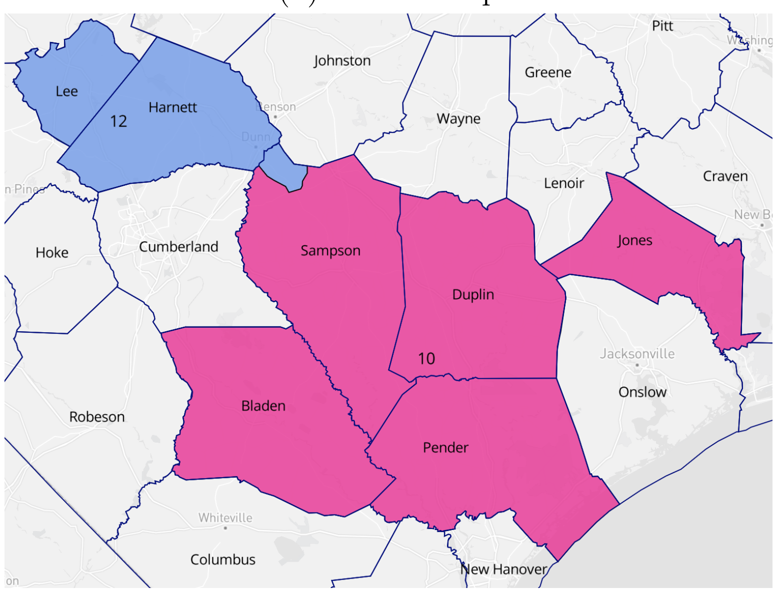


Figure 100: **Map of Enacted Plan in Bladen, Duplin, Harnett, Jones, Lee, Pender, and Sampson Senate County Cluster**

(a) Enacted Map



(b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
9 (10 in Duchin)	0.40	0.41
12	0.41	0.41

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Table 35: Simulation Results by Individual Elections

Bladen, Duplin, Harnett, Jones, Lee, Pender, and Sampson Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

8.4 Brunswick, Columbus, and New Hanover Senate County Grouping

The Brunswick-Columbus-New Hanover Senate county group contains 2 districts. In the Enacted Map these are Districts 7 and 8. The county cluster has an overall partisan index of .45, which is Republican leaning. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 31,037 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 30,499 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 101. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 102.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 103. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 77% of the simulations there is 1 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 1 Democratic leaning district. The Duchin Map also generates 1 Democratic leaning district.

Table 36 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 9 of the 11 individual elections there is agreement between the modal outcome in the simulations and the Enacted Map. In all 11 of the 11 individual elections the Enacted Plan falls within the middle 50% of the simulation results.

Figure 101: **Map of Brunswick, Columbus, and New Hanover Senate County Cluster**

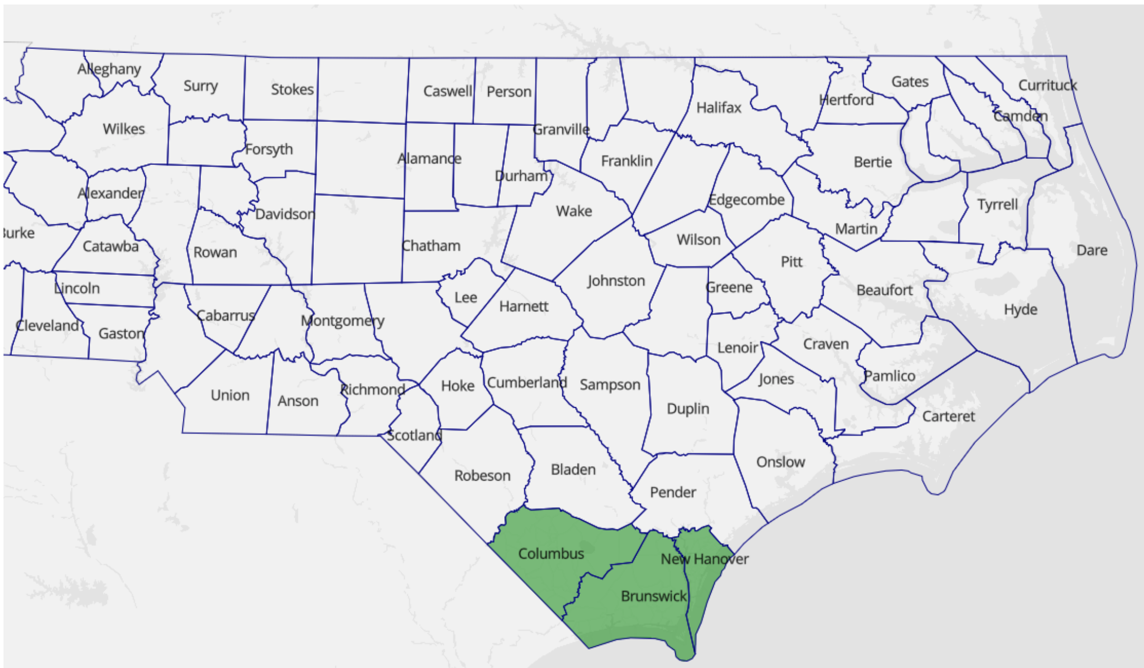
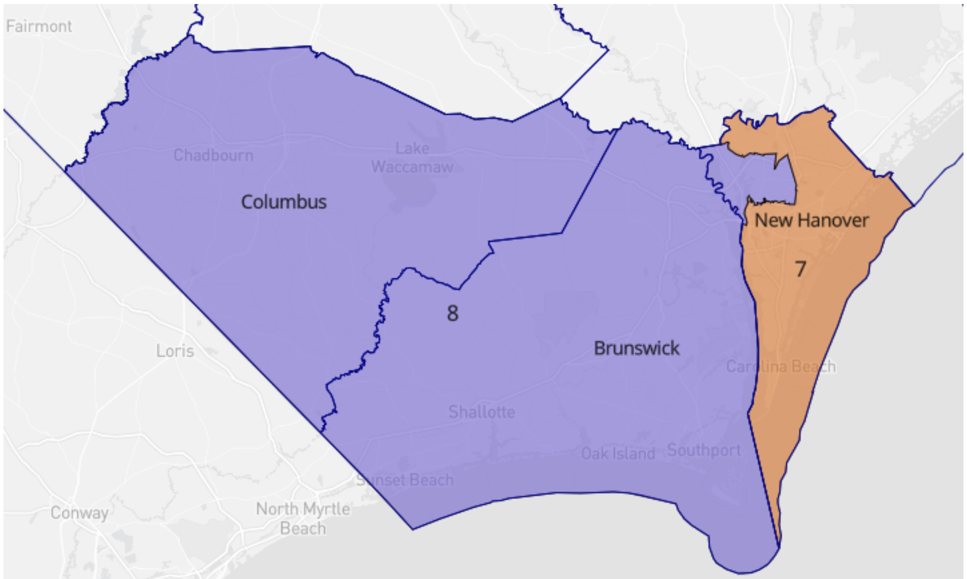
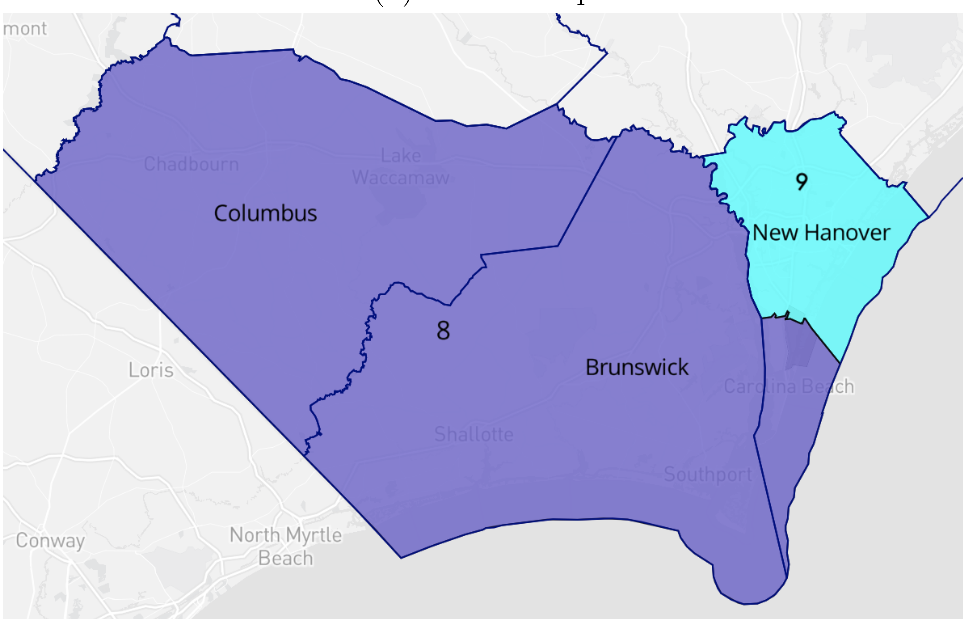


Figure 102: **Map of Enacted Plan in Brunswick, Columbus, and New Hanover Senate County Cluster**

(a) Enacted Map



(b) Duchin Map

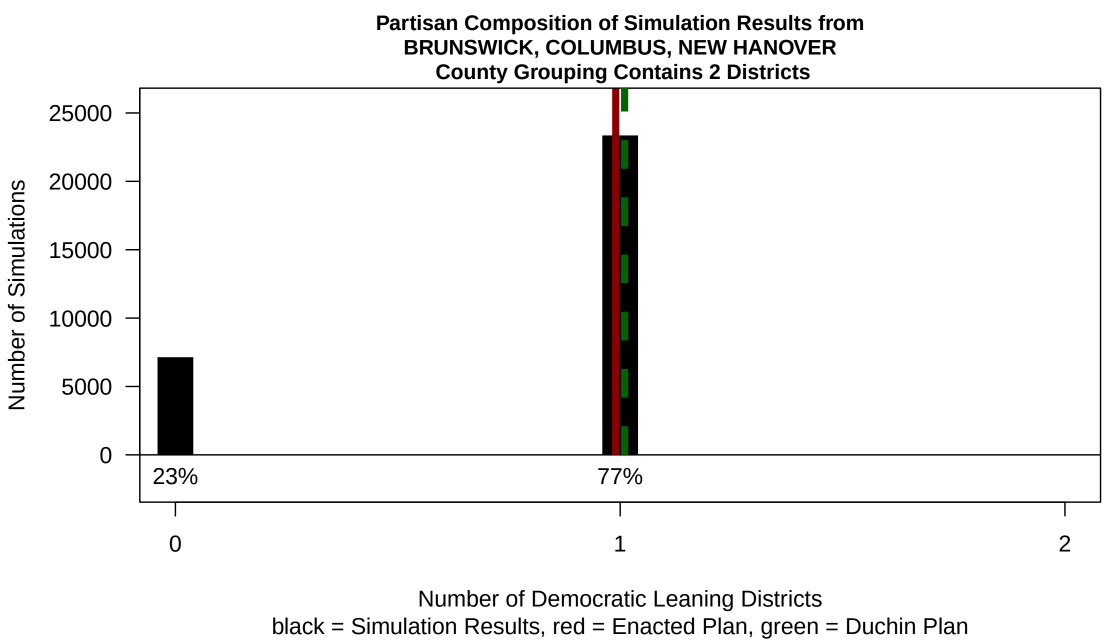


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
7 (9 in Duchin)	0.50	0.52
8	0.39	0.39

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 103: **Distribution of Partisan Districts from Simulations in Brunswick, Columbus, and New Hanover Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 36: Simulation Results by Individual Elections

Brunswick, Columbus, and New Hanover County Senate Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	13%	**87%**	0%
2020 Senate	24%	**76%**	0%
2020 Governor	0%	**100%**	0%
2020 Lt. Governor	**28%**	72%	0%
2020 Attorney General	7%	**93%**	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	3%	**97%**	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	16%	**84%**	0%
2014 Senate	**26%**	74%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 87% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

8.5 Alleghany, Ashe, Avery, Caldwell, Catawba, Cherokee, Clay, Graham, Haywood, Jackson, Macon, Madison, Mitchell, Swain, Transylvania, Watauga, and Yancey Senate County Grouping

The Alleghany-et al. Senate county group contains 3 districts. In the Enacted Map these are Districts 47, 45, and 50. The county cluster has an overall partisan index of .35, which is strongly Republican. After conducting 50,000 initial simulations to create three districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 37,454 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 22,065 simulated maps, each containing three districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 104. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 105.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 106. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic leaning districts. The Duchin Map also generates 0 Democratic leaning districts.

Table 37 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the 11 individual elections there is agreement between the modal outcome in the simulations and the Enacted Map.

Figure 104: **Map of Alleghany, Ashe, Avery, Caldwell, Catawba, Cherokee, Clay, Graham, Haywood, Jackson, Macon, Madison, Mitchell, Swain, Transylvania, Watauga, and Yancey Senate County Cluster**

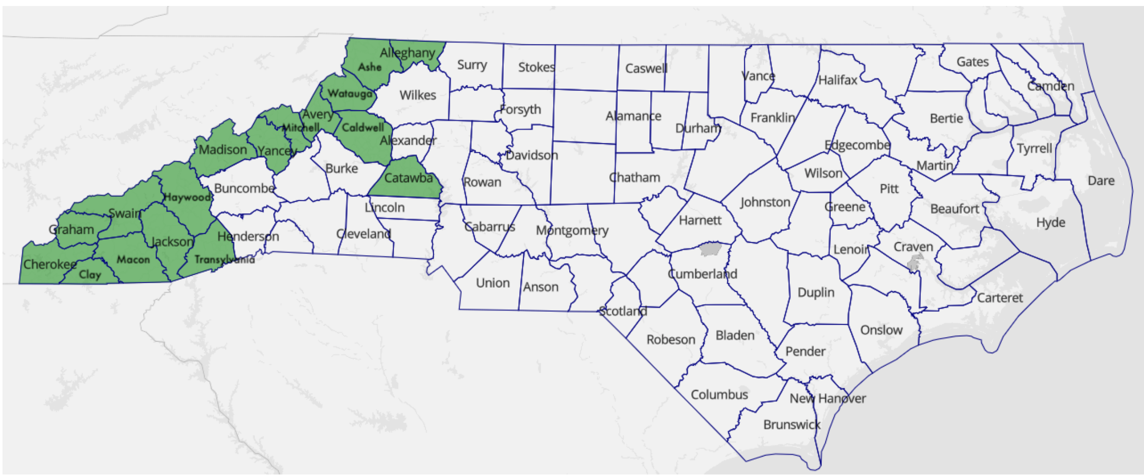
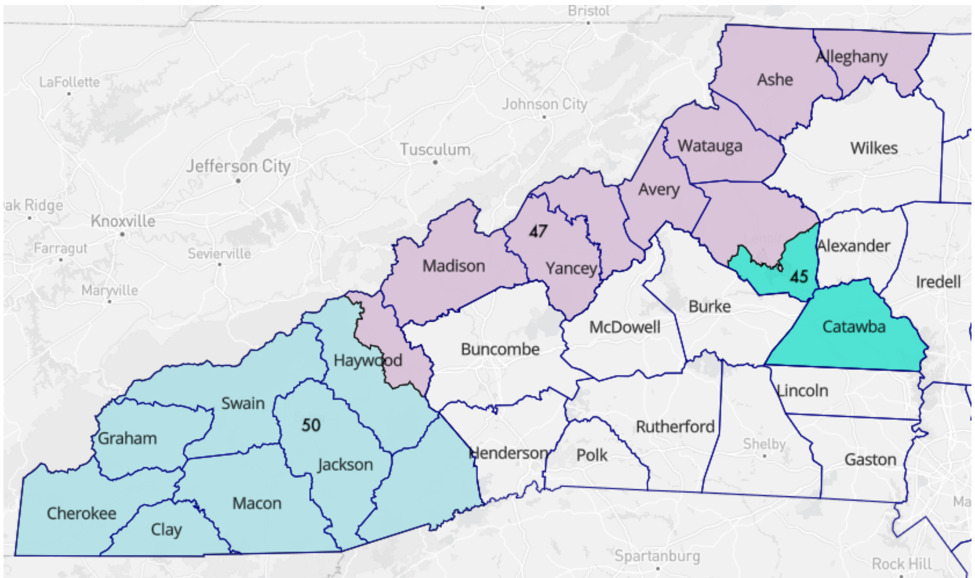
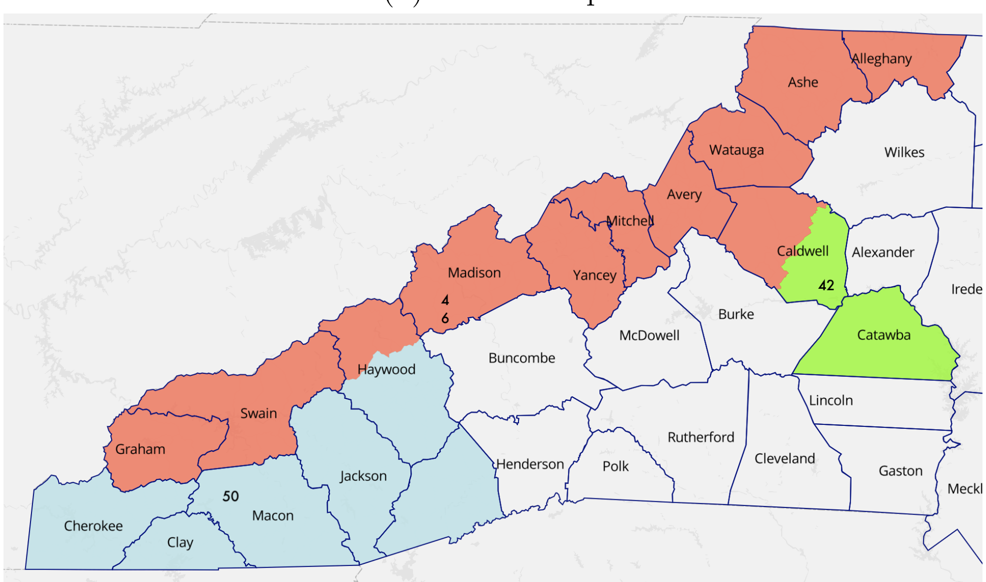


Figure 105: **Map of Enacted Plan in Alleghany, Ashe, Avery, Caldwell, Catawba, Cherokee, Clay, Graham, Haywood, Jackson, Macon, Madison, Mitchell, Swain, Transylvania, Watauga, and Yancey Senate County Cluster**

(a) Enacted Map



(b) Duchin Map

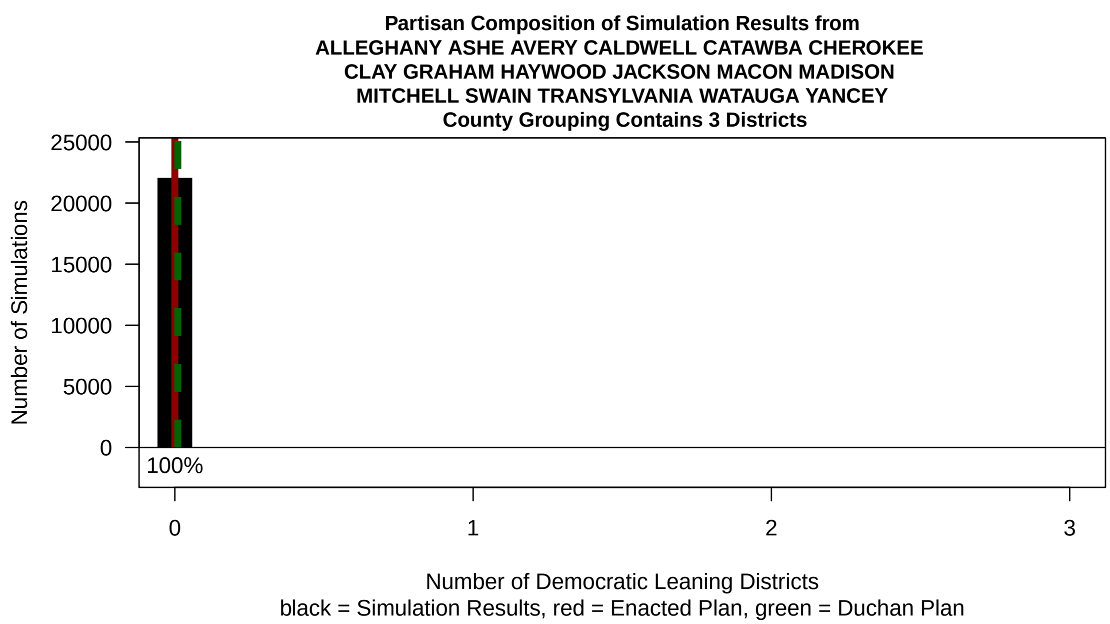


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
45 (42 in Duchin)	0.30	0.30
47 (46 in Duchin)	0.37	0.38
50	0.37	0.37

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 106: **Distribution of Partisan Districts from Simulations in Alleghany, Ashe, Avery, Caldwell, Catawba, Cherokee, Clay, Graham, Haywood, Jackson, Macon, Madison, Mitchell, Swain, Transylvania, Watauga, and Yancey Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 37: Simulation Results by Individual Elections

Alleghany, Ashe, Avery, Caldwell, Catawba, Cherokee, Clay, Graham, Haywood, Jackson, Macon, Madison, Mitchell, Swain, Transylvania, Watauga, and Yancey Senate County Cluster

	Percentage of Simulations			
Number of Democratic Leaning Districts:	0	1	2	3
Individual Elections:				
2020 President	**100%**	0%	0%	0%
2020 Senate	**100%**	0%	0%	0%
2020 Governor	**100%**	0%	0%	0%
2020 Lt. Governor	**100%**	0%	0%	0%
2020 Attorney General	**100%**	0%	0%	0%
2016 President	**100%**	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%
2014 Senate	**100%**	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 Districts' cell is bolded in that row.

8.6 Guilford and Rockingham Senate County Grouping

The Guilford-Rockingham Senate county group contains 3 districts. In the Enacted Map these are Districts 26, 27, and 28. The county cluster has an overall partisan index of .57, which is solidly Democratic. After conducting 50,000 initial simulations to create three districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 37,148 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 24,667 simulated maps, each containing three districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 107. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 108.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 110. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 94% of the simulations there are 2 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 2 Democratic leaning districts. The Duchin Map generates 3 Democratic leaning districts, which only occurs in 6% of the simulations. This is outside the middle 50% of simulations and is a partisan outlier.

Table 39 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Enacted Map.

The Duchin Plan creates three Democratic leaning district by dividing the city of Greensboro, the county seat and largest city in Guilford County, into three relatively equal pieces. The Enacted Plan does not and instead keeps the vast majority of Greensboro in two districts. Most of the Democratic leaning voting in this cluster reside in Greensboro. This "pie" division of Greensboro by the Duchin Plan therefore spread Democratic voters more equally across the three districts. However, it comes at the expense of dividing a city into more districts than necessary. Table 38 shows the division of Greensboro residents across the districts in the two plans. Figure 109 shows a map of the divisions.

Table 38: Division of Greensboro in Enacted Plan and Duchin Plan

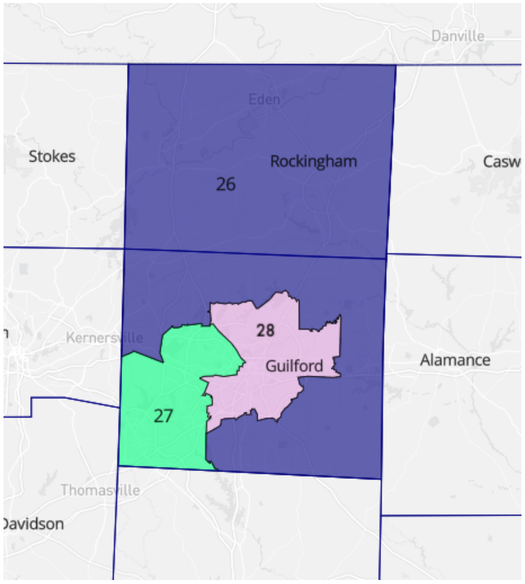
	Percent of Greensboro in district	
District:	Enacted Plan	Duchin Plan
26 (30 in Duchin)	4.3	19.6
27	30.8	20.4
28	64.9	60.0
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: `https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf` Population numbers for city by district for Duchin Plan from Dave's Redistricting online. `https://davesredistricting.org/`

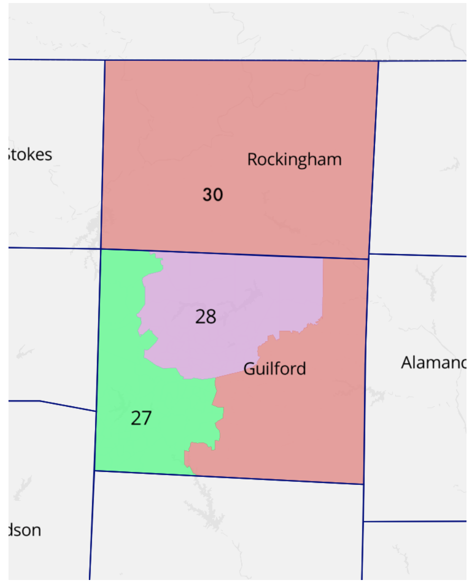
Figure 107: **Map of Guilford and Rockingham Senate County Cluster**

Figure 108: **Map of Enacted Plan in Guilford and Rockingham Senate County Cluster**

(a) Enacted Map



(b) Duchin Map



Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
26 (30 in Duchin)	0.37	0.52
27	0.60	0.58
28	0.77	0.62

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 109: **Map of Greensboro Divisions in Guilford-Rockingham Senate County Cluster**

(a) Enacted Map (b) Duchin Map

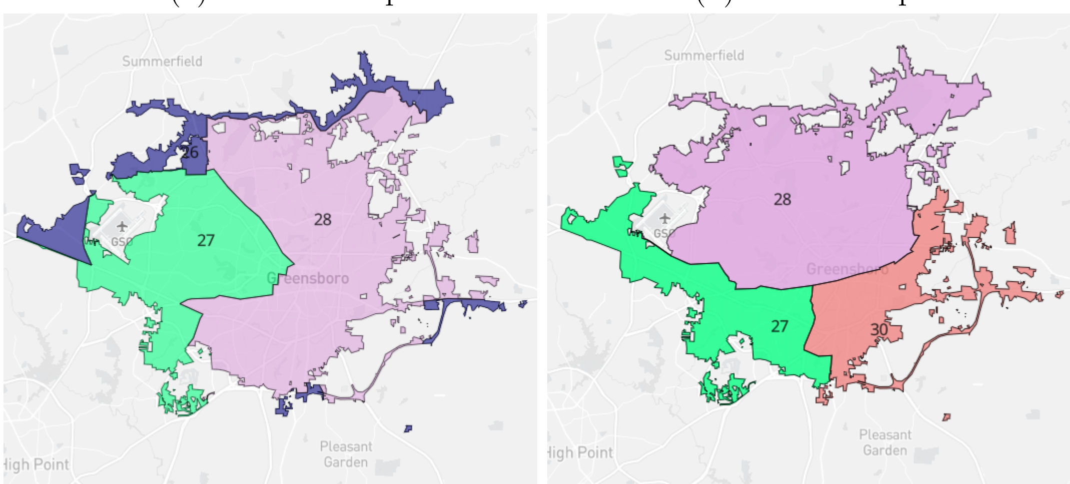
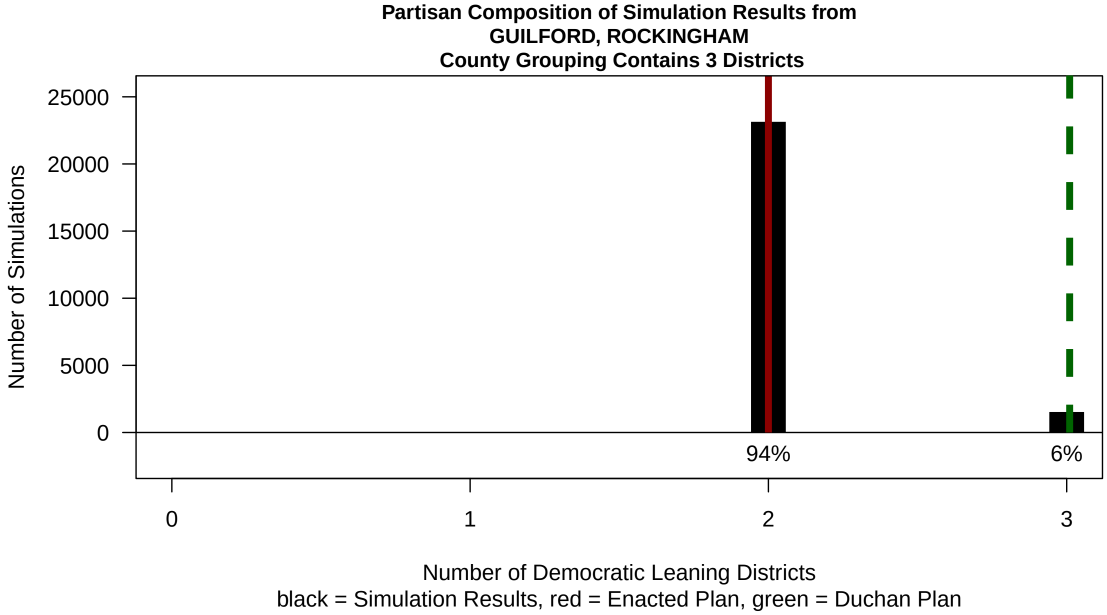


Figure 110: **Distribution of Partisan Districts from Simulations in Guilford and Rockingham Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 39: Simulation Results by Individual Elections

Guilford and Rockingham County Cluster

Number of Democratic Leaning Districts:				
	0	1	2	3
Individual Elections:				
2020 President	0%	0%	**95%**	5%
2020 Senate	0%	0%	**94%**	6%
2020 Governor	0%	0%	**57%**	43%
2020 Lt. Governor	0%	0%	**96%**	4%
2020 Attorney General	0%	0%	**93%**	7%
2016 President	0%	0%	**96%**	4%
2016 Senate	0%	1%	**96%**	3%
2016 Governor	0%	0%	**83%**	17%
2016 Lt. Governor	0%	1%	**96%**	3%
2016 Attorney General	0%	0%	**91%**	9%
2014 Senate	0%	1%	**94%**	5%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 95% of the simulations produce 2 Democratic leaning districts. The Enacted Plan does as well, as the '2 Districts' cell is bolded in that row.

8.7 Alamance, Anson, Cabarrus, Montgomery, Randolph, Richmond, and Union Senate County Grouping

The Alamance-Anson-Cabarrus-Montgomery-Randolph-Richmond-Union Senate county group contains 4 districts. In the Enacted Map these are Districts 25, 29, 34, and 35. The county cluster has an overall partisan index of .38, which is solidly Republican. After conducting 50,000 initial simulations to create four districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 35,298 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 25,747 simulated maps, each containing four districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 111. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 112.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 113. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic leaning districts. The Duchin Map also generates 0 Democratic leaning districts.

Table 40 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Demo-

cratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Enacted Map.

Figure 111: **Alamance, Anson, Cabarrus, Montgomery, Randolph, Richmond, and Union Senate County Cluster**

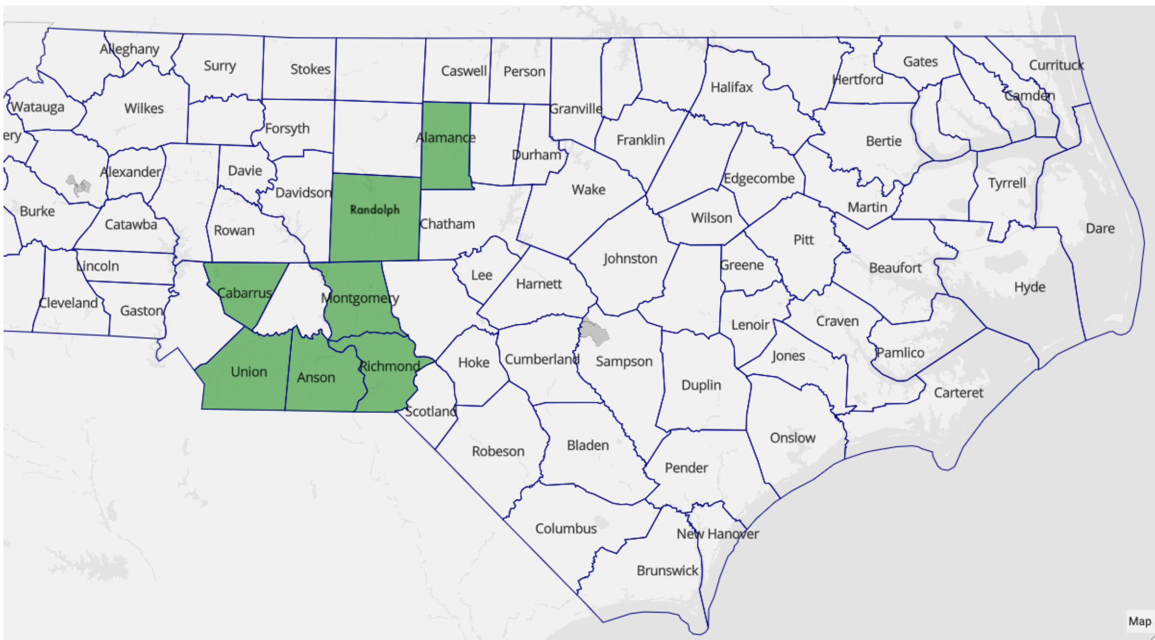
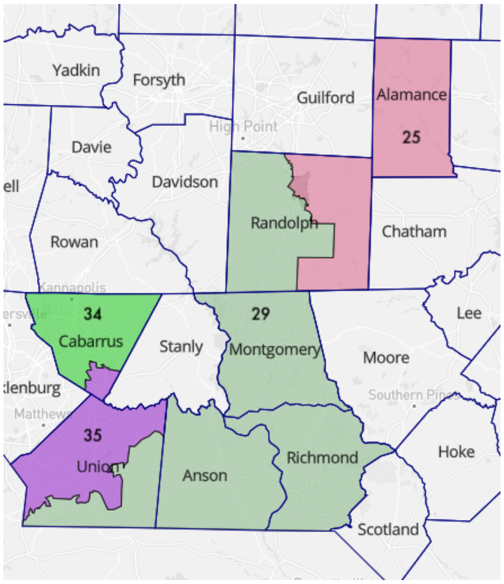
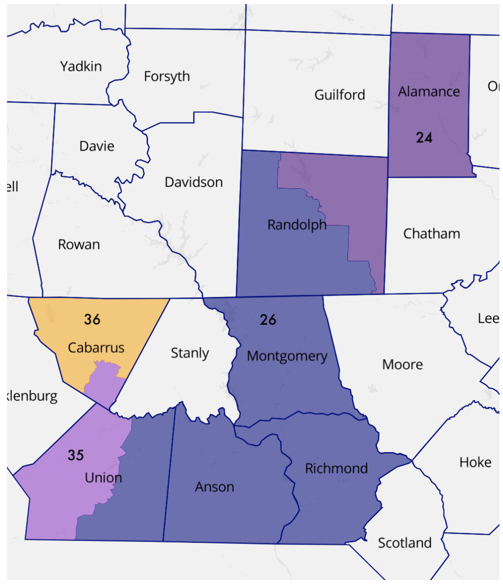


Figure 112: **Map of Enacted Plan in Alamance, Anson, Cabarrus, Montgomery, Randolph, Richmond, and Union Senate County Cluster**

(a) Enacted Map



(b) Duchin Map

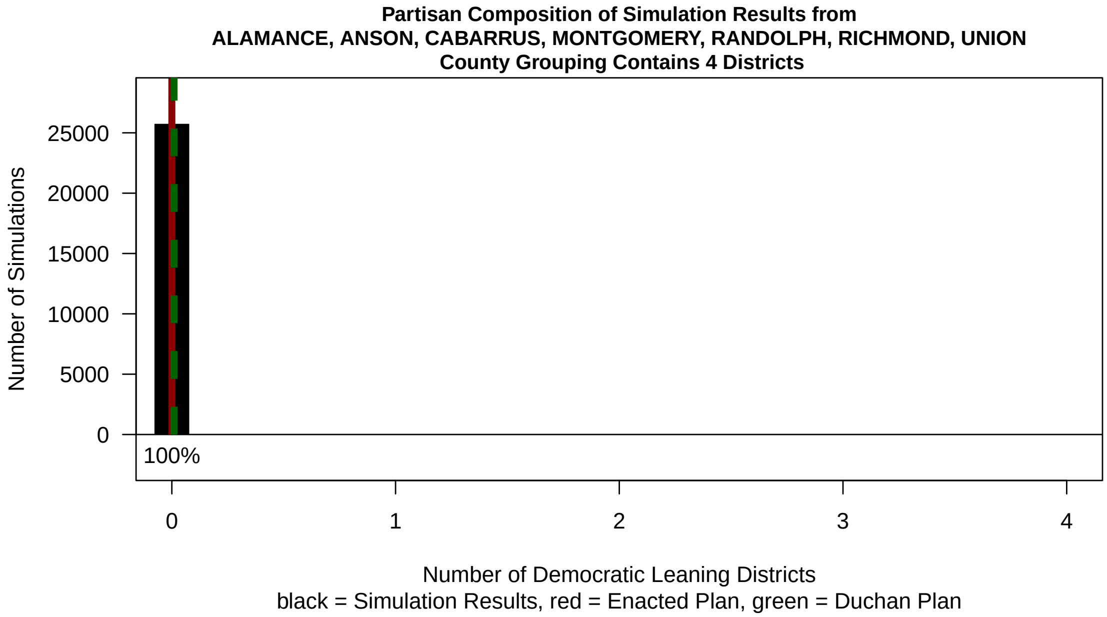


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
25 (24 in Duchin)	0.40	0.40
29 (26 in Duchin)	0.34	0.34
34 (36 in Duchin)	0.44	0.44
35	0.36	0.36

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 113: **Distribution of Partisan Districts from Simulations in Alamance, Anson, Cabarrus, Montgomery, Randolph, Richmond, and Union Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 40: Simulation Results by Individual Elections

Alamance, Anson, Cabarrus, Montgomery, Randolph, Richmond, and Union Senate County Cluster

Number of Democratic Leaning Districts:					
	0	1	2	3	4
Individual Elections:					
2020 President	**100%**	0%	0%	0%	0%
2020 Senate	**100%**	0%	0%	0%	0%
2020 Governor	**100%**	0%	0%	0%	0%
2020 Lt. Governor	**100%**	0%	0%	0%	0%
2020 Attorney General	**100%**	0%	0%	0%	0%
2016 President	**100%**	0%	0%	0%	0%
2016 Senate	**100%**	0%	0%	0%	0%
2016 Governor	**100%**	0%	0%	0%	0%
2016 Lt. Governor	**100%**	0%	0%	0%	0%
2016 Attorney General	**100%**	0%	0%	0%	0%
2014 Senate	**100%**	0%	0%	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 Districts' cell is bolded in that row.

8.8 Granville and Wake Senate County Grouping

The Granville-Wake Senate county group contains 6 districts. In the Enacted Map these are Districts 13, 14, 15, 16, 17, and 18. The county cluster has an overall partisan index of .61, which is solidly Democratic. After conducting 50,000 initial simulations to create six districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 45,850 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 2,835 simulated maps, each containing six districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 114. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 115.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 117. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 1% of the simulations there are 4 Democratic leaning districts. In 24% of the simulations there are 5 Democratic leaning districts, and in 75% of the simulations there are 6 Democratic leaning districts. The Enacted Map generates 4 Democratic leaning districts, which is an outlier from middle 50% of the simulations. The Duchin Map generates 5 Democratic leaning districts and is also classified as a partisan outlier.

Table 42 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Demo-

cratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 10 of the 11 individual elections the Enacted Plan is not in alignment with the middle 50% of the simulation results and is therefore classified as an outlier.

Why is the Enacted Plan such an outlier in this county grouping? There are two factors to consider in explaining this divergence. First, while the Enacted Plan generates 4 solidly Democratic leaning districts, the remaining two districts are not solidly Republican. Instead, they would be best classified as highly competitive. District 13 has a partisan index of 0.481 and District 17 has a partisan index of 0.489. These two districts will likely be very closely decided with candidates from both parties winning them with some regularity, given their narrow margins. This is actually quite close to the partisan lean of the Duchin Plan. While the Duchin Plan creates 5 Democratic leaning districts in the county group, there are also two very competitive districts (District 22 - partisan index of 0.499 and District 17 - partisan index of 0.505). It just happens that one of the competitive districts is just over the .50 line and is classified as Democratic leaning. Thus, both plans generate 4 solidly Democratic districts and 2 highly competitive districts. The Duchin Plan's competitive districts are just slightly more Democratic by roughly 1.7 percentage points.

The second factor to consider is that the Enacted Plan divides the city of Raleigh and groups other municipalities differently from the Duchin Plan, which has the impact of placing a greater share of its residents in fewer districts. For example, District 13 keeps the cities of Wake Forest, Rolesville, and Zebulon together in one district. Additionally, the Enacted Plan places more of Raleigh into fewer districts. This is ideal if one is trying to keep municipalities together and spread across as few districts as possible. However, because the bulk of Democratic leaning voters in this county cluster are also in the city of Raleigh, this will have the effect of creating districts that are more heavily Democratic. This, of course, has the spillover effect of making the districts that do not contain portions of Raleigh to likewise become more Republican. Figure 116 shows how the two different plans divide the city of Raleigh, and Table 41 shows that it is the case the the Duchin Plan spreads the resident of Raleigh out across more districts than does the Enacted Plan. The tactic of dividing Democratic cities in a 'pinwheel' or 'pizza' shape and grouping those 'slices' with more Republican suburban and exurban areas is a classic tactic to generate more Democratic districts and overcome the geographic clustering that is common among Democratic voters. The Enacted Plan keeps much more of Fayetteville within three districts.

Table 41: Division of Raleigh in Enacted Plan and Duchin Plan

	Percent of Raleigh in district	
District:	Enacted Plan	Duchin Plan
13 (22 in Duchin)	1.7	12.3
14	21.1	27.0
15	35.8	39.6
16	0	0
17	0	0
18	41.0	20.8
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: `https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf` Population numbers for city by district for Duchin Plan from Dave's Redistricting online. `https://davesredistricting.org/`

Figure 114: **Granville and Wake Senate County Cluster**

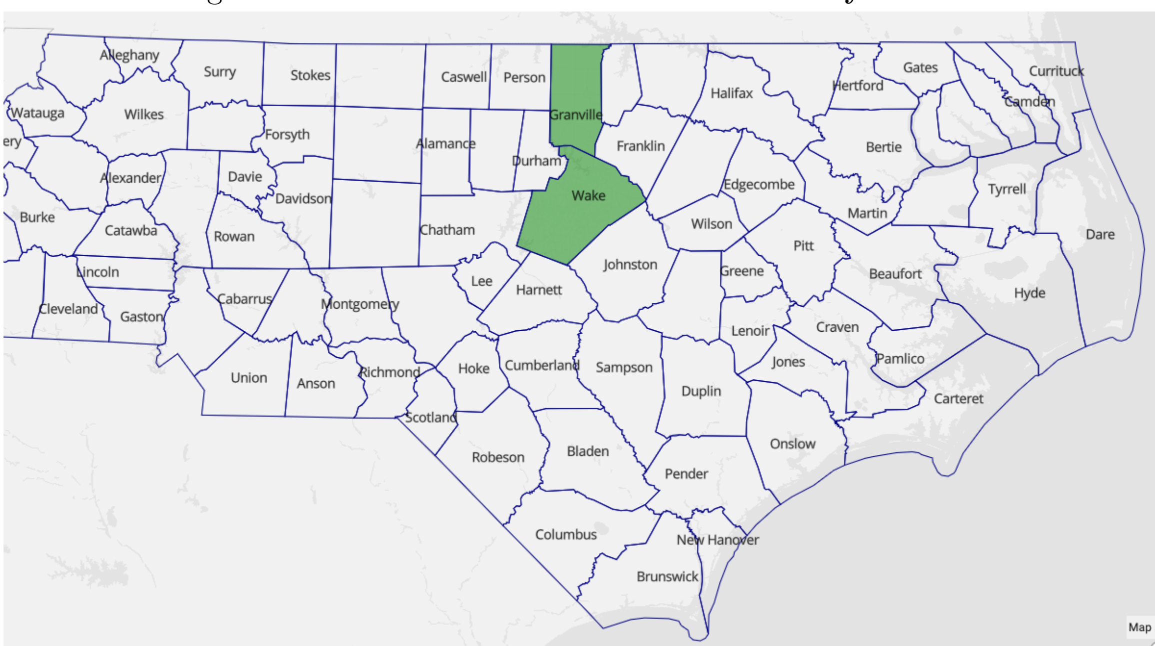
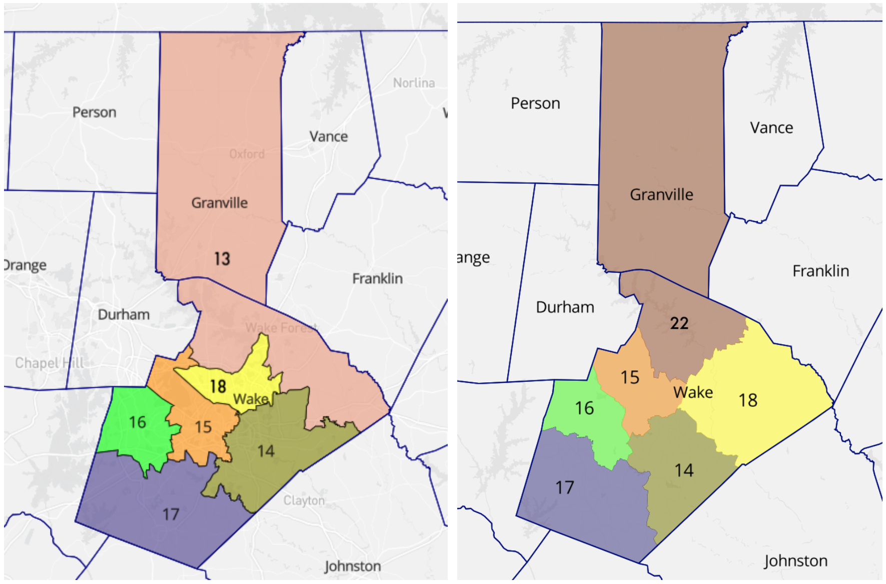


Figure 115: **Map of Enacted Plan in Granville and Wake Senate County Cluster**

(a) Enacted Map (b) Duchin Map



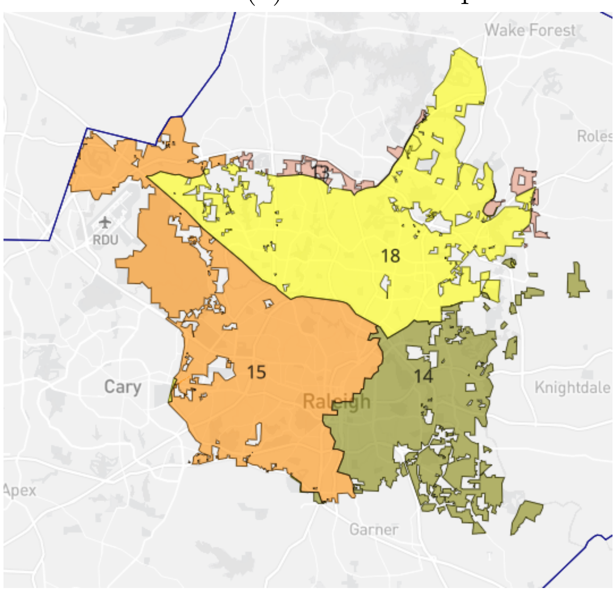
Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
13 (22 in Duchin)	0.48	0.50
14	0.73	0.73
15	0.68	0.64
16	0.63	0.63
17	0.49	0.51
18	0.65	0.65

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 116: **Map of Raleigh Divisions in Wake Senate County Cluster**

(a) Enacted Map



(b) Duchin Map

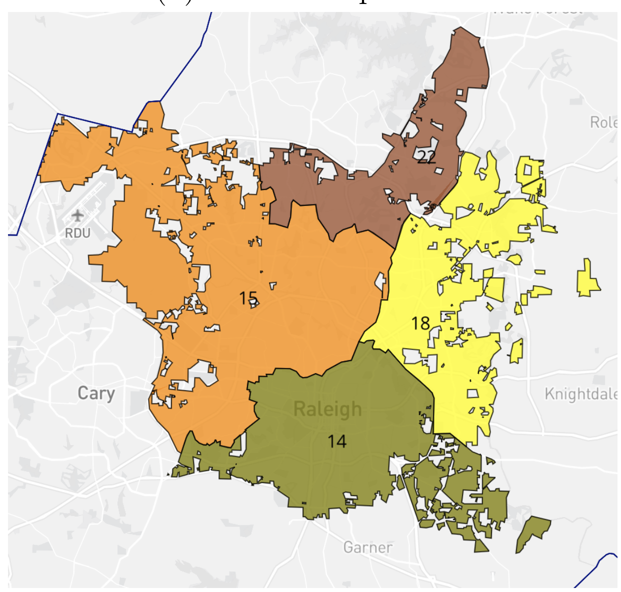
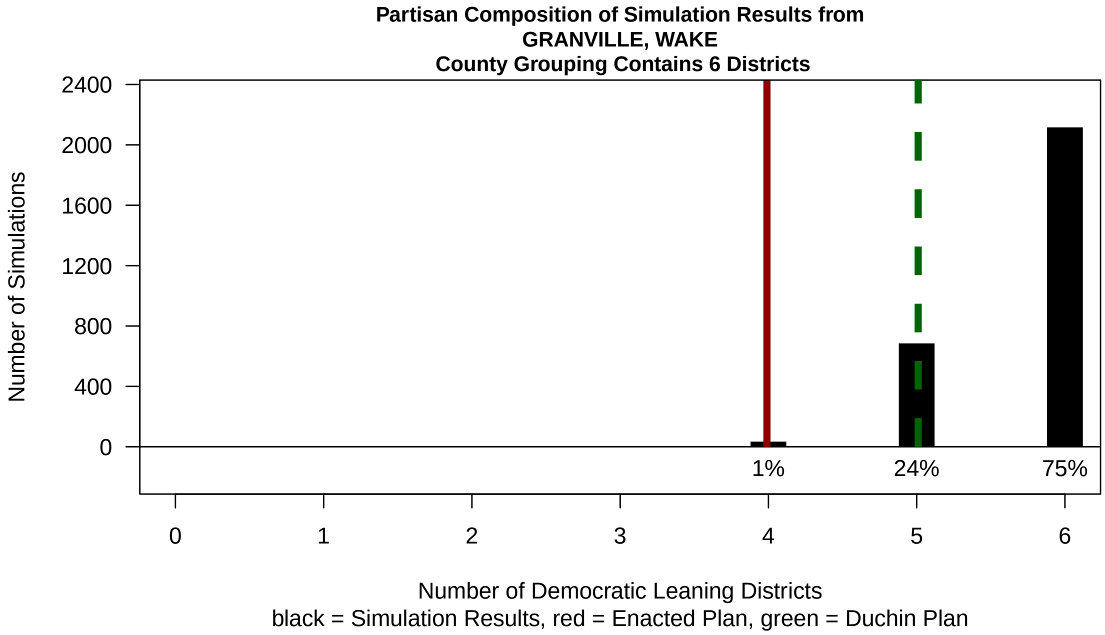


Figure 117: **Distribution of Partisan Districts from Simulations in Granville and Wake Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 42: Simulation Results by Individual Elections

Granville and Wake Senate County Cluster

	Number of Democratic Leaning Districts:						
	0	1	2	3	4	5	6
Individual Elections:							
2020 President	0%	0%	0%	0%	0%	**0%**	100%
2020 Senate	0%	0%	0%	0%	**1%**	24%	75%
2020 Governor	0%	0%	0%	0%	0%	0%	**100%**
2020 Lt. Governor	0%	0%	0%	0%	**1%**	25%	74%
2020 Attorney General	0%	0%	0%	0%	0%	**0%**	100%
2016 President	0%	0%	0%	0%	**4%**	35%	61%
2016 Senate	0%	0%	0%	0%	**19%**	70%	12%
2016 Governor	0%	0%	0%	0%	**1%**	24%	75%
2016 Lt. Governor	0%	0%	0%	11%	**13%**	71%	5%
2016 Attorney General	0%	0%	0%	0%	**1%**	26%	73%
2014 Senate	0%	0%	0%	0%	**9%**	63%	27%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 0% of the simulations produce 5 Democratic leaning districts. The Enacted Plan does, as the '5 Districts' cell is bolded in that row.

8.9 Iredell and Mecklenburg Senate County Grouping

The Iredell-Mecklenburg Senate county group contains 6 districts. In the Enacted Map these are Districts 37, 38, 39, 40, 41, and 42. The county cluster has an overall partisan index of .60, which is solidly Democratic. After conducting 50,000 initial simulations to create six districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. All 50,000 simulations meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 7,700 simulated maps, each containing six districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 118. A map of the Enacted Map's district boundaries and the Duchin Map's district boundaries within this county grouping are shown in Figure 119.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 120. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 5% of the simulations there are 4 Democratic leaning districts. In 95% of the simulations there are 5 Democratic leaning districts. The Enacted Map generates 4 Democratic leaning districts, which is an outlier from middle 50% of the simulations. The Duchin Map also generates 5 Democratic leaning districts.

Table 43 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted

Plan using the equivalent election. In 9 of the 11 individual elections the Enacted Plan is in alignment with the majority outcome of the simulation results.

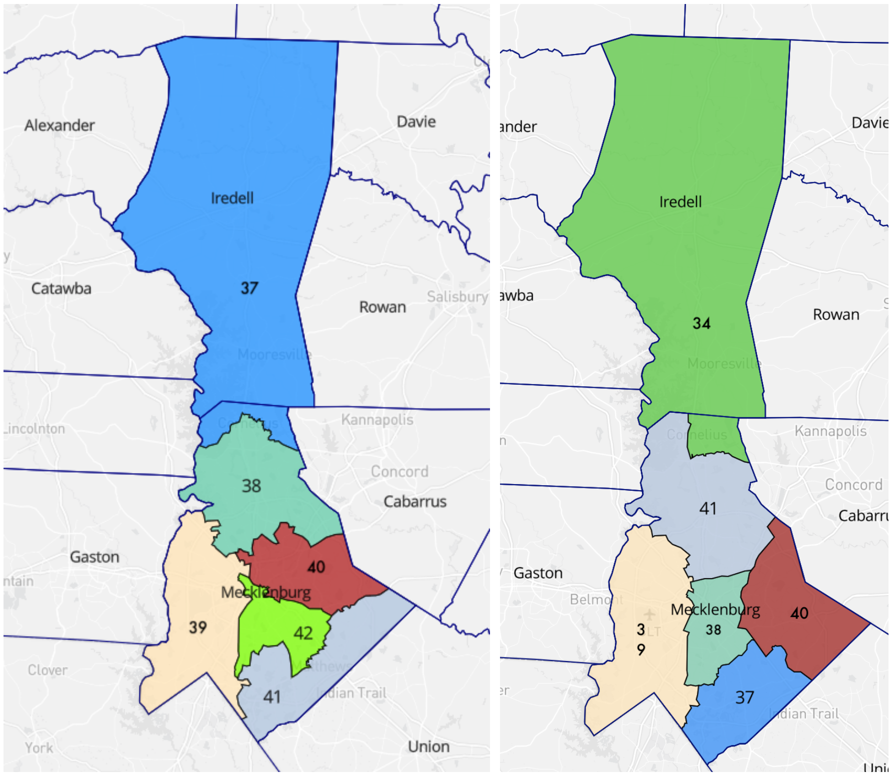
Why is the Enacted Plan an outlier in this county grouping? There are two factors to consider in explaining this divergence. First, while the Enacted Plan generates 4 solidly Democratic leaning districts, the remaining two districts are not solidly Republican. Instead, one is solidly Republican. District 37 in Iredell County has a partisan index of 0.36. The other would be best classified as highly competitive. District 41 has a partisan index of 0.490. This district will likely be very closely decided with candidates from both parties winning them with some regularity, given their narrow margins. This is actually quite close to the partisan lean of the Duchin Plan. While the Duchin Plan creates 5 Democratic leaning districts in the county group, there is also one solidly Republican district. District 34 in Iredell County has a partisan index of 0.36. The other would be best classified as highly competitive. District 37 has a partisan index of 0.526. Thus, both plans generate 4 solidly Democratic districts, 1 solidly Republican district and 1 competitive districts. The Duchin Plan's competitive districts are just slightly more Democratic by roughly 3.6 percentage points.

The second factor to consider is that the partisan index is calculated using elections from 2014-2020. Looking at Table 43 we see that the Enacted Plan is in agreement with 100% of the simulations in the five elections from the most recent election cycle. Given the trend in Mecklenburg towards more support for Democratic candidates, elections conducted under the Enacted Plan will align more consistently with the more recent elections in the index. That is, the Enacted Plan will more often generate 5 Democratic districts as is the case in 2020 than it will generate 4 Democratic districts as it did in the elections in 2016 and earlier.

Figure 118: **Iredell and Mecklenburg County Senate Cluster**

Figure 119: **Map of Enacted Plan in Iredell and Mecklenburg Senate County Cluster**

(a) Enacted Map (b) Duchin Map

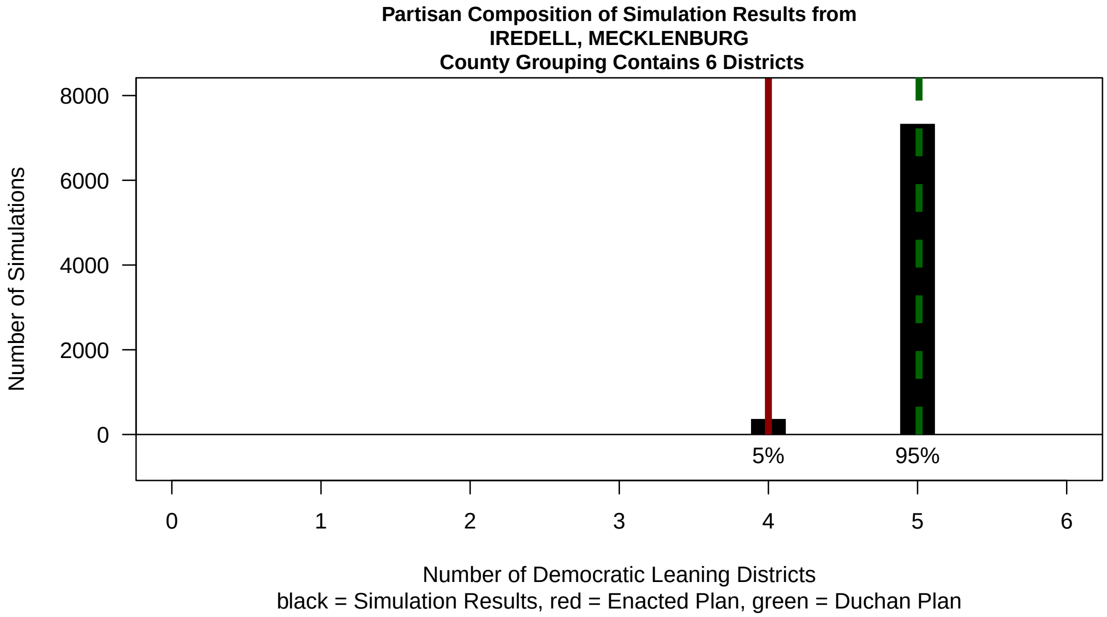


Partisan Lean of Districts

District:	Enacted Plan	Duchin Plan
37 (34 in Duchin)	0.36	0.36
38 (41 in Duchin)	0.65	0.66
39	0.73	0.73
40	0.83	0.72
41 (37 in Duchin)	0.49	0.53
42 (38 in Duchin)	0.65	0.68

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 120: **Distribution of Partisan Districts from Simulations in Iredell and Mecklenburg Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster.

Table 43: Simulation Results by Individual Elections

Iredell and Mecklenburg Senate County Cluster

	Number of Democratic Leaning Districts:						
	0	1	2	3	4	5	6
Individual Elections:							
2020 President	0%	0%	0%	0%	0%	**100%**	0%
2020 Senate	0%	0%	0%	0%	0%	**100%**	0%
2020 Governor	0%	0%	0%	0%	0%	**100%**	0%
2020 Lt. Governor	0%	0%	0%	0%	0%	**100%**	0%
2020 Attorney General	0%	0%	0%	0%	0%	**100%**	0%
2016 President	0%	0%	0%	0%	**5%**	95%	0%
2016 Senate	0%	0%	0%	0%	**96%**	4%	0%
2016 Governor	0%	0%	0%	0%	**7%**	93%	0%
2016 Lt. Governor	0%	0%	0%	0%	**99%**	1%	0%
2016 Attorney General	0%	0%	0%	0%	**51%**	49%	0%
2014 Senate	0%	0%	0%	0%	**99%**	1%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 5 Democratic leaning districts. The Enacted Plan does as well, as the '5 Districts' cell is bolded in that row.

8.10 Buncombe, Burke, and McDowell Senate County Grouping

The Buncombe-Burke-McDowell Senate county group contains 2 districts. In the Enacted Map these are Districts 46 and 49. The county cluster has an overall partisan index of .51, which is very slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 49,161 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 18,137 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 121. A map of the Enacted Map's district boundaries is shown in Figure 122. The Duchin Plan uses an alternative county grouping and is therefore not comparable to this cluster in the Enacted Plan. I analyze the Duchin Plan and the alternative cluster in a later section of this report.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 123. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there is 1 Democratic leaning district. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 1 Democratic leaning district.

Table 44 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Enacted Map.

Figure 121: **Map of Buncombe, Burke, and McDowell Senate County Cluster**

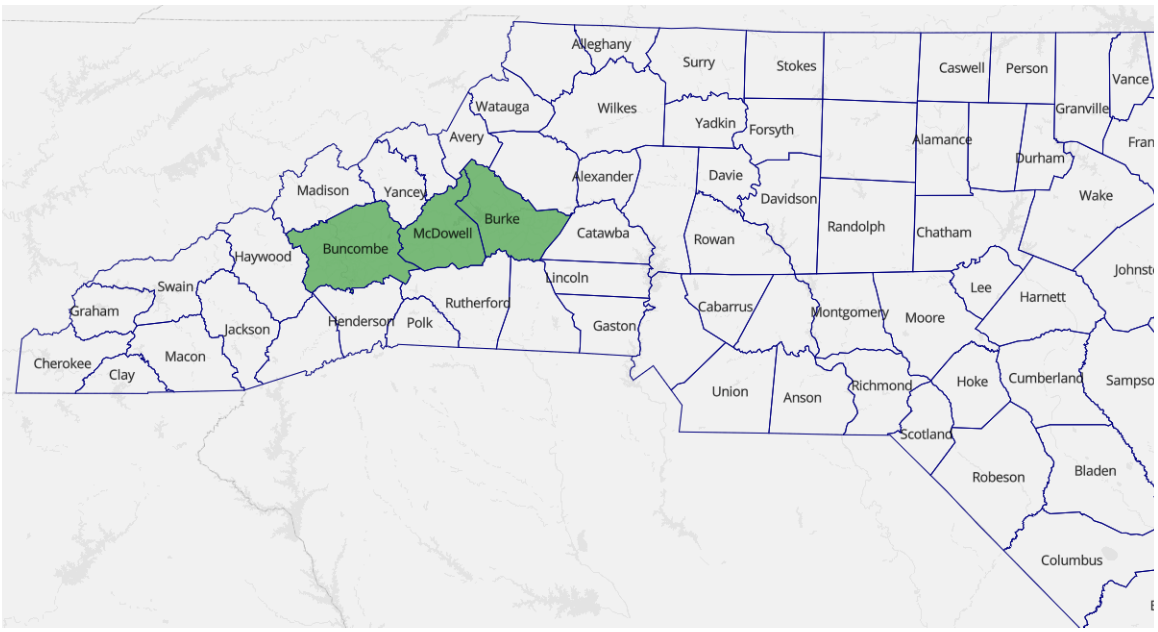
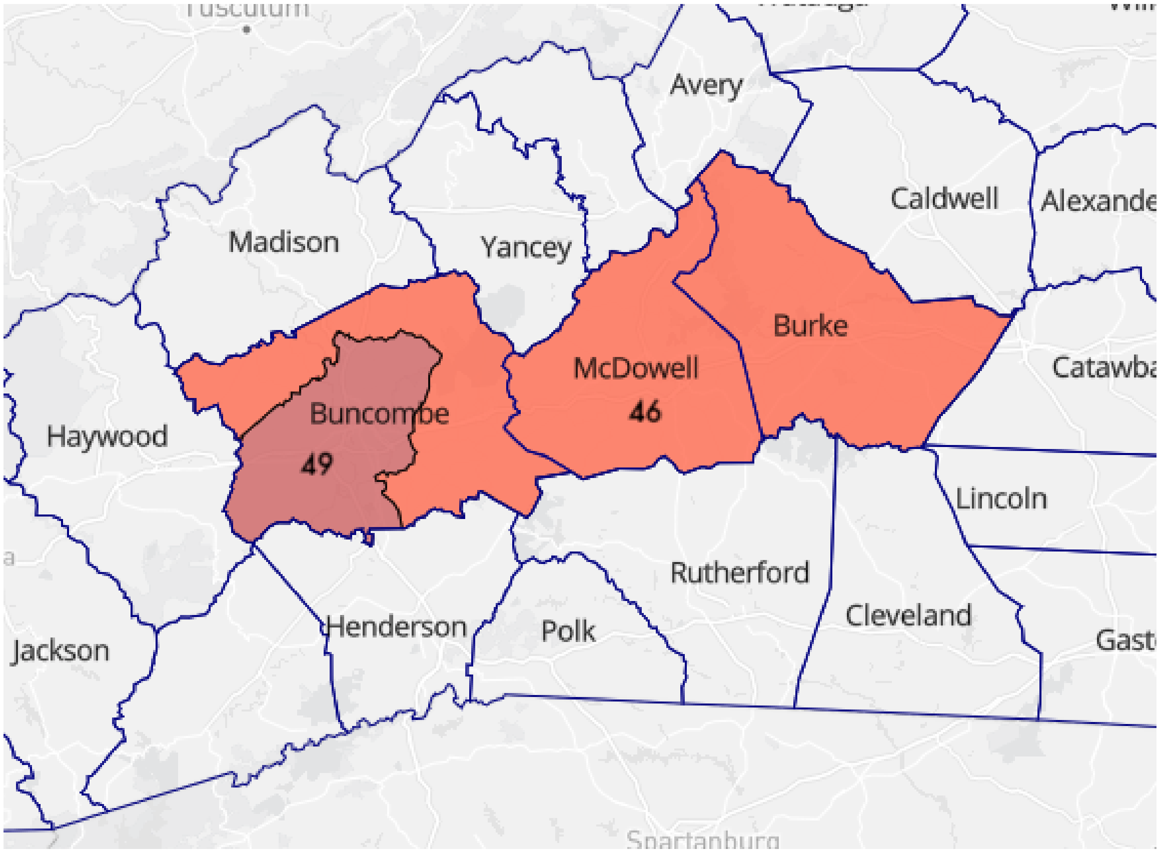


Figure 122: **Map of Enacted Plan in Buncombe, Burke, and McDowell Senate County Cluster**

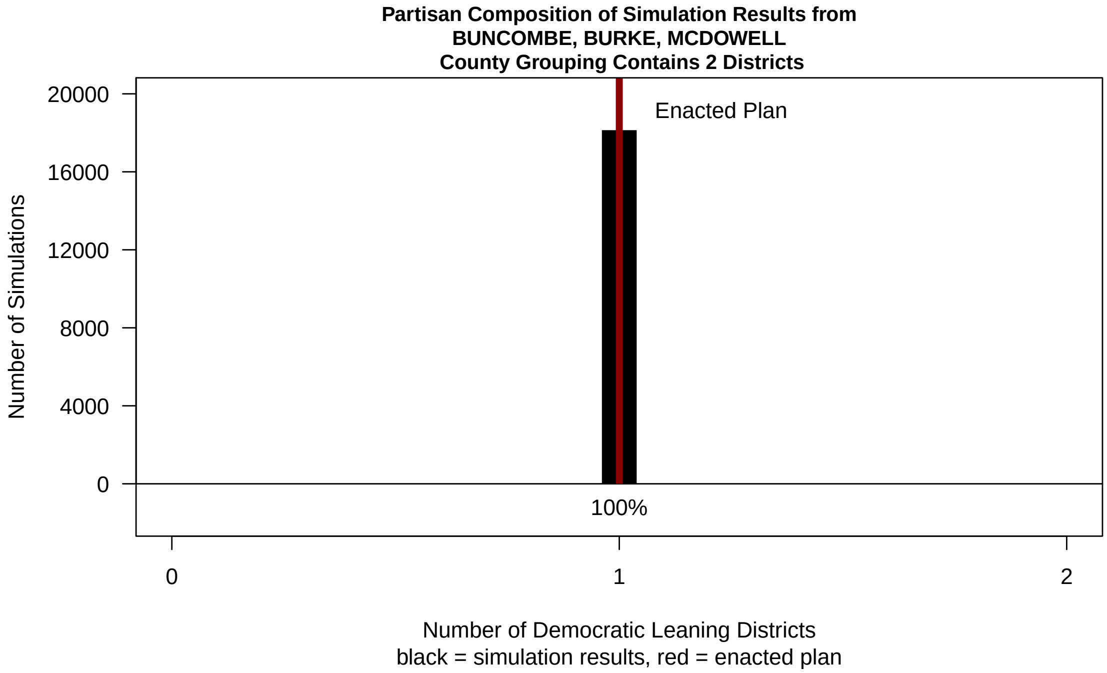


Partisan Lean of Districts

District:	Enacted Plan
46	0.37
49	0.65

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 123: **Distribution of Partisan Districts from Simulations in Buncombe, Burke, and McDowell Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster.

Table 44: Simulation Results by Individual Elections

Buncombe, Burke, and McDowell County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	**100%**	0%
2020 Senate	0%	**100%**	0%
2020 Governor	0%	**100%**	0%
2020 Lt. Governor	0%	**100%**	0%
2020 Attorney General	0%	**100%**	0%
2016 President	0%	**100%**	0%
2016 Senate	0%	**100%**	0%
2016 Governor	0%	**100%**	0%
2016 Lt. Governor	0%	**100%**	0%
2016 Attorney General	0%	**100%**	0%
2014 Senate	0%	**100%**	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

8.11 Cleveland, Gaston, and Lincoln Senate County Grouping

The Cleveland-Gaston-Lincoln Senate county group contains 2 districts. In the Enacted Map these are Districts 43 and 44. The county cluster has an overall partisan index of .34, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 4,074 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves only four unique maps that are as compact as the Enacted Plan.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 124. A map of the Enacted Map's district boundaries is shown in Figure 125. The Duchin Plan uses an alternative county grouping and is therefore not comparable to this cluster in the Enacted Plan. I analyze the Duchin Plan and the alternative cluster in a later section of this report.

Because there are only four maps that fit the criteria I use of equal population, county traversals, and compactness equal to or better than the Enacted Map, I do not present the distribution of district partisanship for the simulations here. It is sufficient to say that in the Enacted Map and the four remaining simulations, all create 2 Republican districts and 0 Democratic leaning districts, regardless of the index or election used. Table 45 shows this below.

Table 45 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In all 11 of the 11 individual elections there is unanimous agreement between the simulations and the Enacted Map.

Figure 124: **Map of Cleveland, Gaston, and Lincoln Senate County Cluster**

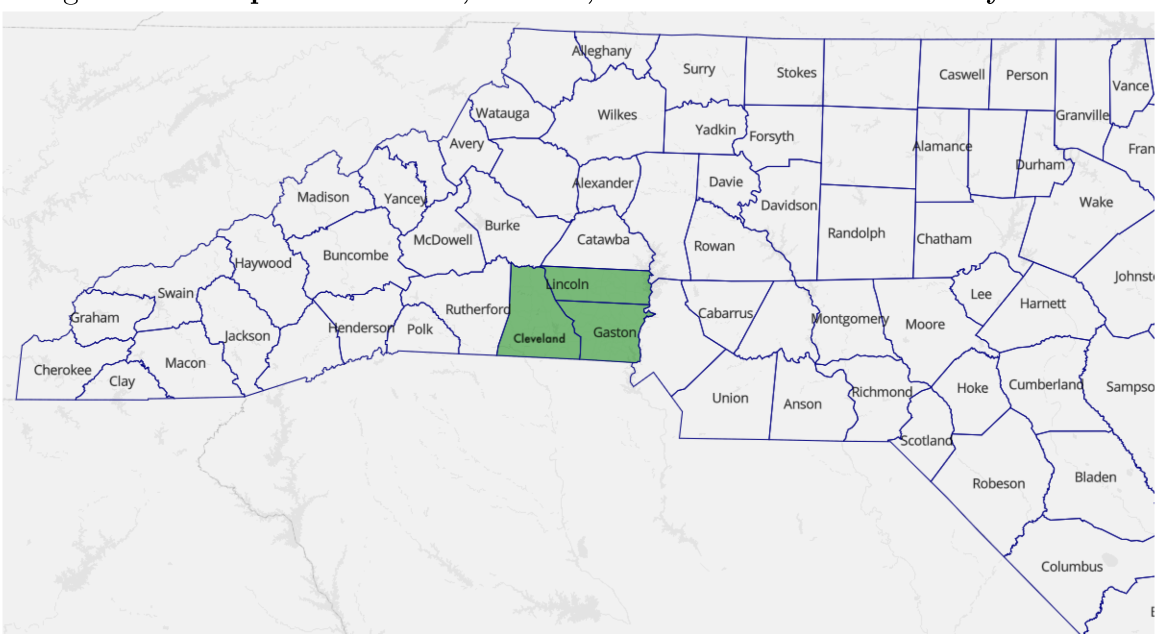
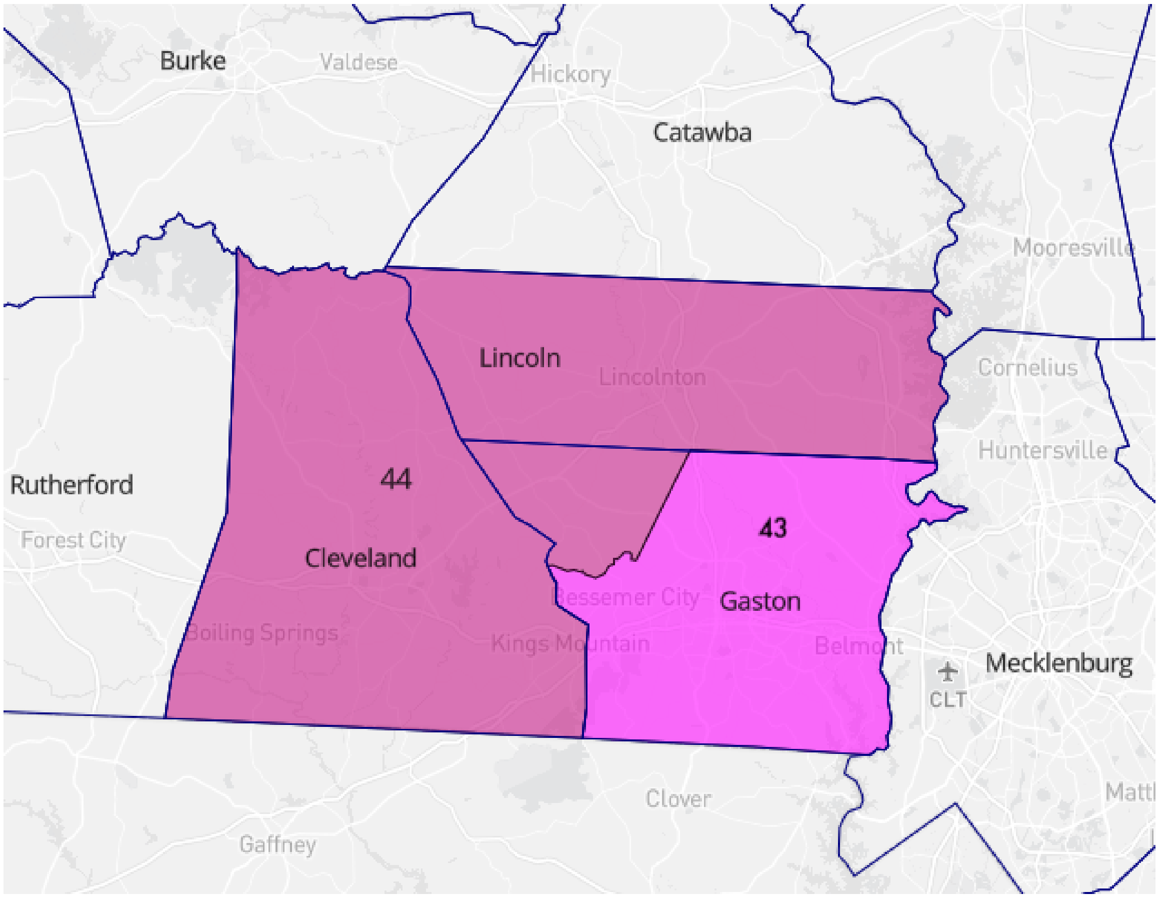


Figure 125: **Map of Enacted Plan in Cleveland, Gaston, and Lincoln Senate County Cluster**



Partisan Lean of Districts

District:	Enacted Plan
43	0.37
44	0.31

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Table 45: Simulation Results by Individual Elections

Cleveland, Gaston, and Lincoln Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Enacted Plan does as well, as the '0 District' cell is bolded in that row.

8.12 Forsyth and Stokes Senate County Grouping

The Forsyth-Stokes Senate county group contains 2 districts. In the Enacted Map these are Districts 31 and 32. The county cluster has an overall partisan index of .52, which is slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Enacted Plan. This leaves 35,085 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Enacted Map. This leaves 9,601 simulated maps, each containing two districts.

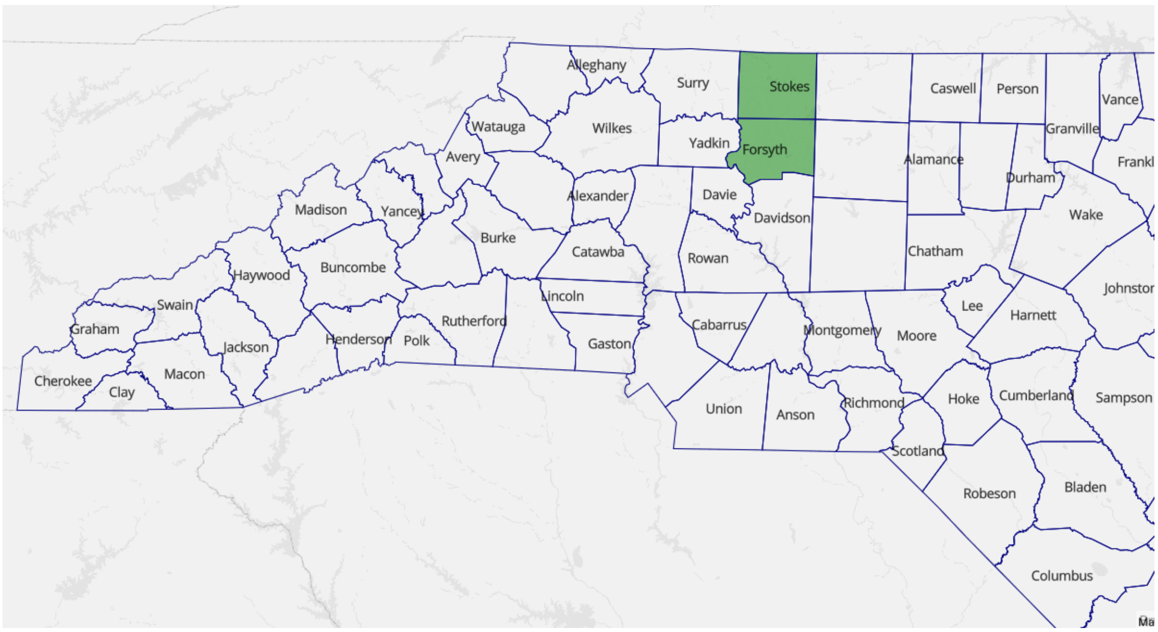
A map of the location of this county cluster in relation to the rest of the state is shown in Figure 126. A map of the Enacted Map's district boundaries is shown in Figure 127. The Duchin Plan uses an alternative county grouping and is therefore not comparable to this cluster in the Enacted Plan. I analyze the Duchin Plan and the alternative cluster in a later section of this report.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 128. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster, and the vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there is 1 Democratic leaning district. The Enacted Map is in alignment with the modal outcome of the simulations by also creating 1 Democratic leaning district.

Table 46 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded

number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. In 8 of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Enacted Map. In 9 of the 11 individual elections the Enacted Map falls inside the middle 50% of simulation results.

Figure 126: **Map of Forsyth and Stokes Senate County Cluster**



Partisan Lean of Districts

District:	Enacted Plan
31	0.38
32	0.69

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 127: **Map of Enacted Plan in Forsyth and Stokes Senate County Cluster**

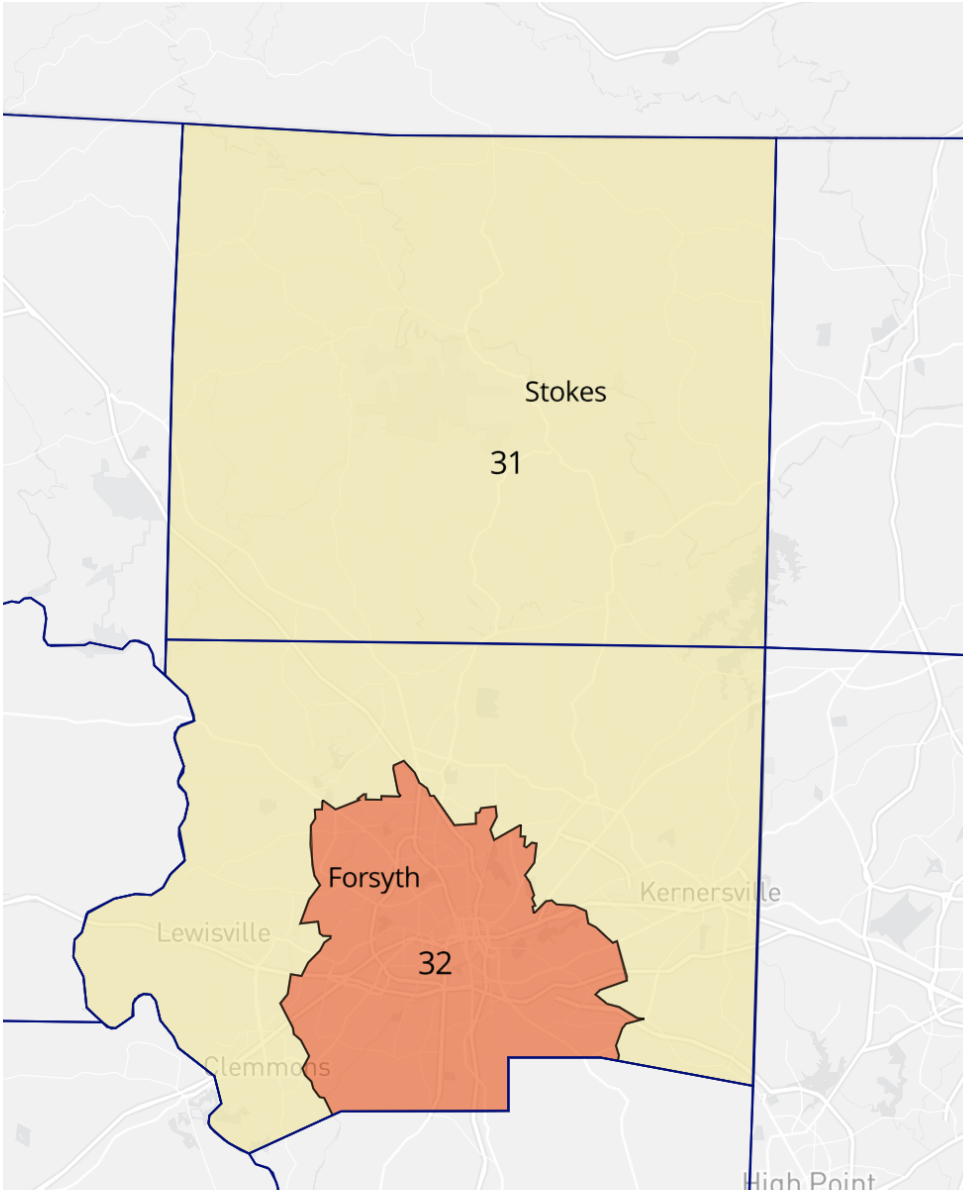
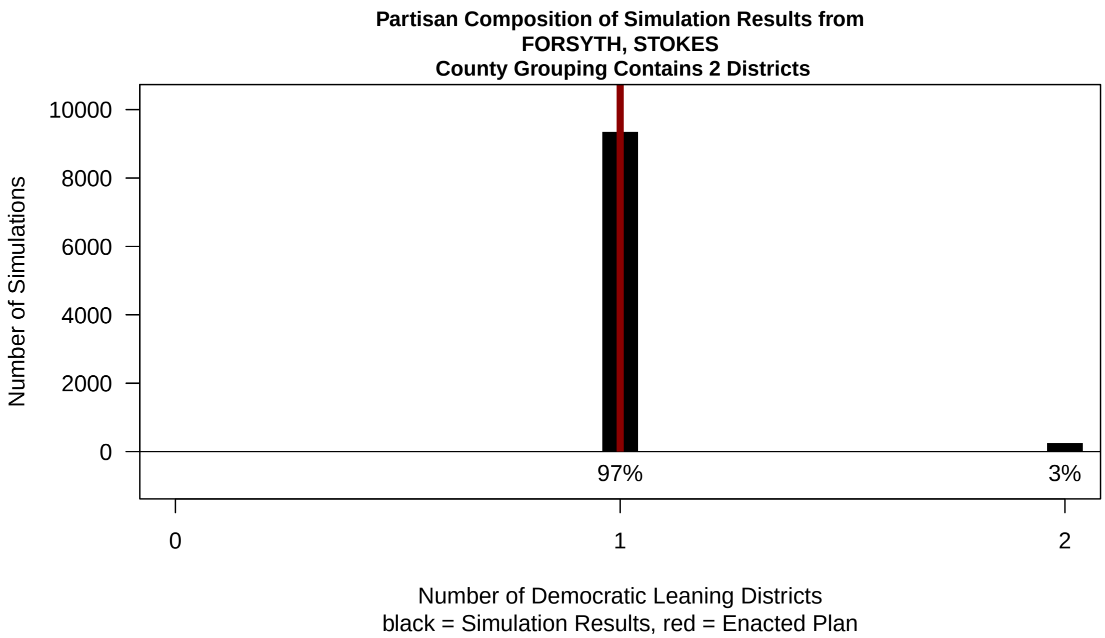


Figure 128: **Distribution of Partisan Districts from Simulations in Forsyth and Stokes Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The red vertical line shows the number of Democratic leaning seats in the Enacted Map in the same cluster.

Table 46: Simulation Results by Individual Elections

Forsyth and Stokes Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	**98%**	2%
2020 Senate	0%	**99%**	1%
2020 Governor	0%	**48%**	52%
2020 Lt. Governor	0%	**99%**	1%
2020 Attorney General	0%	**99%**	1%
2016 President	0%	**98%**	2%
2016 Senate	0%	**6%**	94%
2016 Governor	0%	**51%**	49%
2016 Lt. Governor	0%	**2%**	98%
2016 Attorney General	0%	**72%**	28%
2014 Senate	0%	**94%**	6%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Enacted Plan using the equivalent election. For example, using the 2020 Presidential election 98% of the simulations produce 1 Democratic leaning district. The Enacted Plan does as well, as the '1 District' cell is bolded in that row.

9 Comparison of Alternative Clusters to Those Chosen by the Legislature

In this section I compare the partisan index and simulations for the three alternative clusters chosen by the Duchin Plan and compare them to simulations in those same counties. The alternative clusters are very similar in their partisan indices as well as the partisan lean of the districts that are generated by the Enacted Map and the Duchin Map. This can be seen below in Table 47

Table 47: Senate Alternative County Grouping Analysis Summary

			# of Districts that are Democratic Leaning		
County Cluster	**Cluster Democratic Partisan Index**	**# Districts**	**Enacted Map**	**Duchin Map**	**Simulations**
Clusters Used by Enacted Plan					
Buncombe-Burke-McDowell	0.51	2	1		1
Cleveland-Gaston-Lincoln	0.34	2	0		0
Forsyth-Stokes	0.52	2	1		1
Alternative Clusters Used by Duchin Plan					
Buncombe-Henderson-Polk	0.54	2		1	1
Burke-Gaston-Lincoln	0.34	2		0	0
Forsyth-Yadkin	0.54	2		1	1
Total Enacted:		6	2	2	2
Total Duchin:		6	2	2	2

Note: Number of Democratic leaning districts is measured using the average two-party vote share in each district from the 11 statewide races noted earlier. Simulations range represents the middle 50% of outcomes from the simulations results. Clusters that fall outside of the simulation range are bolded.

9.1 Buncombe, Henderson, and Polk Senate Alternative County Grouping

The Buncombe-Henderson-Polk Senate alternative county group contains 2 districts. In the Duchin Map these are Districts 48 and 49. The county cluster has an overall partisan index of .53, which is slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Duchin Plan. This leaves 25,911 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Duchin Map. This leaves 17,474 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 129. A map of the Duchin Map's district boundaries is shown in Figure 130.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 132. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there is 1 Democratic leaning district. The Duchin Map is in alignment with the modal outcome of the simulations by creating 1 Democratic leaning district.

Table 49 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Duchin Plan using the equivalent election. In 7 of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Duchin Map. In 4 of the 11 individual elections the Duchin Map falls outside the middle 50% of simulation results and would be considered a statistical partisan outlier in these elections.

The Duchin Plan creates a solidly Democratic district and an additional very competitive district by dividing the city of Asheville. The Duchin Plan splits Asheville nearly equally across both districts while the Enacted Plan keeps the entirety of Asheville in one district. The tactic of dividing Democratic cities in a 'pinwheel' or 'pizza' shape and grouping those 'slices' with more Republican suburban and exurban areas is a classic tactic to generate more Democratic districts and overcome the geographic clustering that is common among Democratic voters. The Enacted Plan keeps the entirety of Asheville within one district. Table 48 shows the percent of Asheville voters in each district in each plan. It is clear that the Duchin plan splits Asheville into 2 roughly equal parts while the Enacted Plan places a much larger majority of Asheville into only 1 district. Figure 131 shows this division.

Table 48: Division of Asheville in Enacted Plan and Duchin Plan

	Percent of Asheville in district	
District:	Enacted Plan	Duchin Plan
46 (48 in Duchin)	0	42.8
49	100	57.2
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf Population numbers for city by district for Duchin Plan from Dave's Redistricting online. https://davesredistricting.org/

Figure 129: **Map of Buncombe, Henderson, and Polk Alternative Senate County Cluster**

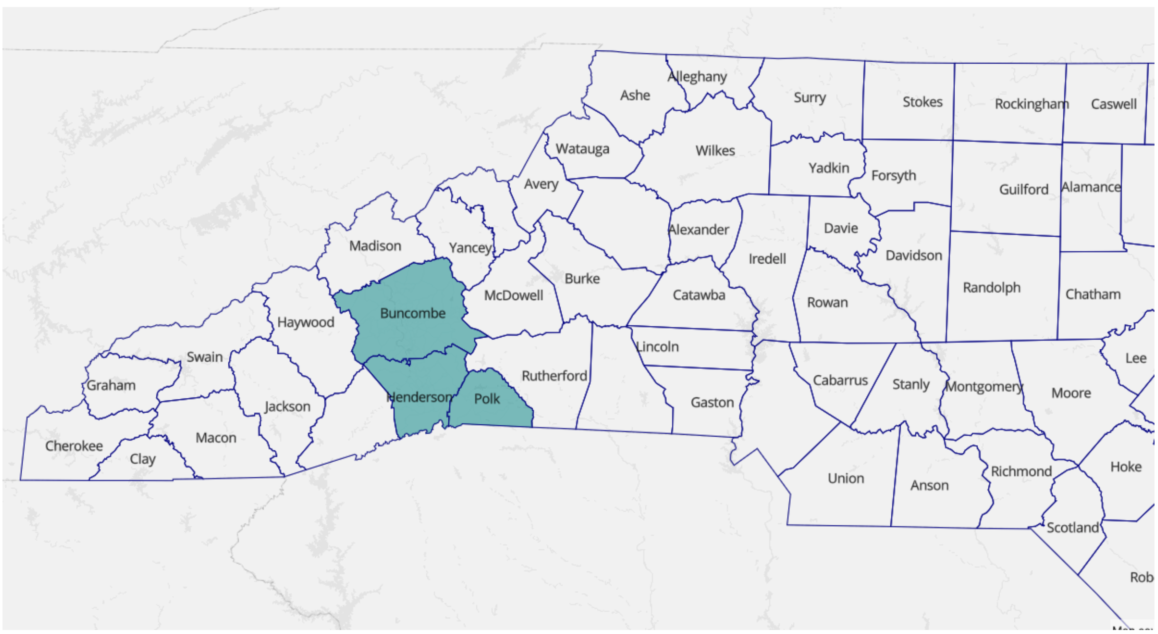
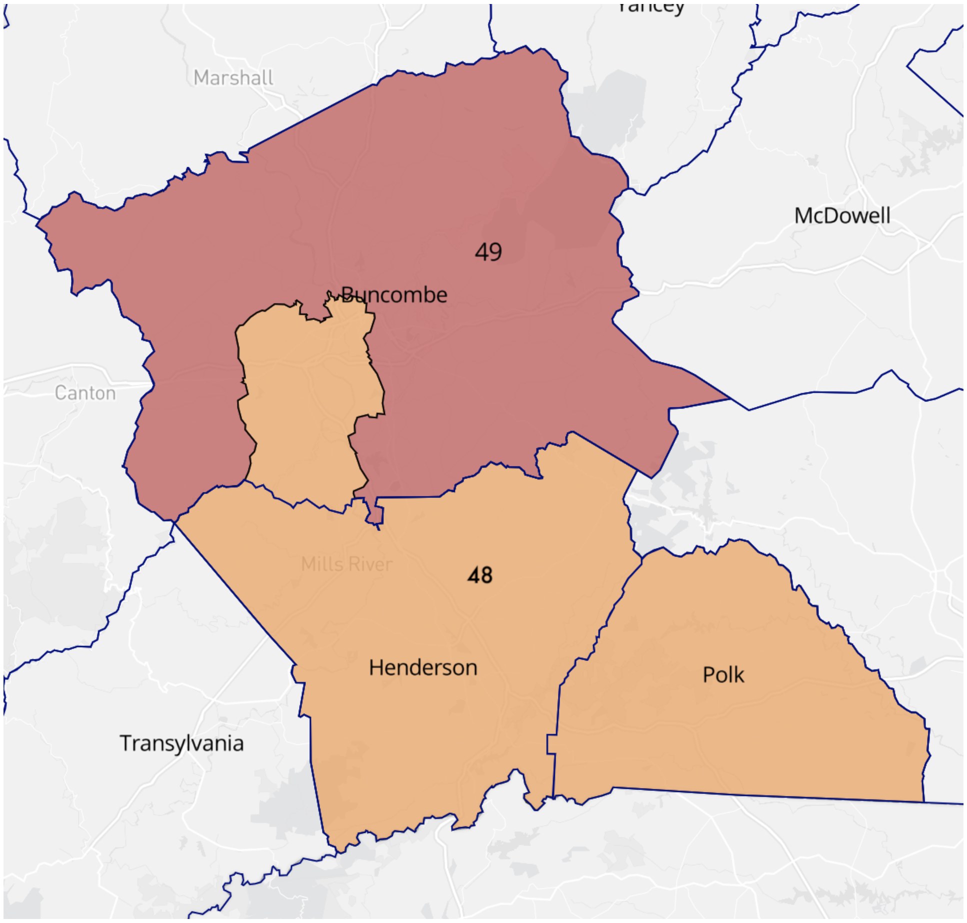


Figure 130: **Map of Duchin Plan in Buncombe, Henderson, and Polk Alternative Senate County Cluster**



Partisan Lean of Districts

District:	Enacted Plan
48	0.49
49	0.56

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 131: **Map of Division of Asheville in Enacted and Duchin Senate Plans**

(a) Enacted Map (b) Duchin Map

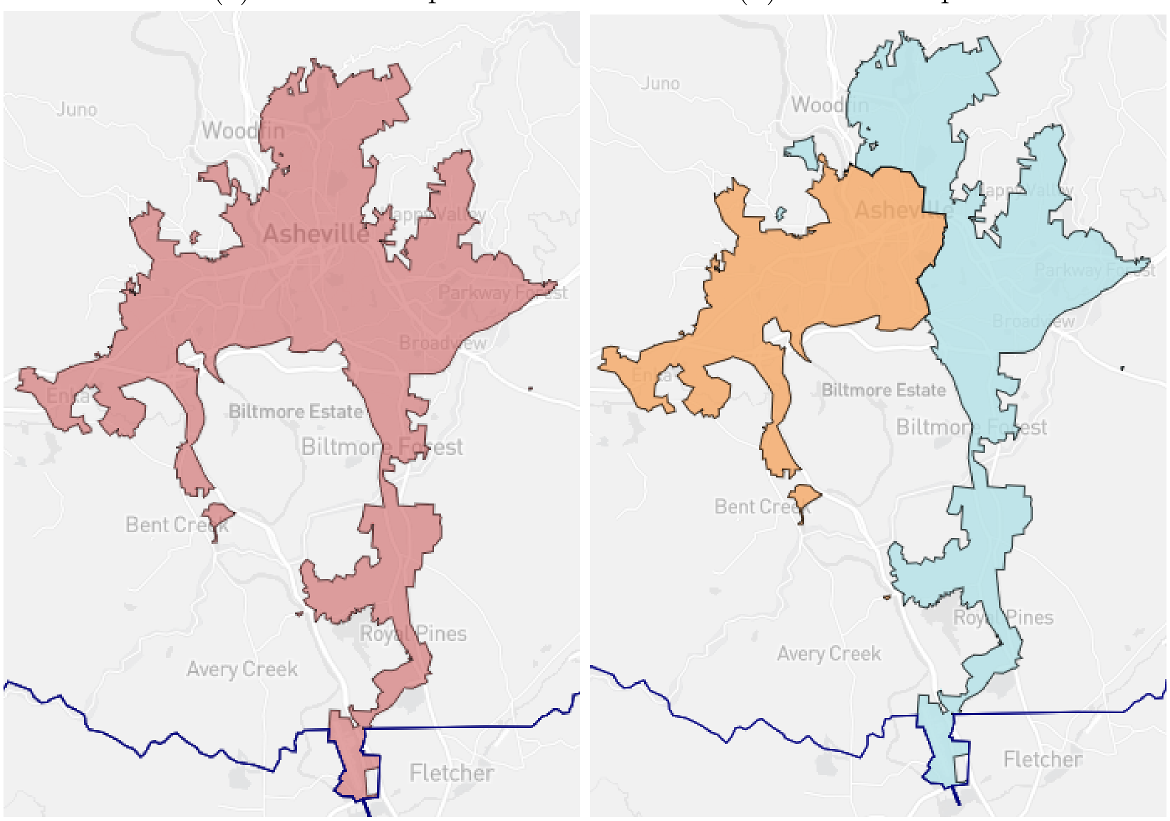
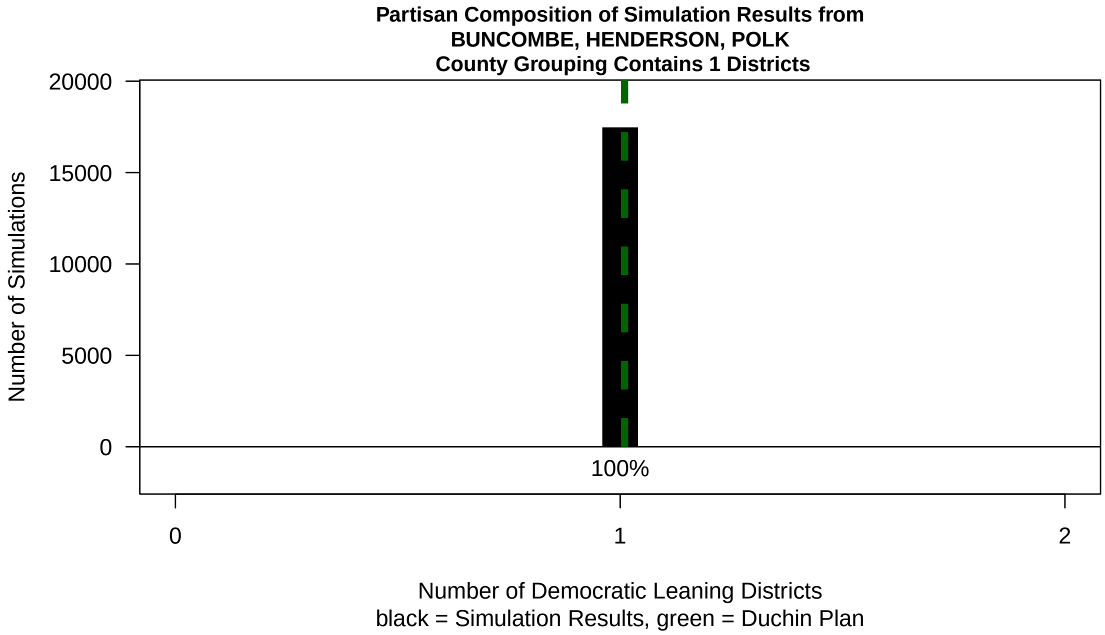


Figure 132: **Distribution of Partisan Districts from Simulations in Buncombe, Henderson, and Polk Alternative Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The green vertical line shows the number of Democratic leaning seats in the Duchin Map in the same cluster.

Table 49: Simulation Results by Individual Elections

Buncombe, Henderson, and Polk Alternative Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	100%	**0%**
2020 Senate	0%	**100%**	0%
2020 Governor	0%	93%	**7%**
2020 Lt. Governor	0%	**100%**	0%
2020 Attorney General	0%	100%	**0%**
2016 President	0%	**100%**	0%
2016 Senate	0%	**100%**	0%
2016 Governor	0%	100%	**0%**
2016 Lt. Governor	0%	**100%**	0%
2016 Attorney General	0%	**100%**	0%
2014 Senate	0%	**100%**	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Duchin Plan using the equivalent election. For example, using the 2020 Presidential election 0% of the simulations produce 2 Democratic leaning district. The Duchin Plan does, as the '2 District' cell is bolded in that row.

9.2 Burke, Gaston, and Lincoln Senate Alternative County Grouping

The Burke-Gaston-Lincoln Senate alternative county group contains 2 districts. In the Duchin Map these are Districts 43 and 44. The county cluster has an overall partisan index of .33, which is strongly Republican. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Duchin Plan. This leaves 15,719 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Duchin Map. This leaves 13,370 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 133. A map of the Duchin Map's district boundaries is shown in Figure 134.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 135. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Duchin Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic leaning districts.

Table 50 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Duchin Plan using the equivalent election. In all of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Duchin Map.

Figure 133: **Map of Burke, Gaston, and Lincoln Alternative Senate County Cluster**

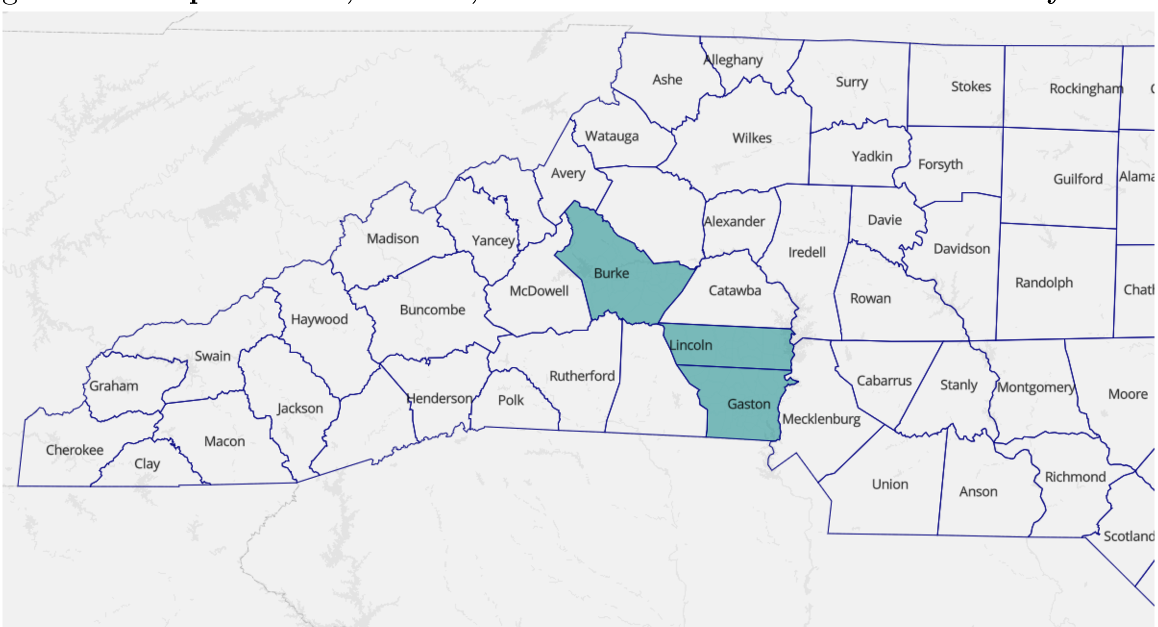
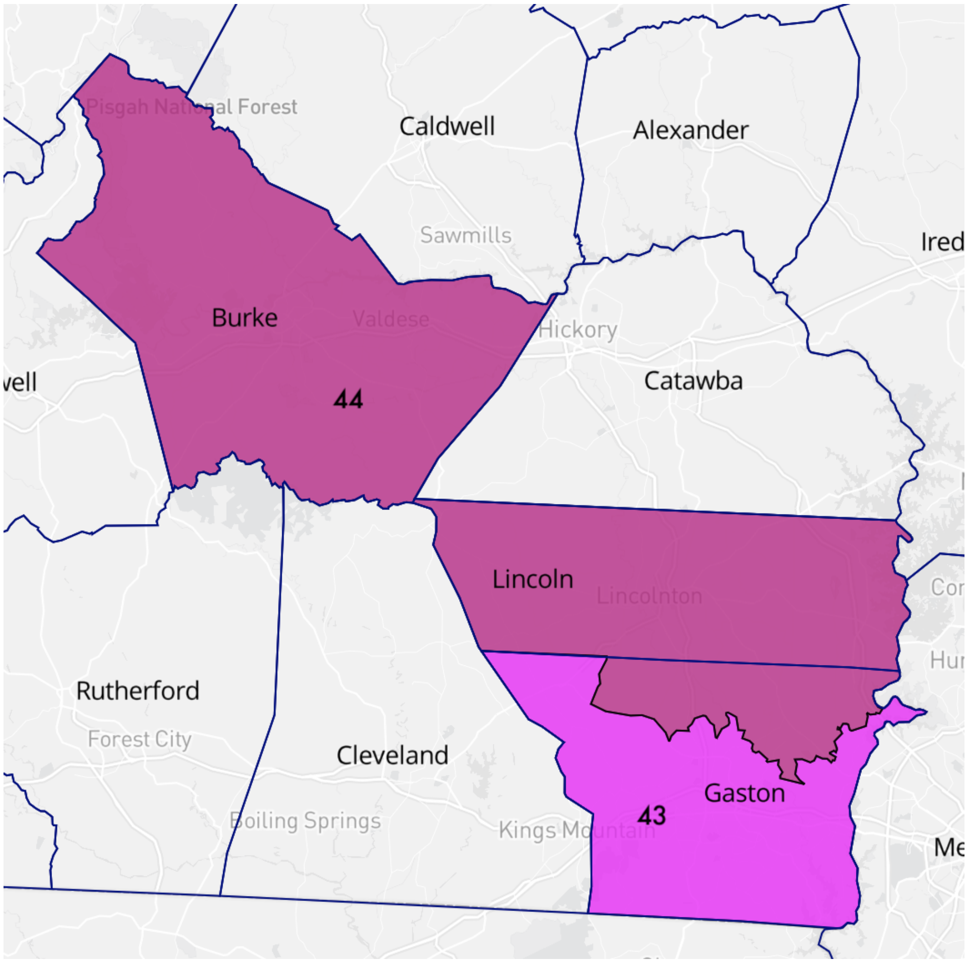


Figure 134: **Map of Duchin Plan in Burke, Gaston, and Lincoln Alternative Senate County Cluster**

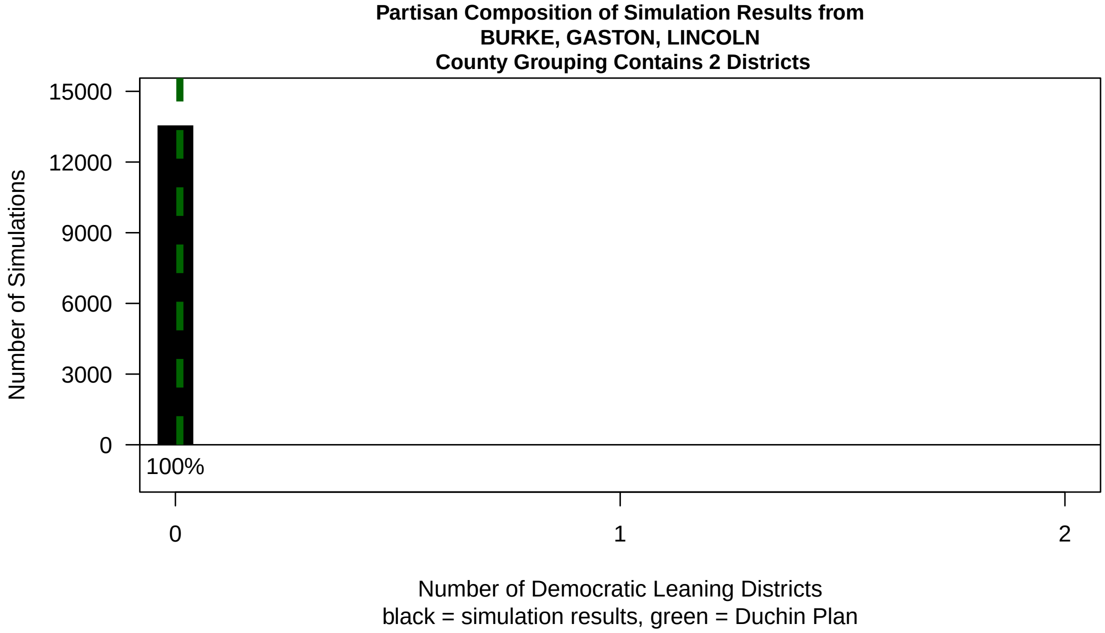


Partisan Lean of Districts

District:	Enacted Plan
43	0.38
44	0.29

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 135: **Distribution of Partisan Districts from Simulations in Burke, Gaston, and Lincoln Alternative Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The green vertical line shows the number of Democratic leaning seats in the Duchin Map in the same cluster.

Table 50: Simulation Results by Individual Elections

Burke, Gaston, and Lincoln Alternative Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	**100%**	0%	0%
2020 Senate	**100%**	0%	0%
2020 Governor	**100%**	0%	0%
2020 Lt. Governor	**100%**	0%	0%
2020 Attorney General	**100%**	0%	0%
2016 President	**100%**	0%	0%
2016 Senate	**100%**	0%	0%
2016 Governor	**100%**	0%	0%
2016 Lt. Governor	**100%**	0%	0%
2016 Attorney General	**100%**	0%	0%
2014 Senate	**100%**	0%	0%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Duchin Plan using the equivalent election. For example, using the 2020 Presidential election 100% of the simulations produce 0 Democratic leaning districts. The Duchin Plan does as well, as the '0 Districts' cell is bolded in that row.

9.3 Forsyth and Yadkin Senate Alternative County Grouping

The Forsyth and Yadkin Senate alternative county group contains 2 districts. In the Duchin Map these are Districts 31 and 32. The county cluster has an overall partisan index of .53, which is slightly Democratic. After conducting 50,000 initial simulations to create two districts in this cluster, I discard any simulations that contain more county traversals than the Duchin Plan. This leaves 48,151 simulations that meet this criteria. Next, I discard any simulations in which the average compactness score of the districts in the simulations is not as large or larger than the compactness score of the Duchin Map. This leaves 19,706 simulated maps, each containing two districts.

A map of the location of this county cluster in relation to the rest of the state is shown in Figure 136. A map of the Duchin Map's district boundaries is shown in Figure 137.

The distribution of district partisanship based on the statewide partisan elections index calculated for each of the simulation results is shown in Figure 139. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The vertical dashed green line shows the number of Democratic leaning seats in the Duchin Map in the cluster. In 100% of the simulations there are 0 Democratic leaning districts. The Duchin Map is in alignment with the modal outcome of the simulations by also creating 0 Democratic leaning districts.

Table 52 breaks apart the partisan index into the 11 constituent elections and shows the distribution of Democratic leaning seats generated if one were to look at each election separately. Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Duchin Plan using the equivalent election. In all of the 11 individual elections there is agreement between the modal (most common) outcome in the simulations and the Duchin Map.

The Duchin Plan creates a solidly Democratic district and an additional very competitive district by dividing the city of Winston-Salem. While Winston-Salem is too large to be a single district, the Duchin Plan splits Winston-Salem nearly equally across both districts while the Enacted Plan keeps a larger share of Winston-Salem in one district. The tactic of dividing Democratic cities in a 'pinwheel' or 'pizza' shape and grouping those 'slices' with more Republican suburban and exurban areas is a classic tactic to generate more Democratic districts and overcome the geographic clustering that is common among Democratic voters. The Enacted Plan keeps much more of Winston-Salem within one district. Table 51 shows the percent of Winston-Salem voters in each district in each plan. It is clear that the Duchin plan splits Winston-Salem into 2 roughly equal parts while the Enacted Plan places a much larger majority of Winston-Salem into only 1 district. Figure 138 shows this division.

Table 51: Division of Winton-Salem in Enacted Plan and Duchin Plan

	Percent of Winston-Salem in district	
District:	Enacted Plan	Duchin Plan
31	16.35	52.3
32	83.65	47.7
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: `https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf` Population numbers for city by district for Duchin Plan from Dave's Redistricting online. `https://davesredistricting.org/`

Figure 136: **Map of Forsyth and Yadkin Alternative Senate County Cluster**

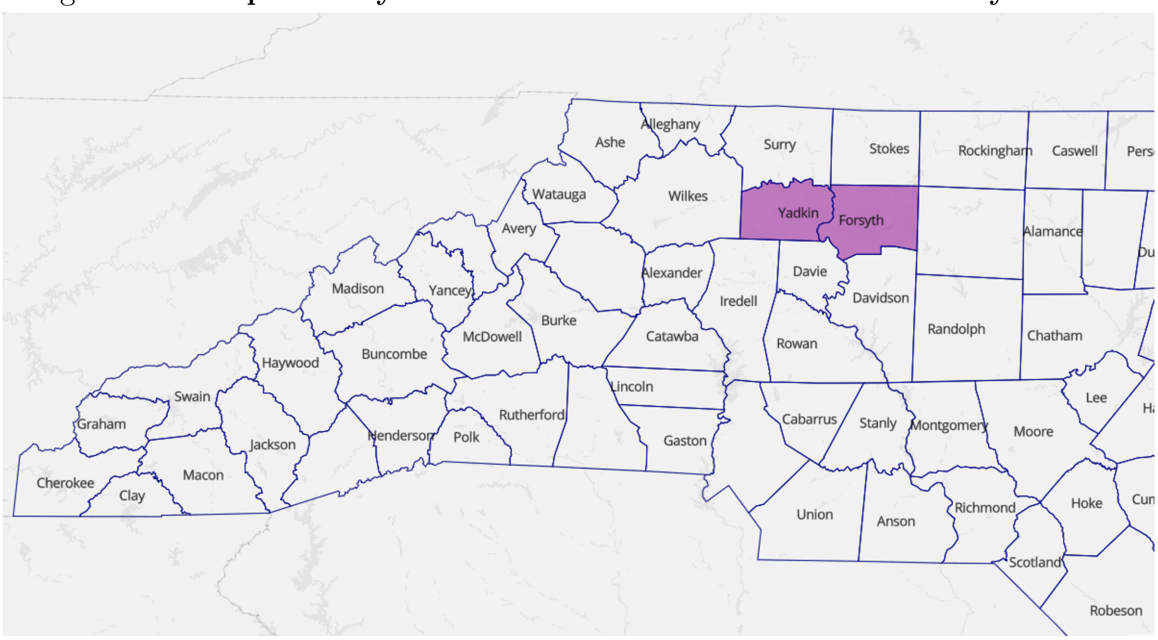


Figure 137: **Map of Duchin Plan in Forsyth and Yadkin Alternative Senate County Cluster**

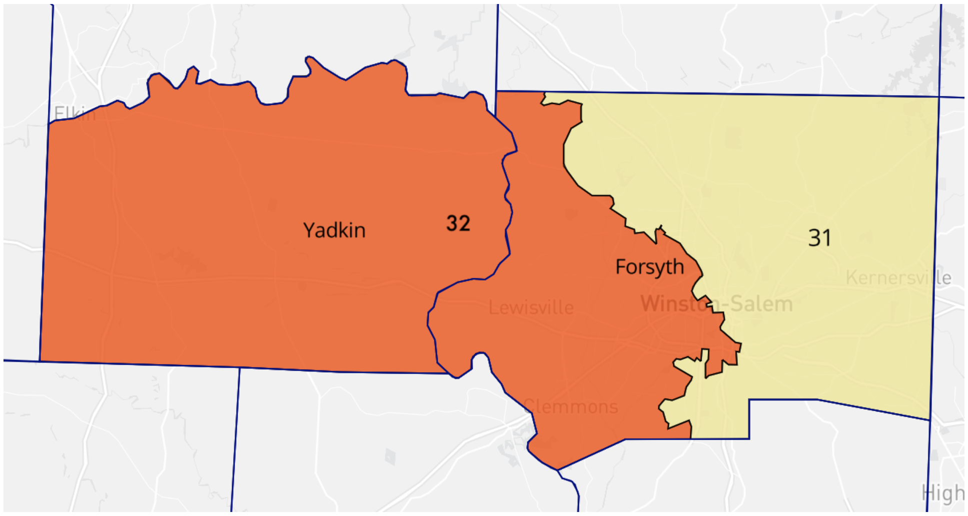
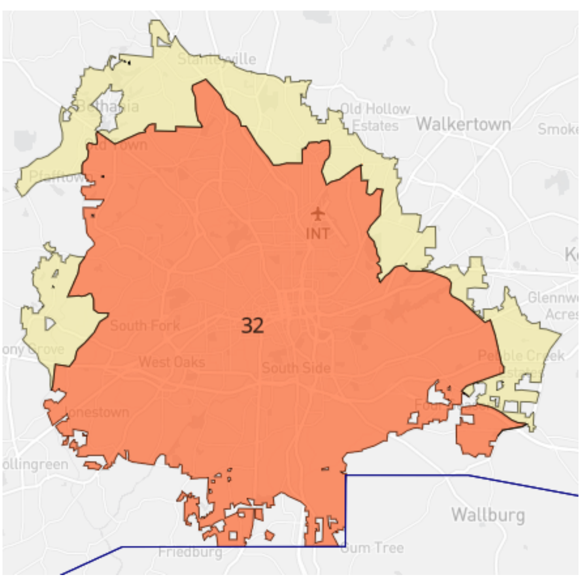
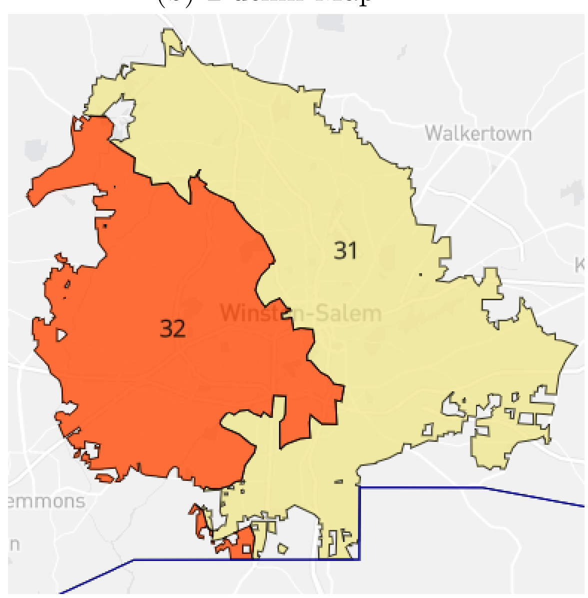


Figure 138: **Map of Division of Winston-Salem in Enacted and Duchin Senate Plans**

(a) Enacted Map



(b) Duchin Map

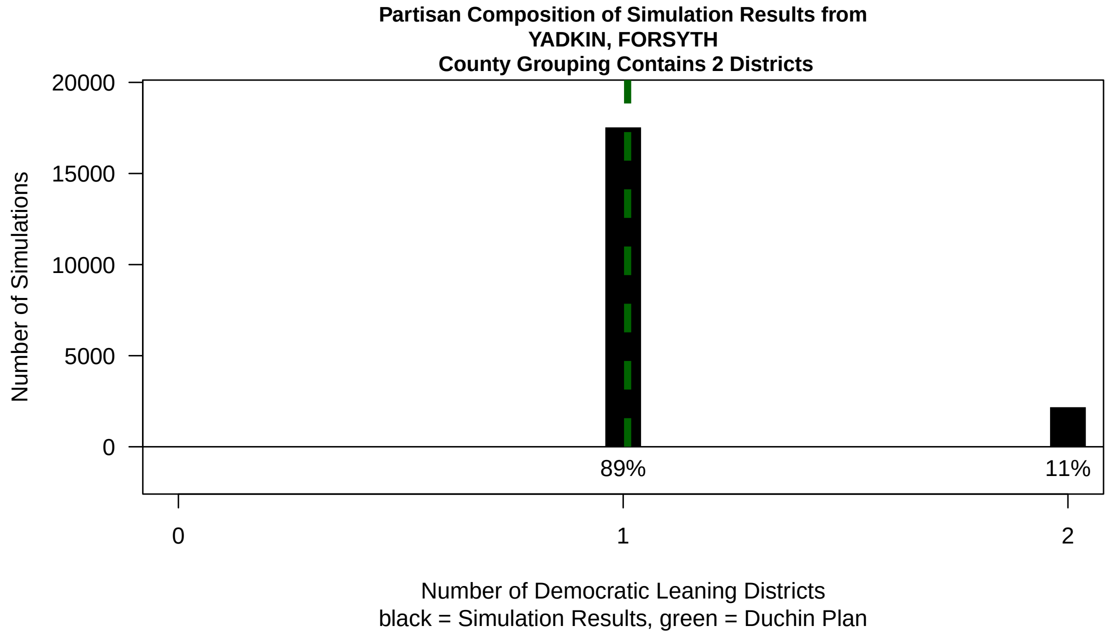


Partisan Lean of Districts

District:	Enacted Plan
31	0.58
32	0.49

Note: Partisan index is based on the two-party vote average of 11 statewide partisan elections between 2014-2020.

Figure 139: **Distribution of Partisan Districts from Simulations in Forsyth and Yadkin Alternative Senate County Cluster**



Note: Distribution of likely district partisanship based on the statewide partisan elections index calculated for each of the simulation results. The black bars show the distribution from the simulation results, with the percentage of simulations that generate each of the various possible number of Democratic seats in the cluster shown below each bar. The green vertical line shows the number of Democratic leaning seats in the Duchin Map in the same cluster.

Table 52: Simulation Results by Individual Elections

Forsyth and Yadkin Alternative Senate County Cluster

Number of Democratic Leaning Districts:			
	0	1	2
Individual Elections:			
2020 President	0%	56%	**44%**
2020 Senate	0%	**77%**	23%
2020 Governor	0%	0%	**100%**
2020 Lt. Governor	0%	**91%**	9%
2020 Attorney General	0%	**86%**	14%
2016 President	0%	**92%**	8%
2016 Senate	4%	**96%**	0%
2016 Governor	0%	62%	**38%**
2016 Lt. Governor	3%	**97%**	0%
2016 Attorney General	0%	**84%**	16%
2014 Senate	0%	**98%**	2%

Note: Each row shows the percent of simulations that produce the number of Democratic leaning districts using the election or election index indicated in the row. The bolded number in each row is the number of Democratic leaning districts produced by the Duchin Plan using the equivalent election. For example, using the 2020 Presidential election 44% of the simulations produce 2 Democratic leaning districts. The Duchin Plan does as well, as the '2 Districts' cell is bolded in that row.

10 Conclusion

Based upon my analysis of North Carolina's recently enacted redistricting plans for the General Assembly and the plans submitted by the North Carolina League of Conservation Voters, it is my opinion that the Enacted Maps are not "extreme partisan gerrymanders" as plaintiffs allege.

I come to this opinion through the use of a redistricting simulation algorithm to generate 50,000 simulated district maps in each county grouping in which there are multiple districts in both the North Carolina House of Representatives and the North Carolina Senate. The redistricting algorithm generates a representative sample of districts by following neutral redistricting criteria without regard to racial or partisan data. In this way, the simulated districts establish a comparison set of plans that use purely non-partisan redistricting inputs. I then compare the simulated plans against the Enacted Plans and the Duchin Plans by reference to election results to assess whether the partisan effects of those plans are consistent with what one would expect to see in a redistricting plan composed without reference to any partisan considerations.

In the House, these simulations show that the Enacted Plans consistently score more often within the range of the non-partisan simulated maps than the Duchin Plans. In addition, the simulations show that the Enacted Plans contain one county grouping, the Guilford County grouping in the House of Representative, that is a partisan outlier. However, this grouping largely follows the boundaries of a 2019 court-approved district plan. In contrast, the Duchin Plans generate partisan outliers in four county groupings.

In the Senate analysis both the Enacted and Duchin plans generate partisan outliers when compared to the simulated district maps in two clusters each. Furthermore, neutral redistricting criteria such as following municipal lines support the decisions by the map drawers in the Enacted Plan in more districts, while in these same districts the Duchin Plan divides Democratic-leaning municipalities into more pieces in order to combine Democratic-leaning voters in cities with Republican voters in suburban and rural parts of North Carolina to create additional competitive or Democratic-leaning districts.

Based on the evidence and analysis presented below, my opinions regarding the 2021 enacted redistricting plans in the North Carolina General Assembly can be summarized as follows:

- The contemporary political geography of North Carolina is such that Democratic majorities are often geographically clustered in the largest cities of the state while Republican voters often dominate the suburban and rural portions of the state.
- This is not the case in the rural northeastern region of the state, where there are also significant Democratic majorities.

- This geographic clustering in cities an in the rural northeast puts the Democratic Party at a natural disadvantage when single-member districts are drawn.
- This is further amplified by the 'county grouping' process that is unique to North Carolina's redistricting process where districts are constrained to remain within county groups.
- This disadvantage partially arises from the difficulty, and in many cases impossibility, of drawing Democratic-leaning districts in many of the county groupings that comply with constitutional requirements, even though Democratic voters make up roughly 40% of voters in these parts of the state.
- Based on a comparison between the Enacted Plan, the Duchin Plan, and a set of 50,000 simulated maps, the Enacted Plan is less of a partisan outlier than the Duchin Plan in the State House.
- In the Senate analysis both the Enacted and Duchin plans generate partisan outliers when compared to the simulated district maps in two clusters each.
- Areas of disagreement between proposed plans often arise because the Duchin plan divides Democratic leaning municipalities into more pieces in order to combine Democratic-leaning voters with Republican voters in suburban and rural parts of the state to create additional competitive or Democratic leaning districts.
- Given these results, as well as the otherwise high degree of agreement between the Enacted and Duchin maps, it is my opinion that the Enacted Maps are not "extreme partisan gerrymanders" as plaintiffs allege.

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CONTACT INFORMATION

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ACADEMIC APPOINTMENTS

Brigham Young University, Provo, UT

August 2020 - present Associate Professor, Department of Political Science
2014 - July 2020 Assistant Professor, Department of Political Science
2014 - present Faculty Scholar, Center for the Study of Elections and Democracy

EDUCATION

Princeton University Department of Politics, Princeton, NJ

Ph.D., Politics, July 2014

- Advisors: Brandice Canes-Wrone, Nolan McCarty, and Kosuke Imai
- Dissertation: "Buying Representation: the Incentives, Ideology, and Influence of Campaign Contributions on American Politics"
- 2015 Carl Albert Award for Best Dissertation, Legislative Studies Section, American Political Science Association (APSA)

M.A., Politics, December 2011

Brigham Young University, Provo, UT

B.A., International Relations - Political Economy Focus, April, 2008

- *Cum Laude*

RESEARCH INTERESTS

American politics, congressional polarization, political ideology, campaign finance, survey research

PUBLICATIONS

19. **"Ideological Disagreement and Pre-emption in Municipal Policymaking"** with Adam Dynes
Forthcoming at *American Journal of Political Science*
18. **"Comparing Campaign Finance and Vote Based Measures of Ideology"**
Forthcoming at *Journal of Politics*
17. **"The Participatory and Partisan Impacts of Mandatory Vote-by-Mail"**, with John Holbein
Science Advances, 2020. Vol. 6, no. 35, DOI: 10.1126/sciadv.abc7685
16. **"Issue Politicization and Interest Group Campaign Contribution Strategies"**, with Mandi Eatough
Journal of Politics, 2020. Vol. 82: No. 3, pp. 1008-1025

15. **“Campaign Contributions and Donors’ Policy Agreement with Presidential Candidates”**, with Brandice Canes-Wrone and Sharece Thrower
Presidential Studies Quarterly, 2019, 49 (4) 770–797
14. **“Conservatism in the Era of Trump”**, with Jeremy Pope
Perspectives on Politics, 2019, 17 (3) 719–736
13. **“Legislative Constraints on Executive Unilateralism in Separation of Powers Systems”**, with Alex Bolton and Sharece Thrower
Legislative Studies Quarterly, 2019, 44 (3) 515–548
Awarded the Jewell-Loewenberg Award for best article in the area of subnational politics published in *Legislative Studies Quarterly* in 2019
12. **“Electoral Competitiveness and Legislative Productivity”**, with Soren Schmidt
American Politics Research, 2019, 47 (4) 683–708
11. **“Does Party Trump Ideology? Disentangling Party and Ideology in America”**, with Jeremy Pope
American Political Science Review, 2019, 113 (1) 38–54
10. **“The Evolution of National Constitutions”**, with Scott Abramson
Quarterly Journal of Political Science, 2019, 14 (1) 89–114
9. **“Who is Ideological? Measuring Ideological Responses to Policy Questions in the American Public”**, with Jeremy Pope
The Forum: A Journal of Applied Research in Contemporary Politics, 2018, 16 (1) 97–122
8. **“Status Quo Bias in Ballot Wording”**, with David Gordon, Ryan Hill, and Joe Price
The Journal of Experimental Political Science, 2017, 4 (2) 151–160.
7. **“Ideologically Sophisticated Donors: Which Candidates Do Individual Contributors Finance?”**, with Brandice Canes-Wrone and Sharece Thrower
American Journal of Political Science, 2017, 61 (2) 271–288.
6. **“Gender Inequalities in Campaign Finance: A Regression Discontinuity Design”**, with Daniel Butler and Jessica Preece
Quarterly Journal of Political Science, 2016, Vol. 11, No. 2: 219–248.
5. **“Representing the Preferences of Donors, Partisans, and Voters in the U.S. Senate”**
Public Opinion Quarterly, 2016, 80: 225–249.
4. **“Donation Motivations: Testing Theories of Access and Ideology”**
Political Research Quarterly, 2016, 69 (1) 148–160.
3. **“Ideological Donors, Contribution Limits, and the Polarization of State Legislatures”**
Journal of Politics, 2016, 78 (1) 296–310.
2. **“Online Polls and Registration Based Sampling: A New Method for Pre-Election Polling”** with Quin Monson, Kelly Patterson and Chris Mann.
Political Analysis 2014, 22 (3) 321–335.
1. **“Causes and Consequences of Political Polarization”** In *Negotiating Agreement in Politics*. Jane Mansbridge and Cathie Jo Martin, eds., Washington, DC: American Political Science Association: 19–53. with Nolan McCarty. 2013.

    - Reprinted in *Solutions to Political Polarization in America*, Cambridge University Press. Nate Persily, eds. 2015
    - Reprinted in *Political Negotiation: A Handbook*, Brookings Institution Press. Jane Mansbridge and Cathie Jo Martin, eds. 2015

AVAILABLE WORKING PAPERS

"Misclassification and Bias in Predictions of Individual Ethnicity from Administrative Records" (Revise and Resubmit at *American Political Science Review*)

"Taking Cues When You Don't Care: Issue Importance and Partisan Cue Taking"
with Jeremy Pope (Revise and Resubmit)

"A Revolution of Rights in American Founding Documents"
with Scott Abramson and Jeremy Pope (Conditionally Accepted)

"410 Million Voting Records Show the Distribution of Turnout in America Today"
with John Holbein (Revise and Resubmit)

"Partisanship and Trolleyology"
with Ryan Davis (Under Review)

"Who's the Partisan: Are Issues or Groups More Important to Partisanship?"
with Jeremy Pope (Revise and Resubmit)

"Race and Realignment in American Politics"
with Jeremy Pope (Revise and Resubmit)

"The Policy Preferences of Donors and Voters"

"Estimating Neighborhood Effects on Turnout from Geocoded Voter Registration Records."
with Kosuke Imai

"Super PAC Contributions in Congressional Elections"

WORKS IN PROGRESS

"Collaborative Study of Democracy and Politics"
with Brandice Canes-Wrone, Gregory Huber, and Joshua Clinton

"Preferences for Representational Styles in the American Public"
with Ryan Davis and Adam Dynes

"Representation and Issue Congruence in Congress"
with Taylor Petersen

"Education, Income, and the Vote for Trump"
with Edie Ellison

INVITED PRESENTATIONS

"Are Mormons Breaking Up with Republicanism? The Unique Political Behavior of Mormons in the 2016 Presidential Election"

- Ivy League LDS Student Association Conference - Princeton University, November 2018, Princeton, NJ

"Issue Politicization and Access-Oriented Giving: A Theory of PAC Contribution Behavior"

- Vanderbilt University, May 2017, Nashville, TN

"Lost in Issue Space? Measuring Levels of Ideology in the American Public"

- Yale University, April 2016, New Haven, CT

"The Incentives, Ideology, and Influence of Campaign Donors in American Politics"

- University of Oklahoma, April 2016, Norman, OK

"Lost in Issue Space? Measuring Levels of Ideology in the American Public"

- University of Wisconsin - Madison, February 2016, Madison, WI

"Polarization and Campaign Contributors: Motivations, Ideology, and Policy"

- Hewlett Foundation Conference on Lobbying and Campaign Finance, October 2014, Palo Alto, CA

"Ideological Donors, Contribution Limits, and the Polarization of State Legislatures"

- Bipartisan Policy Center Meeting on Party Polarization and Campaign Finance, September 2014, Washington, DC

"Representing the Preferences of Donors, Partisans, and Voters in the U.S. Senate"

- Yale Center for the Study of American Politics Conference, May 2014, New Haven, CT

Conference Presentations

Washington D.C. Political Economy Conference (PECO):

- 2017 discussant

American Political Science Association (APSA) Annual Meeting:

- 2014 participant and discussant, 2015 participant, 2016 participant, 2017 participant, 2018 participant

Midwest Political Science Association (MPSA) Annual Meeting:

- 2015 participant and discussant, 2016 participant and discussant, 2018 participant

Southern Political Science Association (SPSA) Annual Meeting:

- 2015 participant and discussant, 2016 participant and discussant, 2017 participant

Teaching Experience

Poli 315: Congress and the Legislative Process

- Fall 2014, Winter 2015, Fall 2015, Winter 2016, Summer 2017

Poli 328: Quantitative Analysis

- Winter 2017, Fall 2017, Fall 2019, Winter 2020, Fall 2020, Winter 2021

Poli 410: Undergraduate Research Seminar in American Politics

- Fall 2014, Winter 2015, Fall 2015, Winter 2016, Summer 2017

Awards and Grants

2019 BYU Mentored Environment Grant (MEG), American Ideology Project, $30,000

2017 BYU Political Science Teacher of the Year Award

2017 BYU Mentored Environment Grant (MEG), Funding American Democracy Project, $20,000

2016 BYU Political Science Department, Political Ideology and President Trump (with Jeremy Pope), $7,500

2016 BYU Office of Research and Creative Activities (ORCA) Student Mentored Grant x 3

- Hayden Galloway, Jennica Peterson, Rebecca Shuel

2015 BYU Office of Research and Creative Activities (ORCA) Student Mentored Grant x 3

- Michael-Sean Covey, Hayden Galloway, Sean Stephenson

2015 BYU Student Experiential Learning Grant, American Founding Comparative Constitutions Project (with Jeremy Pope), $9,000

2015 BYU Social Science College Research Grant, $5,000

2014 BYU Political Science Department, 2014 Washington DC Mayoral Pre-Election Poll (with Quin Monson and Kelly Patterson), $3,000

2014 BYU Social Science College Award, 2014 Washington DC Mayoral Pre-Election Poll (with Quin Monson and Kelly Patterson), $3,000

2014 BYU Center for the Study of Elections and Democracy, 2014 Washington DC Mayoral Pre-Election Poll (with Quin Monson and Kelly Patterson), $2,000

2012 Princeton Center for the Study of Democratic Politics Dissertation Improvement Grant, $5,000

2011 Princeton Mamdouha S. Bobst Center for Peace and Justice Dissertation Research Grant, $5,000

2011 Princeton Political Economy Research Grant, $1,500

Other Scholarly Activities

Expert Witness in Nancy Carola Jacobson, et al., Plaintiffs, vs. Laurel M. Lee, et al., Defendants. Case No. 4:18-cv-00262 MW-CAS (U.S. District Court for the Northern District of Florida)

Expert Witness in Common Cause, et al., Plaintiffs, vs. LEWIS, et al., Defendants. Case No. 18-CVS-14001 (Wake County, North Carolina)

Expert Witness in Kelvin Jones, et al., Plaintiffs, v. Ron DeSantis, et al., Defendants, Consolidated Case No. 4:19-cv-300 (U.S. District Court for the Northern District of Florida)

Expert Witness in Community Success Initiative, et al., Plaintiffs, v. Timothy K. Moore, et al., Defendants, Case No. 19-cv-15941 (Wake County, North Carolina)

Expert Witness in Richard Rose et al., Plaintiffs, v. Brad Raffensperger, Defendant, Civil Action No. 1:20-cv-02921-SDG (U.S. District Court for the Northern District of Georgia)

Georgia Coalition for the People's Agenda, Inc., et. al., Plaintiffs, v. Brad Raffensberger, Defendant. Civil Action No. 1:18-cv-04727-ELR (U.S. District Court for the Northern District of Georgia)

Expert Witness in Alabama, et al., Plaintiffs, v. United States Department of Commerce; Gina Raimondo, et al., Defendants. Case No. CASE No. 3:21-cv-00211-RAH-ECM-KCN (U.S. District Court for the Middle District of Alabama Eastern Division)

Expert Witness in League of Women Voters of Ohio, et al., Relators, v. Ohio Redistricting Commission, et al., Respondents. Case No. 2021-1193 (Supreme Court of Ohio)

ADDITIONAL TRAINING — EITM 2012 at Princeton University - Participant and Graduate Student Coordinator

COMPUTER SKILLS — Statistical Programs: R, Stata, SPSS, parallel computing

Updated December 22, 2021

Rebuttal to report of Michael Barber

Wesley Pegden

December 28, 2021

1 Introduction

In his report, Michael Barber presents the results of simulated district plans as part of an analysis which purports to elicit whether the enacted House and Senate maps of North Carolina are "partisan outliers". Barber makes choices in his analysis that reduce its ability to detect gerrymandering North Carolina clusters; for example, he discusses the partisan bias of the enacted House and Senate maps through the lens of the whole number of "Democratric-lean" districts in one hypothetical election, a lens through which even the effects of extreme gerrymandering in NC county clusters—each with a small number of districts—are made to appear less dramatic.

Nevertheless, his primary analyses (Tables 2 and 32) still find the whole-state House and Senate plans to be partisan outliers compared to his simulated maps, according to the definition he lays out in his report; in particular, he reports the middle-50% of simulated maps to have 46-51 total "Democratic-lean" districts across the House clusters he analyzes, and reports that the enacted map contains 45 such districts. For the Senate he reports a middle-50% range of 19-19 total Democratic-lean districts in his simulations, and that the enacted map contains 16 such districts.

In fact, Barber incorrectly calculated the distribution of Democrat-leaning seats for the whole-state outcomes of his simulation analysis, incorrectly reporting the sums of lower- and upper-quartile seat counts in individual clusters as the lower- and upper-quartile for total statewide seats. When the distribution of "lean Democrat district" counts at the whole-state level are calculated correctly for Barber's simulations (still using the partisan index he defines), one finds that the middle-50% range for Barber's simulated maps in the House is actually 48-50 Democratic-lean districts, not 46-51 as Barber shows, and that **the enacted North Carolina House map lies in the most Republican-biased 00.18% of whole state maps composed of Barber's simulations, and the enacted North Carolina Senate map lies in the most Republican-based 00.39% of whole state maps composed of Barber's simulations.** This computation can be carried out entirely with the figures provided in Barber's report, and uses Barber's simulated maps and Barber's metric of partisan bias (number of lean-Democrat districts), calculated with Barber's own partisan voting index.

Finally, when re-analyzing Barber's simulated maps (as provided in his backup data) to compare their expected performance over a range of electoral outcomes rather than comparing the crude number of "lean Democratic districts" for a fixed election average, the differences between the enacted map and Barber's ensemble of simulated comparison maps becomes more dramatic at the cluster level as well. Through this lens, every cluster which my original analysis found to be optimized for partisanship would qualify as a partisan outlier according to Barber's "middle 50%" criterion, and many are extreme outliers, among the most Republican biased 10%, 1%, or 0.1% of maps, even in clusters where Barber reported that the enacted map was not be a partisan outlier.

2 Barber finds the enacted House and Senate maps to be outliers according to his own definition

On page 29 of his report, in the section on House clusters, Barber writes that he considers a districting plan of North Carolina to be a partisan outlier if it lies outside of the "middle 50%" of simulation results; in Barber's report, the middle 50% are the maps that lie between the 25th and 75th percentiles according to the number of lean-Democrat districts, as measured with the partisan index Barber obtains by averaging election results. He calls this a "conservative definition" of an outlier, noting that "in the social sciences, medicine, and other disciplines it is traditional to consider something an outlier if it falls outside the middle 95% or 90% of the comparison distribution."

In both of his whole-state analysis tables (Table 2 and 32), Barber's own findings report the whole map as falling outside the middle 50% of simulated outcomes for the House and Senate. For example, in the last row, labeled "Total", of Table 2 on page 31, he reports that in the 26 clusters he analyzed, the enacted map contained 45 statewide "lean-Democrat" districts according to his partisan index, while the middle 50% range of the simulated maps for the total number of seats was $46 - 51$. Similarly, in Table 32 for the Senate, he reports the enacted map scored as having a total of 16 lean-Democrat seats in the 12 clusters used by the enacted map he analyzed, while the middle 50% range for his middle 50% range for the total number of seats in his simulated maps was 19-19. By the definition he chose to offer of a partisan outlier, Barber finds the enacted House and Senate plans are partisan outliers.

3 Barber reports incorrect quartiles for totals across clusters

Recall that in his Table 2, in the last column, Barber reports the range of the "middle 50%" for the number of lean-Democratic districts for his simulations in each cluster, and, at the bottom of the column, for the total across clusters (he reports the range for this total as 46-51). Recall that the bottom of the middle-50% range is the lower quartile of the data, and the top of the range is the upper quartile.

For example, in the House:

- for the Buncombe cluster in the House map, Barber reports in Figure 45 that 28% of his simulated maps contained 2 lean-Democrat districts, while 72% contained 3.
- for the Cumberland cluster in the House map, Barber reports in Figure 55 that 82% of his simulated maps contained 3 districts, while 18% contained 4.

I summarize this information in my Table 1, below:

Cluster	0	1	2	3	4
Buncombe			28%	72%	
Cumberland				82%	18%

Table 1: Fraction of maps with various lean-Democrat-district counts, as reported by Barber for Buncombe and Cumberland county districtings.

In his Table 2, Barber correctly summarizes the middle 50% ranges for the data in each of these clusters as 2-3 and 3-3, respectively; in each case, the lower end of the range is the smallest value below which 25% of his simulated maps lie, and the upper end is the smallest value below which 75% lie.

Suppose though, just as an example, that we wished to calculate the distribution of the total number of lean-Democrat districts across just these two clusters according the Barber's simulations; this will also enable us to calculate the middle-50% of outcomes for the total lean-Democrat districts across these two clusters.

Note that for maps of these two clusters composed of maps from Barber'simulations, a total of 5, 6, or 7 lean-Democrat districts are possible. For example, 5 lean-Democrat districts can arise only by having 2 such districts in Buncombe and 3 in Cumberland, and fewer are not possible.

According to Barber's simulations, as summarized in Table 1, 28% of the maps of these two clusters would have 2 lean-Democrat districts in Buncombe, while 82% would have 3 lean-Democrat districts in Cumberland. As the districtings in each cluster can be chosen independently of each other, a total of

$$28\% \times 82\% = 22.96\%$$

of districtings of these two counties would have a total of 5 lean-Democrat districts. (Note that having fewer than 5 lean-Democrat seats happens 0% of the time, according to Barber's simulations.)

6 lean-Democrat districts can arise from having 2 lean-Democrat districts in Buncombe and 4 in Cumberland, or having 3 lean-Democrat districts in Buncombe and 3 in Cumberland. Thus according to Barber's simulation results the frequency of this outcome would be

$$28\% \times 18\% + 72\% \times 82\% = 64.08\%.$$

Finally, the likelihood of 7 lean-Democrat seats, which arise just when there are 3 lean-Democrat districts in Buncombe and 4 lean-Democrat districts in Cumberland, would be

$$72\% \times 18\% = 12.96\%,$$

(Note that altogether, 22.96%+64.08%+12.96%=100%.)

Evidently, the middle-50% range for the total of lean-Democrat seats across these two counties would be 6-6; the 6-lean-Democrat-district maps include the middle-50% of simulated maps. (6 is both the 25th percentile and the 75th percentile of the number of Democratic-lean seats in the simulated maps.)

Under Barber's incorrect approach, he would have simply added the bottom and top of the middle-50% ranges for Buncombe and Cumberland (2-3 and 3-3, respectively) to arrive at a middle-50% range for the total number of lean-Democrat-districts across these two counties; that procedure would produce a range of 5-6, which is wider than the true middle-50% range of the total number of districts across the two counties (namely 6-6), as correctly calculated above.

In general, the magnitude of this error grows larger and larger the more independent cluster-specific results are aggregated by incorrectly summing the lower and upper quartiles as a substitute for a correct calculation of the distribution of total statewide lean-Democrat districts. In Barber's report, he aggregates across 26 clusters in this way. As we will see in the next section, this has the effect of inflating the true middle-50% range of 48-50 to an incorrectly reported range of 46-51.

Technical Remark. Probability generating functions can be used to allow larger calculations of the same type as the one above to be performed using publicly web-based computer algebra systems instead of by programming or using statistical software. Note that precisely the same three calculations above would have been performed if expanding the algebraic expression

$$\begin{aligned}(.28x^2 + .72x^3)(.82x^3 + .18x^4) &= (.28 \times .82)x^5 + (.28 \times .18 + .72 \times .82)x^6 + (.72 \times .18)x^7 \\ &= .2296x^5 + .6408x^6 + .1296x^7.\end{aligned}$$

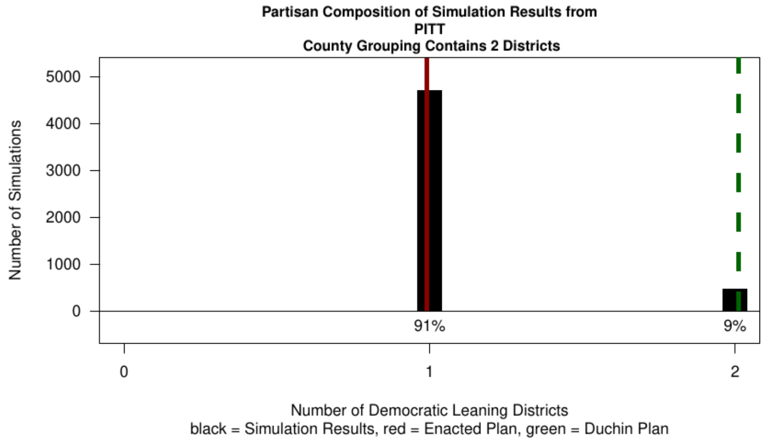
Observe that the polynomial $.28x^2 + .72x^3$ here can be seen as representing the fact that two seats occur in 28% of the maps for Buncombe, while 3 seats occur in 72% of the maps. (Similarly, then, for Cumberland and the polynomial $.82x^3 + .18x^4$.) The same answers that we found above for the fraction of simulated plans with a total of 5, 6, and 7 lean-Democrat districts, respectively, can be read off as the coefficients of x^5, x^6, and x^7, in the resulting expansion.

In the technical remark in the next section, I will point out a similar polynomial expansion which can verify the next section's calculations using public web applications, making the main findings of this rebuttal report easy to independently verify.

4 Correcting Barber's calculations

In my Table 2 on page 13 of this rebuttal report, I report the results of Barber's Figures 11, 14, 17, 20, 25, 28, 31, 34, 37, 45, 48, 51, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, and 88. Each of these figures reports, for one of the clusters Barber analyzes, the fraction of his simulated maps which achieve different numbers of "lean Democrat" districts according to the partisan index he uses. For example, in Figure 14 on page 44, Barber reports that 91% of his simulated maps had one lean-Democrat district, while the remaining 9% had 2, as seen in this reproduction below:

Figure 14: **Distribution of Partisan Districts from Simulations in Pitt House County Cluster**



This information is then reproduced in my Table 2 on page 13, as the following row:

Cluster	0	1	2	3	4	5	6	7	8	9	10	11	12
Pitt		91%	9%										

In particular, everything in my Table 2 (and the corresponding Table 3 for the Senate) is taken directly from Barber's report itself.

The data in Table 2 can then be used to calculate the distribution of the total number of lean-Democrat seats based on Barber's simulations across the 26 clusters, exactly in the same way as we did above for just 2 clusters from the data in Table 1. The result of the same calculation is the histogram shown in Figure 1. In particular, according to Barber's own simulated map set, and using his own measure of the number of lean-Democrat districts under his own partisan index, **the enacted House map exhibits more Republican bias than 99.82% of maps** composed of Barber's simulations, over the clusters Barber analyzes.

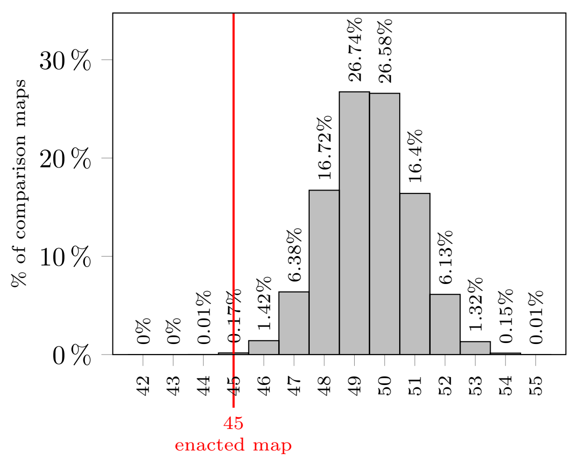


Figure 1: **Total lean-Democrat districts across Barber's House simulations**. This histogram shows the performance of Barber's simulated map set across the total set of House clusters Barber analyzes. It uses Barber's set of simulated maps, Barber's chosen metric (number of lean Democratic seats), calculated using the partisan metric Barber himself calculates in his report. The range 49-50 contains 50% of the simulated maps, the range 48-51 contains 86% of the simulated maps, and the range 47-52 contains more than 98% of the simulated maps. With 45 lean-Democratic districts across these clusters, the enacted map is in the most Republican-biased 0.18% of Barber's simulated maps.

In Table 3 I show Barber's Senate data analogous to the House data I show in Table 2. And in Figure 2, I plot the histogram showing the total of Barber's metric of Democratic-leaning districts across Barber's simulated map set, produced in the same way as I produce Figure 1 for the House. In particular, according to Barber's own simulated map set, and using his own measure of the number of lean-Democrat districts under his own partisan index, **the enacted Senate map exhibits more Republican bias than 99.61% of maps** over the clusters Barber analyzes.

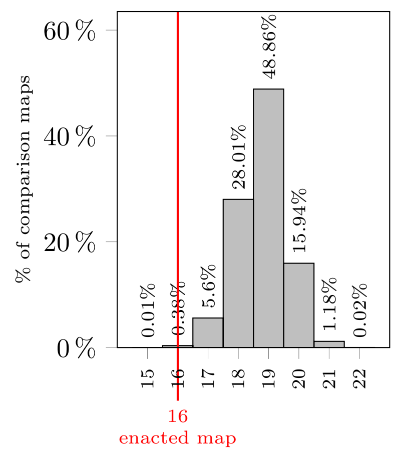


Figure 2: **Total lean-Democrat districts across Barber's Senate simulations**. This histogram shows the performance of Barber's simulated map set across the total set of Senate clusters Barber analyzes. It uses Barber's set of simulated maps, Barber's chosen metric (number of lean Democratic seats), calculated using the partisan metric Barber himself calculates in his report. The range 18-20 contains 93% of the simulated maps, and the range 17-21 contains more than 99% of the simulated maps. With 16 lean-Democrat districts, the enacted map is among the most Republican 00.39% of maps.

Technical Remark. As noted in the earlier Technical Remark, calculating the results of a histogram like Figure 1 is equivalent to expanding a certain polynomial expression. Based on the data in Table 2, (rows with only zero seats possible can be ignored), the polynomial to be expanded is

$$(.91x+.09x^2)(.44+.56x)(x^2)(x^2)(x)(.28x^2+.72x^3)(.82x^3+.18x^4)(x^4)(x)(.33x^2+.5x^3+.17x^4)(.99+.01x^1) \\ \cdots(.18+.82x)(.01x^4+.79x^5+.21x^6)(.01x^{10}+.56x^{11}+.44x^{12})(.02x^{10}+.32x^{11}+.66x^{12})$$

and publicly available tools such as `wolframalpha.com` can be used to verify that this polynomial expands to

$$5.55283\times10^{-7}x^{56}+0.0000685893x^{55}+0.00147488x^{54}+0.0131615x^{53} \\ +0.0612515x^{52}+0.163979x^{51}+0.265839x^{50}+0.267369x^{49}+0.167218x^{48}+0.0637935x^{47}+0.0141775x^{46} \\ +0.00167669x^{45}+0.000089375x^{44}+1.74341\times10^{-6}x^{43}+1.08123\times10^{-8}x^{42}$$

The histogram in Figure 1 can be read off the coefficients in this polynomial. For example, the fact that the coefficient of x^{49} is .267369 corresponds to the fact that Figure 1 reports the fraction of simulated maps with a total of 49 Democrat-leaning districts across the clusters Barber analyzes as 26.74% (rounded to two decimal places).

For the senate, from Table 3, the probability generating function is

$$(.77x+.23x^2)(x^2)(.23+.77x)(.93x^2+.06x^3)(.01x^4+.24x^5+.75x^6)(.05x^4+.95x^5)x(.97x+.03x^2),$$

which expands to

$$0.000227131x^{22}+0.0118152x^{21}+0.159415x^{20}+0.488577x^{19} \\ +0.280141x^{18}+0.0559707x^{17}+0.00377389x^{16}+0.0000807399x^{15} \quad (1)$$

giving the results shown in Figure 2.

5 A more sensitive cluster-by-cluster analysis of Barber's maps

In the previous section, I showed that even against Barber's simulated maps, using the partisan index Barber calculates, and using Barber's preferred metric for partisan bias (the number of lean-Democrat districts using that partisan index), both the enacted House and Senate plans are extreme partisan outliers.

This is true despite the fact that using the number of whole lean-Democrat districts with only a single proxy for partisanship is unlikely to capture the effects even of extreme gerrymandering in North Carolina county clusters, where a small number of seats are at stake in each, and the effects of extreme gerrymandering can be to put one or two seats into play (or take them out of contention), even in cases where districts do not change columns in a single hypothetical election.

In other words, I take Barber's single partisan index (which has a two-party statewide Democratic vote-share of XX), and analyze what would happen under his simulations, on average, if you swung the election results so that Democrats did better or worse by a normally-distributed swing matched to past statewide North Carolina elections. This is the same metric I used in my initial report.

In this section, I re-analyze Barber's results, still using his simulated maps, and still using his partisan index, but comparing maps in each cluster using the seats-expected metric (calculated with respect to that index), which evaluates how a map would be expected to perform under a range of conditions rather than one fixed hypothetical election.

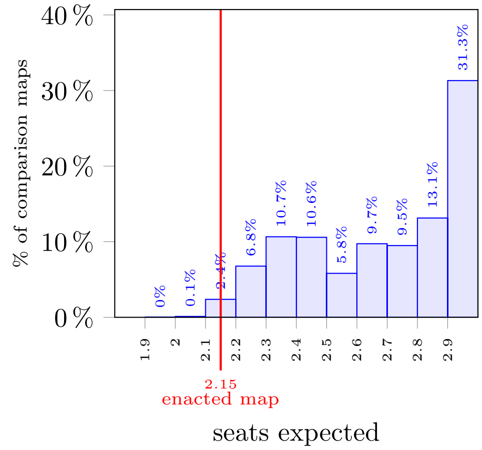
Below, I conduct this analysis for every county cluster I analyzed in my original expert report. In every cluster for which my analysis found the enacted map to be among the most optimized-for-partisanship possible maps (the first six House analyzed in the subsections below, and every Senate cluster analyzed below), Barber finds the map to be a partisan outlier according to the "middle-50%" definition he uses in his report. I summarize the outlier status of these 6+5 House and Senate clusters according to Barber's simulations in the following table:

Cluster	Enacted map among most Republican-biased…
House: Buncombe	00.797%
House: Forsyth-Stokes	00.0805%
House: Guilford	00.00646%
House: Mecklenburg	04.43%
House: Wake	05.78%
House: Pitt	24.2%
Senate: Cumberland-Moore	00.0024%
Senate: Forsyth-Stokes	00.01%
Senate: Granville-Wake	00.035%
Senate: Guilford-Rockingham	00.25%
Senate: Iredell-Mecklenburg	00.1%
	…against Barber's simulations.

Among the four remaining clusters in my report, there are two where the enacted maps are nevertheless extreme outliers against Barber's simulation sets. I summarize the results for these four clusters in the following table:

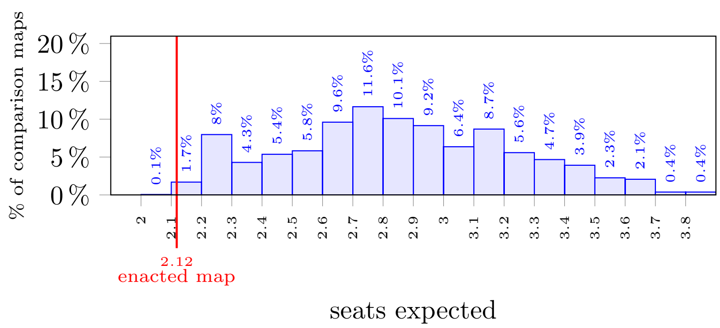
Cluster	Enacted map among most Republican-biased…
House: Alamance	39.4%
House: Brunswick-New Hanover	73.9%
House: Durham-Person	00.00265%
House: Cabarrus-Davie-Rowan-Yadkin	00.352%
	…against Barber's simulations.

5.1 House: Buncombe



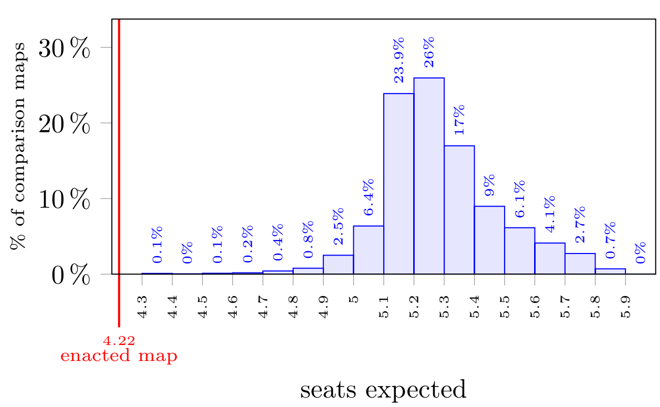
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.797% of maps.

5.2 House: Forsyth-Stokes



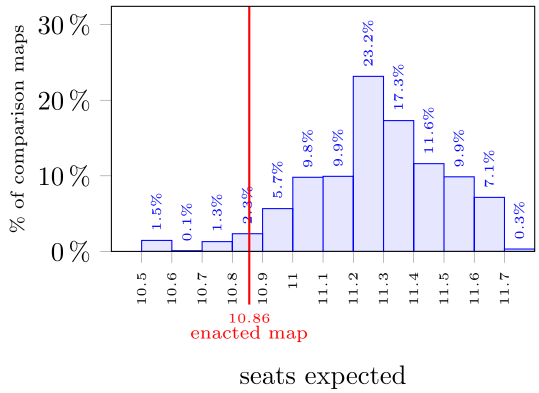
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.0805% of maps.

5.3 House: Guilford



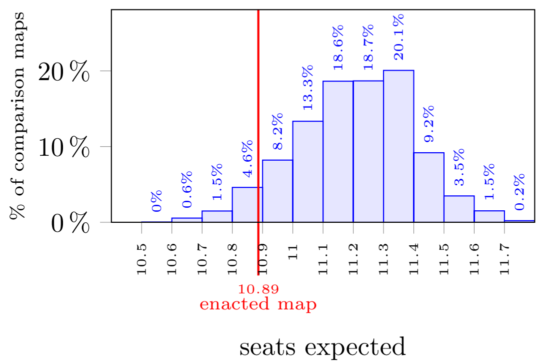
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.00646% of maps.

5.4 House: Mecklenburg



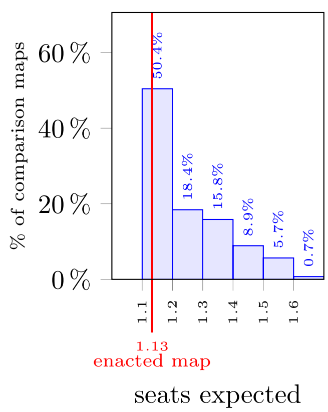
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 4.43% of maps.

5.5 House: Wake



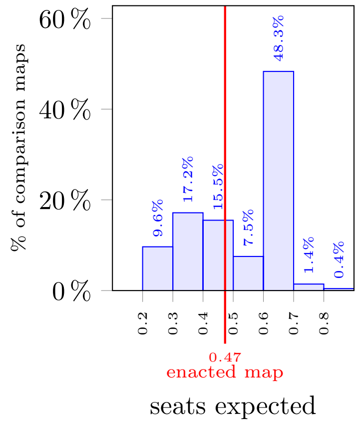
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 5.78% of maps.

5.6 House: Pitt



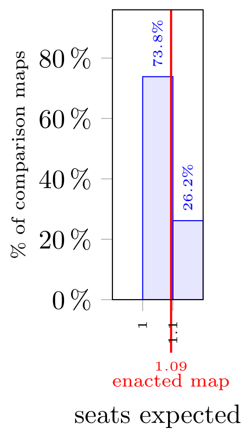
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 24.2% of maps.

5.7 House: Alamance



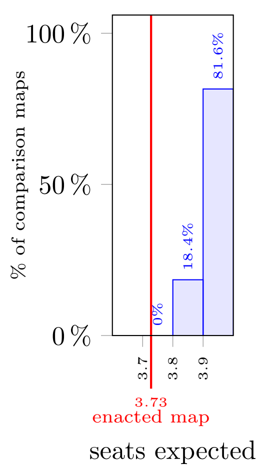
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map is not an outlier.

5.8 House: Brunswick-New Hanover



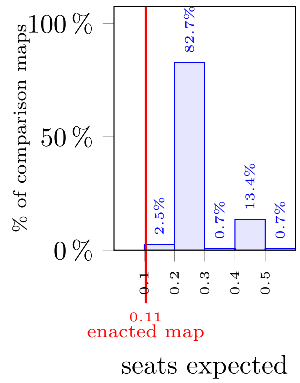
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map is not an outlier.

5.9 House: Durham-Person



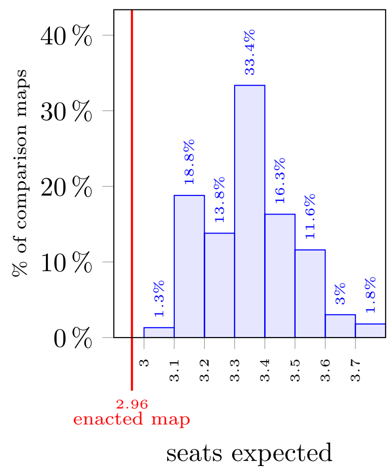
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.00265% of maps.

5.10 House: Cabarrus-Davie-Rowan-Yadkin



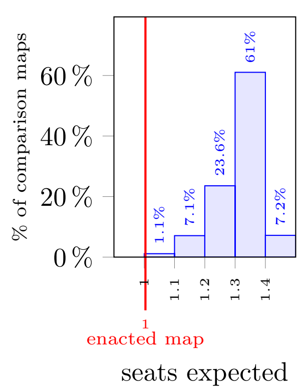
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.352% of maps.

5.11 House: Cumberland



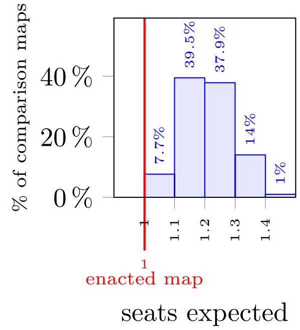
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.0095% of maps.

5.12 Senate: Cumberland-Moore



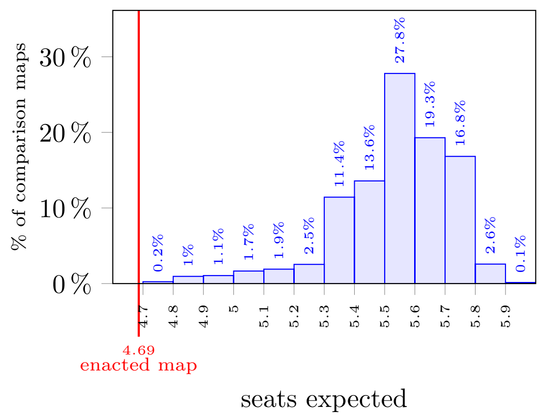
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.00235% of maps.

5.13 Senate: Forsyth-Stokes



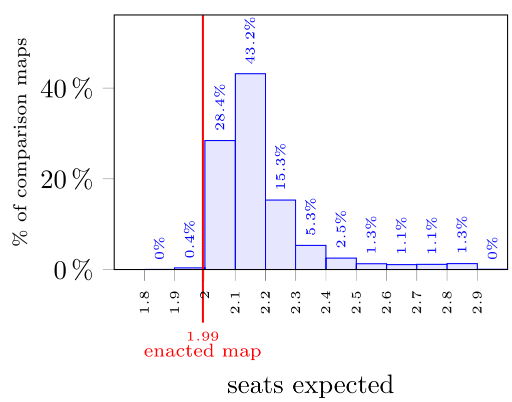
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.0104% of maps.

5.14 Senate: Granville-Wake



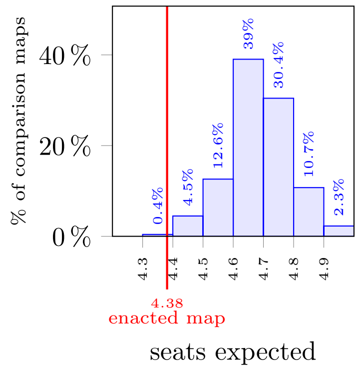
Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.0353% of maps.

5.15 Senate: Guilford-Rockingham



Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.251% of maps.

5.16 Senate: Iredell-Mecklenburg



Against the comparison-set of Barber's simulated maps for this cluster, the enacted map in this cluster is among the most Republican-biased 0.104% of maps.

Cluster	0	1	2	3	4	5	6	7	8	9	10	11	12
Davidson	100%												
Pitt		91%	9%										
Alamance	44%	56%											
Columbus-Robeson	100%												
Carteret-Craven													
Duplin-Wayne	100%												
Nash-Wilson			100%										
Caswell-Orange			100%										
Alexander-Surry-Wilkes	100%												
Franklin-Granville-Vance		100%											
Alleghany-*etc*	100%												
Beaufort-*etc*	100%												
Buncombe			28%	72%									
Anson-Union	100%												
Onslow-Pender	100%												
Cumberland				82%	18%								
Harnett-Johnston	100%												
Catawba-Iredell	100%												
Durham-Person					100%								
Brunswick-New Hanover		100%											
Forsyth-Stokes			33%	50%	17%								
Cabarrus-*etc*	99%	1%											
Chatham-*etc*	18%	82%											
Guilford					1%	79%	21%						
Avery-*etc*	100%												
Mecklenburg											1%	56%	44%
Wake											2%	32%	66%

Table 2: This table collects in one place the fraction of maps in Barber's House simulation sets realizing each number of lean-Democratic seats, as reported by Barber in his Figures 11, 14, 17, 20, 25, 28, 31, 34, 37, 45, 48, 51, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, and 88. He does not present figures for the clusters in Alleghany-Ashe-Caldwell-Watauga and Beaufort-Chowan-Currituck-Dare-Hyde-Pamlico-Perquimans-Tyrrell-Washington clusters because his 0-Democratic-district results for those clusters are based on a very small number of maps. For Carteret-Craven his method does not produce any maps.

Cluster	0	1	2	3	4	5	6
Cumberland-Moore		77%	23%				
Chatham-Durham			100%				
Alleghany-*etc*	100%						
Brunswick-Columbus-New Hanover	23%	77%					
Bladen-*etc*	100%						
Guilford-Rockingham			94%	6%			
Alamance-*etc*	100%						
Granville-Wake					1%	24%	75%
Iredell-Mecklenburg					5%	95%	
Buncombe-Burke-McDowell		100%					
Cleveland-Gaston-Lincoln	100%						
Forsyth-Stokes		97%	3%				

Table 3: This table collects in one place the fraction of maps in Barber's Senate simulation sets realizing each number of lean-Democratic seats, as reported by Barber in his Figures 95, 98, 103, 106, 110, 113, 117, 120, 123, 128. He does not present figures for the Bladen-Duplin-Harnett-Jones-Lee-Pender-Sampson and Cleveland-Gaston-Lincoln clusters because his 0-district results for these clusters are based on a small number of maps.

I hereby certify that the foregoing statements are true and correct to the best of my knowledge, information, and belief.

Wesley Pegden
12/28/2021

Response to Expert Report by Dr. Barber on the North Carolina State Legislature Redistricting Plans

Jonathan C. Mattingly

December 28, 2021

Contents

1 Introduction

The report by Dr. Michael Barber begins with a discussion of the political geography of the state of North Carolina. He emphasizes the heterogeneity of the state. While he points out the strengths of ensemble methods to separate the effect of natural clustering of votes and other effects due to political geography, Dr. Barber limits its use to analysis of the individual county clusters. Similarly, though he uses a collection of election data at the cluster level, he does not consider a diverse collection of election analyses both at the cluster level and when performing his statewide analysis. Rather, he restricts himself to a single summary statistic, namely, counting the number of Democratic-leaning districts at the individual cluster level based primarily on a composite election obtained through averaging several past statewide elections.

We complete the missing parts of Dr. Barber's analysis using data directly from his report when possible. When needed, we augment this data with an ensemble of maps obtained by running Dr. Barber's code. From this completed analysis, we see that Dr. Barber's ensemble shows both the Enacted NC House and the Enacted NC Senate to be extreme partisan outliers with a clear and systematic tilt in favor of electing Republicans.

When we focus on the structure of the enacted maps in the county clusters under Dr. Barber's analysis, we again see the same structures we observed using the Primary Ensembles from our initial report. These structures showed the enacted map to be an extreme outlier. Due to time constraints, we did not complete cluster level analysis on all clusters using Dr. Barber's simulations; we have, however, performed a cluster level analysis on a diverse collection of clusters in the NC House. Our cluster level analysis considers not only seat counts, but also the margins of victory within those seats. By examining the margins, we identify extreme partisan behavior at the cluster level using the very sampling code that Dr. Barber created.

We conclude that Dr. Barber's ensembles provide another independent verification that the enacted plans for the NC House and NC Senate are extreme gerrymanders.

2 Comment on Political Geography of State

In Section 3 of Dr. Barber's report, he discusses the political geography of the state. He made a number of statewide evaluations of the partisan structure using a single average of 11 statewide elections from 2014-2020. As his analysis in later sections makes clear, the political climate varies significantly from year to year and election to election. The average of these elections creates a new set of voting data, possibly quite district from those averaged to create it. I see no reason to elevate the behavior and properties of a map under the one particular political environment signified by this vote over other elections. It is important that the map used to translate our election votes into elected officials act in a non-biased way across a number of elections which represent different political climates seen in North Carolina, not just one.

In the rest of the report, Dr. Barber does switch to considering a number of distinct elections. However, he does not return to any aggregate statewide discussion using these individual elections and the diversity of election environments they represent. He does firmly endorse the use of a computer drawn ensemble of maps to create a base line against which the enacted map can be compared. He correctly represents that this method has the advantage of taking into account all of the political geography of the state, such as the concentrating of particular voters in some regions of the state or the preservation of counties and the like. Hence, when a map is an outlier compared to a computer drawn ensemble, these natural clustering or political geography considerations cannot be the explanation.

Dr. Barber never conducts any statewide analysis under his ensemble using different election results. However, all of the components necessary to perform such analysis are present in his report. Utilizing Dr. Barber's cluster-by-cluster ensembles, we complete the absent statewide analysis to examine the number of Democratic leaning seats under various elections. This analysis demonstrates that the enacted map *is* an extreme outlier when compared to Dr. Barber's ensemble.

3 Nonpartisan Ensemble Generated by Dr. Barber

In analyzing the North Carolina State House and Senate maps, Dr. Michael Barber generates an ensemble of non-partisan redistricting maps via the Sequential Monte Carlo (SMC) procedure in the *redist* R-package developed and maintained by a research group at Harvard University. When used to sample from a known distribution in a moderate sized problem, this method has been shown to faithfully sample the target distribution. This was validated on moderate sized examples using an enumeration algorithm developed by the same group that developed the *redist* R-package at Harvard. The method we used has similarly been validated using this and other methods. Dr. Barber used the ensemble method only at the cluster level and does not use it to perform a statewide analysis based on a statewide ensemble. Rather he just summarizes the cluster by cluster results in a few tables (Table 2 and Table 32) instead of performing any analysis which would show the cumulative effect at the statewide level. The coin flipping analogy we offer below shows why this is so inadequate. In utilizing Dr. Barber's ensemble, we demonstrate that he would have concluded the enacted map was an extreme outlier at the statewide level. This is not an endorsement of any of the particular algorithm choices he has made, but rather to demonstrate that this conclusion is available from his findings.

By taking the percentages in the cluster-by-cluster tables in Dr. Barber's report, we were able to perform the statewide analysis he neglected using his data. We were also able to perform this for the collection of different statewide elections Dr. Barber used in his analysis. This allowed us to see the behavior of the maps under different types of elections. Both of these considerations are important and we briefly discuss them individually before turning to the statewide analysis using Dr. Barber's data.

- **Importance of statewide analysis:** Dr. Barber analyzes each cluster one-by-one and concludes that the majority of them are not extreme outliers so under his election composite the map is not an outlier. However, in almost every case, he finds that the more Republican of the non-outlying options is selected. Consider the following analogy. Someone flips a coin that they claim is fair but is in fact biased to produce heads more often. They flip the coin and produce 40 heads and zero tails. On each flip, the chance of getting a head from a fair coin is 50%. Hence the outcome on each flip is not that surprising. Dr. Barber's analysis is analogous to looking at each flip alone and then claiming that the coin is fair because the outcome was a head and the chance of a fair coin producing a head was reasonable. However, taking a more global view one can an easily see that the chance of getting 40 heads in a row is astronomically small. And thus, one can conclude the coin is biased. This would even be true if there were only 35 heads and 5 tails.

  Analogously, each cluster taken individually might not be an extreme outlier, but it is extremely unlikely that all of these clusters woud exist together in a statewide map drawn without partisan intent.

  We will also see that some of the local clusters are extreme outliers in their own right using Dr. Barber's data and extending his analysis to look at the margins of victory (or the extent of the partisan lean) rather than only focusing on the number of seats won by either party (or the direction of the partisan lean). This extended analysis agrees with the finding in our initial report.

- **Often extreme behavior is apparent in only some elections:** If one wanted to rig a card game by colluding with some of the other players, the group would only need to act when none of the group was going to win. The group need only act when cards were aligned against them. Hence, the behavior of a gerrymandered map might appear typical in settings where the gerrymandering party is content with the outcome that one would typically expect without gerrymandering. Furthermore, it is possible that whatever system the card players are using is not sufficient to counteract some hands. In other words, even a card player that is cheating might not be able to win when their opponent draws a royal flush. Hence, it is not to be expected that in all cases a gerrymandered map is effective in supporting the gerrymandering party.

  In particular, one can not simply declare that a map is not gerrymandered because it is fair in some fraction (even a relatively large fraction) of the election environments. If it is clearly gerrymandered in some reasonable and pertinent election environments, then the map should be seen as gerrymandered. To do otherwise would be to argue that a casino would be happy with card players who only cheated 30% of the time and in particular did not cheat when they were already winning or had an unsalvageable hand.

In addition to generating a statewide analysis using the actual data from Dr. Barber's report, we also employ ensembles generated from the *redist* code base, set up according to Dr. Barber's analysis scripts.[1] We then show that well-established methods of probing for gerrymandering reveal that many of the individual clusters are indeed extreme gerrymanders. In doing so, we consider the partisan seat counts of each party and also extend the analysis to consider *how* the seats are won. The latter is important as it shows the degree that a given district is politically safe as well as determines how future political swings, unseen at present, might affect political outcomes. For example, atypically polarized districts can lead to maps which do not respond to the shifts in the electorate's preferences, and effectively lock in a particular outcome. Additionally, when a map has an extremely partisan structure, this can speak to the intent of the map makers even if the structure would be unlikely to affect some collection of elections such as wave elections in favor of the gerrymandering party.

[1]Dr. Barber did include a R Data file which might have included the maps he generated in his run. However, since our version of R was slightly different than his, it would not load. Hence we were forced to re-run his code.

4 Statewide Analysis of Dr. Barber's Ensemble of NC House Plans

Within each cluster, Dr. Barber presents the fraction of plans in his ensembles that would lead to a certain number of Democratic districts under each set of historic and averaged vote counts. These tables can be used to construct the probability of drawing a non-partisan plan at the statewide level that would yield a certain number of Democratic leaning districts under various elections.

Beginning with his averaged statewide vote counts, we construct the statewide probabilities of electing various numbers of representatives and present them in Figure 1 in terms of the number of Democrats elected. Only 0.177% of all of the plans in Dr. Barber's ensemble elect the same or more Republicans than the enacted plan.

Note that our count of Democrats elected includes the Democrats elected in single-district clusters, which are omitted from Dr. Barber's Table 2. So our Figure 1 reports that the enacted plan elects 49 Democrats under Dr. Barber's composite of elections, which is the four Democrats elected in single-district clusters that Dr. Barber reports in his Table 1 plus the 45 Democrats elected in multi-district clusters that Dr. Barber reports in his Table 2.

We repeat the above analysis with the 2016 and 2020 election data used by Dr. Barber. The only supplemental data we introduce is the number of single district Democratic clusters in each election which we have taken from our previous analysis. We summarize the 10 elections in Figure 2 and Table 1.

As in our previous analysis, we find that the outlier status of the ensemble has a significant impact on the amount of power the Republicans can amass in the House. For example, under the votes of the 2020 Lt. Governor race, 2016 Presidential race, and 2020 US Senate race, the ensemble breaks a Republican supermajority in 99.3937%, 98.976, and 99.992% of the plans in Dr. Barber's ensemble, respectively. However, the enacted plan would elect a Republican supermajority under each of these votes. Similarly, under the 2020 Governor race, the Republican majority would have been broken in 96.42% of the plans in Dr Barber's ensemble, yet they would have maintained the majority using the enacted map under these votes.

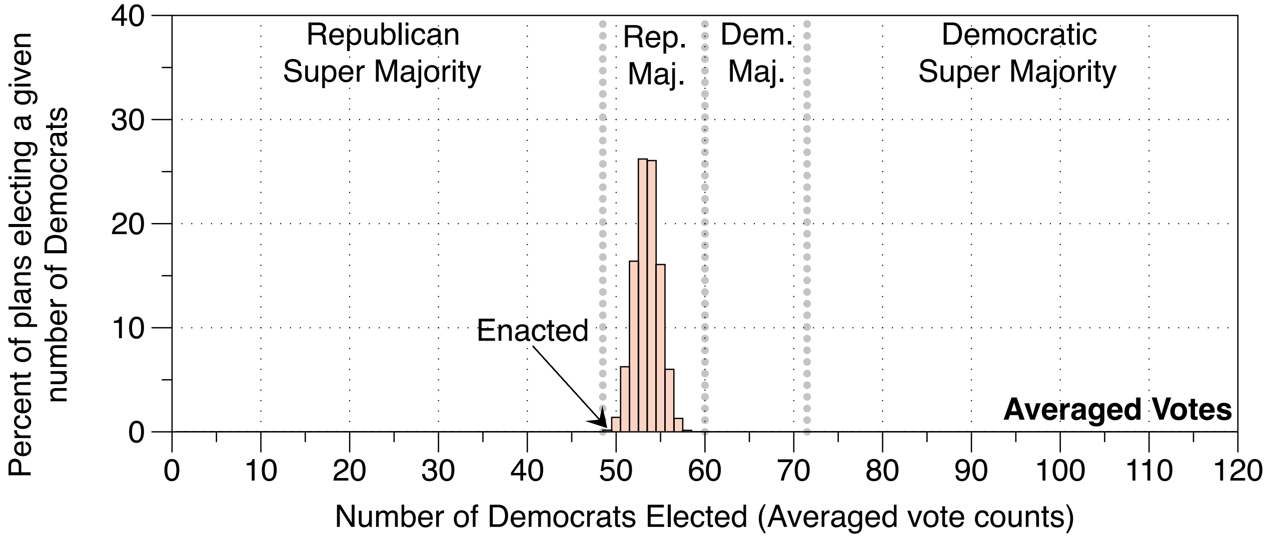


Figure 1: We compare Dr. Barber's statewide ensemble with the enacted plan under the Averaged election results used in his report. We find that only 0.177% of all of the plans in his ensemble would elect the same or more Republicans.

Election	Statewide Dem. Vote	% of Dr. Barber's Plans electing the same or more Republicans than the enacted plan
Barber's Average Vote	-	0.177%
2020 Governor	52.32%	0.204%
2016 Attorney General	50.20%	1.34%
2020 Attorney General	50.13%	0.00684%
2016 Governor	50.047%	0.215%
2020 President	49.36%	0.000146%
2020 Senate	49.14%	0.00804%
2020 Lt. Governor	48.40%	0.000377%
2016 President	48.024%	1.02%
2016 Senate	46.98%	0.223%
2016 Lt. Governor	46.59%	0.518%

Table 1: When considered at the statewide level, the ensembles produced by Dr. Barber are all extreme outliers. The chance that a plan drawn from the ensemble would elect the same or more Republicans as the enacted plan is, at most, 1.34%; in all but three of the elections it is less than 0.25%. We have ordered the elections with the election with the largest Democratic statewide vote fraction at the top and the election with largest Republican statewide vote fraction at the bottom. It is worth noting that many of the most extreme outliers happen for those between 50% and 48%. Looking at Figure 2, we see that this is the range where the Republicans would typically lose the super majority according to Dr. Barber's analysis. Though "Barber's Average Vote" which he used as a partisan index might or might not represent an actual plausible voting pattern, we have included it for comparison.

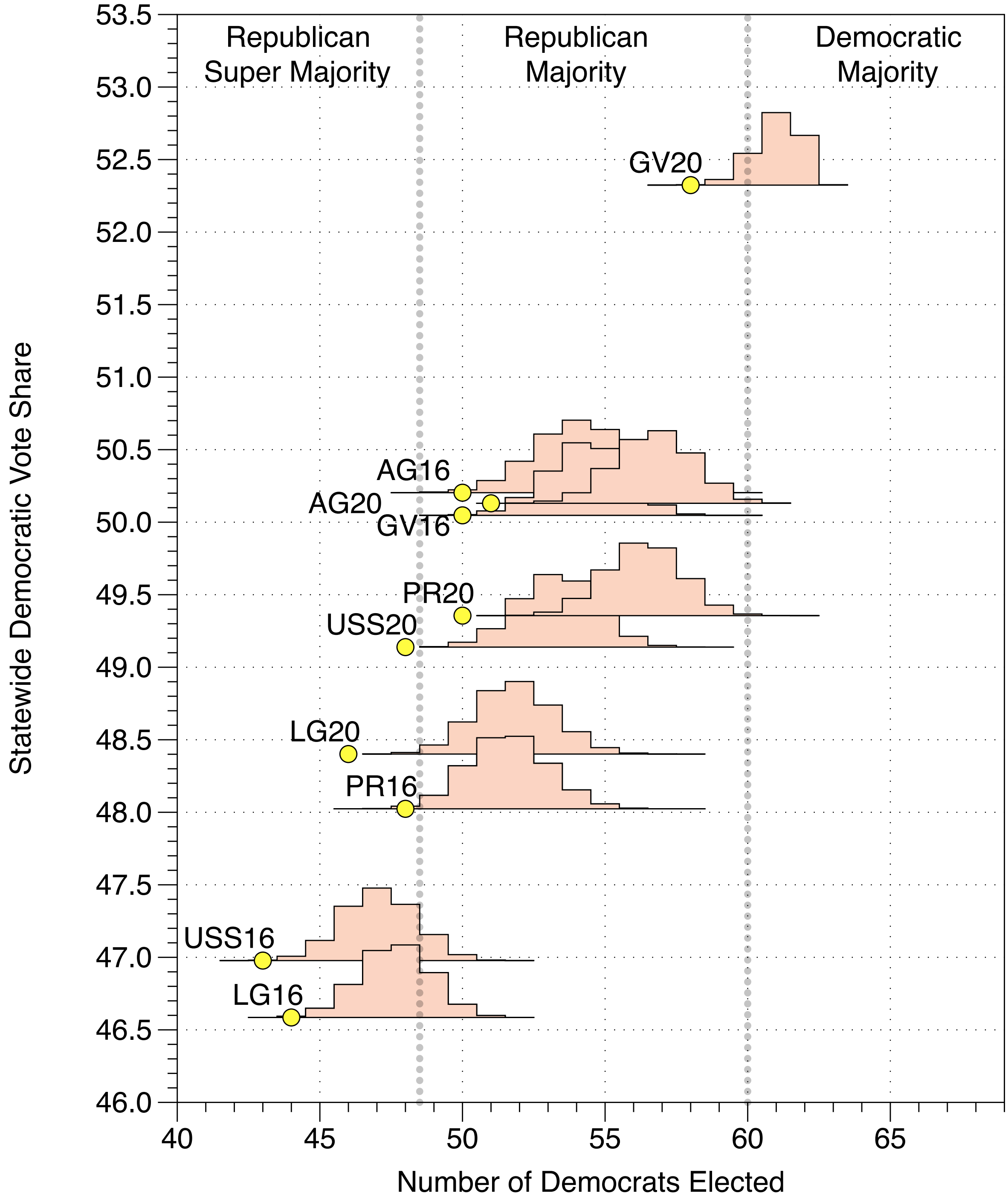


Figure 2: We compare Dr. Barber's statewide ensemble with the enacted plan under the ten 2016 and 2020 elections used in his report. Yellow dots show the result of the enacted plan. The enacted plan is an extreme outlier when considering the same data under a statewide lens. We summarize the numerical extent of the outliers in Table 1. The elections are abbreviated with the last two digits signifying the year, and the first letters representing Lt. Governor (LG), Governor (GV), President (PR), and US Senate (USS).

5 Statewide Analysis of Dr. Barber's Ensemble of NC Senate Plans

Repeating the above analysis for Dr. Barber's ensemble of Senate plans, we begin with the averaged statewide vote counts. We construct the statewide probabilities of electing various numbers of Senators and present them in Figure 3. Once again, our count of Democrats elected includes the Democrats elected in single-district Senate clusters, which are omitted from Dr. Barbers Table 32. So our Figure 3 reports that the enacted plan elects 20 Democrats under Dr. Barbers composite of elections, which is the four Democrats elected in single-district clusters that Dr. Barber reports in his Table 31 plus the 16 Democrats elected in multi-district clusters that Dr. Barber reports in his Table 32. Only 0.00385% of all of the plans in Dr. Barber's ensemble elect the same or more Republicans. Furthermore, this is the percentage of plans that lead to a Republican supermajority under these votes (which the enacted plan would produce as well). In other words, while the enacted plan always produces a Republican supermajority under Dr. Barber's analysis, only .00385% of the non-partisan plans that Dr. Barber simulates would produce a Republican supermajority.

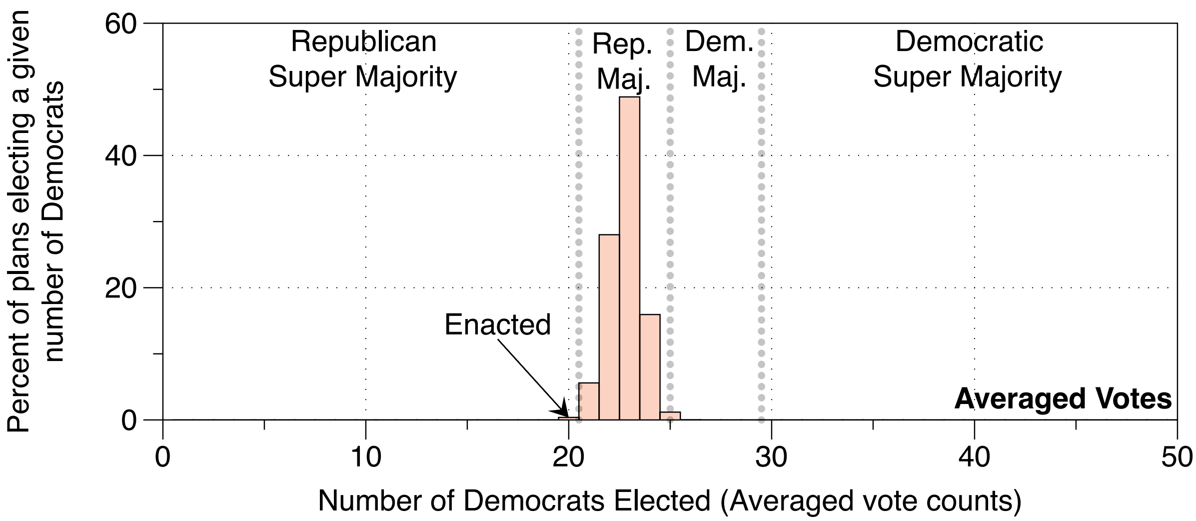


Figure 3: We compare Dr. Barber's statewide ensemble with the enacted plan under the Averaged election results used in his report. We find that only 0.00385% of all of the plans in his ensemble would elect the same or more Republicans than the enacted plan.

We repeat the above analysis with the 2016 and 2020 election data used by Dr. Barber. The only supplemental data we introduce is the number of single district Democratic clusters in each election which we have taken from our previous analysis. We summarize the 10 elections in Figure 4 and Table 2.

Again, we find that the outlier status of the ensemble has a significant impact on the amount of power the Republicans can amass in the Senate. Under the votes of the 2016 Governor race and 2016 Attorney General races, the Republicans lose their supermajority in 99.9544% and 98.9501% of the plans in Dr. Barber's ensemble, respectively. However, the enacted plan would elect a Republican supermajority under each of these voting patterns.

Election	Statewide Dem. Vote	% of Dr. Barber's Plans electing the same or more Republicans than the enacted plan
Averaged	-	0.00385%
2020 Governor	52.32%	1.92%
2016 Attorney General	50.20%	1.05%
2016 Governor	50.047%	0.047%
2020 Attorney General	50.13%	3.74%
2020 President	49.36%	9.92%
2020 Senate	49.14%	5.76%
2020 Lt. Governor	48.40%	0.250%
2016 President	48.024%	0.16%
2016 Senate	46.98%	1.22%
2016 Lt. Governor	46.59%	10.9%

Table 2: When considered at the statewide level, many of the ensembles produced by Dr. Barber are extreme outliers. In six of the ten elections, there is less than a 2% chance that a plan drawn from the ensemble would elect the same or more Republicans as the enacted plan; in three of the ten elections, there is less than a 0.251% chance that a plan drawn from the ensemble would elect the same or more Republicans than the enacted plan. As we have remarked in both our original report and in the analysis below, this *does not* mean that the enacted plan is not an extreme partisan gerrymander under the other four elections; it only indicates that the plan is not as extreme of an outlier in these elections under the particular lens of seat counts.

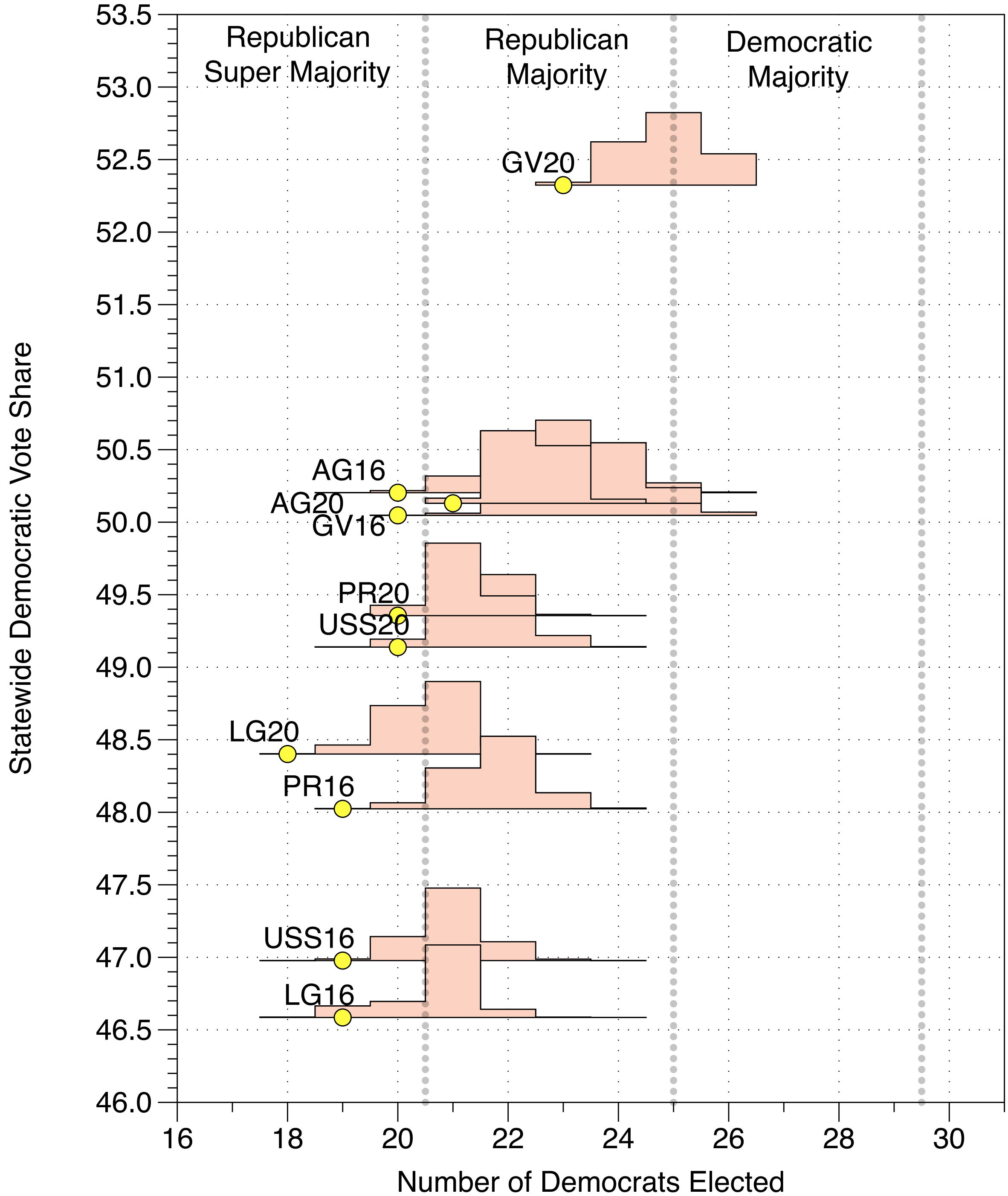


Figure 4: We compare Dr. Barber's statewide ensemble with the enacted plan under the ten 2016 and 2020 elections used in his report. Yellow dots show the result of the enacted plan. The enacted plan is an extreme outlier when considering the same data under a statewide lens. We summarize the numerical extent of the outliers in Table 1. The elections are abbreviated with the last two digits signifying the year, and the first letters representing Lt. Governor (LG), Governor (GV), President (PR), and US Senate (USS).

6 Cluster by Cluster Analysis

We now turn to examining certain clusters presented in Dr. Barber's work. We do not exhaustively examine all of the clusters. Rather, we select certain clusters to demonstrate how the lens that Dr. Barber chooses to use (namely only looking at the number of Democratic districts) yields an incomplete picture of the partisan make up of the districts even with respect to the individual districts.

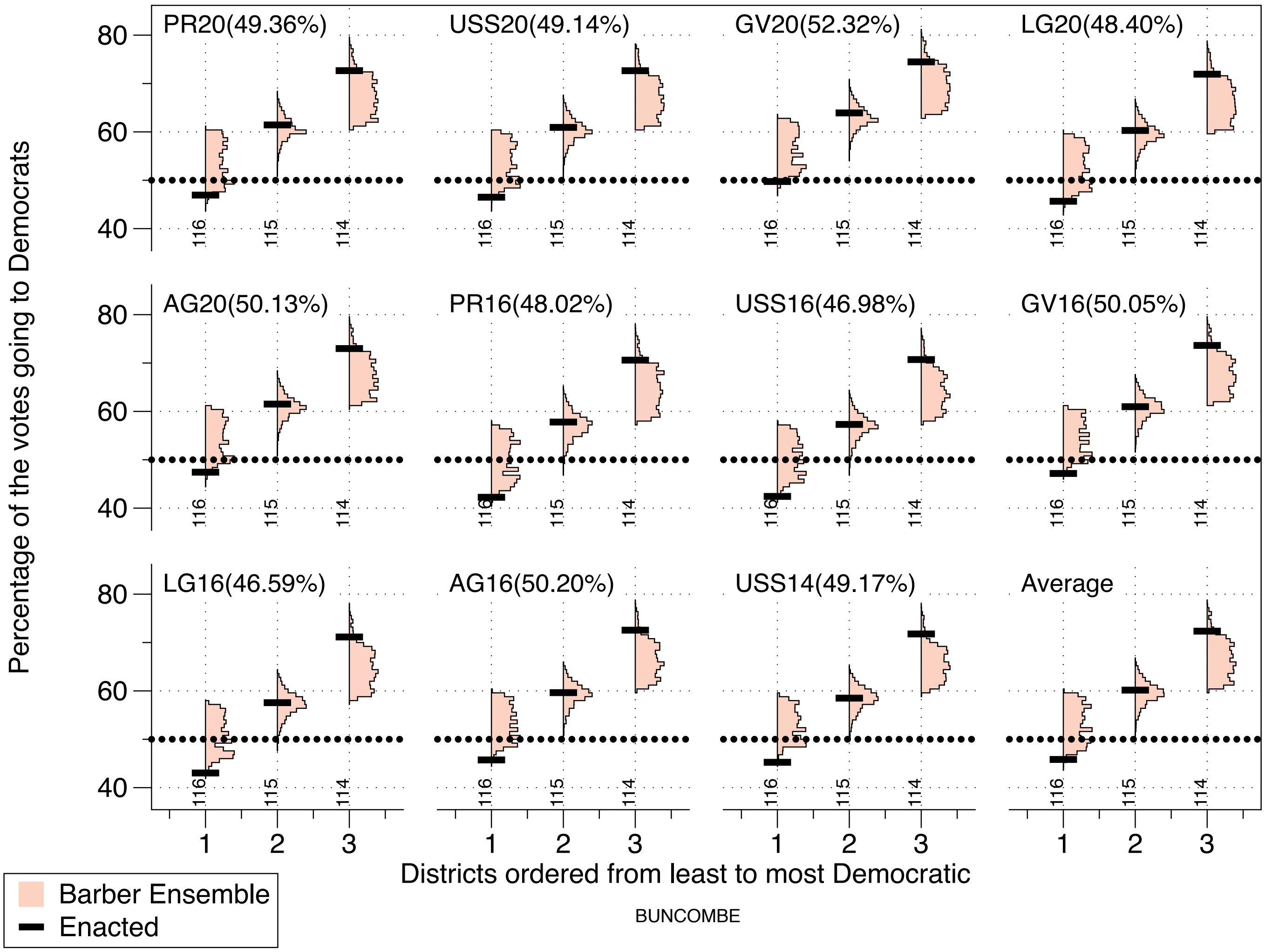
For a more complete picture, one would need to look at the actual partisan make-up of each district within a cluster. In fact, Dr. Barber reported on these values for the enacted plan, but did not compare these values to those found in his ensemble. One way of comparing these numbers is to examine the rank ordered marginal distributions of the vote fraction in each district. To do this, we order the districts from least to most Democratic (what Dr. Barber calls the Partisan Lean of Districts), and then look at the distribution of the most Republican, second most Republican, etc..., all the way until we reach the most Democratic district.

This type of analysis reveals not only how many Democratic leaning districts are within Dr. Barber's ensemble, but also *how much* they lean Democratic (or Republican). As we have demonstrated in our report, this is also relevant at a statewide level.

Note that all of our previous statewide analysis of seat counts simply relied on the numbers presented in Dr. Barber's report, i.e., the exact same ensemble that he relies on. The analysis below uses an ensemble of plans derived from running Dr. Barbers code (we were unable to extract his ensembles he used from the data he provided).[2] However, re-running his same code with his exact same input parameters should produce a comparable ensemble to the one he generated from the report, assuming that his code performs in the way he represents.

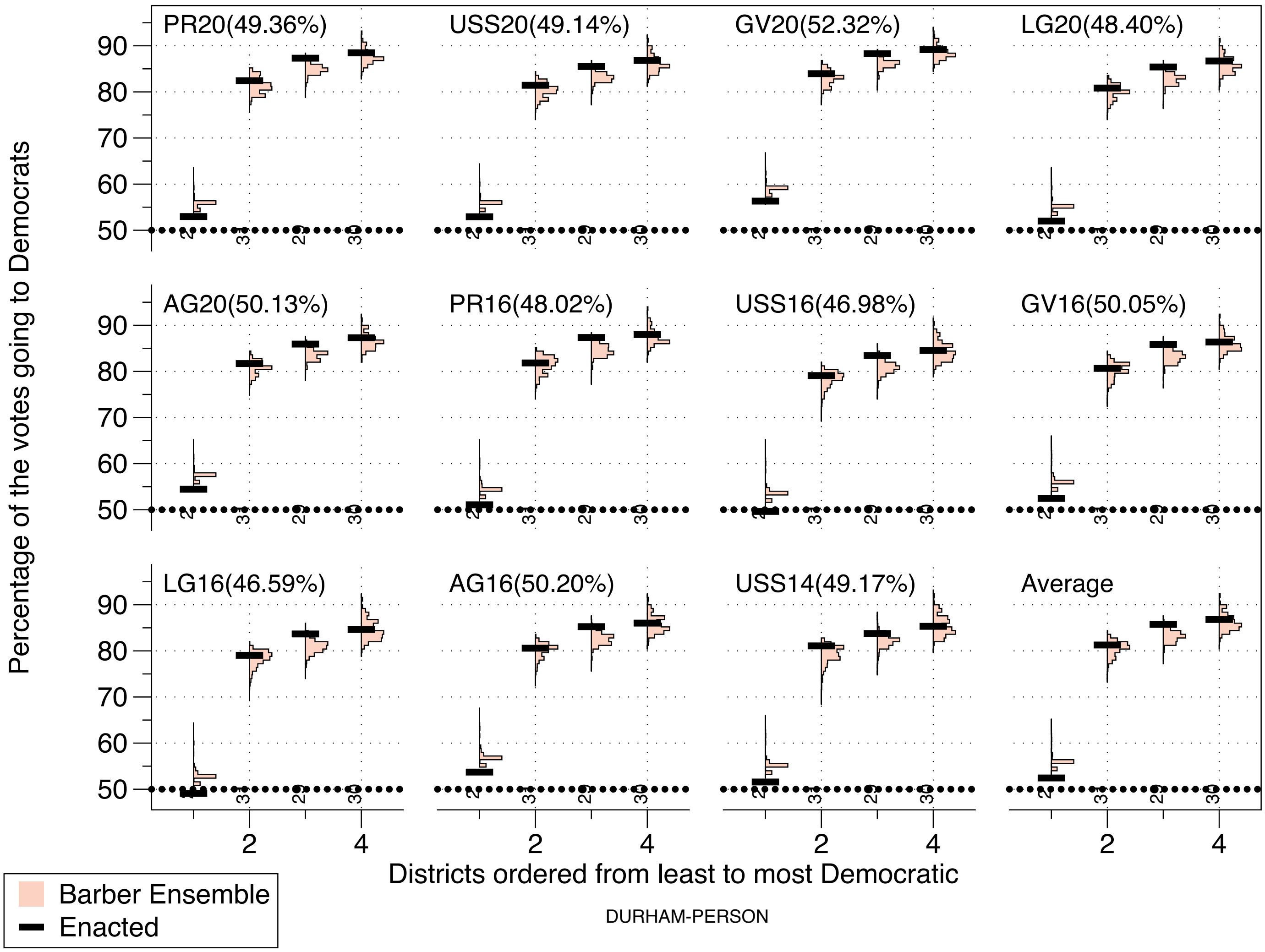
The main conclusion is that when comparing the cluster-by-cluster results from Dr. Barber's ensemble to those in our report, we find the qualitative structure to be the same. We again conclude that the enacted map is an extreme outlier when using Dr. Barber's ensemble with this additional analysis. We include a number of county clusters from the NC House. We make a number of comments in the caption of each figure. We refer the reader to our initial report to the court for a description of these Ranked-Ordered-Marginal-Histograms.

[2]We obtained the ensemble data from runs of Dr. Barber's code from Wes Pegden (CMU) who ran the code on his R installation as we did not have a computing environment able to run the code conveniently during the window when the rebuttal reports were due.



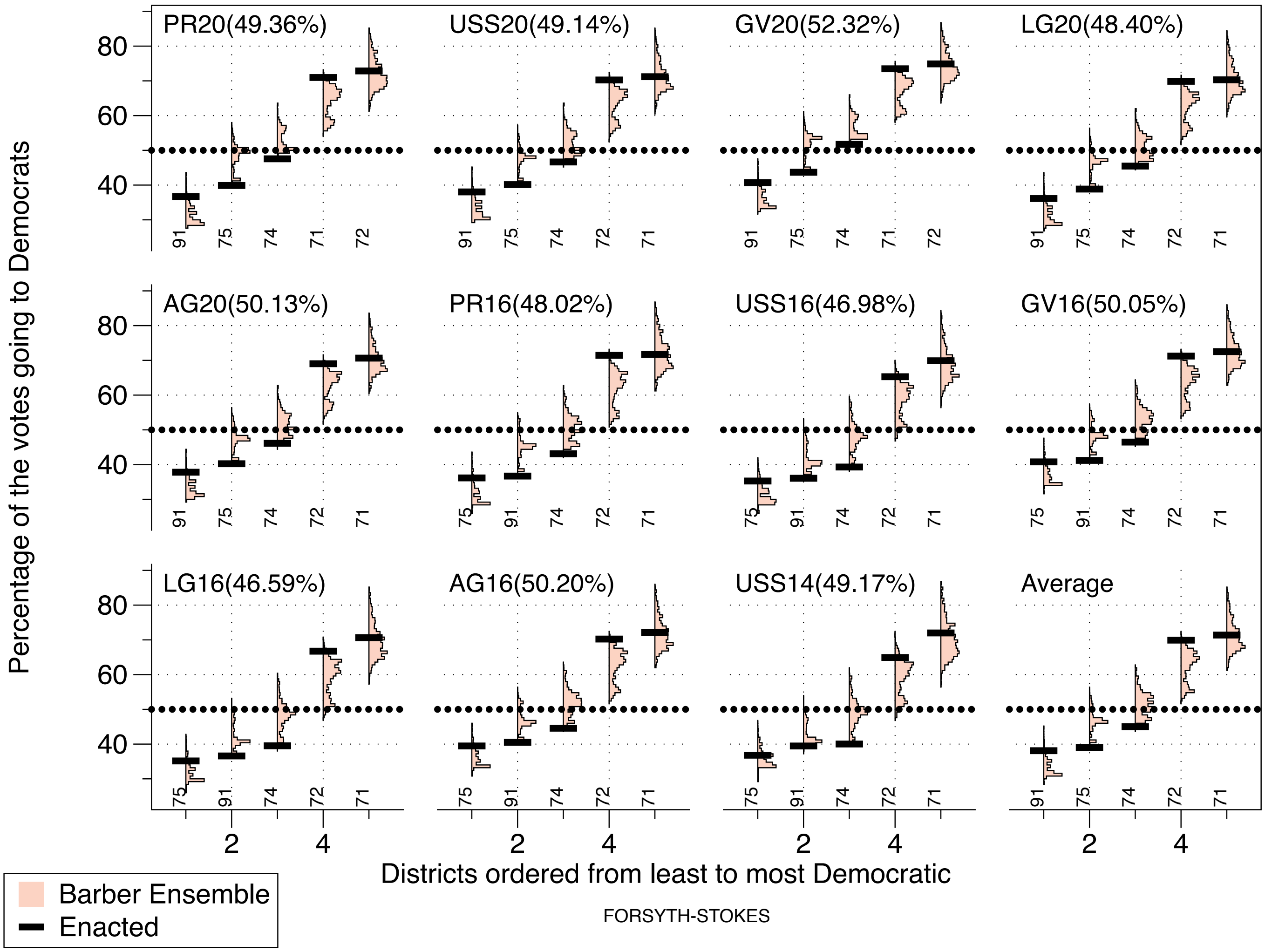
Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	107	0.277	2409	6.23	38664	1	3
PR20	756	1.96	3095	8.0	38664	1	3
USS20	409	1.06	2529	6.54	38664	1	3
GV20	662	1.71	3200	8.28	38664	1	3
LG20	424	1.1	2624	6.79	38664	1	3
AG20	534	1.38	2655	6.87	38664	1	3
PR16	321	0.83	2701	6.99	38664	1	3
USS16	17	0.044	2062	5.33	38664	1	3
GV16	18	0.0466	2067	5.35	38664	1	3
LG16	18	0.0466	1998	5.17	38664	1	3
AG16	17	0.044	1992	5.15	38664	1	3
USS14	3	0.00776	1807	4.67	38664	1	3

Figure 5: In Buncombe County, the Enacted maps is an extreme outlier under Dr. Barber's ensemble. We see the same structure as we saw when compared with the probability ensemble our initial report. The most Republican district in the enacted plan has exceptionally few Democrats while the most Democratic district has exceptionally many Democrats. The result is that the Democrats never win three seats in the enacted plan under any of the elections considered, including Dr. Barber's composite "Averaged Election", even though they would typically do so under a number of elections under Dr. Barber's ensemble.



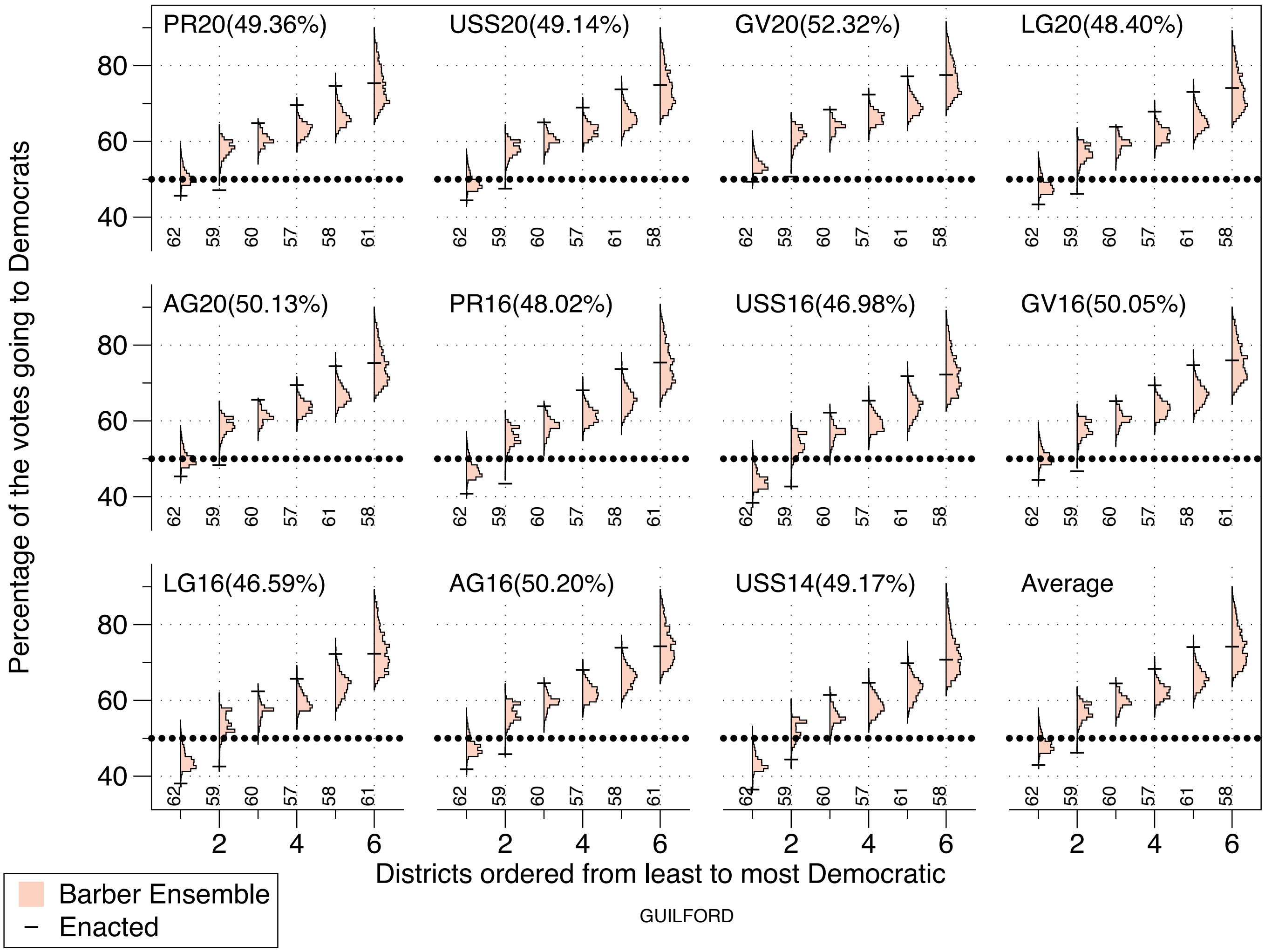
Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	0	0.0	1396	3.69	37800	1	3 4
PR20	0	0.0	790	2.09	37800	1	3 4
USS20	0	0.0	1326	3.51	37800	1	3 4
GV20	0	0.0	1123	2.97	37800	1	3 4
LG20	0	0.0	1199	3.17	37800	1	3 4
AG20	0	0.0	1205	3.19	37800	1	3 4
PR16	0	0.0	1184	3.13	37800	1	3 4
USS16	0	0.0	2932	7.76	37800	1	3 4
GV16	0	0.0	1382	3.66	37800	1	3 4
LG16	0	0.0	2675	7.08	37800	1	3 4
AG16	0	0.0	1931	5.11	37800	1	3 4
USS14	0	0.0	10357	27.4	37800	1	3 4

Figure 6: In the Durham-Person cluster, we the same outlier structure in the enacted map when compared to Dr. Barber's ensemble as when compared to the primary ensemble in our orignal report. We see that the most Republican district has been depleted of Democrates. This makes the district much more competitive than it typically would be under a non-partisan redistricting plan.



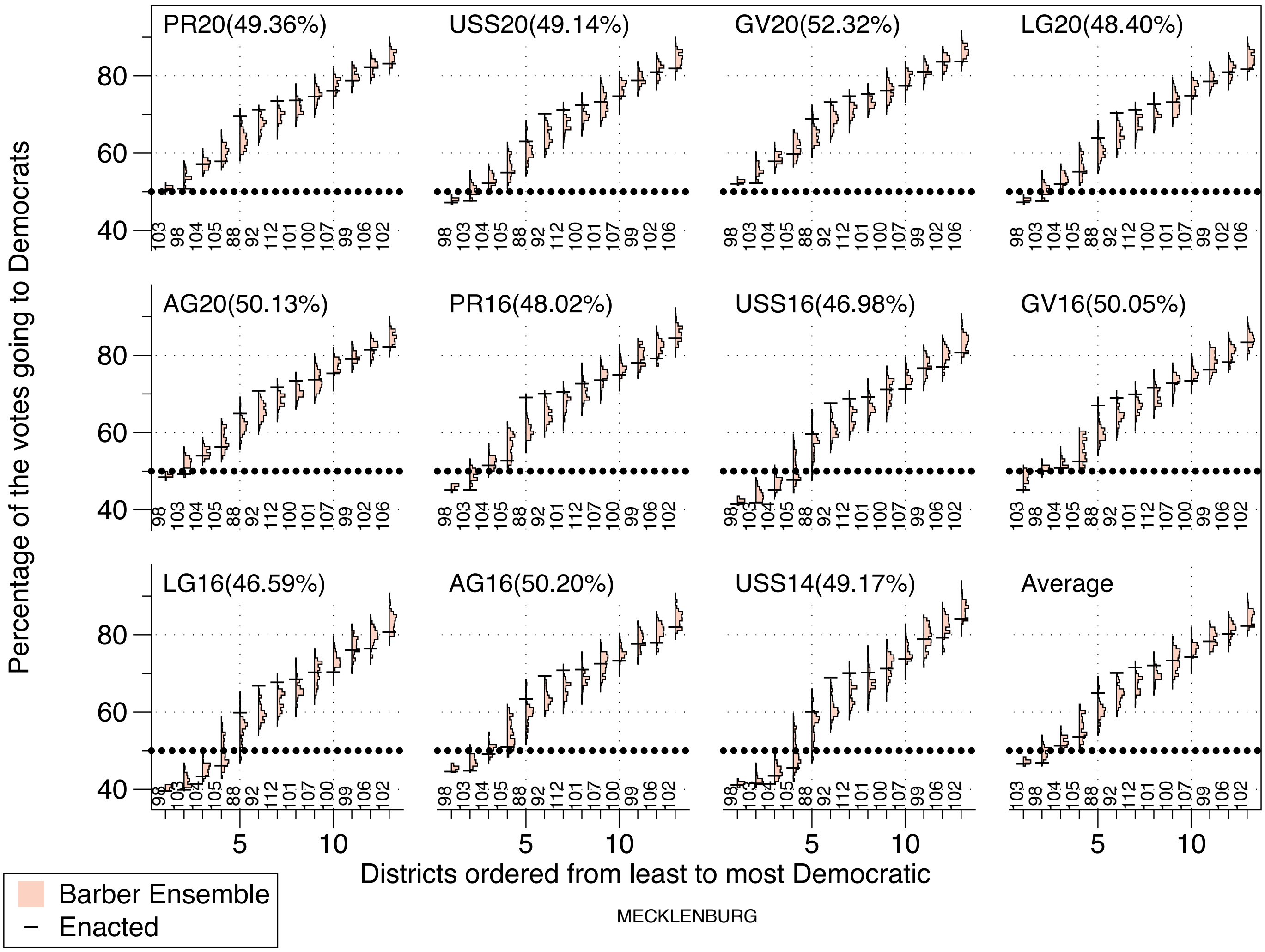
Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	17	0.456	317	8.51	3726	1 2 3	4 5
PR20	4	0.107	349	9.37	3726	1 2 3	4 5
USS20	60	1.61	429	11.5	3726	1 2 3	4 5
GV20	2	0.0537	357	9.58	3726	1 2 3	4 5
LG20	21	0.564	376	10.1	3726	1 2 3	4 5
AG20	47	1.26	395	10.6	3726	1 2 3	4 5
PR16	7	0.188	284	7.62	3726	1 2 3	4 5
USS16	44	1.18	280	7.51	3726	1 2 3	4 5
GV16	11	0.295	292	7.84	3726	1 2 3	4 5
LG16	30	0.805	269	7.22	3726	1 2 3	4 5
AG16	25	0.671	263	7.06	3726	1 2 3	4 5
USS14	13	0.349	351	9.42	3726	1 2 3	4 5

Figure 7: In the Forsyth-Stokes cluster, We again see the same structure in Dr. Barber's ensemble as in the primary ensemble from our initial report. We see abnormally few Democrats in the second and third most Republican districts while we see abnormally many Democrats in the most Republican district and in the two most Democratic districts. The effect is to regularly flip the 3rd most Republican district to the republicans under the enacted map even under elections where many to almost all of the plans in Dr. Barber's ensemble would have awarded the seat to the Democrats.



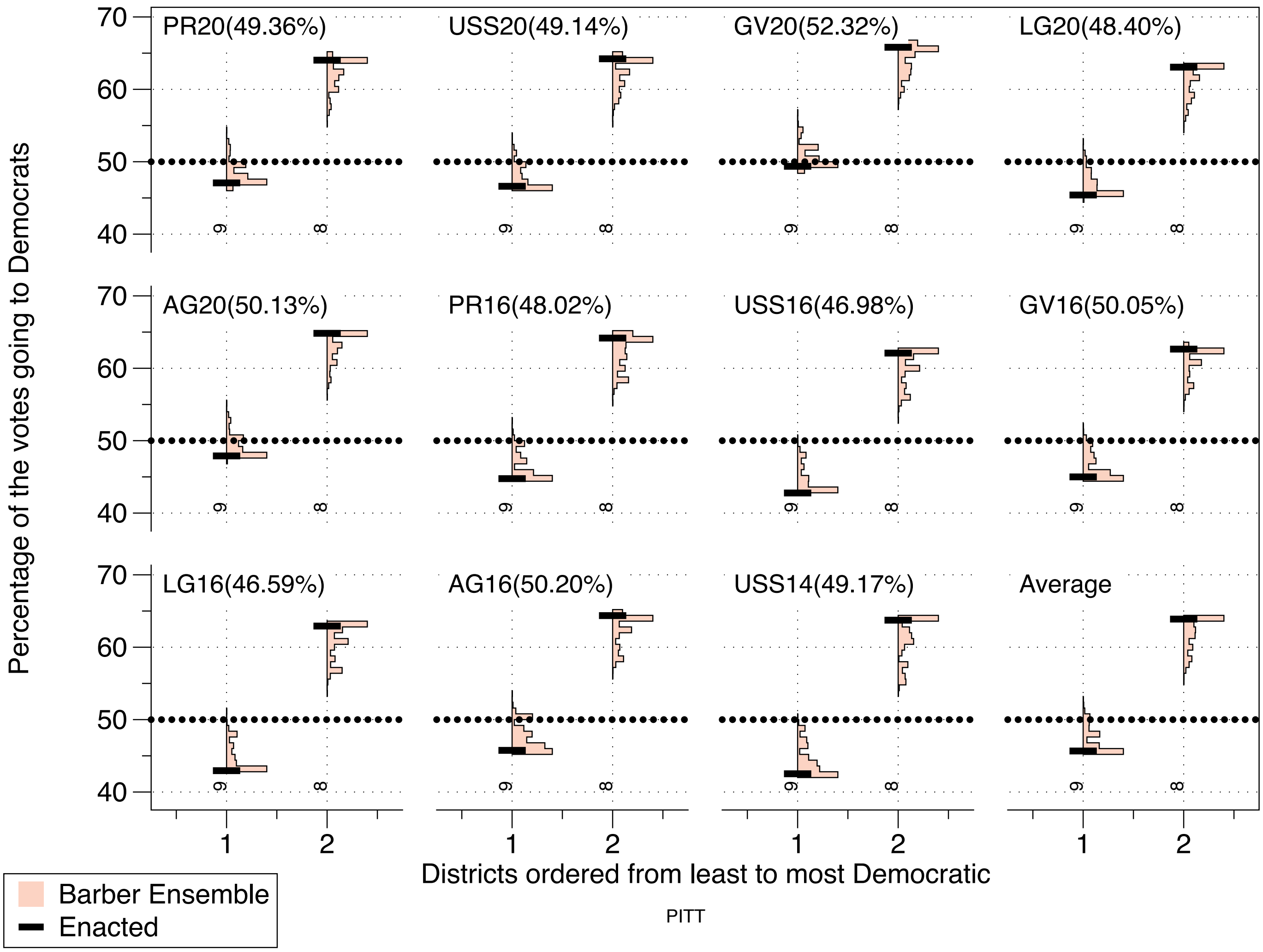
Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	0	0.0	0	0.0	15489	1 2	3 4 5 6
PR20	0	0.0	0	0.0	15489	1 2	3 4 5 6
USS20	0	0.0	0	0.0	15489	1 2	3 4 5 6
GV20	0	0.0	0	0.0	15489	1 2	3 4 5 6
LG20	0	0.0	0	0.0	15489	1 2	3 4 5 6
AG20	0	0.0	0	0.0	15489	1 2	3 4 5 6
PR16	0	0.0	0	0.0	15489	1 2	3 4 5 6
USS16	0	0.0	0	0.0	15489	1 2	3 4 5 6
GV16	0	0.0	0	0.0	15489	1 2	3 4 5 6
LG16	0	0.0	0	0.0	15489	1 2	3 4 5 6
AG16	0	0.0	0	0.0	15489	1 2	3 4 5 6
USS14	0	0.0	0	0.0	15489	1 2	3 4 5 6

Figure 8: Dr. Barber did identify Guilford county as a Republican Gerrymander in the enacted map. The structure which produces this result is clear when compared with this plot of Dr. Barber's ensemble. We see that the two most Republican districts have abnormally few Democrats and the next three Republican districts have abnormally many Democrats. The effect is that the second most Republican seat reliably goes to the Republican party even though in some elections almost all of the maps in Dr. Barber's ensemble would award the seat to the Democrats. This was the same structure seen in the plots of our primary ensemble from our initial report.



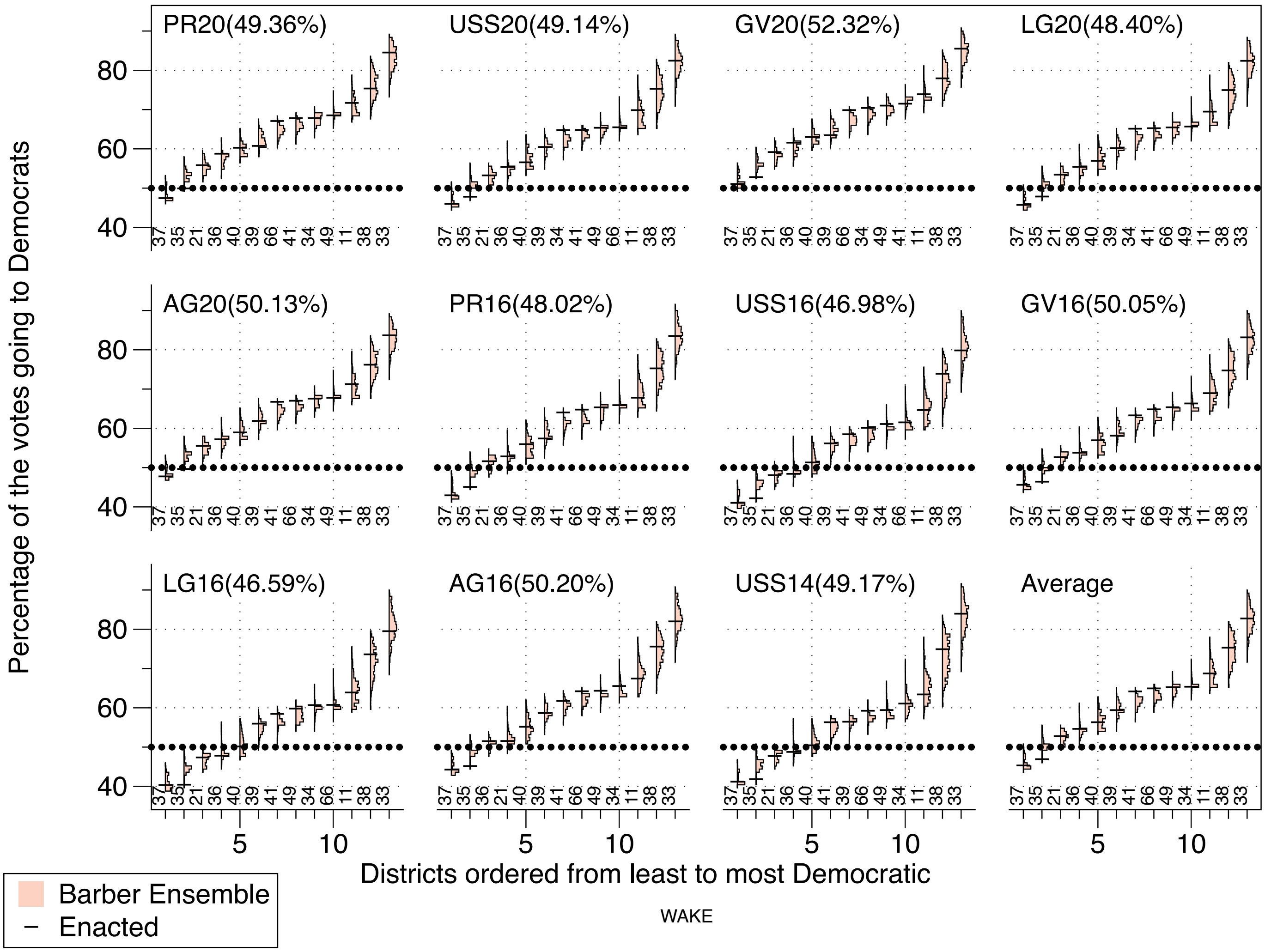
Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	139	4.4	14	0.443	3161	1 2 3 4	5 6 7 8
PR20	105	3.32	18	0.569	3161	1 2 3 4	5 6 7 8
USS20	145	4.59	29	0.917	3161	1 2 3 4	5 6 7 8
GV20	114	3.61	17	0.538	3161	1 2 3 4	5 6 7 8
LG20	117	3.7	17	0.538	3161	1 2 3 4	5 6 7 8
AG20	119	3.76	17	0.538	3161	1 2 3 4	5 6 7 8
PR16	23	0.728	18	0.569	3161	1 2 3 4	5 6 7 8
USS16	74	2.34	15	0.475	3161	1 2 3 4	5 6 7 8
GV16	56	1.77	23	0.728	3161	1 2 3 4	5 6 7 8
LG16	68	2.15	18	0.569	3161	1 2 3 4	5 6 7 8
AG16	52	1.65	15	0.475	3161	1 2 3 4	5 6 7 8
USS14	153	4.84	16	0.506	3161	1 2 3 4	5 6 7 8

Figure 9: In Mecklenburg county, we again have that the four most Republican districts have abnormally few Democrats in them while the next four most Republican districts have abnormally many Democrats. This is the same structure as we saw under our primary ensemble in our initial report. The effect is that in a number of elections the Republican party wins one to two more seats than the typical plan from Dr. Barber's ensemble would award.



Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	314	6.05	1929	37.2	5189	1	2
PR20	1539	29.7	1974	38.0	5189	1	2
USS20	1525	29.4	1929	37.2	5189	1	2
GV20	1556	30.0	1974	38.0	5189	1	2
LG20	1537	29.6	1974	38.0	5189	1	2
AG20	1537	29.6	1974	38.0	5189	1	2
PR16	483	9.31	1929	37.2	5189	1	2
USS16	0	0.0	1660	32.0	5189	1	2
GV16	483	9.31	1929	37.2	5189	1	2
LG16	0	0.0	1660	32.0	5189	1	2
AG16	169	3.26	1660	32.0	5189	1	2
USS14	0	0.0	1660	32.0	5189	1	2

Figure 10: In Pitt county we see that same structure we found in our Primary ensemble repeated in Dr. Barber's ensemble. In particular, we see the districts pulled to the extremes of what is seen in Dr. Barber's ensemble. The depletion of Democrats from the more Republican district protects it from electing a Democrat in the enacted plan even though it would elect a Democrat in many of the plans in Dr. Barber's ensemble in a few of the elections we considered.



Election	No. plans w/ ≤ Dems (First Cluster)	% of plans w/ ≤ Dems (First Cluster)	No. plans w/ ≥ Dems (Second Cluster)	% of plans w/ ≥ Dems (Second Cluster)	Total Plans	First Cluster	Second Cluster
Average	159	1.11	2649	18.5	14305	1 2	3 4 5 6 7 8
PR20	140	0.979	1872	13.1	14305	1 2	3 4 5 6 7 8
USS20	209	1.46	2961	20.7	14305	1 2	3 4 5 6 7 8
GV20	145	1.01	1772	12.4	14305	1 2	3 4 5 6 7 8
LG20	159	1.11	2240	15.7	14305	1 2	3 4 5 6 7 8
AG20	165	1.15	2260	15.8	14305	1 2	3 4 5 6 7 8
PR16	137	0.958	2264	15.8	14305	1 2	3 4 5 6 7 8
USS16	196	1.37	3774	26.4	14305	1 2	3 4 5 6 7 8
GV16	220	1.54	3504	24.5	14305	1 2	3 4 5 6 7 8
LG16	196	1.37	2707	18.9	14305	1 2	3 4 5 6 7 8
AG16	205	1.43	3076	21.5	14305	1 2	3 4 5 6 7 8
USS14	287	2.01	3632	25.4	14305	1 2	3 4 5 6 7 8

Figure 11: In Wake county, we see that the number of Democrats in the first two districts is exceptionally low. Looking across the different Ranked Ordered Marginal Histograms, we see that this increases the electoral environments (as captured in different elections) in which the Republican party wins one of these two districts. In particular, Dr. Barber's ensemble would lead to the Democrats typically winning one of these two districts in cases where the enacted plan does not.

7 Comments on Sampling Methods

We now give some additional details to clarify some of the terms we used and the procedures we followed in sampling of the legislative maps in our original report in light of the discussion in Dr. Barber's report.

We recall that in the Legislative case we used parallel tempering to interpolate between a base measure equal to the uniform measure on spanning forests given the county and population constraints and a measure centered on the districts with a compactness similar to the enacted plan. The Primary ensemble for the legislative ensemble reported in the report was the latter of these two ensembles. The first of these ensembles would be the target distribution of the SMC algorithms from the *rdist* package when it is properly configured with resampling included. We took 4 million steps (proposals the Metropolis-Hastings algorithm) at the spanning tree level and 2 million steps on the other levels. We output maps every 25 steps for a total of 160,000 maps in the 4 million step case and 80,000 map in the 2 million step cases. We interpolated between the different ensembles using between 60 and 100 parallel tempering levels. We proposed switching between the parallel tempering levels every 100 steps. In some cases, we ran a number of clusters together in one sampling run and sometimes we ran them separately or is smaller subgroups in a single run. Generally we ran the larger, more compacted clusters such as Wake or Mecklenburg, in this way.[3] As described in the original report, *independent sample reservoirs* were used to split the 60 to 100 levels into computationally feasible chunks. This also improved the mixing and decorrelation properties of our algorithm. The congressional ensemble was drawn from a measure with a compactness weight against the same tree measure that the resampled rdist algorithm would sample. We used 12 parallel temping levels to move between the distribution without a compactness measure and the finial target distribution with the sampling weight. The number of steps was as specified above. The weights and other parameters used in the different run are specified in the header files of the datasets.

[3] For one run in the Senate, we only ran Granville-Wake for 1 million steps as we had strong evidence that this was sufficient for the parameter values being considered.

I declare under penalty of perjury under the laws of the state of North Carolina that the foregoing is true and correct to the best of my knowledge.

Jonathan Mattingly, 12/28/2021

STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, et al.,

REBECCA HARPER, et al.,

Plaintiffs,

vs.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

Consolidated with
21 CVS 500085

AFFIDAVIT OF MICHAEL BARBER

Now comes affiant Michael Barber, having been first duly cautioned and sworn, deposes and states as follows:

1. I am over the age of 18 and am competent to testify regarding the matters discussed below.

2. For the purposes of this litigation, I have been asked by counsel for Legislative Defendants to analyze relevant data and provide my expert opinions.

3. To that end, I have personally prepared the rebuttal report attached to this affidavit as Exhibit A, and swear to its authenticity and to the faithfulness of the opinions.

FURTHER THE AFFIANT SAYETH NAUGHT.

NCLCV v. Hall
21 CVS 15426
LDTX166

Exhibit #
Barber 4
12/30/2021
exhibitsticker.com

Executed on 28 December, 2021.

Michael Barber

DIGITALLY SIGNED

Michael Barber

STATE OF FLORIDA

COUNTY OF PINELLAS

Sworn to and subscribed before me by online notarization this 28th day of December, 2021, by MICHAEL BARBER, who appeared by way of two-way audio/video communication technology, and he provided his Utah driver's license as identification.

DIGITALLY SIGNED

Cynthia D. Glaros
Notary Public, State of Florida
My Commission Expires: 06/30/2022

Cynthia D. Glaros
Notary Public, State of Florida
Commission # GG228737
My Commission Expires June 30, 2022

Online Notary Public. This notarial act involved the use of online audio/video communication technology.

Reply Report of Michael Barber, PhD

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Contents

1 Introduction and Qualifications

I have been asked by counsel for the Legislative Defendants to analyze and respond to reports submitted by Drs. Magleby, Pegden, Mattingly, and Cooper with regards to their analysis of North Carolina's recently enacted redistricting plans for the General Assembly (the "Enacted Plans").[1]

I do this in the following ways. First, I provide a summary of their conclusions as well as comparisons between their main results and those I produced in my original report. I also consider the specific analysis they produce for several county groupings that are singled out in their reports for additional scrutiny. I also define a measure of substantive significance to determine the degree to which the Enacted Map differs from Dr. Pegden's simulations and subsequent expected seats analysis.

The results show that there is often not agreement, even among the plaintiffs' experts, as to whether or not a county grouping's districts constitute a partisan outlier. In some cases the simulations produced by different experts come to different conclusions, and in other cases some of the experts assert an extreme partisan gerrymander, but in that same grouping the map proposed by the North Carolina League of Conservation Voters (NCLCV Map) exhibits the same qualities as the Enacted Map.

Based on the evidence and analysis presented below, my opinions regarding these reports studying the North Carolina General Assembly can be summarized as follows:

- There is significant agreement between Dr. Magley's simulation results and those produced in my original report with regard to the number of seats carried by Democrats in both the simulations and and the Enacted Plan despite some differences in our particular simulation methods.

- However, Dr. Magleby does not present county grouping by county grouping analyses,

[1] Due to the incredibly tight time constraints between the submission of reports and the deadline for submission of rebuttal reports, I only analyze Dr. Cooper's report in the House clusters and not the Senate clusters. My analysis has been provided to the best of my ability given the time constraints.

so it is not possible to compare his results with mine to identify if there are differences at this more granular level.

- In many of the 12 county groupings considered by Drs. Pegden and Mattingly in the House the Enacted Plan is either not a statistical outlier, is not substantively different from the simulations, or is in agreement with the map proposed by the NCLCV plaintiffs in the districts under dispute. Furthermore, in other cases there are reasonable explanations for the boundaries of the map that are separate from partisanship.

- In the 5 county groupings considered by plaintiffs' experts in the Senate, there is also often disagreement on whether the map constitutes a large outlier. In many of the clusters the Enacted Plan is either not a statistical outlier, is not substantively different from the simulations, or is in agreement with the map proposed by the NCLCV plaintiffs in the districts under dispute.

I am an associate professor of political science at Brigham Young University and faculty fellow at the Center for the Study of Elections and Democracy in Provo, Utah. I received my PhD in political science from Princeton University in 2014 with emphases in American politics and quantitative methods/statistical analyses. My dissertation was awarded the 2014 Carl Albert Award for best dissertation in the area of American Politics by the American Political Science Association.

I teach a number of undergraduate courses in American politics and quantitative research methods.[2] These include classes about political representation, Congressional elections, statistical methods, and research design.

I have worked as an expert witness in a number of cases in which I have been asked to analyze and evaluate various political and elections-related data and statistical methods. Cases in which I have testified at trial or by deposition are listed in my CV, which is attached to the end of this report. I have previously provided expert reports in a number of

[2] The political science department at Brigham Young University does not offer any graduate degrees.

cases related to voting, redistricting, and election-related issues: *Nancy Carola Jacobson, et al., Plaintiffs, vs. Laurel M. Lee, et al., Defendants. Case No. 4:18-cv-00262 MW-CAS (U.S. District Court for the Northern District of Florida); Common Cause, et al., Plaintiffs, vs. Lewis, et al., Defendants. Case No. 18-CVS-14001 (Wake County, North Carolina); Kelvin Jones, et al., Plaintiffs, v. Ron DeSantis, et al., Defendants, Consolidated Case No. 4:19-cv-300 (U.S. District Court for the Northern District of Florida); Community Success Initiative, et al., Plaintiffs, v. Timothy K. Moore, et al., Defendants, Case No. 19-cv-15941 (Wake County, North Carolina); Richard Rose et al., Plaintiffs, v. Brad Raffensperger, Defendant, Civil Action No. 1:20-cv-02921-SDG (U.S. District Court for the Northern District of Georgia); Georgia Coalition for the People's Agenda, Inc., et. al., Plaintiffs, v. Brad Raffensberger, Defendant. Civil Action No. 1:18-cv-04727-ELR (U.S. District Court for the Northern District of Georgia); Alabama, et al., Plaintiffs, v. United States Department of Commerce; Gina Raimondo, et al., Defendants. Case No. CASE NO. 3:21-cv-00211-RAH-ECM-KCN (U.S. District Court for the Middle District of Alabama Eastern Division); League of Women Voters of Ohio, et al., Relators, v. Ohio Redistricting Commission, et al., Respondents. Case No. 2021-1193 (Supreme Court of Ohio); Adams, et al., Relators, v. DeWine, et al., Respondents. Case No. 2021-1428 (Supreme Court of Ohio)*

In my position as a professor of political science, I have conducted research on a variety of election- and voting-related topics in American politics and public opinion. Much of my research uses advanced statistical methods for the analysis of quantitative data. I have worked on a number of research projects that use "big data" that include millions of observations, including a number of state voter files, campaign contribution lists, and data from the US Census. I have also used geographic information systems and other mapping techniques in my work with political data.

Much of this research has been published in peer-reviewed journals. I have published nearly 20 peer-reviewed articles, including in our discipline's flagship journal, *The American Political Science Review* as well as the inter-disciplinary journal,*Science Advances*. My CV, which details my complete publication record, is attached to this report as Appendix A.

The analysis and opinions I provide in this report are consistent with my education, training in statistical analysis, and knowledge of the relevant academic literature. These skills are well-suited for this type of analysis in political science and quantitative analysis more generally. My conclusions stated herein are based upon my review of the information available to me at this time. I reserve the right to alter, amend, or supplement these conclusions based upon further study or based upon the availability of additional information. I am being compensated for my time in preparing this report at an hourly rate of $400/hour. My compensation is in no way contingent on the conclusions reached as a result of my analysis. The opinions in this report are my own, and do not represent the view of Brigham Young University.

2 Review of Dr. Magleby's Report

My review of Dr. Magleby's report shows many areas in which our data and methods are similar and a few important areas where we differ in our methods. I begin with areas of similarity. As my report considered only the state legislative districts and not the congressional districts, I focus on that portion of Dr. Magleby's report as well.

My review of his report over the last several days indicates that our analysis is similar in the following ways:

- We both use a redistricting simulation algorithm to construct hypothetical legislative districts in the NC House and Senate.
- We both use data from historical elections at the level of the VTD to compute the partisan lean of the Enacted Plan as well as the simulated districts.
- We both use statewide election data to compute partisan indices.
- Using the partisan indices, we both compute the number of districts "carried" by

Democrats and Republicans as a measure of the partisan lean of the districts in the Enacted Plan and the set of simulations.

Our analysis differs in the following ways:

- While we both use a redistricting simulation algorithm to construct hypothetical legislative districts in the NC House and Senate, the exact method and computer programs differ in their construction.

- While we both use data from historical elections at the level of the VTD to compute the partisan lean of the Enacted Plan as well as the simulated districts, we use slightly different elections to generate a partisan index for each district. Professor Magleby uses the following elections in 2016 and 2020 in his index: President, US Senate, Governor, Lieutenant Governor, Attorney General, Treasurer, Secretary of State, Auditor, Agriculture Commissioner, Insurance Commissioner, Labor Commissioner, and Superintendent of Public Instruction. I also use elections for President, US Senate, Governor, Lieutenant Governor, and Attorney General. Due to the very tight time constraints of this case I was unable to obtain data for Treasurer, Secretary of State, Auditor, Agriculture Commissioner, Insurance Commissioner, Labor Commissioner, and Superintendent of Public Instruction. I also include the 2014 Senate race. However, the differences in our indices will not make a large difference given the large number of elections included in either index. Any one election carries very little weight. Finally, if the intention of simulations is to compare the Enacted Plan to a set of simulated districts, the more important factor is that the measure by which the Enacted Plan is evaluated is the same as the measure by which the simulated districts are measured. This is true of both sets of simulations.

- Professor Magleby takes a random sample of 1,000 districting plans from a larger set of simulations to use as his comparison set. From the description in his report, it appears that there is no consideration for whether the simulated districts divide more counties or are more or less compact than the Enacted Plan. In my report I only include simulations with as many or fewer county traversals and simulations in which the districts comprising the county grouping have an average compactness score that is as large or larger than the Enacted Plan.

- We both conduct simulations separately for each county grouping, however, Professor Magleby's report does not include them in his report. Because of this, I am unable to identify county groupings where the Enacted Map may differ from the simulated districts.

At the statewide level, our results are quite similar. In the State House Dr. Magelby's index predicts the Enacted Plan to have 48 Democratic districts (see Figure 1 of Magleby report). Dr. Magleby's simulations produce a distribution of seats carried by Democrats, with a peak at 52 seats carried by Democrats for a gap of 4 seats between the Enacted Plan and the modal outcome of the simulations.

My index in the House yields 49 seats carried by Democrats (see Tables 1 and 2 in Barber report). Because I consider each county grouping separately, I do not produce a single statewide histogram of seats carried by Democrats statewide, however, Tables 1 and 2 in my report show the middle 50% range of simulations across all House clusters to be 50-55 Democratic seats, which would include the modal outcome in Dr. Magleby's Figure 1. This produces a gap of 1-6 seats between the Enacted Plan and the middle 50% range of simulated plans.

In the State Senate Dr. Magelby's index predicts the Enacted Plan to have 19 Democratic districts (see Figure 3 of Magleby report). Dr. Magleby's simulations produce a distribution of seats carried by Democrats, with a peak at 22 seats carried by Democrats for a gap of 3 seats between the Enacted Plan and the modal outcome of the simulations.

My index yields 20 seats carried by Democrats in the State Senate (see Tables 31 and 32 in Barber report). Because I consider each county grouping separately, I do not produce a single statewide histogram of seats carried by Democrats statewide, however, Tables 31 and 32 in my report show the middle 50% range of simulations across all clusters to be 23 Democratic seats for a gap of 3 seats between the Enacted Plan and the modal outcome of the simulations.

3 Review of Dr. Cooper's Report

Dr. Cooper provides no quantitative analysis of the Enacted Plan aside from computing a few different partisan indices of the Enacted Plan. He does not compare the plan to any other alternative plan or set of plans, simulated or otherwise. While the partisan indices he uses are quantitative in nature, the analysis he conducts is fundamentally qualitative. For his analysis of the State House and Senate he looks at each county grouping and offers opinions and anecdotes about the boundaries of the districts as well as the supposed intentions of the legislature. However, he offers no evidence aside from his own opinion to support his assertions of the intentions of the legislature when drawing the district boundaries.

There is nothing wrong, per se, with a qualitative approach to evaluating a state's map. However, qualitative research requires the same standards and rigor as quantitative research. King, Keohane, and Verba (2021), arguably the most influential recent work on qualitative research, describe the need for rigorously defined standards in qualitative research as the following:

> We argue that nonstatistical research will produce more reliable results if researchers pay attention to the rules of scientific inference—rules that are sometimes more clearly stated in the style of quantitate research....Indeed the distinctive characteristic that sets social science apart from casual observation is that social science seeks to arrive at valid inferences by the systematic use of well-established procedures of inquiry (pg. 4).[3]

[3]King, Gary., Verba, Sidney., Keohane, Robert O.. Designing Social Inquiry: Scientific Inference in Qualitative Research, New Edition. United States: Princeton University Press, 2021.

From my review of Dr. Cooper's cluster-by-cluster analysis, there is no systematic process by which he determines if a set of districts in a county group constitute a gerrymander or not. Dr. Cooper does not describe any methods or processes that would be consistent with analysis in political science. Instead, I would describe his report as more akin to "casual observation," rather than rigorous social science. Nevertheless, I consider the particular county groups that he identifies and compare his assessment to that of my report and the other plaintiff expert reports.

4 Review of Dr. Pegden's Report

Dr. Pegden provides an analysis of the districts in the State House and Senate, as well as the congressional maps. However, I only consider the State House and Senate portion of his report. My understanding of his analysis is that he performs something akin to a simulation analysis, but in a slightly different way. Through a series of very large number of small perturbations to the existing districts that adhere to the redistricting criteria in North Carolina he creates a large set of comparison maps. He then compares the Enacted Map to this set of comparison maps using the 2020 Attorney General election as a "proxy for partisan voting patterns (pg. 9)" in two ways.

Unlike myself, Professor Magleby, and Professor Mattingly, Dr. Pegden only considers one election instead of an index or series of elections. It is unclear to me why he makes this choice since using any individual election as a proxy for future state legislative election results will be subject to the idiosyncrasies (candidate-related factors, issues specific to the office and campaign, campaign spending/advertising, etc) of the particular election chosen. While he provides alternative elections in the Appendix of his report for the 2020 Presidential election, the 2020 Lieutenant Governor election, and the 2020 Governor election, these are only included for the statewide analysis and do not look at specific county groupings in a group-by-group analysis, like is done earlier in his report.

The first analysis Dr. Pegden conducts is to determine the proportion of maps that are more "partisan" than the set of comparison maps. This fraction is treated throughout the report in a similar fashion to a reported p-value in other quantitative research in the social sciences. As Dr. Pegden states: "My method produces a rigorous p-value (statistical significance level) which precisely captures the confidence one can have in the findings of my "second level" analyses. In particular, for my statewide analyses, my second-level claims are all valid at a statistical significance of p = .002 (pg. 6)."

He also produces an additional analysis for each county grouping in which he computes the expected seat share for the Enacted Plan and compares this to the expected seat share of the set of comparison maps he produces. As he states: "When I am evaluating the partisanship of a comparison districting (to compare it to the Enacted Plan), I am interested in the number of seats we expect Democrats might win in the districting, given unknown shifts in partisan support. In particular, the metric I use is: How many seats, on average, would Democrats win in the given districting, if a random uniform swing is applied to the historical voting data being used?" This comparison is akin to a measure of substantive significance, as it helps us to understand the substantive difference between the Enacted Map and the set of comparison maps generated by Dr. Pegden's algorithm.

Substantive significance is a way of measuring the "practical significance" of a statistical finding. Gross (2015) states, "The function of statistical tests is merely to answer: Is the variation great enough for us to place some confidence in the result; or, contrarily, may the latter be merely a happenstance of the specific sample on which the test was made? The question is interesting, but it is surely secondary, auxiliary, to the main question: Does the result show a relationship which is of substantive interest because of its nature and its magnitude? "[4] As an example, suppose a drug trial discovers a drug to reduce blood pressure that produces a statistically significant effect in a randomized controlled trial. However,

[4]Gross, Justin H. "Testing What Matters (If You Must Test at All): A Context-Driven Approach to Substantive and Statistical Significance." American Journal of Political Science 59, no. 3 (2015): 775-788. quoting Kish, Leslie. 1959. "Some Statistical Problems in Research Design." American Sociological Review 24(3):328–38.

suppose that the substantive impact of this drug on patients' blood pressure remains very small. Given this, it may not be in the interests of the company to produce the drug given other considerations such as cost, potential side effects, and the opportunity costs of other activities. This would be an example of a difference between statistical and substantive significance.

The previous paragraph is relevant to Dr. Pegden's analysis because the first and second level analyses he provides are akin to measures of statistical significance while the expected seat share he computes is akin to a measure of substantive significance. Various measures of redistricting have been created and used, but agreement on any one particular measure as the ideal is lacking. Furthermore, even when a particular measure is agreed upon, what constitutes a substantively significant difference using that measure is even rarer.[5] Cain et al. summarise this issue well when they state, "Any partisan gerrymandering doctrine that the Court adopts will presumably allow states to draw maps that deviate some from the counterfactual plans. Strict adherence is not likely to be required. The critical question in applying this method then becomes: How much deviation is too much?"[6]

Given this, agreement on a strict definition of substantive significance is vanishingly rare. As a guidepost, I look at the expected seat share between the Enacted Plan and the expected seat share of the middle 50% of Dr. Pegden's simulations (in other words, the simulations which constitute the 25th to the 75th percentile). I then calculate how this difference would translate into an expectation for a party to pick up an additional seat over the 5 legislative elections that would take place over the decade in which the plan would be in place.[7] A redistricting plan is in place for a decade, so it makes sense to consider the

[5]Herschlag, Gregory, Han Sung Kang, Justin Luo, Christy Vaughn Graves, Sachet Bangia, Robert Ravier, and Jonathan C. Mattingly. "Quantifying gerrymandering in North Carolina." Statistics and Public Policy 7, no. 1 (2020): 30-38.; Stephanopoulos, Nicholas O., and Eric M. McGhee. "The measure of a metric: The debate over quantifying partisan gerrymandering." Stan. L. Rev. 70 (2018): 1503.; Warrington, Gregory S. "A comparison of partisan-gerrymandering measures." Election Law Journal: Rules, Politics, and Policy 18, no. 3 (2019): 262-281.

[6]Cain, Bruce E., Wendy K. Tam Cho, Yan Y. Liu, and Emily R. Zhang. "A Reasonable Bias Approach to Gerrymandering: Using Automated Plan Generation to Evaluate Redistricting Proposals." William & Mary Law Review 59, no. 5 (2018): 1521.

[7]I also use the middle 50% standard in my own analysis when looking at whether the Enacted Plan is

substantive differences over that time period.

5 Review of Dr. Mattingly's Report

Dr. Mattingly also produces a set of simulated districting plans and compares the Enacted Plan to this set of comparison maps. Dr. Mattingly does not produce an election index, but instead analyzes separately the results in 12 or 16 different elections in 2016 and 2020. In his statewide analysis he includes 2020: Attorney General, United States Senate, Commissioner of Insurance, Lieutenant Governor, Governor, State Treasurer, Secretary of State, State Auditor, Commissioner of Agriculture, Commissioner of Insurance, and US President; 2016: Commissioner of Agriculture, Governor, Lieutenant Governor, US Senate, and President. In his cluster-by-cluster analysis these elections are 2020: Attorney General, United States Senate, Commissioner of Insurance, Lieutenant Governor, Governor, State Treasurer, Secretary of State, State Auditor, Commissioner of Agriculture, and United States President; 2016: Lieutenant Governor and President. It is unclear to me why he does not include the other 2020 races in the cluster-by-cluster analysis.

In his analysis of the State House Dr. Mattingly produces two different "ensembles" or sets of simulations. The first set he describes as "matched" in that the simulations match the criteria used to draw the Enacted Plan. However, this is often not the case in the cluster-by-cluster analyses where the simulations often do not match the degree to which the Enacted Plan follows these criteria (See, for example, Figures 6.1.3, 6.1.9, 6.1.12, 6.1.21, 6.1.24, 6.1.27, 6.1.30, 6.1.33, 6.1.36 where the Enacted Plan splits fewer municipalities or has fewer ousted voters than a substantial number of the simulations). The simulations are often higher than the Enacted Plan in number of municipalities split, number of voters "ousted" from a district (see pg. 9 of the Mattingly report for a description of ousted voters), and the average compactness of the simulated districts is also often lower than the Enacted Plan (see

an outlier from the simulation results. This interquartile range is a commonly used measure of the central range of expected outcomes in a distribution.

Figure 7.3.1 in Mattingly Report.) Given this, I analyze the results of Dr. Mattingly's second set of simulations that are more strict regarding municipal splits and district compactness and do not consider the first set of simulations especially helpful in analyzing the Enacted Plan.

In his analysis of the State Senate the opposite is true. As in the House Dr. Mattingly produces two different "ensembles" or sets of simulations. The first set he describes are "matched" in that the simulations match the criteria used to draw the Enacted Plan. Here Dr. Mattingly notes, "We will see that the enacted NC Senate preserves municipalities to a high degree; in a way consistent with the most municipality preserving distributions we could produce. Hence, we also provide a Secondary Ensemble for the NC Senate which does not explicitly preserve municipalities (though compactness and the county preservation lead to a degree of municipality preservation.) It coincides with the primary ensemble properties in other resects" (pg. 6). Given the stated interests of the legislature in keeping municipalities whole, it is unclear to me why it would be useful to produce an analysis that intentionally violates this principle.[8] As such, I focus my comparisons on the first set of simulations in the Senate.

6 Disagreement Among Plaintiffs' Experts in House County Groupings

In this section I consider the county groupings that are singled out in the various expert reports submitted by the plaintiffs as being especially egregious examples of gerrymandering. However, as I will show, there is often disagreement even among the plaintiffs' own experts as to the presence, degree, and extent of the problem.

[8]For example, the committee hearing transcripts state: "We honored municipal boundaries. The chair made every effort to keep municipalities whole throughout the draw." See 9:43:00-9:45:00 in the committee hearing https://www.youtube.com/watch?v=7pyfVT6VOc4&t=34565s&ab_channel=NCGARedistricting and https://www.youtube.com/watch?v=GOVerOsNMm4&ab_channel=NCGARedistricting in the Senate.

6.1 Pitt House County Grouping

The Pitt county grouping contains two districts. The largest city in the cluster is Greenville, with a population of 87521, or nearly 1 district exactly (the target district population in the House is 208,788). However, creating a district that is entirely Greenville with the second district constituting everything in Pitt County that is not Greenville would create a district that resembles a donut hole (in other words, an embedded district). This type of district is also not proposed in the NCLCV proposed map. Given this, to avoid a "donut hole" scenario requires connecting the district that incorporates the majority of Greenville to the edge of the county so as to make sure this district is no longer embedded in the outer district. Simply adding a VTD to the district is not possible since no single VTD can be added without making the population of the district too large and the district highly non-compact. Thus, extending the boundaries of the district to the edge of the county necessitates splitting Greenville. The legislature chose to do this in a relatively east-west direction with northern Greenville in HD-8 and southern Greenville in HD-9.

Dr. Pegden's report states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 11% of all alternative districting satisfying my districting criteria (in other words, 89.1% are less optimized-for-partisanship)...(pg. 21)". 11% would not constitute a statistical outlier in a traditional scientific study.

With regards to substantive significance, Dr. Pegden's analysis predicts the expected seats from a range of uniform swings in election outcomes in the Enacted Plan in this cluster to be 1.3 Democratic seats. To gauge the substantive significance of this result, I compare it to the 25th percentile outcome of the simulations on the same metric. This yields an expected seats of between 1.45 Democratic districts, for a difference of between .15 districts. In other words, in a series of 5 elections with varying electoral environments (some good for Democrats and some good for Republicans) in each district in the cluster, we would expect the Enacted Map to elect an additional Democrat in the county group less than 1 time, on average, than the simulated maps would do.

In Dr. Mattingly's report, all 12 elections he considers generate a strongly Democratic district (HD-8). In only 3 of the 12 elections he considers a majority of the simulations create a second Democratic district while in 9 of the 12 elections the majority of the simulations generate a Republican district. In Figure 6.1.23 the Enacted Plan agrees with the majority outcome of the simulations in 10 of the 12 elections he considers.

These results are similar to those contained in my original report. In 10 of the 11 elections I include a majority of simulations generate one Democratic District and one Republican leaning district. In 10 of the 11 elections, the Enacted Plan agrees with the majority outcome of the simulated maps.

The overall picture here is one of agreement that in the majority of cases the Enacted Plan and the simulations generate one Democratic-leaning district and one Republican-leaning district.

Dr. Cooper does not provide any analysis of the Enacted Plan aside from calculating a partisan index of the districts. However, Dr. Cooper notes that Pitt County is currently represented by two Democrats, Kandie Smith and Brian Farkas. Dr. Cooper fails to note the old (2020) districting arrangement had 3 districts in Pitt County with the third district (District 12) extending into Lenoir County and being represented by Republican Chris Humphrey.

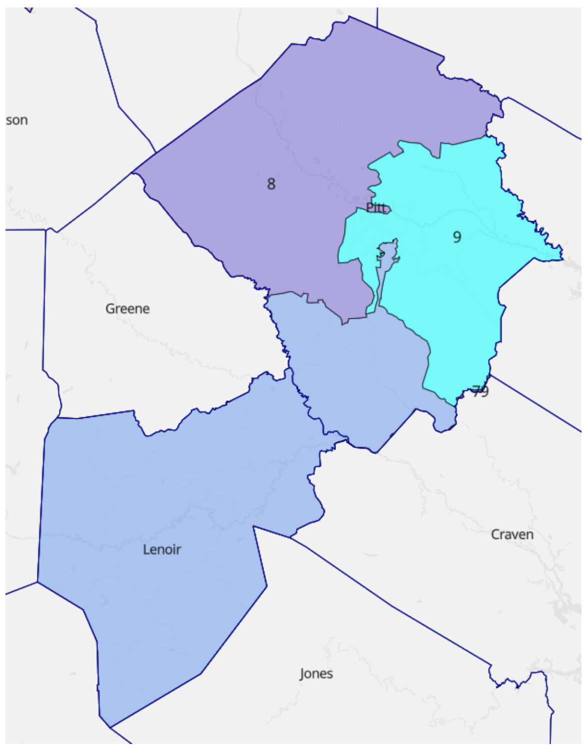


Figure 1: **2020 Districts in Pitt County**

6.2 Alamance House County Grouping

The Alamance County grouping contains two districts, HD-63 and HD-64. In this county there is disagreement between plaintiffs' experts as to whether or not the Enacted Map constitutes a gerrymander. Drs. Pegden and Mattingly do not find the map to be a partisan outlier, while Dr. Cooper objects to the particular shape of the districts.

Dr. Pegden's analysis places the Alamance County plan among the lowest quarter of districtings. He states, "In every run, the districting was in the most partisan 74% of districtings (in other words, 26.3% were less partisan, in every run) (pg. 23)." Because of this, he further states, "The Enacted Map is not unusual enough in the first-level analysis to enable a statistically significant second-level analysis of this cluster (pg. 23)." Looking at the range of expected Democratic seats in this county, the Enacted Plan is actually *more* Democratic than the median simulation in Dr. Pegden's report.

Dr. Mattingly also agrees that this plan is not an outlier. He states, "From Figure 6.1.25, we see that thought [sic] the Enacted Map tends have more Democrats in the more Democratic district and less in the less democratic [sic] district it not [sic] an outlier on its own (pg. 46)."

The simulations in my initial report also agree with this assessment. In 10 of 11 elections I analyze, the partisan lean of the districts in the Enacted Plan agree with the partisan lean of the majority of the simulations run. In 6 of the 11 elections a Democrat won a majority of the two-party vote in District 63 while in 5 of the elections the Republican candidate won the majority of the votes.

However, Dr. Cooper notes the unusual shape of the district but does not mention that this shape is largely the same (different by only 2.5 precincts) as the 2019 court-approved maps.

6.3 Duplin-Wayne House County Grouping

The Duplin-Wayne County grouping contains two districts, HD-4 and HD-10.

Dr. Pegden does not provide an analysis of this county. He states, "For this cluster, my conservative approach (as discussed in Section 4.3.2) does not allow my algorithm to generate any comparison maps other than the map itself." This is interesting as it aligns with my simulations in which I found no alternative maps that had an equal (or fewer) number of county traversals and were as compact or more compact than the Enacted Plan (see pg. 58 of Barber original report).

Dr. Mattingly does not find the map to be a partisan outlier in his analysis. He states, "In the Duplin-Wayne county cluster the two districts are safely Republican under the elections considered. The Enacted Map is typical, falling in the middle of the observed democratic [sic] fraction on the Histograms (pg. 42)."

However, the proposed NCLCV Map generates one consistently Democratic-leaning district across all 11 election that I analyze. This constitutes a partisan outlier in all 11 elections I consider and would also fall outside the majority of the simulation results in all comparable elections in Dr. Mattingly's simulations as well.[9]

[9]While we do not use the same elections Dr. Mattingly and I both use the 2016 Lieutenant Governor, 2016 President, 2020 Lieutenant Governor, 2020 US Senate, 2020 President, 2020 Attorney General, and 2020 Governor races.

6.4 Buncombe House County Grouping

The Buncombe County grouping contains three districts, HD-114, HD-115, and HD-116. In this county there is agreement among experts that the Enacted Map in this county grouping generally creates two Democratic seats and 1 Republican-leaning seat. The degree to which this is a partisan outlier is less certain.

Dr. Pegden reports that the Enacted Map in this county "was in the most partisan 0.020% of districtings (in other words, 99.979% were less partisan, in every run) (pg. 16)." This is a statistically significant result. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 2.26 seats while the 25th percentile plan has an expected Democratic seats of 2.85. This leads to a substantive difference of 0.59 expected Democratic seats. Put another way, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 2 rather than 3 in this cluster) than the 25th percentile simulation roughly 3 additional times.

Dr. Mattingly's presents simulations in which the Enacted Map and the simulations agree on the creation of 2 Democratic districts in the cluster (HD-114 and HD-115). In all 12 elections considered the Enacted Map and the simulations are in agreement on the partisan lean of these two ditricts. The third district, HD-116, is the source of the disagreement. In 10 of the 12 simulations HD-116 in the Enacted Plan does not agree with the majority of the simulations in Dr. Mattingly's report (see Figure 6.1.14).

Dr. Cooper offers his assessment by saying "By shifting the current district lines where the districts meet in Asheville, however, the Enacted Map packs as many Democrats as possible into HD-114, while HD-115 stays relatively constant in terms of predicted vote share. The C-shaped HD-116 now includes most of the Republican-leaning VTDs in Buncombe..." Dr. Cooper appears to imply that a more appropriate orientation of the district lines would be to place a substantial portion of Asheville into each of the three districts.

In other words, across all three experts, the disagreement with the Enacted Plan centers on district HD-116. The "C" shape in District HD-116, as noted by Dr. Cooper, is the result of a decision to minimize the division of the city of Asheville. With a population of 94,589, the city will need to be split into two different districts, but not necessarily three. The Enacted Plan does this by placing approximately 87 percent of the city population in two districts, HD-114 and HD-115, leaving HD-116 to wrap around the the city and largely avoid its boundaries. This, however, creates the "C" shape of the district.

Finally, Dr. Cooper states, "Soon after the maps were passed, all three Democratic incumbents announced that they would be retiring and not running for office in these newly drawn districts." It is unclear to me how this fact is relevant to the shape of the new districts. If the Enacted Map create two strong Democratic districts, how is the announced retirement of all three Democratic incumbents in any way a result of the districting process, as Dr. Cooper implies? Dr. Cooper does not offer any other evidence that something else related to the new districts may have been the cause, such as double bunking, or a dramatic shift in the composition of each district from the old (2020) districts.

6.5 Cumberland House County Grouping

The Cumberland County group contains four districts, HD-42, HD-43, HD-44, and HD-45. In this cluster there is disagreement between the experts as to whether this county constitutes an extreme gerrymander.

Dr. Pegden's analysis contend the that the Enacted Plan is neither a statistically significant nor substantively significant outlier. He states, "In every run, the districting was in the most partisan 16% of districtings (in other words, 83.5% were less partisan, in every run)...The Enacted Map is not unusual enough in the first-level analysis to enable a statistically significant second-level analysis of this cluster (pg. 27)."

Beyond not being statistically unique, the substantive difference in the number of expected Democratic seats is very small. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 3.21 seats while the 25th percentile plan has an expected Democratic seats of 3.25. This leads to a substantive difference of between 0.04 expected Democratic seats. In other words, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 3 rather than 4 in this cluster) than the 25th percentile simulation less than 1 additional time.

Dr. Mattingly's presents analysis in which the simulations generate two solidly Democratic districts (HD-44 and HD-42) and two districts that are closer to the .50 line with HD-43 being Democratic-leaning and HD-45 being Republican-leaning (see Figure 6.1.29 in Mattingly Report). Regarding this outcome he states, "In an ensemble that better preserves municipalities, the most Republican district is typically more republican [sic] and the second most Republican district more Democratic. This makes the Enacted Plan which squeezes the two together with an [sic] large outlier."

A closer look at Figure 6.1.29 shows that the Enacted Plan is an outlier not because it favors one party over the other, but rather because it creates more competitive races than the majority of Dr. Mattingly's simulations. While Dr. Mattingly's simulations produce a reliably Republican district in HD-45 and a reliably Democratic district in HD-43, the Enacted Plan creates neither and instead generates two very competitive districts. This produces a responsive map in which the partisanship of legislators elected to these two districts will likely shift frequently with shifting electoral preferences, something Dr. Mattingly notes is a desirable feature of a districting plan in other portions of his report (see pg. 3 and 4 of Mattingly Report).

Dr. Cooper agrees with this this when he states, "The Enacted Map creates two extremely competitive districts, HD-43 and HD-45 (with CCSC scores of D+1,334 and D+663, respectively) by splitting the Democratic-leaning City of Fayetteville into all four districts in the cluster." While his assessment of the competitiveness of these two districts is correct, he is incorrect as to the reason. Fayetteville has a population of 208,501 and as such is required to be divided into at least three districts, but not four. And while the Enacted Plan does draw parts of Fayetteville into all four districts, only 7.3 % of Fayetteville's population is placed in District 45.

Furthermore, the Enacted Plan places a much smaller proportion of Fayetteville in to the 45th district than NCLCV plaintiff's proposed map does. If Dr. Cooper's objections to dividing municipalities more than necessary is applied to this map, then plaintiff's map fares much worse than the Enacted Map. The table and figure below shows the comparison of how Fayetteville is divided in the two plans, which is also shown as Table 18 and Figure 54 in my original report.

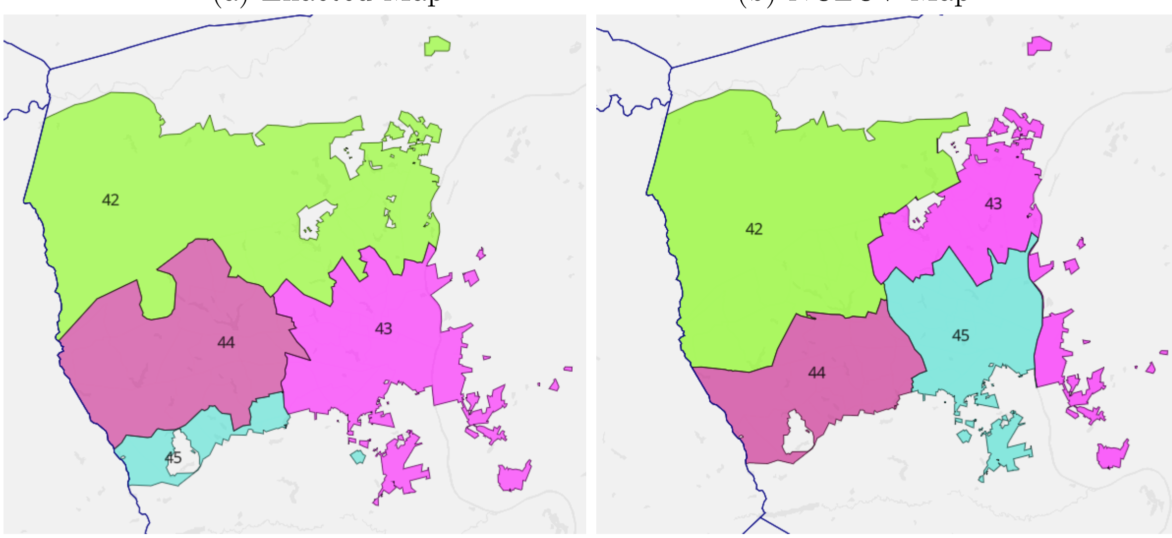
Table 1: Division of Fayetteville in Enacted Plan and NCLCV Plan

	Percent of Feyetville in district	
District:	Enacted Plan	NCLCV Plan
42	31.4	33.4
43	21.4	21.5
44	39.9	26.8
45	7.3	18.3
Total:	100%	100%

Note: Population number for city by district for Enacted Plan from: `https://ncleg.gov/Files/GIS/Plans_Main/Senate_2021/SL%202021-173%20Senate%20-%20StatPack%20Report.pdf` Population numbers for city by district for NCLCV Plan from Dave's Redistricting online. `https://davesredistricting.org/`

Figure 2: **Map of Fayetteville Divisions in Cumberland County Cluster**

(a) Enacted Map (b) NCLCV Map



6.6 Durham-Person House County Grouping

The Durham-Person County grouping contains 4 districts, HD-2, HD-29, HD-30 and HD-31. In this cluster there is disagreement with one district in particular, HD-2, which takes in the entirety of Person County to the north and the northern and eastern portions of Durham county.

Dr. Pegden's analysis of this county cluster yields the following results. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.20% of all alternative districtings satisfying my districting criteria (in other words, 99.79% are less optimized-for-partisanship)" (pg. 25).

However, the substantive effect of this difference is very small. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 3.87 seats while the 25th percentile plan has an expected Democratic seats of 3.95. This leads to a substantive difference of between 0.08 expected Democratic seats. Put another way, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 3 rather than 4 in this cluster) than the 25th percentile simulation less than 1 additional time.

Dr. Mattingly's simulations reveal three highly Democratic districts and one district that is more competitive. In the three highly Democfatic district (HD-31, HD-29, and HD-30), the Enacted Plan and the simulations are in agreement in all 12 of the 12 elections considered. In 10 of the 12 elections he considers the Enacted Plan agrees with the majority of simulations on the partisanship of the more competitive district, HD-2 (see Figure 6.1.23 of Mattingly Report).

Dr. Cooper simultaneously criticizes the map for dividing Durham across all four district while also packing Democratic into three of the four districts. He states, "The Enacted Map splits the City of Durham across all four districts but packs Democratic voters in HDs 29, 39, and 31; there is not a single Republican or competitive VTD in those districts (pg. 84)." This is a confusing complaint to offer since there are nearly no Republican VTDs in Durham County (if any at all when looking at Map 40 in Dr. Cooper's report), so it comes as no surprise that the three districts that are entirely contained in Durham County would contain no Republican-leaning VTDs. Furthermore, Dr. Cooper notes that the city of Durham is included in all four districts. However, remedying this by making sure District 2 contained no portion of Durham would only further make District 2 more Republican as the most Democratic VTDs in District 2 are those within the Durham city limits. Furthermore, the population of Durham is 283,506, which means it is large enough that it is absolutely necessary to include parts of Durham in all four districts.

6.7 Brunswick-New Hanover House County Grouping

The Brunswick-New Hanover County grouping contains 4 districts, HD-17, HD-18, HD-19, and HD-20. In this case, there is disagreement between experts as to whether this cluster constitutes an extreme gerrymander.

Dr. Pegden's analysis contends that the Enacted Plan is not a significant outlier, statistically or substantively. He states, "In every run, the districting was in the most partisan 11% of districtings (in other words, 89.4% were less partisan, in every run). The Enacted Map is not unusual enough in the first-level analysis to enable a statistically significant second-level analysis of this cluster (pg. 24)."

Beyond not being unusual in comparison to the simulations to perform a statistically significant second-level analysis, the substantive difference in the expected Democratic seat share is also very small. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 1.25 seats while the 25th percentile plan has an expected Democratic seats of 1.25. This leads to a substantive difference of between 0.00 expected Democratic seats. In other words, across 5 hypothetical elections of each district in the cluster, we would not expect the Enacted Map to differ from the 25th percentile simulation at all, on average.

Dr. Mattingly argues on the other hand that the cluster is problematic. Specifically, he locates the problem in District 20. He states of this district, "The Republican party typically wins the second most democratic [sic] district [HD-20] in the Enacted Plan even though it would go to the Democrats under a number of elections when the neutral maps in the primary ensemble are used." Looking at Figure 6.1.35 in Dr. Mattingly's report we see that in 5 of the 12 elections the Enacted Plan agrees with the majority of simulations on the partisan lean of HD-20.

Dr. Cooper does not offer much by way of exposition in this cluster other than to claim that District 18 is packing Democratic voters "in and around Wilmington" and that "[t]he heavily Republican HD-19 also ensnares a Democratic-leaning VTD south of Wilmington, which keeps that VTD out of competitive HD-20 (pg. 95).” Another way to consider the “packing” referred to by Dr. Cooper is to note that District 18 keeps the communities of Hightsville, Wrightsboro, Skippers Corner, Castle Hayne, Blue Clay Farms, Northchase, Murraysville, and Kings Grant — all municipalities in and around Wilmington — together. Secondly, the “ensnared” VTD that Dr. Cooper refers to is only moderately Democratic (.56 in the 2020 Presidential election) and would make only the slightest difference in the overall partisan lean of HD-20 were it to somehow capture it from HD-19.

6.8 Forsyth-Stokes House County Grouping

The Forsyth-Stokes County grouping contains 5 districts, HD-91, HD-71, HD-72, HD-74, and HD-75. In this county there is agreement among experts that the Enacted Map in this county grouping generally creates two Democratic seats and 2 Republican-leaning seats. The partisan lean of the middle district in the Enacted Plan, HD-74, is in dispute.

Dr. Pegden's analysis contends that the Enacted Plan is a significant outlier, statistically and substantively. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.26% of all alternative districtings satisfying my districting criteria (in other words, 99.73% are less optimized-for-partisanship) (pg. 18)."

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 2.18 seats while the 25th percentile plan has an expected Democratic seats of 2.85. This leads to a substantive difference of 0.67 expected Democratic seats. Stated differently, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 2 rather than 3 in this cluster) than the 25th percentile simulation roughly 3 additional times.

Dr. Mattingly's presents simulations that contain two districts that are consistently Democratic leaning (HD-71 and HD-72) and two districts in which the distribution of simulation results are nearly always Republican leaning (HD-91 and HD-75). Thus, the outlier in his analysis lies with HD-74 where the simulations often generate both Republican and Democratic leaning districts and the Enacted Plan is more consistently Republican leaning.

However, the Enacted Plan's District 74 is very similar in shape and partisan lean to the NCLCV "optimized map." A map of the similarities in these districts is presented in Figure 69 of my original report. The partisan lean of District 74 using the election index in my original report is 0.45 while the partisan lean of District 74 in the NCLCV map is 0.46. Thus, if the Enacted Map is an extreme gerrymander due to the boundaries and partisan lean of District 74, then this criticism would also apply to the proposed NCLCV map as well.

Finally, Dr. Cooper notes of this district, "The splits of Winston-Salem do not make sense without reference to the anticipated voting behavior of the VTDs arranged into each district." However, this is not the case. The splits of Winston-Salem are largely the same as the 2020 maps, which were approved by a court in 2019. To a large degree the legislature appears to have chosen to leave the district boundaries much the same as the previous court-approved maps. Figure 69 in my original report presents this comparison between the current maps and the old maps in this cluster.

6.9 Cabarrus-Davie-Rowan-Yadkin House County Grouping

The Cabarrus County grouping contains 5 districts, HD-73, HD-76, HD-77, HD-82, and HD-83.

The layout of districts in this cluster is largely determined by the geography of the four counties in the cluster. Yadkin and Davie are sparsely populated and as such must constitute a portion of a single district (HD-77). This district then extends south into northern Rowan County, where it borders Davie County. Rowan County has a larger population - enough to sustain 1.68 districts. To minimize county traversals in the group, this implies creating a single district that is entirely contained within Rowan county and then another district that spans Rowan County and extends into northern Cabarrus County. Finally, Cabarrus County is the most populated county of the group (population = 225,804) with a population large enough to support 2.6 districts. This means that there will be two districts entirely contained in Cabarrus County with a partial district that spans Rowan and Cabarrus Counties. Because the county grouping is arranged in a linear North/South axis, this layout of districts - 1 in Yadkin and Davie, and partially in Rowan, 1 in Rowan, 1 spanning Rowan/Cabarrus, and 2 entirely in Cabarrus is the only arrangement that complies with the rules requiring the minimization of county traversals.

Thus, complaints of the districts are limited to the particular boundaries of the two and a half districts in Cabarrus county (HD-73, HD-82 and HD-83).

Dr. Pegden does not find the Enacted Plan to be a significant outlier. He states, "In every run, the districting was in the most partisan 12% of districtings (in other words, 87.7% were less partisan, in every run). The Enacted Map is not unusual enough in the first-level analysis to enable a statistically significant second-level analysis of this cluster (pg. 26)."

Beyond not being unusual in comparison to the simulations to perform a statistically significant second-level analysis, the substantive difference in the expected Democratic seat share is also very small. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 0.33 seats while the 25th percentile plan has an expected

Democratic seats of 0.45. This leads to a substantive difference of 0.12 expected Democratic seats. In other words, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 0 rather than 1 in this cluster) than the 25th percentile simulation less than 1 additional time.

Dr. Mattingly's simulations produce 4 very Republican districts and one district that generates both Republican and Democratic outcomes (HD-82), depending on the election one uses to measure partisanship. He states, "In the Cabarrus-Davie-Rowan-Yadkin county cluster, there are abnormally few Democrats in the most Democratic district (district 82)." In 4 of the 12 elections he considers the Enacted Plan agrees with the majority of the simulations on the partisanship of this swing district.

One important thing to note is that the proposed NCLCV map performs worse than the Enacted Plan by this metric described by Dr. Mattingly. The most Democratic district in this plan is actually *less* Democratic than the Enacted Plan (0.43 in the NCLCV plan compared to 0.41 in the Enacted Plan using the partisan index in my original report). Thus, by Dr. Mattingly's argument, this would place the NCLCV map as more of a partisan outlier than the Enacted Plan in this county cluster.

6.10 Guilford County House County Grouping

The Guilford County grouping contains 6 districts, HD-57, HD-58, HD-59, HD-60, HD-61, and HD-62.

Dr. Pegden's analysis contends that the Enacted Plan is a significant outlier. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.000089% of all alternative districtings satisfying my districting criteria (pg. 19)."

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 4.46 seats while the 25th percentile plan has an expected Democratic seats of 5.45. This leads to a substantive difference of 0.99 expected Democratic seats. In other words, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 4-5 rather than 5-6 in this cluster) than the 25th percentile simulation every time, on average.

Dr. Mattingly states of his simulations in this county: "The ensemble reliably has four democratic districts and a 5th which typically leans Republican but sometimes is competitive. Yet, the Enacted Plan gives one clearly Republican district and one which is often safely Republican and at times competitive (pg. 36)." District 59 is the district in question. Excluding HD-59, in 12 of the 12 elections the Enacted Plan agrees with the majority of Dr. Mattingly's simulations on the partisanship of the remaining 5 districts in the cluster. Thus the discussion of a potential gerrymander is focused on the composition of HD-59.

This also conforms with the simulation results in my original report. In 11 of the 11 elections I consider, the partisan lean of the districts in the Enacted Plan is one Democratic district short of the outcome in the majority of the simulations run.

However, one factor to consider is that District 59's boundaries are identical to the court-approved 2019 map's boundaries, but for one precinct, G53 (See Figure 78 in my original report for a map of the district under the two plans). District 59's population would be is too large if the map were to use the exact boundaries from 2019 based on the updated 2020 census population numbers. At the same time, District 61 and 58 are within the new population thresholds based on the new census numbers. Thus, it makes perfect sense to move one precinct from 57 into either 61 or 58 to equalize the population of these districts. Precinct G53 may have been chosen because it contains the right population size and is nearly entirely within the city of Greensboro, allowing a larger share of Greensboro to be contained within fewer districts.

6.11 Mecklenburg County House County Grouping

The Mecklenburg County cluster contains 13 districts, HD-88, HD-92, HD-98, HD-99, HD-100, HD-101, HD-102, HD-103, HD-104, HD-105, HD-106, HD-107, and HD-112.

Dr. Pegden's analysis contends that the Enacted Plan is a outlier, but not to the degree of other clusters discussed above. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 5.0% of all alternative districtings satisfying my districting criteria (in other words, 95.0% are less optimized-for-partisanship) (pg. 20)." In a traditional scientific study, the 5% boundary represents the line of a statistically significant outlier.

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 11.56 seats while the 25th percentile plan has an expected Democratic seats of 11.95. This leads to a substantive difference of 0.39 expected Democratic seats. Put another way, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 11-12 rather than 12-13 in this cluster) than the 25th percentile simulation in approximately 2 of these 5 elections, on average. In other words, the difference across this range of electoral environments is Republicans picking up an additional seat about 2 in 5 times.

Dr. Mattingly's presents simulation analysis that present the partisan distributions of the different districts and where, specifically, an outlier might occur. Figure 6.1.2 of Dr. Mattingly's report shows that in the 10 most Democratic districts in the cluster, the Enacted Plan agrees with the majority of simulations in 12 of the 12 elections considered. Both the simulations and the Enacted Plan contain 9 comfortably Democratic districts and a 10th district that is Democratic in 11 of the 12 elections considered. In the 2 most Republican districts (HD-98 and HD-103), the Enacted Plan agrees with the majority of simulations in 12 of the 12 elections considered. These two districts occasionally lean Democratic and occasionally lean Republican, but in all 12 elections the Enacted Plan's partisan lean aligns with the partisan lean of the majority of the simulations. This leaves one districts in dispute - HD-104. In District 104, the Enacted Plan agrees with the majority of the simulations in 11 of the 12 elections considered. Thus, across the 13 different districts in 12 different elections, the Enacted Plan is in alignment with the majority of the simulation results in all but 1 election (Figure 6.1.2 shows a misalignment of HD-104 with the majority of the simulations in the 2020 Commissioner of Agriculture election).

Dr. Cooper states that, "[t]he Enacted Map places no Republican VTDs in HDs 92, 99, 100, 101, 102, 106, 107, and 112, leaving every Republican-leaning VTD in HDs 88, 103, 104, and 105." Dr. Cooper omits here that there are very few Republican leaning VTDs at all on his map to begin with, they tend to be close to one another, and are concentrated in northern and southeastern Mecklenburg County. Thus it is not surprising that they are placed in relatively few of the districts given the desire for geographically compact districts. He notes the partisan composition of HDs 98 and 103 as being "carved out of the pockets of Republican voters in the north and southeast portions of the county... (pg. 68)." However, this assessment ignores the partisan geography of the cluster. District 98 is geographically compact and avoids traversing into the Charlotte city limits. Furthermore, District 103 in the southeast of the county keeps the cities of Mint Hill (there are 6 voters from this city not in District 103) and Matthews whole and together in one district.

6.12 Wake County House County Grouping

The Wake County cluster contains 13 districts, HD-11, HD-21, HD-33, HD-34, HD-35, HD-36, HD-37, HD-38, HD-39, HD-40, HD-41, HD-49, and HD-66.

Dr. Pegden's analysis contends that the Enacted Plan is a statistical outlier. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 2.2% of all alternative districtings satisfying my districting criteria (in other words, 97.8% are less optimized-for-partisanship) (pg. 22)."

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 11.62 seats while the 25th percentile plan has an expected Democratic seats of 11.85. This leads to a substantive difference of 0.23 expected Democratic seats. In other words, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 11-12 rather than 12-13 in this cluster) than the 25th percentile simulation in approximately 1 of these 5 elections, on average.

Dr. Mattingly's simulation analysis presents the partisan distributions of the different districts and where specifically an outlier might occur. Figure 6.1.5 of Dr. Mattingly's report shows that in the 10 most Democratic districts in the cluster, the Enacted Plan agrees with the majority of simulations in 12 of the 12 elections considered. In the most Republican district (HD-37), the Enacted Plan agrees with the majority of simulations in 9 of the 12 elections considered. This leaves two districts - HD-35 and HD-21. In District 35, the Enacted Plan agrees with the majority of the simulations in 7 of the 12 elections considered, and in HD-21 the Enacted Plan agrees with the majority of the simulations in 10 of the 12 elections considered. However, in the 2 elections where it is in disagreement, the Enacted Plan actually creates a *Democratic* leaning district where the majority of simulations create a Republican leaning district. Thus, the results in this cluster are mixed. Some of the Enacted Plan's districts are more Republican, on average, than the simulations and in other cases the Enacted Plan's districts are more Democratic. And in most cases there is agreement.

7 Disagreement Among Plaintiff Experts in Senate County Groupings

7.1 Cumberland and Moore Senate County Grouping

The Cumberland and Moore Senate county grouping contains two districts, SD-19 and SD-21.

Dr. Pegden's analysis contend that the Enacted Plan is a statistical outlier. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.000015% of all alternative districtings satisfying my districting criteria (in other words, 99.999984% are less optimized-for-partisanship) (pg. 28)."

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 1.01 seats while the 25th percentile plan has an expected Democratic seats of 1.35. This leads to a substantive difference of 0.34 expected Democratic seats. Put differently, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 1 rather than 2 in this cluster) than the 25th percentile simulation in approximately 1-2 of these 10 elections, on average. In other words, the difference across this range of electoral environments is Republicans picking up an additional seat less than 2 in 5 times.

Dr. Mattingly states of the result of the simulations in this cluster, "The districts in the enacted are chosen to maximize the number of Democrats in the more democratic district and the number of republicans in the most Republican district. The map is an extreme outlier in both of these regards. The effect is a maximally non-responsive map." It is noteworthy that in other clusters Dr. Mattingly criticizes the map for being overly responsive (see Cumberland House grouping discussion). Despite this critique, from Figure 6.2.10 we see that in all 12 elections the Enacted Map agrees with the majority of the simulations in all districts. In not a single election do a majority of the simulations produce two Democratic seats.

It is also noteworthy that the NCLCV plaintiff's proposed plan is identical to Enacted Plan in this cluster.

7.2 Fosyth-Stokes Senate County Grouping

The Forsyth and Stokes Senate county grouping contains two districts, SD-31 and SD-32.

Dr. Pegden's analysis contend that the Enacted Plan is a statistical outlier. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.0051% of all alternative districtings satisfying my districting criteria (in other words, 99.9947% are less optimized-for-partisanship) (pg. 29)."

However, in this cluster the substantive difference in the expected Democratic seat share is nearly zero. This is a particularly good example of the importance of distinguishing between statistical and substantive significance. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 1.00 seats while the 25th percentile plan has an expected Democratic seats of 1.05. This leads to a substantive difference of 0.05 expected Democratic seats. Put another way, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 1 rather than 2 in this cluster) than the 25th percentile simulation in approximately 0 of these 5 elections, on average. In other words, the difference between the Enacted Plan and the simulations results across this range of electoral environments is effectively zero in this cluster.

Dr. Mattingly states of the result of the simulations in this cluster, "The districts in the enacted are chosen to maximize the number of Democrats in the more democratic district and the number of republicans [sic] in the most Republican district. The map is an extreme outlier in both of these regards. The effect is a maximally non-responsive map (pg. 61)." This is similar to his objection to the Cumberland-Moore cluster above, and is again noteworthy that in other clusters Dr. Mattingly criticizes the map for being overly responsive (see Cumberland House grouping discussion). Despite this critique, from Figure 6.2.7 we see that in all 12 elections the Enacted Map agrees with the majority of the simulations in all districts. In not a single election do the simulations produce two Democratic seats.

7.3 Guilford-Rockingham Senate County Grouping

The Guilford and Rockingham Senate county grouping contains 3 districts, SD-26, SD-27, and SD-28.

Dr. Pegden's analysis contend that the Enacted Plan is a statistical outlier. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.00012% of all alternative districtings satisfying my districting criteria (in other words, 99.99987% are less optimized-for-partisanship) (pg. 31)."

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 2 seats while the 25th percentile plan has an expected Democratic seats of 2.25. This leads to a substantive difference of 0.25 expected Democratic seats. Put differently, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 2 rather than 3 in this cluster) than the 25th percentile simulation in approximately 1-2 of these 10 elections, on average. In other words, the difference across this range of electoral environments is Republicans picking up an additional seat less than 2 in 5 times.

Dr. Mattingly's summary of the simulations results in this cluster are as follows: "The three districts in the Guilford-Rockingham cluster are constructed to pack an exceptional number of democrats [sic] in the most democratic [sic] district (district 28) and exceptionally few Democrats in the most Republican district (district 26). The effect is to ensure a Republican victory in the district 26, when in some elections the most republican [sic] district would be at risk of going to the Democratic Party (pg. 63)." However, in 11 of the 12 elections the Enacted Map's least Democratic district (SD-26) agrees with the majority of the simulations by electing a Republican. In only 1 of the 12 elections do the majority of his simulations produce 3 Democratic districts while the Enacted Plan produces only 2. SD-26 is less competitive (i.e. more Republican leaning) than the majority of simulations, but the inverse is also true of SD-27, which is competitive in many of the simulations and in a few rare cases elects a Republican but is more Democratic and always elects a Democrat in the Enacted Plan.

7.4 Granville-Wake Senate County Grouping

The Granville and Wake Senate county cluster contains 6 districts, SD-13, SD-14, SD-15, SD-16, SD-17, and SD-18.

Dr. Pegden's analysis contend that the Enacted Plan is a statistical outlier. He states, "My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.000030% of all alternative districtings satisfying my districting criteria (in other words, 99.999969% are less optimized-for-partisanship) (pg. 30)."

The substantive difference in the expected Democratic seat share is as follows: The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 5.13 seats while the 25th percentile plan has an expected Democratic seats of 5.75. This leads to a substantive difference of 0.62 expected Democratic seats. Put another way, across 6 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 5 rather than 6 in this cluster) than the 25th percentile simulation in approximately 3 of these 5 elections, on average. In other words, the difference across this range of electoral environments is Republicans picking up an additional seat roughly 3 in 5 times.

Dr. Mattingly's presents simulations that contain four districts that are solidly Democratic in which no simulation nor the Enacted Plan produce a Republican-leaning seat (see Figure 6.2.4 in Dr. Mattingly's report). The simulations also contain two seats (SD-13 and

SD-17) in which a majority of the simulations produce a Republican-leaning seat (4 of the 12 elections considered) and in other elections produce a Democratic-leaning seat (5 of the 12 elections considered). In some cases the majority of simulations in SD-13 and SD-17 diverge with one district being majority Republican and the other producing a majority of the simulations generating a Democratic district (3 of the 12 elections). In the most Republican district the Enacted Plan agrees with the majority of the simulations in 10 of the 12 elections considered and in the second most Republican district there is agreement in 9 of the 12 elections considered.

7.5 Iredell-Mecklenburg Senate County Grouping

The Iredell and Mecklenburg Senate county cluster contains 6 districts, SD-37, SD-38, SD-39, SD-40, SD-41, and SD-42.

Dr. Pegden's analysis contend that the Enacted Plan is a statistical outlier. He states, 'My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.0057% of all alternative districtings satisfying my districting criteria (in other words, 99.9943% are less optimized-for-partisanship) (pg. 32)."

However, the substantive difference in the expected Democratic seat share is much smaller. The Enacted Map has an expected Democratic seats generated from the uniform swing analysis of 4.67 seats while the 25th percentile plan has an expected number of Democratic seats of 4.85. This leads to a substantive difference of 0.18 expected Democratic seats. In other words, across 5 hypothetical elections of each district in the cluster, we would expect the Enacted Map to elect one fewer Democrat (meaning 4 rather than 5 in this cluster) than the 25th percentile simulation in approximately 1 of these 5 elections, on average. Put another way, the difference across this range of electoral environments is Republicans picking up an additional seat roughly 1 in 5 times.

Dr. Mattingly's simulations in this cluster contain four districts that are solidly Democratic in which no majority of his simulations nor the Enacted Plan produce a Republican-leaning seat (see Figure 6.2.1 in Dr. Mattingly's report). The simulations also contain one seat (SD-37) in which a majority of the simulations produce a heavily Republican-leaning seat in all 12 elections. The Enacted Plan is in total agreement with the majority of simulations in these districts. This leaves SD-41, which is a more competitive seat in the simulations. In 9 of the 12 elections considered the partisan outcome in the Enacted Plan matches the partisan outcome in the majority of the simulations by producing a majority of the two-party vote share for the Democratic candidate.

Appendix A: Curriculum Vitae

Michael Jay Barber

Contact Information

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Academic Appointments

Brigham Young University, Provo, UT

August 2020 - present Associate Professor, Department of Political Science
2014 - July 2020 Assistant Professor, Department of Political Science
2014 - present Faculty Scholar, Center for the Study of Elections and Democracy

Education

Princeton University Department of Politics, Princeton, NJ

Ph.D., Politics, July 2014

- Advisors: Brandice Canes-Wrone, Nolan McCarty, and Kosuke Imai
- Dissertation: "Buying Representation: the Incentives, Ideology, and Influence of Campaign Contributions on American Politics"
- 2015 Carl Albert Award for Best Dissertation, Legislative Studies Section, American Political Science Association (APSA)

M.A., Politics, December 2011

Brigham Young University, Provo, UT

B.A., International Relations - Political Economy Focus, April, 2008

- *Cum Laude*

Research Interests

American politics, congressional polarization, political ideology, campaign finance, survey research

Publications

19. **"Ideological Disagreement and Pre-emption in Municipal Policymaking"** with Adam Dynes
Forthcoming at *American Journal of Political Science*
18. **"Comparing Campaign Finance and Vote Based Measures of Ideology"**
Forthcoming at *Journal of Politics*
17. **"The Participatory and Partisan Impacts of Mandatory Vote-by-Mail"**, with John Holbein
Science Advances, 2020. Vol. 6, no. 35, DOI: 10.1126/sciadv.abc7685
16. **"Issue Politicization and Interest Group Campaign Contribution Strategies"**, with Mandi Eatough
Journal of Politics, 2020. Vol. 82: No. 3, pp. 1008-1025

15. **“Campaign Contributions and Donors’ Policy Agreement with Presidential Candidates”**, with Brandice Canes-Wrone and Sharece Thrower
Presidential Studies Quarterly, 2019, 49 (4) 770–797
14. **“Conservatism in the Era of Trump”**, with Jeremy Pope
Perspectives on Politics, 2019, 17 (3) 719–736
13. **“Legislative Constraints on Executive Unilateralism in Separation of Powers Systems”**, with Alex Bolton and Sharece Thrower
Legislative Studies Quarterly, 2019, 44 (3) 515–548
Awarded the Jewell-Loewenberg Award for best article in the area of subnational politics published in *Legislative Studies Quarterly* in 2019
12. **“Electoral Competitiveness and Legislative Productivity”**, with Soren Schmidt
American Politics Research, 2019, 47 (4) 683–708
11. **“Does Party Trump Ideology? Disentangling Party and Ideology in America”**, with Jeremy Pope
American Political Science Review, 2019, 113 (1) 38–54
10. **“The Evolution of National Constitutions”**, with Scott Abramson
Quarterly Journal of Political Science, 2019, 14 (1) 89–114
9. **“Who is Ideological? Measuring Ideological Responses to Policy Questions in the American Public”**, with Jeremy Pope
The Forum: A Journal of Applied Research in Contemporary Politics, 2018, 16 (1) 97–122
8. **“Status Quo Bias in Ballot Wording”**, with David Gordon, Ryan Hill, and Joe Price
The Journal of Experimental Political Science, 2017, 4 (2) 151–160.
7. **“Ideologically Sophisticated Donors: Which Candidates Do Individual Contributors Finance?”**, with Brandice Canes-Wrone and Sharece Thrower
American Journal of Political Science, 2017, 61 (2) 271–288.
6. **“Gender Inequalities in Campaign Finance: A Regression Discontinuity Design”**, with Daniel Butler and Jessica Preece
Quarterly Journal of Political Science, 2016, Vol. 11, No. 2: 219–248.
5. **“Representing the Preferences of Donors, Partisans, and Voters in the U.S. Senate”**
Public Opinion Quarterly, 2016, 80: 225–249.
4. **“Donation Motivations: Testing Theories of Access and Ideology”**
Political Research Quarterly, 2016, 69 (1) 148–160.
3. **“Ideological Donors, Contribution Limits, and the Polarization of State Legislatures”**
Journal of Politics, 2016, 78 (1) 296–310.
2. **“Online Polls and Registration Based Sampling: A New Method for Pre-Election Polling”** with Quin Monson, Kelly Patterson and Chris Mann.
Political Analysis 2014, 22 (3) 321–335.
1. **“Causes and Consequences of Political Polarization”** In *Negotiating Agreement in Politics*. Jane Mansbridge and Cathie Jo Martin, eds., Washington, DC: American Political Science Association: 19–53. with Nolan McCarty. 2013.

    - Reprinted in *Solutions to Political Polarization in America*, Cambridge University Press. Nate Persily, eds. 2015
    - Reprinted in *Political Negotiation: A Handbook*, Brookings Institution Press. Jane Mansbridge and Cathie Jo Martin, eds. 2015

Available Working Papers

"Misclassification and Bias in Predictions of Individual Ethnicity from Administrative Records" (Revise and Resubmit at *American Political Science Review*)

"Taking Cues When You Don't Care: Issue Importance and Partisan Cue Taking"
with Jeremy Pope (Revise and Resubmit)

"A Revolution of Rights in American Founding Documents"
with Scott Abramson and Jeremy Pope (Conditionally Accepted)

"410 Million Voting Records Show the Distribution of Turnout in America Today"
with John Holbein (Revise and Resubmit)

"Partisanship and Trolleyology"
with Ryan Davis (Under Review)

"Who's the Partisan: Are Issues or Groups More Important to Partisanship?"
with Jeremy Pope (Revise and Resubmit)

"Race and Realignment in American Politics"
with Jeremy Pope (Revise and Resubmit)

"The Policy Preferences of Donors and Voters"

"Estimating Neighborhood Effects on Turnout from Geocoded Voter Registration Records."
with Kosuke Imai

"Super PAC Contributions in Congressional Elections"

Works in Progress

"Collaborative Study of Democracy and Politics"
with Brandice Canes-Wrone, Gregory Huber, and Joshua Clinton

"Preferences for Representational Styles in the American Public"
with Ryan Davis and Adam Dynes

"Representation and Issue Congruence in Congress"
with Taylor Petersen

"Education, Income, and the Vote for Trump"
with Edie Ellison

Invited Presentations

"Are Mormons Breaking Up with Republicanism? The Unique Political Behavior of Mormons in the 2016 Presidential Election"

- Ivy League LDS Student Association Conference - Princeton University, November 2018, Princeton, NJ

"Issue Politicization and Access-Oriented Giving: A Theory of PAC Contribution Behavior"

- Vanderbilt University, May 2017, Nashville, TN

"Lost in Issue Space? Measuring Levels of Ideology in the American Public"

- Yale University, April 2016, New Haven, CT

"The Incentives, Ideology, and Influence of Campaign Donors in American Politics"

- University of Oklahoma, April 2016, Norman, OK

"Lost in Issue Space? Measuring Levels of Ideology in the American Public"

- University of Wisconsin - Madison, February 2016, Madison, WI

"Polarization and Campaign Contributors: Motivations, Ideology, and Policy"

- Hewlett Foundation Conference on Lobbying and Campaign Finance, October 2014, Palo Alto, CA

"Ideological Donors, Contribution Limits, and the Polarization of State Legislatures"

- Bipartisan Policy Center Meeting on Party Polarization and Campaign Finance, September 2014, Washington, DC

"Representing the Preferences of Donors, Partisans, and Voters in the U.S. Senate"

- Yale Center for the Study of American Politics Conference, May 2014, New Haven, CT

Conference Presentations

Washington D.C. Political Economy Conference (PECO):

- 2017 discussant

American Political Science Association (APSA) Annual Meeting:

- 2014 participant and discussant, 2015 participant, 2016 participant, 2017 participant, 2018 participant

Midwest Political Science Association (MPSA) Annual Meeting:

- 2015 participant and discussant, 2016 participant and discussant, 2018 participant

Southern Political Science Association (SPSA) Annual Meeting:

- 2015 participant and discussant, 2016 participant and discussant, 2017 participant

Teaching Experience

Poli 315: Congress and the Legislative Process

- Fall 2014, Winter 2015, Fall 2015, Winter 2016, Summer 2017

Poli 328: Quantitative Analysis

- Winter 2017, Fall 2017, Fall 2019, Winter 2020, Fall 2020, Winter 2021

Poli 410: Undergraduate Research Seminar in American Politics

- Fall 2014, Winter 2015, Fall 2015, Winter 2016, Summer 2017

Awards and Grants

2019 BYU Mentored Environment Grant (MEG), American Ideology Project, $30,000

2017 BYU Political Science Teacher of the Year Award

2017 BYU Mentored Environment Grant (MEG), Funding American Democracy Project, $20,000

2016 BYU Political Science Department, Political Ideology and President Trump (with Jeremy Pope), $7,500

2016 BYU Office of Research and Creative Activities (ORCA) Student Mentored Grant x 3

- Hayden Galloway, Jennica Peterson, Rebecca Shuel

2015 BYU Office of Research and Creative Activities (ORCA) Student Mentored Grant x 3

- Michael-Sean Covey, Hayden Galloway, Sean Stephenson

2015 BYU Student Experiential Learning Grant, American Founding Comparative Constitutions Project (with Jeremy Pope), $9,000

2015 BYU Social Science College Research Grant, $5,000

2014 BYU Political Science Department, 2014 Washington DC Mayoral Pre-Election Poll (with Quin Monson and Kelly Patterson), $3,000

2014 BYU Social Science College Award, 2014 Washington DC Mayoral Pre-Election Poll (with Quin Monson and Kelly Patterson), $3,000

2014 BYU Center for the Study of Elections and Democracy, 2014 Washington DC Mayoral Pre-Election Poll (with Quin Monson and Kelly Patterson), $2,000

2012 Princeton Center for the Study of Democratic Politics Dissertation Improvement Grant, $5,000

2011 Princeton Mamdouha S. Bobst Center for Peace and Justice Dissertation Research Grant, $5,000

2011 Princeton Political Economy Research Grant, $1,500

Other Scholarly Activities

Expert Witness in Nancy Carola Jacobson, et al., Plaintiffs, vs. Laurel M. Lee, et al., Defendants. Case No. 4:18-cv-00262 MW-CAS (U.S. District Court for the Northern District of Florida)

Expert Witness in Common Cause, et al., Plaintiffs, vs. LEWIS, et al., Defendants. Case No. 18-CVS-14001 (Wake County, North Carolina)

Expert Witness in Kelvin Jones, et al., Plaintiffs, v. Ron DeSantis, et al., Defendants, Consolidated Case No. 4:19-cv-300 (U.S. District Court for the Northern District of Florida)

Expert Witness in Community Success Initiative, et al., Plaintiffs, v. Timothy K. Moore, et al., Defendants, Case No. 19-cv-15941 (Wake County, North Carolina)

Expert Witness in Richard Rose et al., Plaintiffs, v. Brad Raffensperger, Defendant, Civil Action No. 1:20-cv-02921-SDG (U.S. District Court for the Northern District of Georgia)

Georgia Coalition for the People's Agenda, Inc., et. al., Plaintiffs, v. Brad Raffensberger, Defendant. Civil Action No. 1:18-cv-04727-ELR (U.S. District Court for the Northern District of Georgia)

Expert Witness in Alabama, et al., Plaintiffs, v. United States Department of Commerce; Gina Raimondo, et al., Defendants. Case No. CASE No. 3:21-cv-00211-RAH-ECM-KCN (U.S. District Court for the Middle District of Alabama Eastern Division)

Expert Witness in League of Women Voters of Ohio, et al., Relators, v. Ohio Redistricting Commission, et al., Respondents. Case No. 2021-1193 (Supreme Court of Ohio)

Additional Training EITM 2012 at Princeton University - Participant and Graduate Student Coordinator

Computer Skills Statistical Programs: R, Stata, SPSS, parallel computing

Updated December 22, 2021

STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,

REBECCA HARPER, et al.,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

AFFIDAVIT OF PROFESSOR MOON DUCHIN

NCLCV v. Hall
21 CVS 15426
LDTX167

Exhibit #
Barber 5
12/30/2021
exhibitsticker.com

I, Dr. Moon Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.
2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.
4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.
5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.
6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.
7. A current copy of my full CV is attached to this report.
8. I am compensated at the rate of $400 per hour.

Rebuttal Report

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

December 28, 2021

1 Background and Introduction

I have previously submitted expert reports in NCLCV vs. Hall. I have been asked by counsel to respond to the report of Dr. Michael Barber, examining his study design and his conclusions.

1.1 Summary of Barber report

In Dr. Barber's report, he uses a new statistical sampling method called Sequential Monte Carlo (SMC) to produce a large collection (called an *ensemble*) of alternative districting plans for both bodies of the North Carolina state legislature—state Senate and state House. SMC is a method based on ideas developed in my research group,[1] but which has not been supported by any peer-reviewed publications.

Dr. Barber proceeds to build ensembles of districting plans for the purposes of comparison, but primarily does so individually on small pieces of the state: groups of counties (often called "county clusters") that correspond to groupings in the Senate and House plans recently enacted in North Carolina (SL-173 and SL-175).

- For legislative redistricting, the Barber report discusses the clusters only on an individual basis, neglecting to assemble them into the big picture for the whole state.
- Dr. Barber omits an ensemble comparison for the enacted Congressional plan, SL-174.

1.2 Summary of findings

- When assembling the statistics from Dr. Barber's own ensembles—completely granting him all methodological choices for algorithm selection and specifications—the enacted House plan is shown to be a major partisan outlier, while the NCLCV alternative plans are not (Figure 6).
- In exactly the same way, the enacted Senate plan is likewise shown to be a major partisan outlier, while the NCLCV alternative plans are not (Figure 5).
- Finally, I was able to run Barber's code to create a Congressional ensemble in the same fashion as his legislative ensembles. Here, too, the enacted plan is a significant outlier in a direction of partisan advantage that is not justified by any good-government goal (Figure 3).

[1] The McCartan–Imai article introducing SMC [5] acknowledges Deford–Duchin–Solomon [3] for "pioneer[ing] the spanning tree-based proposal used in the merge-split algorithm."

2 Ensembles and outliers

Today, the dominant method in computational redistricting analysis is to employ Markov chains to generate ensembles of thousands or millions of alternative valid redistricting plans against which to compare a given proposed plan. When a quantity of interest is measured over the ensemble, it frequently forms a "bell curve" of values, and we can then examine whether the proposed plan falls in the thick of the observed values or whether it is an extreme outlier, falling in one of the tails. If this exercise is carried out with respect to each party's representation, a telltale sign of a partisan gerrymander is when the seat share for a proposed plan falls (a) far from the corresponding vote share, and (b) far to the side of advantage for the party that controlled the line-drawing process. This is particularly problematic in a politically competitive "purple" state like North Carolina.

It is important to note that outlier status is a flag of intentionality, but not necessarily a smoking gun of wrongdoing. Being in a tails of a distribution that was created around certain design principles can often provide persuasive evidence that other principles or agendas were in play. For example, a map might be an outlier as the most compact, or the map that gives minority groups the greatest chance to elect their candidates of choice—these kinds of outlier status would not be marks of a bad plan. But being an outlier can indeed be a sign of problems, as when a plan systematically converts close voting to lopsided seat shares for the party that controls the process.

2.1 Barber methods

The creation and use of districting ensembles in the Barber report can be summarized as follows.

Step 1 *Fix a set of clusters.* Barber focuses on the county clustering found in the enacted plan, not exhaustively considering the dozens of other possibilities.

Step 2 *Partition each cluster.* Split each multi-district cluster into the corresponding number of districts using Sequential Monte Carlo sampling. Create 50,000 partitions (i.e., districting plans) for each cluster.

Step 3 *Winnow.* Selectively discard some of the partitions. Barber uses two statistics from the enacted plan (average Polsby-Popper score and county traversals) as the cutoff for inclusion.

Step 4 *Create an election index.* Barber blends the 11 up-ballot elections since 2014 into a single vote index rather than considering them one at a time. In particular, he sums the votes over all elections before taking shares, which does not control for turnout differences across elections.

Step 5 *Plot histograms and declare outliers.* Barber forms histograms counting "Democratic-leaning districts" for individual clusters, and does not present an overall compilation. His non-standard definition of "outlier" includes a full 50% of the ensemble.

In my opinion, better and more reliable results would have been obtained if several of the choices required in this study design were executed differently.

One glaring omission from Barber's methods is any consideration of the State's obligations under the Voting Rights Act of 1965, which could impact the partisan bottom line.[2] A non-exhaustive list of other potential flaws in Dr. Barber's methods includes the following.

- *Failure to consider all alternative clusterings.*
North Carolina law dictates that districts be drawn within groupings or clusters of counties from which several districts will be formed. Sometimes, however, the General Assembly has a choice and can pick multiple groupings consistent with North Carolina law. Dr. Barber only gives cursory attention to alternative clusterings.

- *Use of sampling methodology not vetted by peer review.*
Even when an idea is promising, peer review is an essential component of vetting. A method may appear promising in concept, but not work in practice. A method may work at small tasks—like the 34-map dataset used for testing in [5]—but not scale well to the enormous sizes needed for realistic problems. Peer review helps surface those issues, which is why the scientific community regards peer review as a mark of reliability.

- *Use of bright-line thresholds for compactness and traversals.*
Dr. Barber's code already samples with a preference for compactness, and is fully capable of handling traversals in a similar manner.[3] Imposing sharp cutoffs for these at the level of the enacted plan creates highly misleading results.[4]

- *Use of election data in a blended rather than serial fashion.*
If Barber records a Democratic share of 49% in his outputs, that is likely to reflect a Democratic win in some of the 11 elections and a Republican win in others—this is obscured when the results are blended to a single number. By the same token, a Democratic share of 45% in the blended election index might downplay a map that favors Republicans 11 out of 11 times, which entrenches an advantage.[5]

- *Employing a highly unconventional use of the* "*outlier*" *label.*
As Dr. Barber himself puts it, "I consider a plan to be a partisan outlier if the number of Democratic districts generated by the plan falls outside the middle 50% of simulation results [sic]. This is a conservative definition of an outlier. In the social sciences, medicine, and other disciplines it is traditional to consider something an outlier if it falls outside the middle 95% or 90% of the comparison distribution." As I will show below in my whole-state comparisons, the enacted plans are outliers at any of these levels of significance, while the NCLCV alternative plans are not.

I will discuss the thresholding question further in §2.3. For the remainder of the report, I will set aside the other concerns and will simply assess Dr. Barber's outputs within his own methodological framework.

[2] Robust VRA consideration is fully compatible with computational redistricting, as is shown in [1].

[3] A preference for compactness is coded in the `smc_redist` parameterization in `house_clusters.R`, lines 354–356 and `senate_clusters.R`, lines 349–351.

[4] The imposition of cutoffs, which Dr. Barber calls "culling," occurs in two stages. Stage 1 (country traversals) is found in `house_clusters.R`, lines 531–536 and `senate_clusters.R`, lines 539–544. Stage 2 (average Polsby-Popper) is found in `house_clusters.R`, line 543–564 and `senate_clusters.R`, lines 552–573. An ad hoc adjustment in the Duplin and Wayne House County Grouping is found in lines 566-568 of the House code.

[5] The 49% Democratic lean occurs, for instance, in the NCLCV alternative maps in the Onslow/Pender House cluster. Vote averaging is found in the Barber replication materials in `house_clusters.R` lines 18-28 and `senate_clusters.R` lines 18-29.

2.2 Analysis methods

Reading Dr. Barber's report, it is striking that he only reported that the enacted plan often performed within the middle 50% of each small comparison while never evaluating how the individual choices aggregate at the level of the map as a whole. After all, if moderate partisan advantage is secured over and over again, it may well accrue to extreme advantage overall. In the context of a state legislature, the overall results are crucial: they determine who controls the chamber. Pursuing this in the Barber materials, I found that this is exactly what happens.

First, I was able to extract Dr. Barber's raw statistical outputs for legislative runs from his materials obtained by counsel.[6] With those, I was able to assemble his ensembles for individual clusters into a compiled ensemble for the entire state. The histogram of Senate outcomes can be found in Figure 6 and the histogram of House outcomes can be found in Figure 5. Second, I was able to run Dr. Barber's code to create an ensemble of alternative Congressional plans with exactly the algorithm and with similar specifications to those he used for his legislative demonstrations.[7] A corresponding plot of Congressional outcomes can be found in Figure 3. For all phases of analysis, Dr. Barber pulled electoral data from a free webapp called Dave's Redistricting App (`davesredistricting.org`). In replicating his analysis, I used the same data source in the same manner.

2.3 Filtered and unfiltered results

As I described above, Dr. Barber took his raw districting plan samples (50,000 maps created for each of 12 Senate cluster ensembles and 26 House cluster ensembles) and aggressively filtered them, applying a cutoff that sometimes left under ten maps out of the original set of 50,000. In fact, when Dr. Barber's filtering rule was applied in the Duplin and Wayne House County Grouping (§6.6 on p.58 of Barber Report), *zero* maps were left, because none of the randomly constructed maps had an average compactness score to match the enacted plan in that cluster. Since this is blatantly unworkable for comparison purposes, Dr. Barber made the ad hoc decision to loosen the rule to retain 2704 maps. Other cluster ensembles were filtered down to leave only 4, 6, or 2 out of 50,000 alternatives and did not receive an adjustment. The "outlier" label was then applied to these tiny sets.

To illustrate why this is methodologically unreasonable, consider JaVale McGee, a basketball center who recently signed with the Phoenix Suns of the NBA on a one-year, $5 million contract. If McGee wanted to argue that he is not unusually wealthy, he could choose to restrict the universe of comparison to Americans at least as tall as he is. Since he is 7 feet tall, this would greatly restrict the comparison pool to a relatively tiny group that also includes Mo Bamba (Orlando Magic), Joel Embiid (Philadelphia 76ers), and Brook Lopez (Milwaukee Bucks), all of whom make more money than he does. Not satisfied with this comparison, he could keep increasing the requirements by insisting on comparing to people who don't speak any more languages than he does, are no older than he is, and have lived in at least as many different cities. Eventually he will narrow the pool enough that he doesn't look like an outlier anymore.

Dr. Barber's filtering skews his sample in a similar way, because he effectively insists that maps have a statistic matching or exceeding the enacted map in every cluster—and then uses that pool to compare the enacted map. Overall, this reduces the number of plans under consideration by a factor of over 500 trillion. And it excludes options that may be better than the enacted plan overall but are less compact or have more traversals in a particular cluster.

Generally, if you are trying to argue that you look typical of a range of alternatives, it is obviously unreasonable to first require the alternatives to look like you in dozens of independent ways (i.e., in each cluster individually).

[6] His materials include the numerical outputs from his runs, but as far as I can determine he does not seem to have saved the district assignments for the individual plans in the ensemble.

[7] To be precise, the ensemble was generated at the state level for Congress, since the concept of county clusters is not applicable, and without the compactness and traversal thresholds. I ran the code exactly as Dr. Barber did, except tightening the allowed population deviation to 1% from ideal instead of 5% as in legislative maps. All other choices are identical. My congressional ensemble includes 20,000 maps rather than 50,000 just because of time limitations.

3 Findings

In this section, I will present the full histograms (or "bell curves") of all the results from Dr. Barber's methodology, compiled to the state level and shown without filtering. (Filtered ensembles can be seen in Appendix A, for comparison purposes.)

By Dr. Barber's own constructs, all three levels of districting show that **the enacted plans are partisan outliers and the NCLCV alternative plans are not**.

In the House, the enacted map is in the most extreme 0.00133 fraction of the Barber ensemble—well under 1 percent of sampled House plans are as extreme as SL-175. By contrast, the NCLCV alternative plan is in the upper .2516 share of the ensemble, not an outlier even by the Barber standard.

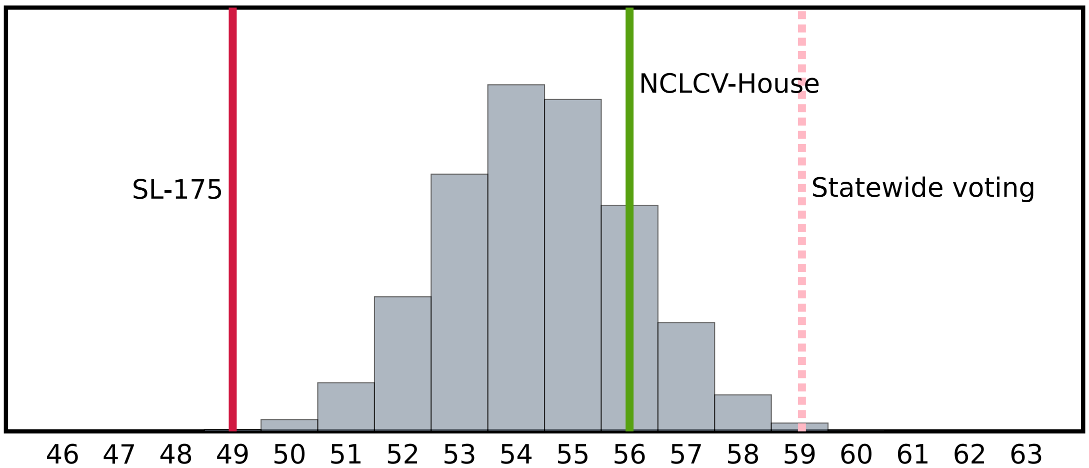

Figure 1: "Democratic-leaning seats" in Dr. Barber's House district ensemble.

At the Senate level, the enacted map is in the most extreme .007 fraction of the Barber ensemble—again, less than 1 percent of sampled plans are as extreme as SL-173. By contrast, the NCLCV alternative map is in the upper .2787 share of ensemble, not an outlier even by the Barber standard.

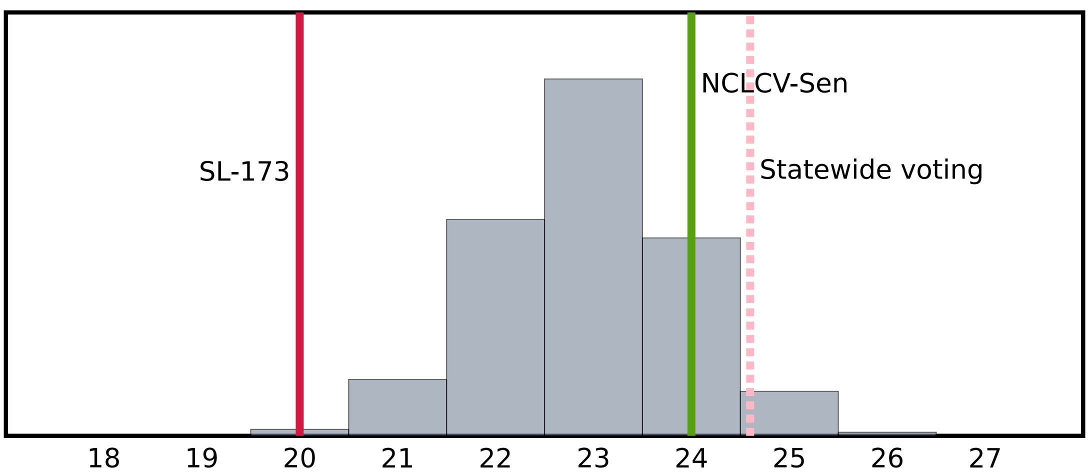

Figure 2: "Democratic-leaning seats" in Dr. Barber's Senate district ensemble.

The Congressional picture, omitted from the Barber report, is likewise crystal clear. The enacted plan is in the most extreme 0.0056 fraction of this Barber-style ensemble, while the NCLCV alternative map is very near the ensemble center—0.5620 share of the ensemble (more than half of randomly constructed maps) has an equal or greater Democratic lean.

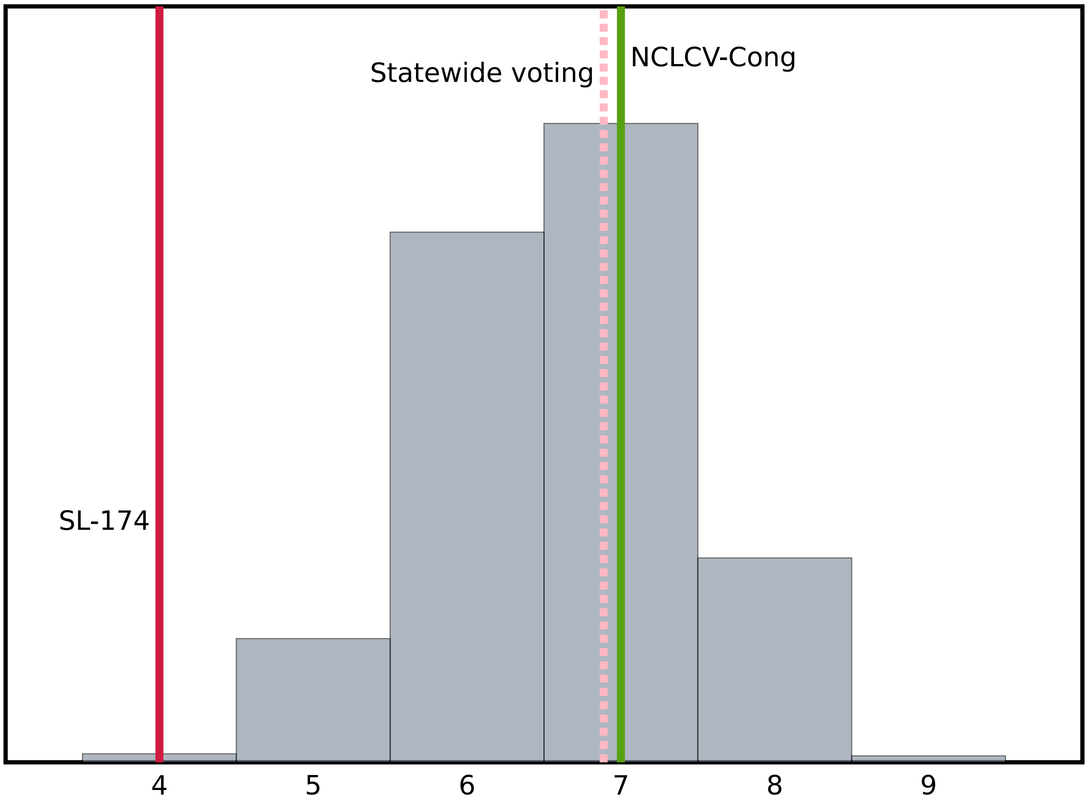

Figure 3: "Democratic-leaning seats" in a Congressional ensemble created with Dr. Barber's code, following his specifications.

4 Conclusion

Granting Dr. Barber all of his methodological choices, the enacted maps are extreme partisan outliers at all three levels, while the NCLCV alternative maps are not.

References

[1] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, *Computational redistricting and the Voting Rights Act.* Election Law Journal. `Available online`.

[2] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, *NC General Assembly County Clusterings from the 2020 Census.* `sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf`

[3] Daryl DeFord, Moon Duchin, and Justin Solomon, *Recombination: A Family of Markov Chains for Redistricting*, Harvard Data Science Review. Issue 3.1, Winter 2021. `Available online`.

[4] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, *Locating the Representational Baseline: Republicans in Massachusetts.* Election Law Journal, Volume 18, Number 4, 2019, 388–401. `Available online`.

[5] Cory McCartan and Kosuke Imai, *Sequential Monte Carlo for Sampling Balanced and Compact Redistricting Plans*, preprint. Available at `arxiv.org/abs/2008.06131`.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 28 day of December, 2021.

Professor Moon Duchin

Sworn and subscribed before me
this the 28 of December, 2021

Notary Public

Name: Simarjit Manhas

My commission expires: 02/02/2024

A notary public or other officer completing this certificate verifies only the identity of the individual who signed the document to which this certificate is attached, and not the truthfulness, accuracy, or validity of that document.

State of California, County of Alameda
Subscribed and sworn to (or affirmed) before me
on this 28 day of December, 2021,
by Moon Duchin,
proved to me on the basis of satisfactory evidence to be the person(~~s~~) who appeared before me.

Signature:

SIMARJIT MANHAS
COMM. #2317737
Notary Public - California
Contra Costa County
My Comm. Expires Feb. 2, 2024
NR01

Appendix A: Filtering comparison

To illustrate the skewing effects of the thresholds applied by Dr. Barber, consider a single example: the Pitt House County Cluster, where the number of Democratic-leaning seats in the sample is either 1 or 2. By thresholding compactness and traversals at the level of the enacted map, Dr. Barber is able to drop the frequency of the 2-seats outcome from roughly 25% of the sample to just 9%.

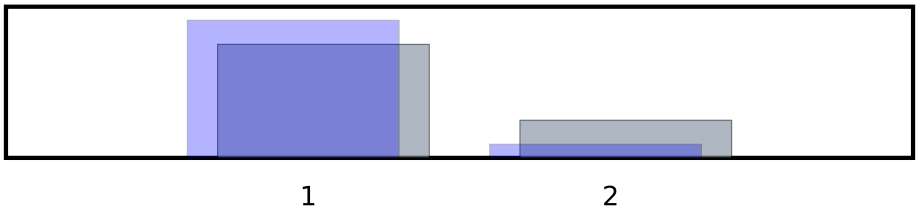

Figure 4: Just focusing on the Pitt House County Cluster (Barber report, p.42), we see that the filtering changes the outcome of 2 "Democratic-leaning seats" from occurring in roughly 25% of the full set of sampled maps (gray) to only occurring in 9% of the reduced sample (blue).

The effects of this cluster-by-cluster restriction do not wash out when aggregated to the full state, but instead add up to a noticeable shift toward the enacted plan, as demonstrated in the House and Senate figures below.

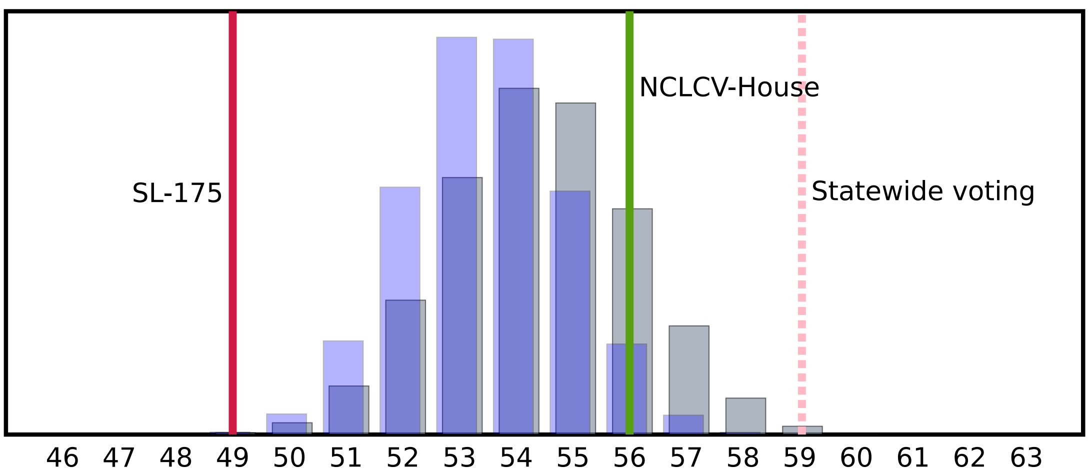

Figure 5: "Democratic-leaning seats" in Dr. Barber's House district ensemble. The unfiltered ensemble (gray) includes $50,000^{26} \approx 1.5 \cdot 10^{122}$ maps; the filtered ensemble (blue) is smaller by a factor of octillions.

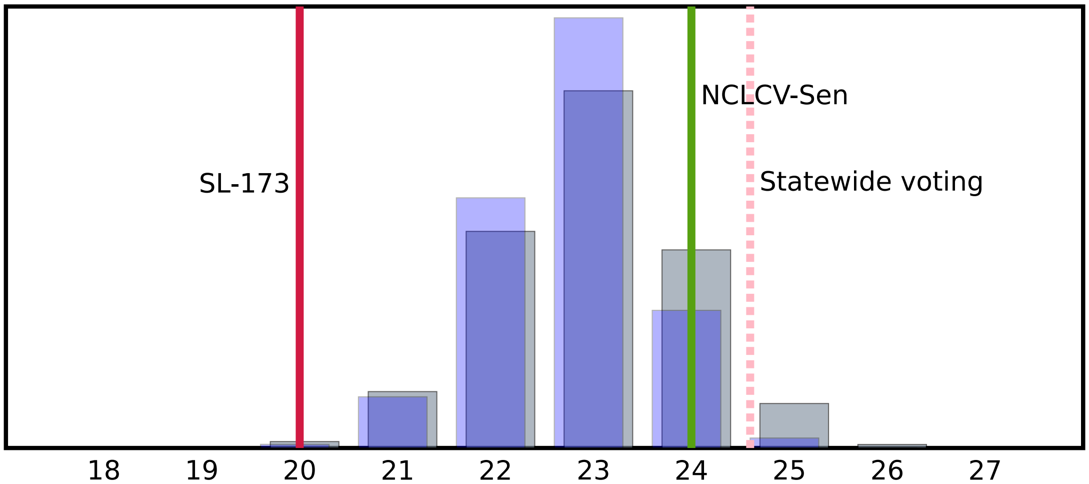

Figure 6: "Democratic-leaning seats" in Dr. Barber's Senate district ensemble. The unfiltered ensemble (gray) includes $50,000^{12} \approx 2.4 \cdot 10^{56}$ maps; the filtered ensemble (blue) is smaller by a factor of trillions.

Significantly, even the subsets of alternative plans that have been heavily limited by the cluster-by-cluster thresholds—that is, the blue bell curves instead of the gray—still show the enacted plans to be extreme outliers, while the NCLCV alternative plans are both far less extreme and comport with statewide voting.

Moon Duchin

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Education

University of Chicago MS 1999, PhD 2005
Mathematics
Advisor: Alex Eskin Dissertation: *Geodesics track random walks in Teichmüller space*

Harvard University BA 1998
Mathematics and Women's Studies

Appointments

Tufts University
Professor of Mathematics 2021—
Assistant Professor, Associate Professor 2011–2021

Director | Program in Science, Technology, & Society 2015–2021
(on leave 2018–2019)

Principal Investigator | MGGG Redistricting Lab 2017—

Senior Fellow | Tisch College of Civic Life 2017—

University of Michigan
Assistant Professor (postdoctoral) 2008–2011

University of California, Davis
NSF VIGRE Postdoctoral Fellow 2005–2008

Research Interests

Data science for civil rights, computation and governance, elections, geometry and redistricting.
Science, technology, and society, science policy, technology and law.
Random walks and Markov chains, random groups, random constructions in geometry.
Large-scale geometry, metric geometry, isoperimetric inequalities.
Geometric group theory, growth of groups, nilpotent groups, dynamics of group actions.
Geometric topology, hyperbolicity, Teichmüller theory.

Awards & Distinctions

Research Professor - MSRI Program in Analysis and Geometry of Random Spaces Spring 2022
Guggenheim Fellow 2018
Radcliffe Fellow - Evelyn Green Davis Fellowship 2018–2019
Fellow of the American Mathematical Society elected 2017
NSF C-ACCEL (PI) - Harnessing the Data Revolution: Network science of Census data 2019–2020
NSF grants (PI) - CAREER grant and three standard Topology grants 2009–2022
Professor of the Year, Tufts Math Society 2012–2013
AAUW Dissertation Fellowship 2004–2005
NSF Graduate Fellowship 1998–2002
Lawrence and Josephine Graves Prize for Excellence in Teaching (U Chicago) 2002
Robert Fletcher Rogers Prize (Harvard Mathematics) 1995–1996

Mathematics Publications & Preprints

The (homological) persistence of gerrymandering
Foundations of Data Science, online first. (with Thomas Needham and Thomas Weighill)

You can hear the shape of a billiard table: Symbolic dynamics and rigidity for flat surfaces
Commentarii Mathematici Helvetici, to appear. arXiv:1804.05690
(with Viveka Erlandsson, Christopher Leininger, and Chandrika Sadanand)

Conjugation curvature for Cayley graphs
Journal of Topology and Analysis, online first. (with Assaf Bar-Natan and Robert Kropholler)

A reversible recombination chain for graph partitions
Preprint. (with Sarah Cannon, Dana Randall, and Parker Rule)

Recombination: A family of Markov chains for redistricting
Harvard Data Science Review. Issue 3.1, Winter 2021. online. (with Daryl DeFord and Justin Solomon)

Census TopDown: The impact of differential privacy on redistricting
2nd Symposium on Foundations of Responsible Computing (FORC 2021), 5:1–5:22. online.
(with Aloni Cohen, JN Matthews, and Bhushan Suwal)

Stars at infinity in Teichmüller space
Geometriae Dedicata, Volume 213, 531–545 (2021). (with Nate Fisher) arXiv:2004.04321

Random walks and redistricting: New applications of Markov chain Monte Carlo
(with Daryl DeFord) For edited volume, Political Geometry. Under contract with Birkhäuser.

Mathematics of nested districts: The case of Alaska
Statistics and Public Policy. Vol 7, No 1 (2020), 39–51. (w/ Sophia Caldera, Daryl DeFord, Sam Gutekunst, & Cara Nix)

A computational approach to measuring vote elasticity and competitiveness
Statistics and Public Policy. Vol 7, No 1 (2020), 69–86. (with Daryl DeFord and Justin Solomon)

The Heisenberg group is pan-rational
Advances in Mathematics **346** (2019), 219–263. (with Michael Shapiro)

Random nilpotent groups I
IMRN, Vol 2018, Issue 7 (2018), 1921–1953. (with Matthew Cordes, Yen Duong, Meng-Che Ho, and Ayla Sánchez)

Hyperbolic groups
chapter in *Office Hours with a Geometric Group Theorist*, eds. M.Clay,D.Margalit, Princeton U Press (2017), 177–203.

Counting in groups: Fine asymptotic geometry
Notices of the American Mathematical Society **63**, No. 8 (2016), 871–874.

A sharper threshold for random groups at density one-half
Groups, Geometry, and Dynamics **10**, No. 3 (2016), 985–1005.
(with Katarzyna Jankiewicz, Shelby Kilmer, Samuel Lelièvre, John M. Mackay, and Ayla Sánchez)

Equations in nilpotent groups
Proceedings of the American Mathematical Society **143** (2015), 4723–4731. (with Hao Liang and Michael Shapiro)

Statistical hyperbolicity in Teichmüller space
Geometric and Functional Analysis, Volume 24, Issue 3 (2014), 748–795. (with Howard Masur and Spencer Dowdall)

Fine asymptotic geometry of the Heisenberg group
Indiana University Mathematics Journal 63 No. 3 (2014), 885–916. (with Christopher Mooney)

Pushing fillings in right-angled Artin groups
Journal of the LMS, Vol 87, Issue 3 (2013), 663–688. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

Spheres in the curve complex
In the Tradition of Ahlfors and Bers VI, Contemp. Math. **590** (2013), 1–8. (with Howard Masur and Spencer Dowdall)

The sprawl conjecture for convex bodies
Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122. (with Samuel Lelièvre and Christopher Mooney)

Filling loops at infinity in the mapping class group
Michigan Math. J., Vol 61, Issue 4 (2012), 867–874. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

The geometry of spheres in free abelian groups
Geometriae Dedicata, Volume 161, Issue 1 (2012), 169–187. (with Samuel Lelièvre and Christopher Mooney)

Statistical hyperbolicity in groups
Algebraic and Geometric Topology **12** (2012) 1–18. (with Samuel Lelièvre and Christopher Mooney)

Length spectra and degeneration of flat metrics
Inventiones Mathematicae, Volume 182, Issue 2 (2010), 231–277. (with Christopher Leininger and Kasra Rafi)

Divergence of geodesics in Teichmüller space and the mapping class group
Geometric and Functional Analysis, Volume 19, Issue 3 (2009), 722–742. (with Kasra Rafi)

Curvature, stretchiness, and dynamics
In the Tradition of Ahlfors and Bers IV, Contemp. Math. **432** (2007), 19–30.

Geodesics track random walks in Teichmüller space
PhD Dissertation, University of Chicago 2005.

Science, Technology, Law, and Policy Publications & Preprints

Models, Race, and the Law
Yale Law Journal Forum, Vol. 130 (March 2021). Available online. (with Doug Spencer)

Computational Redistricting and the Voting Rights Act
Election Law Journal, Available online. (with Amariah Becker, Dara Gold, and Sam Hirsch)

Discrete geometry for electoral geography
Preprint. (with Bridget Eileen Tenner) arXiv:1808.05860

Implementing partisan symmetry: Problems and paradoxes
Political Analysis, to appear. (with Daryl DeFord, Natasha Dhamankar, Mackenzie McPike, Gabe Schoenbach, and Ki-Wan Sim) arXiv:2008:06930

Clustering propensity: A mathematical framework for measuring segregation
Preprint. (with Emilia Alvarez, Everett Meike, and Marshall Mueller; appendix by Tyler Piazza)

Locating the representational baseline: Republicans in Massachusetts
Election Law Journal, Volume 18, Number 4, 2019, 388–401.
(with Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen)

Redistricting reform in Virginia: Districting criteria in context
Virginia Policy Review, Volume XII, Issue II, Spring 2019, 120–146. (with Daryl DeFord)

Geometry v. Gerrymandering
The Best Writing on Mathematics 2019, ed. Mircea Pitici. Princeton University Press.
reprinted from Scientific American, November 2018, 48–53.

Gerrymandering metrics: How to measure? What's the baseline?
Bulletin of the American Academy for Arts and Sciences, Vol. LXII, No. 2 (Winter 2018), 54–58.

Rebooting the mathematics of gerrymandering: How can geometry track with our political values?
The Conversation (online magazine), October 2017. (with Peter Levine)

A formula goes to court: Partisan gerrymandering and the efficiency gap
Notices of the American Mathematical Society **64** No. 9 (2017), 1020–1024. (with Mira Bernstein)

International mobility and U.S. mathematics
Notices of the American Mathematical Society **64**, No. 7 (2017), 682–683.

Graduate Advising in Mathematics

Nate Fisher (PhD 2021), Sunrose Shrestha (PhD 2020), Ayla Sánchez (PhD 2017),
Kevin Buckles (PhD 2015), Mai Mansouri (MS 2014)

Outside committee member for Chris Coscia (PhD 2020), Dartmouth College

Postdoctoral Advising in Mathematics

Principal supervisor Thomas Weighill (2019–2020)

Co-supervisor Daryl DeFord (MIT 2018–2020), Rob Kropholler (2017–2020), Hao Liang (2013–2016)

Teaching

Courses Developed or Customized

Mathematics of Social Choice | sites.tufts.edu/socialchoice
Voting theory, impossibility theorems, redistricting, theory of representative democracy, metrics of fairness.

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Social history of mathematics, organized around episodes from antiquity to present. Themes include materials and technologies of creation and dissemination, axioms, authority, credibility, and professionalization. In-depth treatment of mathematical content from numeration to cardinal arithmetic to Galois theory.

Reading Lab: Mathematical Models in Social Context | sites.tufts.edu/models
One hr/wk discussion seminar of short but close reading on topics in mathematical modeling, including history of psychometrics; algorithmic bias; philosophy of statistics; problems of model explanation and interpretation.

Geometric Literacy
Module-based graduate topics course. Modules have included: p-adic numbers, hyperbolic geometry, nilpotent geometry, Lie groups, convex geometry and analysis, the complex of curves, ergodic theory, the Gauss circle problem.

Markov Chains (graduate topics course)
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Fuchsian Groups (graduate topics course)
Continued Fractions and Geometric Coding (undergraduate topics course)
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Standard Courses

Discrete Mathematics, Calculus I-II-III, Intro to Proofs, Linear Algebra, Complex Analysis, Differential Geometry, Abstract Algebra, Graduate Real Analysis, Mathematical Modeling and Computation

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BMC/BAMC Public Lecture Joint British Mathematics/Applied Mathematics Colloquium, Glasgow, Scotland	April 2021 *online (COVID)*
AMS Einstein Public Lecture in Mathematics Southeastern Sectional Meeting of the AMS, Charlottesville, VA	[March 2020] *postponed*
Gerald and Judith Porter Public Lecture AMS-MAA-SIAM, Joint Mathematics Meetings, San Diego, CA	January 2018
Mathematical Association of America Distinguished Lecture MAA Carriage House, Washington, DC	October 2016
American Mathematical Society Invited Address AMS Eastern Sectional Meeting, Brunswick, ME	September 2016

Named University Lectures

- Parsons Lecture | UNC Asheville	October 2020
- Loeb Lectures in Mathematics | Washington University in St. Louis	[March 2020]
- Math, Stats, CS, and Society | Macalester College	October 2019
- MRC Public Lecture | Stanford University	May 2019
- Freedman Memorial Colloquium | Boston University	March 2019
- Julian Clancy Frazier Colloquium Lecture | U.S. Naval Academy	January 2019
- Barnett Lecture | University of Cincinnati	October 2018
- School of Science Colloquium Series | The College of New Jersey	March 2018
- Kieval Lecture | Cornell University	February 2018
- G. Milton Wing Lectures | University of Rochester	October 2017
- Norman Johnson Lecture | Wheaton College	September 2017
- Dan E. Christie Lecture | Bowdoin College	September 2017

Math/Computer Science Department Colloquia

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- Georgetown (CS)	Sept 2020
- Santa Fe Institute	July 2020
- UC Berkeley	Sept 2018
- Brandeis-Harvard-MIT-NEU	Mar 2018
- Northwestern University	Oct 2017
- University of Illinois	Sept 2017
- University of Utah	Aug 2017
- Wesleyan	Dec 2016
- Worcester Polytechnic Inst.	Dec 2016
- Université de Neuchâtel	Jun 2016
- Brandeis University	Mar 2016
- Swarthmore College	Oct 2015
- Bowling Green	May 2015
- City College of New York	Feb 2015
- Indiana University	Nov 2014
- the Technion	Oct 2014
- Wisconsin–Madison	Sept 2014
- Stony Brook	March 2013

Minicourses

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- Workshop in geometric topology (main speaker, three talks) | Provo, UT — June 2017
- Growth in groups (two talks) | MSRI, Berkeley, CA — August 2016
- Hyperbolicity in Teichmüller space (three talks) | Université de Grenoble — May 2016
- Counting and growth (four talks) | IAS Women's Program, Princeton — May 2016
- Nilpotent groups (three talks) | Seoul National University — October 2014
- Sub-Finsler geometry of nilpotent groups (five talks) | Galatasaray Univ., Istanbul — April 2014

Science, Technology, and Society

- The Mathematics of Accountability | Sawyer Seminar, Anthropology, Johns Hopkins — February 2020
- STS Circle | Harvard Kennedy School of Government — September 2019
- Data, Classification, and Everyday Life Symposium | Rutgers Center for Cultural Analysis — January 2019
- Science Studies Colloquium | UC San Diego — January 2019
- Arthur Miller Lecture on Science and Ethics | MIT Program in Science, Tech, and Society — November 2018

Data Science, Computer Science, Quantitative Social Science

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- Privacy Tools Project Retreat | Harvard (virtual) — May 2020
- Women in Data Science Conference | Microsoft Research New England — March 2020
- Quantitative Research Methods Workshop | Yale Center for the Study of American Politics — February 2020
- Societal Concerns in Algorithms and Data Analysis | Weizmann Institute — December 2018
- Quantitative Collaborative | University of Virginia — March 2018
- Quantitative Social Science | Dartmouth College — September 2017
- Data for Black Lives Conference | MIT — November 2017

Political Science, Geography, Law, Democracy, Fairness

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- "The New Math" for Civil Rights | Social Justice Speaker Series, Davidson College — November 2020
- Math, Law, and Racial Fairness | Justice Speaker Series, University of South Carolina — November 2020
- Voting Rights Conference | Northeastern Public Interest Law Program — September 2020
- Political Analysis Workshop | Indiana University — November 2019
- Program in Public Law Panel | Duke Law School — October 2019
- Redistricting 2021 Seminar | University of Chicago Institute of Politics — May 2019
- Geography of Redistricting Conference Keynote | Harvard Center for Geographic Analysis — May 2019
- Political Analytics Conference | Harvard University — November 2018
- Cyber Security, Law, and Society Alliance | Boston University — September 2018
- Clough Center for the Study of Constitutional Democracy | Boston College — November 2017
- Tech/Law Colloquium Series | Cornell Tech — November 2017
- Constitution Day Lecture | Rockefeller Center for Public Policy, Dartmouth College — September 2017

Editorial Boards

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Selected Professional and Public Service

Amicus Brief of Mathematicians, Law Professors, and Students 2019
principal co-authors: Guy-Uriel Charles and Moon Duchin
Supreme Court of the United States, in Rucho v. Common Cause - cited in dissent

Committee on Science Policy 2020–2023
American Mathematical Society

Program Committee 2020–2021
Symposium on Foundations of Responsible Computing

Presenter on Public Mapping, Statistical Modeling 2019, 2020
National Conference of State Legislatures

Committee on the Human Rights of Mathematicians 2016–2019
American Mathematical Society

Committee on The Future of Voting: Accessible, Reliable, Verifiable Technology 2017–2018
National Academies of Science, Engineering, and Medicine

Visiting Positions and Residential Fellowships

Visiting Professor Department of Mathematics — Fall 2021
Boston College | Chestnut Hill, MA

Fellow Radcliffe Institute for Advanced Study — 2018–19
Harvard University | Cambridge, MA

Member Center of Mathematical Sciences and Applications — 2018–19
Harvard University | Cambridge, MA

Visitor Microsoft Research Lab — 2018–19
MSR New England | Cambridge, MA

Research Member Geometric Group Theory program — Fall 2016
Mathematical Sciences Research Institute | Berkeley, CA

Research Member Random Walks and Asymptotic Geometry of Groups program — Spring 2014
Institut Henri Poincaré | Paris, France

Research Member Low-dimensional Topology, Geometry, and Dynamics program — Fall 2013
Institute for Computational and Experimental Research in Mathematics | Providence, RI

Research Member Geometric and Analytic Aspects of Group Theory program — May 2012
Institut Mittag-Leffler | Stockholm, Sweden

Research Member Quantitative Geometry program — Fall 2011
Mathematical Sciences Research Institute | Berkeley, CA

Postdoctoral Fellow Teichmüller "project blanc" — Spring 2009
Agence Nationale de la Recherche (Collège de France) | Paris, France

ELECTION LAW JOURNAL
Volume 17, Number 1, 2018

DOI: 10.1089/elj.2016.0392

Original Articles

Considering the Prospects for Establishing a Packing Gerrymandering Standard

Robin E. Best, Shawn J. Donahue, Jonathan Krasno, Daniel B. Magleby, and Michael D. McDonald

ABSTRACT

Courts have found it difficult to evaluate whether redistricting authorities have engaged in constitutionally impermissible partisan gerrymandering. The knotty problem is that no proposed standard has found acceptance as a convincing means for identifying whether a districting plan is a partisan gerrymander with knowable unconstitutional effects. We review five proposed standards for curbing gerrymandering. We take as our perspective how easily manageable and effective each would be to apply at the time a redistricting authority decides where to draw the lines or, post hoc, when a court is asked to decide whether an unconstitutional gerrymander has been enacted. We conclude that, among the five proposals, an equal vote weight standard offers the best prospects for identifying the form of unconstitutional gerrymanders that all but ensure one party is relegated to perpetual minority status.

Keywords: gerrymander, vote dilution, efficiency gap, partisan symmetry

Partisan gerrymandering has become such a dark art that retired Justice John Paul Stevens proposed a constitutional amendment to curb it (Stevens 2014). After the 2000 round of redistricting, David Mayhew pointed to five cases of deft gerrymandering—Florida, Michigan, Ohio, Pennsylvania, and Texas (Mayhew 2011, 24; see also Toobin 2003), to which three others could have been added—California, Illinois, and South Carolina (McDonald and Best 2015, 321). After the 2012 round of redistricting, credible gerrymandering allegations have been leveled at no fewer than ten states: Florida, Georgia, Illinois, Louisiana, Maryland, North Carolina, Ohio, Pennsylvania, Tennessee, and Texas (Fang 2014). One could likely add Michigan and Wisconsin without any stretch of credibility. In all these cases the party in power is suspected of designing districts to perpetuate their majority control of a congressional delegation or state legislative chamber almost regardless of what a majority of voters would decide were they not pre-organized in clusters favoring the party in power. The artistry, of this sordid sort, is accomplished through so-called packing gerrymanders. Very many partisans of one stripe are crammed into a small number of districts while partisans of the other stripe are given strong but not overwhelming majorities in the larger number of remaining districts.

Justice Stevens' call for a constitutional amendment comes in the face of two frustrations. Only a few states have shown a willingness to police partisan gerrymandering on their own, and courts have been unable to craft a diagnostic standard that identifies whether a districting plan produces constitutional harm. Needless to say, the wait for a constitutional amendment requires as much patience as the wait for states to adopt rules themselves. Instead

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of waiting, we ask whether any of five recent proposals to assess partisan gerrymandering might be able to supply redistricting authorities in the first instance or courts, if needed later, with a manageable and effective diagnostic tool.

The five proposals are

(1) an *efficiency gap* test (Stephanopoulos and McGhee 2014);
(2) a test *comparing seats won to neutral expectations* (Chen and Rodden 2013a);
(3) an *equal vote weight* test (McDonald and Best 2015);
(4) a *partisan symmetry* test (Grofman and King 2007); and
(5) a *three-prong* test (Wang 2016).

Manageability refers to the clarity and ease with which an analyst can observe a standard's proposed showing of effect. Why? Absent a clear and easily observed effect, debatable aspects of the principal facts leave a conclusion in doubt. *Effectiveness* refers to the accuracy by which a standard's proposed showing of effect identifies gerrymandering as the cause of violating a constitutionally protected right. Why? Absent an accurate assessment of gerrymandering as the cause, doubts about the possibility of false negative or false positive inferences overtake a conclusion.

The next section lays a conceptual foundation by using the language of the Supreme Court to identify the constitutional harm packing gerrymanders can inflict. The third section, first, details the principles of manageability and effectiveness we use to evaluate each proposed standard and, next, describes the types of vote dilution the different standards are designed to uncover. The fourth section describes the reasoning associated with each of the five standards and, through a series of hypotheticals, offers preliminary evaluations of their manageability and effectiveness. Because hypotheticals are useful for illustrating general principles but are prone to doubts about how they operate in actual applications, the fifth section extends the evaluations by applying each standard to state senate districting plans in North Carolina and Iowa. North Carolina is a case where the intention to gain partisan advantage is acknowledged; Iowa is the poster child for a districting process that has neither the intent nor the effect of producing a partisan gerrymander. Thus, reliance on these two cases provides opportunities to check for false negative (North Carolina) and false positive (Iowa) readings.

While arguably manageable, we find that counting wasted votes (aka, the efficiency gap test) relies on a dubious definition of wasted votes and is decidedly ineffective because wasted votes occur for reasons other than gerrymandering. Comparing seats won to neutral expectations requires a set of neutrally drawn districts, a process that can encounter manageability problems due the black-box computer algorithms they require, and they can suffer effectiveness problems because a disadvantaged party hamstrung by a cracking gerrymander can win seats at or even above expectations when its votes amount to less than a majority. The equal vote weight test is manageable and mostly effective but not as aggressive as might be preferred. Testing for partisan symmetry is mostly effective but not entirely manageable because its reading of gerrymanders requires reliance on nonfactual hypotheticals. Finally, the three-prong approach fails on its own terms because the prongs do not fit together as a coherent whole and, worse, the prongs can operate at cross-purposes. All in all, the reviews lead to this conclusion: the equal vote weight standard is the most easily manageable and effective at identifying packing gerrymandering as the cause of a constitutional harm: diluting the votes of one set of partisans.

PARTISAN GERRYMANDERS OF THE PACKING VARIETY

All five proposed standards have been aimed at identifying packing gerrymanders.[1] As remarked, packing gerrymanders concentrate a large number of the disadvantaged party's voters in a small number of districts. When one party's voters are packed

[1]Wasted votes were the primary evidence of effect in a Wisconsin State Senate challenge (*Whitford v. Gill* 2016). Comparing wins was used in a challenge to Florida's congressional districts (*Romo v. Detzner* 2014). The equal vote weight standard was proposed by amici (Hebert and Lang 2015) at the remedy stage of the Virginia litigation that earlier found the State's congressional districts to be an unconstitutional racial gerrymander (*Page v. Virginia State Board of Elections* 2014). Seat-denominated symmetry was proposed to the Supreme Court by amici (King et al. 2005) for consideration in *LULAC v. Perry* (2006). One of the three prongs was proposed by amici (Wang 2015) in *Harris v. Arizona Redistricting Commission* (2016).

into a few districts, the packed partisans hold overwhelming majorities in those districts. Packing gerrymanders also serve to spread the packed party's remaining voters over a large number of districts where they constitute sizable but ineffective minorities.[2] By way of example, a competitive jurisdiction with 10 districts and a vote typically expected to split 52 percent Democrat and 48 percent Republican might enact a packing gerrymander by granting Republicans two districts that are 100 percent Republican and next set up the remaining eight so that they split 35 versus 65, Republican versus Democrat. The result is two safe Republican seats and eight safe Democratic seats, a seat split that would likely hold even if votes shifted substantially in the Republicans' favor. Notice that packing uses cracking at a second step. In the example, two districts are packed with Republicans; this recasts the system-wide percentages among the other eight, which are then cracked, safely for Democrats, so they all divide 35–65.

In theory an optimal partisan gerrymander can be shown to involve pure cracking (Freidman and Holden 2008), but as Owen and Grofman have shown, for reasons both of a party's desire for legislative majority control and of it and its individual candidate's risk aversion, an optimal gerrymander under competitive circumstances relies on packing (Owen and Grofman 1988; see also Gul and Pesendorfer 2010).[3] In any case, as we have noted (fn. 1), the five proposed standards have been aimed at packing gerrymanders and so, too, has the Supreme Court's attention in three major partisan gerrymandering decisions, *Davis v. Bandermer* (1986), *Veith v. Jubelirer* (2004), and *LULAC v. Perry* (2006).[4]

Justice Scalia, announcing the Court's judgment in *Veith*, defined gerrymandering as "[t]he practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give a political party an unfair advantage by diluting the opposition's voting strength" (*Vieth v. Jubelirer*, 2004, 271 n. 1, quoting *Black's Law Dictionary* 1999, 696). Finding intention and observing weirdly shaped districts are seldom difficult (as in *Davis v. Bandemer* 1986; *Veith v. Jubilier* 2004, *LULAC v. Perry* 2006), but finding a standard that identifies a party's unfair advantage because the opposition party's votes have been diluted has proved elusive.

In *Bandemer*, Justice White explained the Court majority's holding of justiciability of partisan gerrymandering in response to a caution from Justice O'Connor. She worried that judicial attempts to police partisan gerrymandering would have courts give preference to proportionality. Justice White and the majority disagreed; justiciability of packing forms of partisan gerrymandering rests on the Court's preference not for proportionality but, rather, for ensuring that popular "majorities are not consigned to minority status" (*Davis v. Bandemer* 125, n. 9).[5] Such majority-to-minority consignment would signal vote dilution because turning a majority into a minority occurs only if the votes of those in the vote majority count less than those in the vote minority.

The Court's disagreement with Justice O'Connor came in a context of whether its approach to racial gerrymandering could also apply to partisan gerrymandering. It can, but with an important

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[2]Gerrymandering is a term used to cover a large range of electoral manipulations. Aside from the packing gerrymander focus under review here, pure cracking gerrymanders spread one party's votes evenly across districts so that they constitute sizable but losing minorities in all districts. These are most effective, least risky, in jurisdictions with lopsided competition. At-large and multi-member district plurality elections with their super-majoritarian effects are referred to as institutional gerrymandering (Dixon 1971, 54). Creating under-populated districts for one versus the other partisan group is a form of malapportionment gerrymandering (Brunell 2012; *see also Harris v. Arizona Redistricting Commission* 2016). Creating a district adverse to or favorable to particular candidates are "personalized" gerrymanders or, when the candidates in question are incumbents, "incumbent-displacement" gerrymanders (Owen and Grofman 1988, 14–16). Each has its own means and methods for accomplishing its manipulation and thus is best approached with its own form of precisely aimed standard for detection.

[3]Freidman and Holden's terminology can be misleading in that their title advises never cracking. Notice, however, they have in mind an uncommon meaning of cracking. They come at the issue from an approach that assigns individuals to districts and from there advises placing (packing in their meaning) the most staunch opposition partisans in districts with one's own staunch supporters. "Intuitively, extreme Democrats can be neutralized by matching them with a slightly larger mass of extreme Republicans" (Freidman and Holden 2008, 115). Discussions of gerrymandering normally refer to this as cracking or dispersal gerrymanders—spreading opposition partisans over many districts to deny them majority control in as many as possible (*see, e.g.*, Owen and Grofman 1988, 6).

[4]The Court considered allegations of a different form of partisan manipulation in *Harris v. Arizona Redistricting Commission* (2016). There, as remarked on in note 2, *supra*, the issue was neither packing nor cracking, as such, but malapportionment partisan manipulation by systematically underpopulating districts favoring Democrats (*see* Brunell 2012 for a general discussion of this form of manipulation).

[5]In relation to purely cracking forms of gerrymander, Justice White refers to the Court's concern for ensuring "significant minority voices are heard" (*Davis v. Bandemer* 1986, n. 9).

qualifying complication. In the same term that *Bandemer* was decided, the Court spelled out a three-prong test for racial gerrymandering (*Thornburg v. Gingles* 1986). While the allegation of racial vote dilution involved several of North Carolina's multi-member districts, the *Gingles* standard could be (and later was) extended to strictly single-member district plans (*Growe v. Emison* 1993; *Voinovich v. Quilter* 1993; *Johnson v. DeGrandy* 1994). It calls for comparing the actual number of majority-minority districts to the number that could reasonably be expected to exist when a fair set of single-member districts is drawn.[6]

On its face, it would appear simple to transfer that diagnostic to partisan gerrymandering. One could ask whether Democrats and Republicans have won a number of districts compared to what could be expected under a fair set of compact and contiguous single-member districts. The resemblance is not quite as straightforward as it appears, however. Unlike counting people based on race or language minority status, where the relevant number is determined and essentially fixed by census count, vote counts vary from one election to another. In a packing gerrymander, an unfair allocation of seats of, say, 40 percent when a party wins 50 percent of the vote is readily apparent. However, when the same party receives only 40 percent of the vote and wins the same 40 percent of the seats, the plan would appear eminently fair. This sort of variable result could occur in a packing gerrymander precisely because a packing gerrymander is designed to grant the disadvantaged party some minority percentage of seats over a wide range of vote percentages. As we shall demonstrate, taking account of this understanding of how packing gerrymanders operate in differential ways when votes vary between low and high is a difficult problem that the five standards propose to but sometimes fail to resolve.

EVALUATIVE FRAMEWORK

We are looking for an easily manageable and effective standard for identifying packing gerrymanders that dilute the voting weights of one party's voters. Easy manageability refers to a diagnostic method that calls for a clear and self-evident observation of the facts as the basis upon which the ultimate inference is to rest. The more directly observable the facts, the more indisputable are the foundation stones of what everyone observes. Indubitably, such transparency fades to ambiguity the more the prescribed method requires leveraging assumptions. The fourth section identifies assumptions each standard relies on to establish the factual underpinning it calls for.

Effectiveness refers to a diagnostic method that avoids errors. A false negative error occurs when a method fails to identify a gerrymander even though the choice of where to place the district lines actually caused vote dilution. A false positive error occurs in either of two ways: a proposed standard identifies vote dilution when there is none, or it identifies gerrymandering as the cause of vote dilution when the cause is attributable to something else. In addition to highlighting assumptions relevant to manageability, the fourth section identifies possible reasons to be concerned about inferential errors. Because possible reasons for doubt are potentially more hypothetical than real, the fifth section evaluates effectiveness in two applications. If we accept that North Carolina's senate districts are a partisan gerrymander, which the state acknowledges, and Iowa's senate districts are not a partisan gerrymander, which most observers acknowledge, then a standard that fails to identify North Carolina's gerrymander or misidentifies Iowa's districts as a gerrymander is committing error. Moreover, if a standard sometimes identifies the same set of districts as a gerrymander with respect to some elections and a non-gerrymander with respect to other elections, we know with assurance it is committing errors.

As for the concept of vote dilution, it must be said that four of the five standards have in mind their own particular meaning. The discussions and analyses accept each standard's definition, and thus we evaluate manageability and effectiveness on each standard's own terms of what it means to dilute votes.

Comparing parties' wasted votes considers dilution to occur when one party's voters cast more

[6]Justice Brennan explained the Court's rationale this way. "The reason that a minority group making such a challenge must show, as a threshold matter, that it is sufficiently large and geographically compact to constitute a majority in a single-member district is this: Unless minority voters possess the potential to elect representatives in the absence of the challenged structure or practice, they cannot claim to have been injured by that structure or practice" (*Thornburg v. Gingles* 1986, 50 n. 17).

unneeded votes in the senses that they go to loosing candidates or exceed what is necessary to win a seat. If votes for one party are more likely to count for nothing, that party has more votes with zero weight and thus more votes that are diluted to a maximum extent. The comparison of wins standard sees dilution as existing to the extent that one set of partisan votes do not count as much as they should because they elect fewer of their party's candidates than would be expected under neutrally drawn districting procedures. This is the direct analogue to the approach taken by the Court in racial gerrymandering. The equal vote weight standard is a vote-denominated symmetry idea that says vote dilution is foretold by comparing the median district to mean district vote percentage. If all votes count the same, the median and mean have the same numerical value; if the median and mean differ, votes for the two major parties count differently as a consequence of being divided into districts. The partisan symmetry standard aims at non-dilution in the sense that whatever seat percentage one party wins with a given vote percentage, the other party is expected to win that same percentage of seats with that same percentage of votes. The idea here is that the same resources, votes, reap the same rewards, seats; otherwise, the two sets of voters are not counting equally. The three-prong test has more expansive interests that include vote dilution but carry concerns beyond just that concept. Its focus includes (1) seat-vote outcomes that hue towards proportional representation; (2) seat shifts that are responsive to vote shifts; and, (3) depending on competitiveness, a non-gerrymandered plan that either preserves symmetry or ensures the predominant party's district vote percentages are not too similar.

FIVE STANDARDS

Efficiency gap

Counting and comparing wasted votes is the basis for the efficiency gap standard proposed by Stephanopoulos and McGhee (2015; see McGhee 2014 for the underlying social science thinking). The approach proceeds from the insight that both winners and losers "waste" votes by inefficient allocation in an election. That is, any votes above the 50% +1 for the winner plus all votes for the loser are wasted in that they contribute nothing of determinative importance to deciding who wins. In a single-district election decided by a 60–40 margin, the winner wastes 10 percentage points above 50% (setting aside ties for the sake of simplicity), while the loser wastes all 40 percentage points. Comparing the magnitude of the waste on both sides, 10 versus 40, shows an efficiency gap (of 30 points) favoring the winner. McGhee and Stephanopoulos argue that in a non-gerrymandered system both sides waste the same number of votes, so ideally the efficiency gap should equal zero.

Their claim has an appealing label along with a seemingly simple, straightforward, and intuitive procedure for calculating a numerical indicator. Nevertheless, it runs into manageability difficulties in two regards: (1) it assumes wasted votes are to be counted in an odd way, and (2) it has no secure baseline for establishing the degree of wasted votes that indicates a gerrymander. Effectiveness difficulties arise for three reasons: (1) votes are wasted for reasons other than gerrymandering; (2) the wasted vote gap co-varies with a party's vote percentage; and (3) the method seeks to cover both cracking and packing gerrymanders in one calculation and thereby can allow some amount of cracking to disguise an undue amount of packing.

Even though the arithmetic required is simple, and in that sense would seem to clear the manageability bar, the efficiency gap's definition of votes wasted by the winning candidate is disputable.[7] In particular, decades ago Andrew Hacker, who refers to the winner's wasted votes as *excess* votes, defines them as one more than the votes received by the losing candidate (Hacker 1964, 55–7). McGhee (2014) and Stephanopoulos and McGhee (2015) define a winner's excess/surplus/wasted votes as votes beyond 50% +1. It runs into a second manageability problem when deciding how many wasted votes signal a gerrymander. Because no democratic or legal principle answers the question of how many wasted votes are needed to say a plan is a gerrymander, the approach calls for comparisons to the historical record in the same jurisdiction and contemporaneous results in other jurisdictions. Such relative baselines beg the question of whether what occurred previously in the same jurisdiction or

[7]Judge Greisbach, dissenting in *Whitford*, goes so far as to call the efficiency gap's method of counting excess wasted votes "absurd" (*Whitford v. Gill* 2016, 150).

is occurring contemporaneously in other jurisdictions are results contaminated by gerrymandering.[8]

The efficiency gap runs into three problems related to its effectiveness. First, and simply, under single-member district rules votes are wasted for reasons other than gerrymandering. One needs to look no further than a simple example of a congressional district in a one-district state such as Delaware to see this. Unless the vote splits 75–25, one party wastes more votes than the other, this despite the fact that a gerrymander is impossible in a one-district state. Maybe the efficiency gap is useful only in multi-district situations, but that can't be true either. Therein resides the efficiency gap's second effectiveness problem. In a three-district state, a symmetrical distribution of 48–52–56 has a gap of +8.3 in favor of the majority party and is, by the eight-point criterion, a gerrymander. Of course, if the vote shifts uniformly to 46–50–54, there is no gerrymander, even though it is the same districting plan. Then, if votes shift another two points to 44–48–52, the gerrymander would be said to run in the direction opposite of what was inferred from the original 48–52–56 distribution. In this scenario, the relative distribution of partisan voters did not change—neither party became relatively more (or less) packed—and yet the efficiency gap registered a substantial shift in partisan advantage. In fewer words, reading a gerrymander from the efficiency gap can and often will vary depending on the underlying percentage level of the votes a party receives.

A third effectiveness problem has to do with the translation of votes to seats, the seat-vote ratio. Assuming equal turnout in all districts, a majoritarian seat-vote ratio of two to one is sufficient for equalizing wasted votes—i.e., having a seat percentage in excess of 50 equal to two times the vote percentage in excess of 50 produces an equal number of wasted votes (McGhee 2014, 79–80; Stephanopoulos and McGhee 2015, 853). For example, winning 60 percent of the seats (10 points above 50) in association with winning 55 percent of the votes (five points above 50) indicates there is no gerrymander. However, that is not necessarily so. A majoritarian seat-vote correspondence of two-to-one can occur even when a packing gerrymander is in place. Hence, a two-to-one seat-vote ratio is not a sufficient condition to conclude there is no gerrymander. For example, consider a 40–40–60–65–70 vote distribution. The distribution is asymmetrical (median 60 and mean 55), but the efficiency gap shows an equal number of wasted votes. Votes are five points above 50, and seats are ten points above 50; the majoritarian ratio is two-to-one even though the distribution is asymmetrical. Thus, despite its proponents' claims to the contrary, the efficiency gap standard does not comport with nor arise from the idea of partisan symmetry.[9]

The wasted vote approach has clear intuitive appeal. Nevertheless, it has several downsides. One, its computation poses a manageability problem because it relies on a shaky definition of what it means to waste a vote, given the alternative way of counting excess votes (as in Hacker 1964; *Whitford v. Gill*, 2016, 150–2, Greisbach dissenting). Two, it underachieves on the question of manageability because evaluation of the wasted vote computation requires using a relative comparison to the historical record of elections in the same jurisdiction or to elections in other jurisdictions. A historical comparison is liable to perpetuate gerrymanders in earlier years; comparison to other jurisdictions leaves one wondering whether the baseline involves a mix of fair and unfair outcomes elsewhere. What's more, it can under-reach and overreach on questions of effectiveness for three reasons, each functionally related to its implications that single-member district elections are fair if and only if they operate with a seat-vote majoritarian ratio of two to one. Underreaching occurs when it offers a false negative reading of gerrymandering because, despite substantial packing, the majoritarian ratio is two to one. It overreaches when it offers a false positive reading of gerrymandering by indicting a districting plan as a gerrymander because it has many competitive districts that slightly favor one party and thus a majoritarian ratio greater than two to one.

Comparing wins

This approach identifies diluted votes as winning fewer seats than expected in districting plans

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[8]In some applications an efficiency gap beyond ± 8 indicates a gerrymander (Stephanopoulos and McGhee 2015, 831). In other applications, a gap beyond ± 7 is deemed indicative (Jackman 2015, 5). As applied to congressional districts, it is designed to be applied only to delegations of eight or more members; in this context a gerrymander is indicated, not by any particular magnitude of the gap, but when one party would have been expected to win two or more seats than it actually did win (Stephanopoulos and McGhee, 2015, 835–6).

[9]*See* Stephanopoulos and McGhee (2015, 834 and passim) for claims about the relationship between symmetry and the efficiency gap.

produced through partisan blind line-drawing. If an enacted plan is an outlier in a partisan-blind null set's expected seat distribution, one can infer that it was probably intended to hold a partisan advantage. This closely aligns with the Court's racial gerrymandering standard that asks for a comparison between how many districts a group actually wins and how many the group would win under a fairly drawn single-member district plan. Its manageability problem arises in association with the black-box nature of the computer algorithm needed to establish the factual baseline for comparison. Its effectiveness can be left wanting because the match of observed versus expected wins (or districts carried) depends on the vote percentage a party wins.

The basic idea behind generating the comparisons is to use a computer to draw a large number of districting plans. Using computers for this purpose is an idea that has been floated at least since William Vickrey made the point more than a half-century ago (Vickrey 1961). A few pioneers succeeded in advancing the idea in modest ways in the 1960s and 1970s (Nagel 1965; Engstrom and Wildgen 1977); then, with advances in processing speed, the approach was ready for a full-scale application years later (e.g., Cirincione, Darling, and O'Rouke, 2000; Altman and McDonald 2011; Chen and Rodden 2013a)—at least it seemed ready in the run up to the Florida proceedings involving the State's congressional districts. Both Thomas Darling along with Jowei Chen and Jonathan Rodden produced null sets in advance of the Florida trial (see Darling 2013; Chen and Rodden 2013b; 2014), and Rodden testified at length. In the end, however, neither the reports nor Rodden's testimony received any mention by the trial court or in subsequent court decisions (*Romo v. Detzner* 2014; *League of Women Voters of Florida v. Detzner* 2015).

For what it says about manageability, the Florida courts' silence is disquieting. It may have been benign. In the face of the smoking gun evidence of partisan maneuvering that violated Florida's newly operative state-constitution intent standard, the court might well have reasoned that nothing as sophisticated as a computer-generated null set was needed.[10] Perhaps, however, the court was dissuaded from crediting the method with probative value because one report identified a few contiguity problems (Hodge 2013) and another report, plus testimony, questioned whether the Chen-Rodden null set was randomly generated since no one can know the characteristics of the population of all possible plans (McCarty 2013; 2014). Or, perhaps and more simply, the black-box nature of the method left the court unsure what reliable conclusions could be drawn.

Because the null set approach has yet to be tried and tested in a full form application, questions about its effectiveness are open. Still, this much can be said. Not enough thought has gone into how the null set could be used to detect gerrymandering beyond forming a baseline to say whether an enacted plan is an outlier in the null set distribution and, on that basis, probably indicates a gerrymander. Engstrom and Wildgen (1977, 469–70) evaluate a plan in regard to how many competitive districts it contains. Cirincione et al. (2000), Darling (2013), along with Chen and Rodden (2013a, 2014), evaluate a plan in regard to the number of districts in which each racial group or political party holds a majority. We have to suppose that focusing solely on the central tendency is not enough. Why? Depending on the vote percentage won by a disadvantaged party, the expected number of competitive districts or of majority-held districts varies and might well include seat outcomes that square with the expectation—i.e., the central tendency—but involve packing.

As an example of the problem associated with a focus on seats won (more precisely, districts carried), consider Chen and Rodden's attempt to indicate a gerrymander by counting President Bush's 2000 or John McCain's 2008 district wins across Florida, in their academic and trial-related work, respectively (Chen and Rodden 2013a, 2013b, 2014). As noticed and noted by both Darling (2013) and McCarty (McCarty 2013, 2014), a match or mismatch between expected and observed number of districts carried is not a per se robust and structural feature of a districting plan. The match or mismatch varies depending on the vote percentage won. A packing gerrymander that all but guarantees that a party win, say, 40 percent of the districts whether it wins, say, 40, 50, or 60 percent of the vote—which is the type of result a packing gerrymander can and often does produce—will sometimes match the expected number of districts carried and

[10]The facts revealed such damning evidence as Republican legislators and their operatives enlisting mapmaking confederates to submit "citizen constructed plans" under fake names and writing scripts for "concerned citizens" to present the operatives' ideas at public meetings (*Romo v. Detzner* 2014, 20–31).

other times will not. In different words, the contours of a districting plan interact with a party's system-wide level of vote support to produce more, equal, or fewer than expected wins. As a consequence, the interaction produces variable readings of gerrymandering under the expected wins standard.[11]

Using computer-generated districts to form a null set holds promise. It removes all but inadvertent partisan effects in its construction of a null set and thus supplies a strong basis for probabilistic inferences about intentions. One problem it has to overcome is making the computer processing more intuitive and transparent. Another pressing matter is choosing a benchmark other than the expected number of competitive districts or the number of district wins. The approach supplies a useful tool, but we need to figure out how to make it transparent and how to use it effectively.

Equal vote weight

The *equal vote weight standard* relies on two observed facts: (1) compare the median district vote percentage to the mean district vote percentage received by the party, and (2) check whether majority rule is violated. When one group of partisans is relatively more packed than the other, a districting plan has the potential to violate the widely embraced principle of equal vote weights and, from the unequal weights, to entrench one party in majority status. Manageability of the equal vote weight standard is straightforward inasmuch as the essential facts are directly observable. Its effectiveness can be challenged, however, because its requirement to observe a violation of majority rule is not as assertive as some ideas about gerrymandering might require.

In all, the standard for a factual identification of a gerrymander rests on three manageable ideas.

(1) *Leading indicator:* Asymmetrical packing exists when the median district vote percentage for one party is persistently lower than its mean district vote percentage.

(2) *Objectionable harm:* A vote weight inequality is clearly identifiable when one set of partisan voters casts a majority of the votes but carries less than a majority of the districts, because violating majority rule occurs only when all votes do not count equally.[12]

(3) *Cause:* District line placements are the known cause of the unequal vote weights. Votes counted system-wide contribute equally to the count. Counting votes after division into districts changes only the manner of counting. To the extent the two forms of counting do not produce the same result, the difference must be caused by the line placements.

Manageable as it is with respect to the required facts, tying its focus to violating majority rule is an arguable shortcoming of its potential effectiveness. Equal median and mean district vote percentages indicate only average symmetry, not full-scale symmetry. Reaching for a full- or at least a fuller-scale approach would be more aggressive. For example, a five-district plan applied to two-party competition that has (expected) Republican district vote percentages of 44, 46, 51, 52, and 62 is symmetrical via the equal vote weight standard but asymmetrical under a full-scale symmetry requirement (i.e., as recorded by partisan symmetry considered next—see below). The median and mean are both 51. Thus, average symmetry is upheld inasmuch as deviations above and below the mean of 51 both average six. Majority rule is also preserved; the vote majority holds a three-to-two seat majority. Full-scale symmetry goes wanting, however, because something like uniform vote swings would result in Republicans winning only three seats with 52 percent of the vote—an upward shift of one point resulting in a 45, 47, 52, 53, 63 distribution—but Democrats win four seats when they have 52 percent of the vote—after a downward shift of three points resulting in a 41, 43, 48, 49, 59 distribution. While majority rule is maintained under both vote swings, the idea of equality is not as aggressive as it might be in the sense that different rewards (seats) can be acquired from the same resources (votes).

[11]Darling analyzed his 5,000-map null set for nine pre-2012 statewide Florida elections in addition to the McCain-Obama presidential contest. For the McCain-Obama contest he found, as did Chen and Rodden, the expected number of McCain wins under the 2012 lines was 14, whereas the enacted districting plan had McCain winning 17—a result observed in less than one percent of the null set plans. However, Darling's analysis of the nine other elections showed the actual versus expected wins either matched (three elections), differed by one in favor of Republicans (three elections), or differed by one or two in favor of Democrats (three elections)—*see* Darling (2013, 16).

[12]As McDonald and Best point out, violation of majority rule is evaluated against the two-party statewide vote percentage and not the district mean vote percentage, in order to ensure that the evaluation does not conflate a violation due to turnout bias with a violation due to gerrymandering bias (McDonald and Best 2015, 318).

The equal vote standard has pros and cons. Its required factual finding is easily observed: compare the median and mean district percentages and check for violations of majority rule. However, it is not as aggressively effective as some might demand. It can be charged with under-reaching by not accounting for situations when vote shifts produce different seat outcomes while winning the same vote percentage.

Partisan symmetry

A proposal for a partisan symmetry constructed on the basis of fair seat-vote translations at various levels of vote splits goes back decades (Gelman and King 1994). It has found favor among political scientists (e.g., Engstrom 2013; McGann at al. 2015, 2016). To some extent it has also found favor among members of the Supreme Court in *LULAC v. Perry* (2006; for a detailed discussion of the Justices' reactions see Grofman and King 2007, 1–6). Its effectiveness would not be much in doubt were it not for the assumptions required to establish baseline hypothetical seat results for making comparisons between the two parties.

The approach, which could be called a seat-denominated symmetry standard, relies on an equal opportunity notion of fairness. Within practical and probabilistically knowable limits, each party is expected to win the same seat percentage for the same vote percentage. Suppose Democrats win 35 of 50 seats, 70 percent, with 55 percent of the vote. Seat-denominated symmetry requires that Republicans win 70 percent of the seats (35 of 50) when they win 55 percent of the vote. This notion of a partisan symmetry standard shares the same concern for asymmetry that violates majority rule as the equal vote weight approach, but it adds a requisite symmetrical operation of the swing ratio. At an even 50:50 vote split, seats should split 50:50, and in the competitive range of two-party vote splits, perhaps inside the 40 to 60 range, if Democrats win five more seats with 53 percent of the vote, then Republicans should be expected to add five seats when their vote is three points above 50. Its attention to the swing ratio bears a similarity to the wasted vote approach; however, it differs by being agnostic about the magnitude of the ratio, provided that the effect of the swing is symmetric.

One way to see the standard's manageability problem is from the example used to point to a shortcoming of the equal vote weight approach. There we had a five-district Democratic two-party vote percentage distribution of 44, 46, 51, 52, and 62. The median and mean are equal, and therefore a vote-denominated indicator of asymmetry is missing. However, as discussed, a three-point uniform shift in favor of the Republicans, moving the median and mean to 54, leaves them with three district wins, while a three-point swing in favor of Democrats leads to four district wins. That, of course, depends on the uniformity of the vote swing. If the swing is non-uniform—i.e., if it is mixed in the sense that some districts swing more than others—we need to know more, much more. Getting an assured handle on what else we need to know was the apparent stopping point for Justice Kennedy when he remarked favorably on the partisan symmetry approach but said courts are "wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs" (*LULAC v. Perry* 2006, 420).

The partisan symmetry standard is more comprehensive than the equal vote weight standard. To realize the added value of it comprehensiveness, however, it can under reach in practice by requiring a supporting analysis that makes some decision makers wary of relying on it because it requires leveraging a variety of not easy to evaluate assumptions embedded in computationally intensive analysis of vote swings.

Three prongs

Because gerrymandering is a complex concept, it might seem to be a good idea to use multiple criteria to evaluate whether one has been enacted. Such is the apparent thought standing behind Samuel Wang's proposed three-prong test (Wang 2016). The three prongs are grounded in concerns for (a) a less than justifiable degree of seat-vote proportionality, (b) under-responsiveness of seat shifts to vote shifts, and (c) asymmetry in the vote distribution.

(1) *Excess seat test:* Seat-to-vote responsiveness is within a range between proportionality and what could be expected from the seat-vote relationship in other states (plus allowance for random variation).
(2) *Lopsided outcomes test:* Unequal average lopsidedness in the vote distribution is evaluated by comparing average values of each party's

winning margin above 50 (plus allowance for random variation).

(3) *Reliable wins test (two forms):* In a competitive jurisdiction a party's median district percentage equals its mean district percentage (plus allowance for random variation); in a non-competitive jurisdiction the dominant party's standard deviation of the vote percentages equals the standard deviation of the party's vote from simulations based on other jurisdictions (plus allowance for random variation).

Having three prongs gives the appearance of a more comprehensive set of concerns than the preceding four approaches. That much can be granted, but having three prongs creates at least two manageability problems. One is reliance on election results from other jurisdictions as a basis for comparison. As with the wasted vote approach, an external standard begs the question of whether what occurs in the jurisdiction in question is the consequence of something particular to the jurisdiction other than the manner in which the jurisdiction was divided into districts. Second, Wang advises that the three prongs can be used "separately or combined" (Wang 2016, 1308). Questions naturally follow: Is satisfying one of the prongs enough to say no gerrymander exists? Is violating one of the prongs enough to say a gerrymander has been enacted?

Wang's advice to use his three prongs independently or in combination also carries with it an effectiveness problem. The different prongs can provide indications running in opposite directions. For example, a five-district distribution of 40, 40, 60, 60, 60 satisfies both proportionality (prong 1) and equal average lopsidedness (prong 2) but fails the symmetry standard of prong 3 (median 60 and mean = 52). Likewise, a swing ratio could reside within the bounds of acceptable proportionality but fail on both lopsidedness and symmetry. And a districting plan could fail the lopsidedness test simply because an election-swing moves the vote percentage away from 50 percent even in the absence of gerrymandering. A second effectiveness problem also relates to a lack of clarity regarding which prongs apply. Requiring failure on all three prongs simultaneously leaves an opportunity for mapmakers to satisfy any one prong while enacting a gerrymander that would be indicated by either or both of the other two prongs. In all, and in other words, the three prongs lack a coherent framework that allows them to work together.

Evaluating gerrymanders through three different tests has an intuitive appeal. Nevertheless, it raises difficult questions for both manageability and effectiveness because, as it stands, no compelling coordinating principle supplies clarity about whether a gerrymander exists according to any or all three prongs.

TWO APPLICATIONS

Argument is instructive but not enough when evaluating standards to be applied not just in theory but also in fact. Below we put all five standards to the test in the contexts of North Carolina's and Iowa's post-2011 enacted state senate districts. We want to see whether any of the five produce false negative or false positive diagnoses.

We select North Carolina and Iowa because one case is rather assuredly a gerrymander (North Carolina) and the other is rather assuredly not (Iowa). That's because North Carolina's post-2011 districts are acknowledged by the state itself, assembly members, and, later, the courts to have been drawn with pro-Republican partisan advantage as one goal (*Dickson v. Rucho* 2014, 3). Iowa's redistricting process is often held up as an exemplar of neutral redistricting. Thus, we have opportunities to check on false negative (North Carolina) and false positive (Iowa) readings.

North Carolina

The North Carolina State Senate is a 50-member body elected every two years from 50 single-member districts. Following the 2010 elections, Republicans took control of the state senate and house for the first time since 1870. The 2010 census data were delivered in March 2011, and in July the legislature passed bills establishing state senate districts for the 2012 elections.[13] Those elections saw Republicans win 66 percent of the senate seats (33 of 50) with 52.8 percent of the vote. Two years

[13]While a Democrat, Beverly Perdue, occupied the governor's office, North Carolina's redistricting bills are not subject to gubernatorial veto.

later, 2014, Republicans won 70 percent of the seats with 54.9 percent of the vote.[14] Both are substantial seat victories, 16 to 20 points in seats beyond 50 percent for votes just three to five percentage points beyond 50. But important facts militate against reading too much into the senate results by themselves. Forty percent of the seats went uncontested by one or the other major parties: 19 of 50 in 2012 and 21 of 50 in 2014. This sort of non-competitiveness, we have to think, reflects anticipated wins/losses as a consequence of the way the district lines were drawn in the first place, more so than a statement of accurate fact about the partisan disposition of the districts. More generally, prospective candidates in each of the various districts have to be thought to take account of their prospects of winning, in part—likely in substantial part—depending on a district's partisan leanings.

We can avoid the problem of district-by-district state senate election competition being endogenous to the enacted lines by turning to elections for statewide office (often referred to as *exogenous* elections) aggregated into separate counts within each of the 50 districts. The North Carolina General Assembly provides election returns for each of nine statewide offices elected in 2012 (the nine are identified in Table 1) aggregated to U.S. Census Defined Block Groups.[15] All nine elections resulted in vote percentage splits within a reasonably competitive range. We use these nine as the elections holding the most probative value for revealing whether the district lines are a pro-Republican gerrymander. In addition, with the state board supplying election returns for all nine election results disaggregated to the precinct level, we can run a large number of null set applications to generate expectations based on 50 districts drawn through a partisan-blind procedure.[16] This has a direct benefit for evaluating the observed versus expected district wins. In relation to two other proposed standards (not including the partisan symmetry and the three-prong tests) it has two additional benefits. The expectations provide a baseline for what partisan residential patterns alone could be expected to produce in regard to wasted votes and equal vote weights.

As a visual prelude, Figure 1 presents two histograms, one for the gubernatorial election, the least competitive of our nine elections, and the other for the lieutenant governor, the most competitive of our nine elections. Both distributions are bimodal. Just about two-thirds of the districts reside at percentages favorable to the Republicans regardless of whether Democrats won 44.2 or 49.9 percent of the vote. Indeed, when the vote percentage shifts in the Democrats' favor by 5.7 points, from 44.2 Democratic percent for governor to 49.9 percent Democratic for lieutenant governor, the gain in districts carried by the Democratic candidate is a mere one district. The electoral playing field is tilted substantially in favor of Republicans, leaving Democrats with a rather steep hill to climb before having any realistic prospect of winning a majority of districts.

Table 1 reports the Democratic two-party vote percentage for the nine statewide offices (column #1) and the relevant numbers for the five proposed standards (columns #2 through #6). The competitiveness noted above can be seen in the vote percentages; they range between 44.2–55.8 and 54.2–45.8, Democrat-Republican, two-party splits.

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[14]Data from North Carolina State Board of Elections Nov 6, 2012 General Election Official Results and November 4, 2014 Official General Election Results are posted on the State Board of Elections (SBoE) website.

[15]We rely on the North Carolina General Assembly's (NCGA) 2016 Redistricting Base Data provided through the NCGA's website (NCGA.net). The state provides returns for statewide contests for the 2008 through 2014 general elections. These data are collected at the voter tabulation district (VTD) level (a Bureau of the Census term for a polling area such as a precinct) level; however, several VTDs in close proximity to military bases in North Carolina reported unusually high numbers of votes and contained unusually high numbers of residents. These extremely large VTDs caused problems for our development of a null set of neutral maps because districts containing extremely these large VTDs were liable to exceed reasonable levels of population parity. To circumvent this problem, we disaggregate the returns reported by the NCGA to census blocks. We achieve this by using the spatial join utility in the QGIS software package to determine into which VTD a census block falls (Quantum GIS Development Team 2016). We then assigned votes to a block according to the proportion of the VTD population that resides within the block. We then re-aggregate block level returns to the block groups.

[16]We use a neutral redistricting algorithm proposed by Daniel Magleby and Daniel Mosesson to draw a null set of maps of legislative districts for both North Carolina and Iowa (Magleby and Mosesson 2016). The null set we develop is partisan blind in that the maps that make up the distribution were drawn without reference to any factors besides geographic contiguity and population parity. The analysis uses a graph partitioning algorithm to randomly group geographic units (block groups in North Carolina and VTDs in Iowa). While maintaining district contiguity, it then uses a second algorithm to shift geographic units randomly between districts until all districts in a given plan have roughly equal populations. We repeat the process to draw 50,000 maps of North Carolina and Iowa's state senate districts. For the analysis presented here, we utilize the 25,000 maps with the lowest difference in population across districts. Among the maps included in our sample, the maximum population deviation is within ± 4.5%.

Table 1. Results of Applying Five Standards for Evaluating Whether North Carolina's Senate Districts Are a Gerrymander

Office	*#1 Obs Dem 2-pty vote %*	*#2 Wasted votes*		*#3 District wins*		*#4 Equal vote weight*		*#5 Partisan symmetry*	*#6 3-prong test*	
		Obs	*Exp*	*Obs*	*Exp*	*Obs*	*Exp*	*Dem Seat Advantage*	*Prong 1*	*Prong 2*
Governor	44.2	6.8	13.2 (2.9)	16	15.3 (1.40)	−5.8	−1.6 (.91)	−8.5	2.02	−.44 (−0.22)
Lt Gov	49.9	16.5	5.8 (3.0)	17	21.5 (1.44)	−5.7	−2.0 (.95)	−9.5	1.80	9.22 (5.02)
Auditor	53.7	14.8	−1.6 (2.8)	21	26.9 (1.41)	−5.2	−1.8 (.99)	−8.2	1.72	11.36 (5.72)
Agri Comm	46.8	10.2	12.5 (2.8)	17	16.9 (1.35)	−7.1	−2.8 (.90)	−10.0	1.95	3.25 (1.74)
Insur Comm	51.9	16.2	2.3 (2.9)	19	24.1 (1.40)	−6.4	−2.2 (.98)	−9.5	1.81	10.11 (5.15)
Labor Comm	46.7	11.7	11.7 (2.9)	16	17.3 (1.39)	−6.1	−2.5 (.76)	−9.2	2.09	4.31 (2.33)
Sec of State	53.8	13.3	−3.1 (2.8)	22	27.7 (1.40)	−4.7	−1.8 (.82)	−8.5	1.97	10.49 (4.76)
Supt Pubic Ed	54.2	10.0	−3.9 (2.7)	24	28.3 (1.36)	−4.7	−1.7 (.88)	−8.1	1.91	9.38 (4.09)
Treasurer	53.8	15.1	−1.2 (2.9)	21	26.8 (1.45)	−5.3	−2.1 (.96)	−8.7	1.99	8.48 (3.86)

#1 = Percentages are for the statewide two-party vote.
#2 = Wasted votes are the difference in Dem vs Rep votes cast for a losing candidate plus votes above 50% +1 as a percentage of total two-party votes—i.e., {(Dem wasted – Rep wasted) / Total two-party votes} * 100. Positive numbers indicate more Dems wasted more votes.
#3 = District wins are the number of districts carried by the Dem candidate, observed and expected, with expectations based on 25,000 computer-generated results. Numbers in parentheses are the standard deviation of expectations among the 25,000 neutral plans.
#4 = Equal vote weights record the difference between the median district two-party Dem percentage and the mean two-party district Dem percentage. Negative numbers indicate Dem disadvantage, with the magnitude indicating approximately the percentage points above 50 Dems would need to carry a majority of districts. The column of expected results is the median-mean difference attributable to residential patterns, with standard deviations in parentheses.
#5 = Partisan symmetry is the average difference in Dem–Rep expected number of seats won in a competitive range of vote percentage (40 to 60) if each party won the same vote percentage. Negative numbers indicate Dems are expected to win fewer seats with the same vote percentage as Reps.
#6 = Prong 1 of the three-prong test is the estimated seat-vote swing ratio—e.g., a 2.02 value means a vote gain of one point brings a seat gain of 2.02 points. Prong 2 is the difference between Dem and Rep vote percentages above 50% in districts won by Dems vs Reps. Negative numbers indicate Dems have more extreme lopsided winning percentages. Numbers in parentheses are *t*-test values; values above 1.68 are statistically significant at $p < .05$, one-tail.

Efficiency gap. Applying the efficiency gap calculations produces mixed results for detecting a gerrymander. Eight of the nine elections show wasted vote percentage magnitudes exceeding the suggested demarcation line of 8.0, with the gubernatorial election falling below that line. What is one to say of these results? Sometimes the North Carolina senate districts appear to be a gerrymander, but once in a while they don't. The conclusion depends on which election one looks to as evidence. Notice,

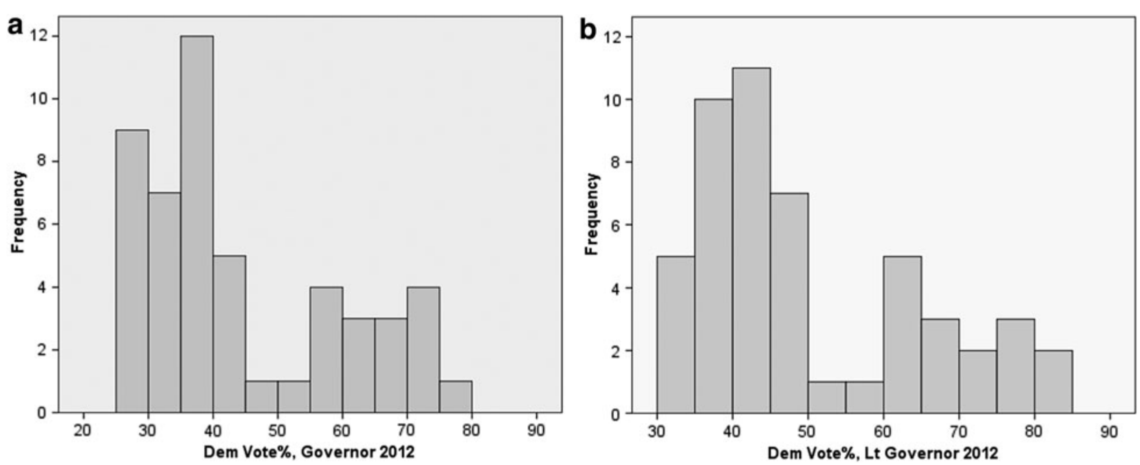

FIG. 1. Distribution of Democratic two-party vote percentages among North Carolina's state senate districts: 2012 governor and lieutenant governor elections. **(a)** *Left panel:* Dem Statewide % = 44.2; Dem Mean % = 44.4; Dem Median % = 38.6; Std. Dev. = 15.6; Dem Vote % > 50 = 16 of 50. **(b)** *Right panel:* Dem Statewide % = 49.9; Dem Mean % = 50.0; Dem Median % = 44.3; Std. Dev. = 15.0; Dem Vote % > 50 = 17 of 50.

also, the expected values rise and fall depending on the levels of the two-party vote. That is a serious problem because it tells us the magnitude of the wasted vote calculations depend on the vote percentage and not just whether the districts are gerrymandered. And notice that, despite being above the 8.0 threshold, two elections (commissioners of agriculture and labor) are not statistically distinguishable from expectations drawn for neutral plans.

What gives rise to the false negative reading from the gubernatorial election? The reason is directly related to the wasted vote requirement of a responsiveness ratio (aka, swing ratio) in the neighborhood of 2.0. When, as in North Carolina's gubernatorial election, Democrats win 44.2 percent of the vote, the wasted vote requirement for fairness is to have the Democrats winning 38.4 percent of the seats—i.e., the vote difference from 50 is 44.2 – 50 = –5.8. Two times that difference is –5.8 * 2 = –11.2, and an equal number of wasted votes would require that Democrats win 38.4 percent of the districts, since −11.6 + 50 = 38.4. Adding or subtracting the standard's requirement to be within eight points of the "fair" outcome implies that seat percentages in the range of 30.4 to 46.4 (38.4 ± 8) indicate no gerrymander effect. Given that a packing gerrymander might well be designed to grant Democrats some outcome in the vicinity of a third of the seats for a range of vote percentages, weak Democratic vote performances can fall within the safe-harbor range of the wasted vote standard. On the flip side, when Democrats receive something close to or exceeding 50 percent of the vote, a gerrymander effect becomes apparent, because seats are restricted to something such as 30 to 45 percent even when Democrats' votes approach or go above a majority. In short, the wasted vote standard can provide false negative readings in certain circumstances precisely because a gerrymander has been fashioned to allow one party to win a circumscribed minority number of districts unless and until it can win especially large vote majorities.

Comparing wins. The standard of counting the number of district wins suffers from the same shortcoming as the wasted vote standard. We see in Table 1 that in the three elections Democrats won with between 44 and 47 percent of the vote (governor, commissioner of agriculture, and commissioner of labor), they won close to the number of districts expected. When Democrats win votes in the vicinity of a majority or above, their shortfalls in seats are clear to see—just as when using the wasted vote standard. Put differently, when Democrats cast a minority of votes below 47, the safe seats granted to them by the gerrymander disguise the fact of the gerrymander. In short, comparing observed and expected district wins is subject to false negative readings under some circumstances.

Equal vote weights. This standard shows a consistent bias against Democrats. The median-mean differences run between 4.7 and 7.1 points adverse to Democrats, implying they would need something approaching 54.7 to 57.1 percent of the vote in order to carry a majority of districts—i.e., (50 + 4.7) to (50 + 7.1). Among the five elections when Democrats actually won a statewide vote majority, these various statewide candidates never carried a majority of the districts.[17] And, while the column of numbers on median-mean difference expectations shows Republicans have a natural 1.5- to 3.0-point advantage simply due to residential patterns, observed advantages attributable to gerrymandering fall far outside those expectations. Indeed, in none of the nine elections is the observed median-mean difference anywhere close to expectations. In the best-case circumstances, the secretary of state election, only 3 of 25,000 neutral maps (.012%, twelve-thousands of one percent) have a median-mean difference as large as the actual –4.7 value. In four elections, no expected value, among the 25,000 per election, is as large as the one observed. All indications from the equal vote weights standard indicate a rather harsh gerrymander favorable to Republicans, adverse to Democrats.

Partisan symmetry. As Justice Kennedy stated in *Veith*, the partisan symmetry standard runs into manageability problems because it relies on hypothetical estimates for the number of seats that would be won were one versus the other party to win the same vote percentage. We address the seat-denominated symmetry question in two ways, one more and one less factual. The facts from among our nine elections show that in the lieutenant governor's election the vote splits 49.9 to 50.1. Partisan symmetry would expect Democrats to win 24 or 25

[17]Turnout bias never exceeds 0.8 percent, and among the nine elections it averages 0.17 percent favoring Democrats.

seats for such an evenly split vote. They actually won only 17 districts. Furthermore, in three elections that Democrats won with 53.7 or 53.8 vote percentages (auditor, secretary of state, and treasurer), they won 21 or 22 seats. By way of contrast, in close to comparable circumstances, when Republicans won 53.2 or 53.3 percent of the vote (agriculture and labor commissioners), they won 33 or 34 seats. Clearly, large discrepancies in equal opportunities exist in the seat-vote relationship. Very similar resources (vote percentages) carry with them hugely different seat rewards. Through this more factual version of applying the seat-denominated symmetry standard we arrive at a clear indication of gerrymandering. Democrats win far fewer seats than Republicans when they win something close to the same vote percentages.

The less factual analysis takes a form more closely aligned with that described by Grofman and King (2007). We construct it through four steps: (1) accept as given the vote percentages and the number of districts won for each of our nine elections, (2) allow for hypothetical uniform vote swings so that they range from 40 and 60, (3) record the number of districts carried by Democrats at each of the 21 percentage points, and (4) compare the differences when both Democrats and Republicans won 40, 41, 42, … , 60 percent of the vote. The seat-denominated column in Table 1 records the results. On average, across the 21 percentage points, Democrats are at an eight- to nine-seat disadvantage despite, hypothetically, winning the same vote percentages as Republicans. Moreover, were we to restrict the comparisons to a vote range of 45 to 55, the Democrats' seat disadvantage runs, on average, between 13 and 15 districts. By this second form of analysis, too, the partisan standard indicates a substantial pro-Republican gerrymander.

Three prongs. Vote-denominated symmetry is the third prong in the proposed test. As discussed, by that prong we see an indication of a pro-Republican gerrymander.

Prong 1, the excess seats test, calls for calculating "whether the outcome … was disproportional relative to the seats/votes curve" by checking whether "the actual seats and the simulated number of seats" correspond beyond chance deviations (see Wang 2016, 1306). One method of checking is to revisit the district wins comparison in the null set test. That would tell us that in some elections district wins are in line with expectations but some are not. Another check is through a simulated seats/votes curve based on the simulation analysis we described for the less factual version of the partisan symmetry analysis but, here, by reporting the seat/vote slope value. Those results show seat/vote relationships between 1.7 and 2.1 (column 5 of Table 1). All results are within the range of one and three, which the standard supposes indicates no gerrymander (Wang 2016, 1286–89).

The reason for the sometime false negative readings from comparing actual and expected seat results is similar to the reasons we reported for the wasted votes and null set comparisons. The expectation ebbs and flows depending on the level of the vote, and when the disadvantaged party's votes are below 47, the districts the gerrymander grants to that party turn out to be about as expected in a non-gerrymandered plan. As the disadvantaged party votes rise to something approaching or beyond a majority, however, few additional districts are won. In fewer words, North Carolina created an effective packing gerrymander, and an associated consequence of packing gerrymanders is to reduce seat responsiveness toward proportional seat-to-vote results. The disadvantaged party wins its granted set of packed districts with relatively small statewide vote percentages, but as its vote percentages approach and go above 50, to say 54 or 55, the seats gains respond only modestly. All in all, therefore, we have to conclude the prong 1 test cannot be considered an effective standard by which to evaluate whether a packing gerrymander was enacted in North Carolina. It is prone to false negative readings because the standard it sets for a non-gerrymander is actually an outcome we expect a gerrymander to produce.

Prong 2 also runs into a problem, where again the problem is a failure to take account of how a gerrymander functions as vote percentages for the disadvantaged party vary between low versus high. It calls for a comparison of average vote percentages above 50 for districts won by Democrats compared to districts won by Republicans. To check whether the comparisons show systematic differences going beyond mere chance, prong 2 applies t-tests for the differences between two means. In contradiction of a pro-Republican gerrymander that North Carolina enacted, applying prong 2 to the Governor's election shows a difference slightly adverse to Republicans, not Democrats. The difference is not statistically significant, and therefore the inference indicated from the gubernatorial election is that there is no gerrymander. Put differently, the prong 2 results tell us that sometimes the North Carolina senate districts appear to be a gerrymander, but sometimes they do not. The conclusion depends on which election is analyzed.

North Carolina Summary. North Carolina's senate districts were drawn for the purpose, in part, of providing Republicans with electoral advantage. Prong 1 of the three-prong standard misses that fact completely. The wasted vote, district wins, and prong 2 of the three-prong standard are not fully reliable indicators of that advantage. More often than not they indicate a Republican advantage, but depending on the size of statewide vote percentage they can, and in North Carolina do, give false negative readings. At the very least we have to conclude that indicators of gerrymandering that vary depending on how the vote splits are undesirable. More to the point, the false negatives exist because packing gerrymanders are intended to produce the seat outcome that the standards misidentify—i.e., packing gerrymanders grant the disadvantaged party some minority number of seats whether their vote percentage is small or substantial. The two symmetry standards, on the other hand, provide consistent indicators of North Carolina's designed partisan advantage. No false negatives appear. Thus, in application to North Carolina the symmetry standards are the dependable indicators, at least in the sense of avoiding false negatives.

Iowa

The Iowa Senate is a 50-member body elected to four-year terms from 50 single-member districts. Elections are staggered, with 25 members elected in presidential years and 25 elected in presidential midterms. Iowa's Legislative Service Agency (LSA) and its subordinate affiliated redistricting commission serve in an advisory capacity by presenting congressional and state legislative districts for the legislature's approval/disapproval, subject to veto by the governor.[18] The LSA is required to ignore partisan-related information of party registration, voting patterns, incumbency, candidate residences, and the like. The process has long drawn praise for its fair-mindedness (*Economist* 2002; Martin 2016).

Following the 2010 round of redistricting, the combined 2012 and 2014 senate elections saw the Democrats win 52 percent of the seats (26 of 50) with only 46.5 percent of the vote. As we noted in regard to North Carolina, however, the senate elections themselves do not offer especially probative evidence because the choices by candidates about whether and how to compete depend on where the lines are located. In Iowa, for instance, nearly one-third of all districts (16 of 50) went uncontested. Among the 34 districts contested by major-party candidates, Democrats cast 51.2 percent of the vote and won 20 districts. Thus, as with North Carolina, the more probative evidence is drawn from analyses of Iowa's statewide elections, here ten of them between 2008 and 2012.

As prelude, Figure 2 presents two vote percentage histograms: one for the secretary of state and the other for the treasurer, the two most competitive elections among our ten. The obvious fact apparent in both graphs is that Iowa has a large number of competitive districts. The numbers of districts in a competitive vote percentage range between 45 and 55 are 26 (secretary of state) and 27 (treasurer). Notice, also, a difference of just 4.4 vote points is associated with seat splits of 17 Democratic and 33 Republican versus 38 Democratic and 12 Republican. Small vote shifts apparently bring large district win rewards.

The numbers relevant to evaluating the five standards are reported in Table 2. Our various analyses track the same path as those reported and discussed for the North Carolina application.

Efficiency gap. The news about whether the wasted vote standard provides the correct reading of no gerrymander in Iowa is mixed. Nine of ten values exceed the suggested line of demarcation for distinguishing a gerrymander from a non-gerrymander, i.e., a value below −8 or above +8. If analysts rely on just one exogenous election to evaluate a gerrymandering allegation, they are likely to arrive at a false positive conclusion. If, however, two or more elections are investigated and each party wins a vote majority in at least one of the elections, it would be possible to see that the wasted votes rise and fall depending on whether a party receives a vote majority or minority. In Iowa, Democrats

[18]If disapproved, the Legislative Service Agency (LSA) is required to draw new maps. After three disapprovals, the legislature is allowed to draw new maps, but this has not occurred since implementation in the 1980 round of redistricting.

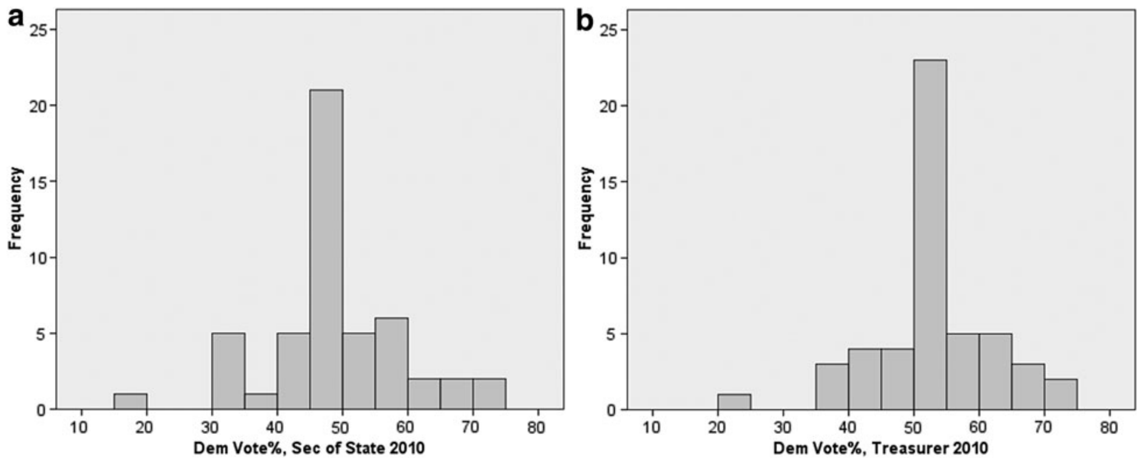

FIG. 2. Distribution of Democratic two-party vote percentages among Iowa's state senate districts: 2010 secretary of state and treasurer elections. **(a)** *Left panel:* Dem Statewide % = 48.5; Dem Mean % = 48.7; Dem Median % = 48.3; Std. Dev. = 10.0; Dem Vote % > 50 = 17 of 50. **(b)** *Right panel:* Dem Statewide % = 52.9; Dem Mean % = 53.0; Dem Median % = 52.8; Std. Dev. = 8.8; Dem Vote % > 50 = 38 of 50.

waste fewer votes than Republicans (indicated by the negative values in column 2) when they win a vote majority but waste more votes (positive values in column 2) when Republicans win a vote majority.

Comparing wins. Comparing actual district wins to expected wins from maps drawn using a neutral process comes close to getting to the right conclusion that Iowa's senate districts are not a gerrymander. The observed results are never too far

Table 2. Results of Applying 5 Standards for Evaluating Whether Iowa's Senate Districts Are a Gerrymander

Office	*#1 Obs Dem 2-pty vote%*	*#2 Wasted votes*		*#3 District wins*		*#4 Equal vote weight*		*#5 Partisan symmetry*	*#6 3-prong test*	
		Obs	*Exp*	*Obs*	*Exp*	*Obs*	*Exp*	*Dem Seat Disdvantage*	*#1*	*#2*
Pres 2012	53.0	−9.6	−8.6 (2.8)	33	32.4 (1.37)	.47	0.1 (.48)	.2	4.60	1.26 (0.71)
Pres 2008	54.8	−7.8	−12.7 (2.7)	34	36.4 (1.32)	.40	−0.3 (.50)	−.2	4.98	4.87 (2.75)
U.S. Senate 10	34.1	14.4	9.0 (1.2)	2	2.3 (0.63)	−.88	−1.14 (.55)	.2	4.82	−11.20 (−1.99)
U.S. Senate 08	62.7	−22.6	−24.6 (1.2)	49	48.6 (0.59)	.47	0.4 (.46)	0	5.91	2.00 (0.29)
Governor	45.0	17.1	15.9 (2.1)	12	12.6 (1.04)	.42	−0.5 (.44)	.2	4.63	−0.60 (−.29)
Sec of State	48.5	13.1	8.7 (3.2)	17	19.3 (1.60)	−.38	−0.3 (.43)	−.2	5.15	2.20 (1.07)
Treasurer	52.9	−20.8	−17.4 (3.1)	38	35.0 (1.53)	−.25	0.1 (.39)	−.9	5.50	−1.42 (−0.67)
Auditor	43.5	22.7	25.0 (2.5)	11	11.1 (1.14)	−.11	−0.1 (.61)	.7	4.36	−3.41 (−1.55)
Sec of Agri	37.1	15.7	15.0 (1.8)	5	5.00 (1.01)	−1.93	−1.6 (.63)	1.1	3.90	−9.39 (−2.57)
Atty Gen	55.6	−21.7	−18.7 (2.6)	41	39.5 (1.28)	−.11	0.2 (.42)	−.6	5.20	0.78 (0.33)

#1 = Percentages are for the statewide two-party vote.
#2 = Wasted votes are the difference in Dem vs Rep votes cast for a losing candidate plus votes above 50% +1 as a percentage of total two-party votes—i.e., {(Dem wasted – Rep wasted) / Total two-party votes} * 100. Positive/negative numbers indicate more Dems/Reps wasted more votes.
#3 = District wins are the number of districts carried by the Dem candidate, observed and expected, with expectations based on 25,000 computer generated results. Numbers in parentheses are the standard deviation of expectations among the 25,000 neutral plans.
#4 = Equal vote weights record the difference between the median district two-party Dem percentage and the mean two-party district Dem percentage. Negative numbers indicate Dem disadvantage, with the magnitude indicating approximately the percentage points above 50 Dems would need to carry a majority of districts. The column of expected results is the median-mean difference attributable to residential patterns, with standard deviations in parentheses.
#5 = Partisan symmetry is the average difference in Dem–Rep expected number of seats won in a competitive range of vote percentage (40 to 60) if each party won the same vote percentage. Negative numbers indicate Dems are expected to win fewer seats with the same vote percentage as Reps.
#6 = Prong 1 of the three-prong test is the estimated seat-vote swing ratio—e.g., a 4.60 value means a vote gain of one point brings a seat gain of 4.60 points. Prong 2 is the difference between Dem and Rep vote percentages above 50% in districts won by Dems vs Reps. Negative numbers indicate Dems have more extreme lopsided winning percentages. Numbers in parentheses are *t*-test values; values above 2.02 are statistically significant at $p < .05$, two-tails.

off expectations. For six of ten elections, the difference is just a fraction of one seat. The one hitch is that two elections are statistically significantly different from expectations (i.e., more than 1.65 standard deviations removed from expectations). Because the differences run in both partisan directions—once with Democrats carrying fewer than expected (treasurer) and once with Republicans carrying fewer (president 2008)—an evaluation of several elections could be used to demonstrate no systematic favoritism serving to advantage one but not the other party. So, even though the comparison of wins standard generally avoids false positives more often than not, the statistical significance consideration is a reminder that it is worthwhile to apply the standard to more than one exogenous election.

Equal vote weight. The equal vote weight standard (aka vote-denominated symmetry) reaches the correct conclusion of no Iowa gerrymander. The median-mean differences are small; they run in different directions (six negative versus four positive); and never is majority rule violated.[19] All this leaves the no gerrymander conclusion on secure footing.

Partisan symmetry. Seat-denominated symmetry involves a degree of ambiguity but essentially reaches the right conclusion. By the method that pairs comparable situations where Democrats and Republicans win the same vote percentage, four comparisons come close to filling the bill: (1) President 2008 vs Governor, (2) Attorney General vs Governor, (3) Treasurer vs Secretary of State, and (4) U.S. Senator vs Secretary of Agriculture. In order, respectively,

(1) D vote % 54.8 and R vote % 55.0 → D seats = 34 vs R seats = 38
(2) D vote % 55.6 and R vote % 55.0 → D seats = 41 vs R seats = 38
(3) D vote % 52.9 and R vote % 51.5 → D seats = 38 vs R seats = 33
(4) D vote % 62.7 and R vote % 62.9 → D seats = 49 vs R seats = 45

The results in any one election are three, four, or five seats off—hence the ambiguity—but one election shows a Republican advantage and the other three a Democratic advantage. In other words, there is no indication of a persistent partisan advantage running in one direction. Alternatively, applying the less factual, simulation analysis reported in Table 2's column 6 (see the details of how this approach works in our discussion of the North Carolina analysis, above), we see mostly fractional seat differences with none amounting to as many as two seats. On this evidence, seat-denominated symmetry indicates about as little of a gerrymandering seat effect as one might imagine in a fair set of districts, but with a touch of ambiguity.

Three prongs. The third prong of the three-prong test has already been covered as it repeats the calculation of the equal vote weight test. On that score, the test indicates no gerrymandering. One version of evaluating the first prong, from the standpoint of a party winning more or fewer seats than expected, also indicates there is no gerrymander inasmuch as that is what the district wins test indicates (i.e., from column 3). That follows, however, when the expectation is based on the null set. Compared to outcomes in other elections nationwide (Wang 2016, 1289–92), the rather large seat swings in response to vote shifts might very well lead to a different conclusion. As can be seen in the prong 1 column of the three-prong test, simulated seat-vote relationships have values above 3.90. All ten simulated slopes are beyond the test's zone of acceptability (Wang 2016, 1286). Taking all of these considerations on board makes it difficult to say what conclusion should be drawn from the prong 1 test.

Finally, prong 2 offers mixed readings. Two of ten differences in the lopsidedness of district-win percentages are statistically significant—viz., president 2008 and secretary of agriculture. On the one hand, because one significant result shows a Democratic win is too lopsided and the other shows a Republican win is too lopsided, one could conclude the lopsidedness shows no partisan favoritism and thus no gerrymandering. On the other hand, the results more generally show that comparing lopsidedness is not a reliable indicator of gerrymandering in any case. Large vote percentage outcomes for a party, as in Iowa's 2010 U.S. Senate and secretary of agriculture elections, can produce disparities in lopsidedness as the result of the vote percentages, not as a result of gerrymandering.

[19] As is true for North Carolina (fn. 17), turnout bias in Iowa does not amount to much. It favors Democrats in all ten elections but never exceeds 0.6 percent and averages just 0.22 percent.

Iowa summary. Iowa's senate districts are widely viewed as fair. All five standards could be made to confirm that they are. Three of the five arrive at that conclusion only as contingencies, however. By way of counting wasted votes in any one election, the results actually look like a gerrymander. The important fact revealed by this contingency is that counting wasted votes and checking whether they exceed the proposed threshold of ± 8 is not anything close to a standard for identifying a gerrymander because wasted votes exceed the threshold for reasons other than gerrymandering. In Iowa they occur in nine of ten elections because many senate districts are highly competitive, something that is neither an ill in and of itself nor something that operates to the detriment of only one party. That same high degree district competitiveness hampers prong 1 of the three-prong approach, and prong 2 is subject to false positives simply when one party wins considerably more votes than the other. Comparing observed to expected wins fares better. It usually arrives at the right conclusion, though it is subject to possible false positive reading as in two of ten elections when the differences are not large but nevertheless statistically significant. Both the equal vote weight and partisan symmetry standards offer credible readings of Iowa's non-gerrymander. One finds no indication of a gerrymander from the equal vote weight standard and, at most, not so much a false positive reading as a degree of ambiguity from the partisan symmetry standard. In all, on questions of avoiding false positives, just as with avoiding false negatives, the two symmetry standards are the dependable indicators, one slightly more so (equal vote weight) and the other slightly less so (partisan symmetry).

DISCUSSION

What have we learned? The two symmetry standards hold the best prospects for identifying a packing gerrymander that dilutes the votes of one party's voters relative to the vote weight enjoyed by the other party's voters. Between the two, the equal vote weight standard is the more convincing as it more readily meets manageability and effectiveness considerations. Considered as matters of principle and checked against hypotheticals, the equal vote weight standard is faulted only for not being aggressive enough to cover the contingency that, while a districting plan is fair in the sense of not violating majority rule, it could miss the fact that one party can expect more seats when it wins a vote majority with X percent of the vote compared to when the other party wins the same X percent of the vote. This lack of aggression has to be balanced against the less manageable partisan symmetry standard, which relies on observed outcomes where the votes are mirror images—e.g., 45–55 and 55–45—or engages in hypothetical projections of what reasonably could be expected to result were votes to shift in some particular way. Also, as the Iowa application illustrates, the equal vote weight standard avoids a few of the modest ambiguities that arise when the partisan symmetry standard is applied.[20]

The three other standards leave much to be desired. Each suffers manageability problems: wasted votes for both its arguable counting procedure and its need to look externally to create a relative metric by which to say whether a gerrymander exists; comparing observed versus expected wins for its black box computer algorithms; and the three-prong test for its possible internal contradictions. All three also suffer effectiveness problems, each and all, in essence, because their results vary depending on the level of the vote each party receives. Their missing effectiveness is especially damning because it means these three approaches misapprehend a key feature of how packing gerrymanders work. Packing gerrymanders grant the disadvantaged party some number of seats that can look fair when that party wins a modest vote percentage but is clearly unfair when the same or similar limited number of seats is all it wins with vote totals approaching or exceeding a majority. The series of false negative readings in the North Carolina applications make this shortcoming ever so clear. To be sure, each of the three can be saved from full-scale rejection. When applied to the "right" mix of elections each can be argued to come to the right conclusion. At that juncture, however, there is nothing to be gained over applying the symmetry standards and

[20]In application, the choice does not need to be treated as a stark either/or. The equal vote choice is easier to manage and, in most cases, is highly likely to reach the same conclusion were one, instead, to apply the partisan symmetry standard. When and where circumstances warrant, a need for the greater aggressiveness of the partisan symmetry approach can be explained and the case for its broader notion of vote dilutions can be pressed.

something to be lost by doubts and arguments about just what is the "right" mix of elections.

CONCLUSION

The ballot box is the essential institution of any democracy, with more than a few thousand up through hundreds of millions of people coming together to exercise self-government. It is remarkable that centuries beyond the widespread recognition that gerrymandering can be and has been used to distort the self-governing process we are still struggling to find ways to identify and combat it. Our evaluation of five proposals for curbing packing gerrymanders reveals both the difficulties and possibilities.

Our focus has been on packing, as it is the most commonly alleged form. Its clear harm to democratic principles protected by the U.S. Constitution is unequal treatment of voters by implicitly assigning them different vote weights. Its contra-democratic systemic consequence is relegation of a popular majority to minority status. The three proposals of computing the efficiency gap, comparing wins, and applying a three-prong test encounter manageability problems. More damning, the three ask for evidence of gerrymandering that, when the specified evidence does not appear, can actually be absent because a gerrymander has been wrought—i.e., the false negative readings North Carolina's senate districts. Just as damning for two of the three proposals, not including comparing wins, is their asking for evidence that when it does appear it is for reasons other than gerrymandering—i.e., the false positive readings of Iowa's senate districts. The two symmetry-based standards, equal vote weights and partisan symmetry, are both more or less easily manageable—the equal vote weight test is the more manageable of the two. By argument and confrontation with evidence we have shown both to be effective at identifying when the placement of lines is the cause of diluting votes—here, again, with the equal vote weight standard providing more clarity—i.e., avoiding the arguable claims that could be focused on why a party did not win more seats at each and various level of its votes. On this review, it is clear that the equal vote weight symmetry standard offers the best prospects for redistricting authorities and courts to confront the perniciousness we know as packing partisan gerrymanders.

REFERENCES

Altman, Micah and Michael P. McDonald. 2011. "BARD: Better Automated Redistricting." *Journal of Statistical Software* 42: 2–36.

Brunell, Thomas L. 2012. "The One Person One Vote Standard in Redistricting: The Uses and Abuses of Population Deviations in Legislative Redistricting." *Case Western Reserve Law Review* 62: 1057–77.

Chen, Jowei and Jonathan Rodden. 2013a. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures." *Quarterly Journal of Political Science* 8: 239–69.

Chen, Jowei and Jonathan Rodden. 2013b. "Report on Computer Simulation of Florida Congressional Districting Plans" (February 15). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

Chen, Jowei and Jonathan Rodden. 2014. "Response to Expert Report of Stephen W. Hodge" (February 18). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

Cirincione, Carmen, Thomas A. Darling, and Timothy O'Rourke. 2000. "Assessing South Carolina's 1990 Congressional Districting." *Political Geography* 19: 189–211.

Darling, Thomas A. 2013. "Report of Thomas A. Darling in *Romo v. Detzner*" (April 8). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

Davis v. Bandemer. 1986. 478 U.S. 109.

Dickson v. Rucho. 2014. 766 S.E.2d 238, vacated and remanded, 575 U. S. ___, aff'd on remand, 781 S.E.2d 404 (2015), cert. pending, No. 16–24.

Dixon, Robert G. Jr. 1971. "The Court, the People, and 'One Man, One Vote.'" In Nelson W. Polsby (ed.). *Reapportionment in the 1970s*. Berkeley, CA: University of California Press, 7–55.

Economist. 2002. "In Praise of Iowa." *The Economist* (October 17).

Engstrom, Erik J. 2013. *Partisan Gerrymandering and the Construction of American Democracy*. Ann Arbor, MI: University of Michigan Press.

Engstrom, Richard L. and John K. Wildgen. 1977. "Pruning Thorns from the Thicket: An Empirical Test of the Existence of Racial Gerrymandering." *Legislative Studies Quarterly* 2: 465–79.

Fang, Lee. 2015. "Gerrymandering Rigged the 2014 Elections for Republican Advantage." *The Nation* (November 5) <www.the.nation.com/article/gerrymandering-rigged-2014-elections-republican-advantage/>.

Friedman, John N. and Richard T. Holden. "Optimal Gerrymandering: Sometimes Pack, but Never Crack." *American Economic Review* 98: 113–144.

Gelman, Andrew and Gary King. 1994. "A Unified Method of Evaluating Electoral Systems and Redistricting Plans." *American Journal of Political Science* 38: 514–54.

Grofman, Bernard N. and Gary King. 2007. "The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after *LULAC v. Perry*." *Election Law Journal* 6: 2–35.

Growe v. Emison. 1993. 507 U.S. 25.

Gudgin, Graham and Peter J. Taylor. 1979. *Seat, Votes, and the Spatial Organisation of Elections*. London, UK: Pion.

Gul, Faruk and Wolfgang Pesendorfer. 2010. "Strategic Redistricting." *American Economic Review* 100: 1616–1641.

Hacker, Andrew. 1964. *Congressional Districting: The Issue of Equal Representation* (revised ed.). Washington, DC: Brookings.

Harris v. Arizona Independent Redistricting Commission. 2016. 578 U.S. ___.

Hebert, J. Gerald and Danielle Lang. 2015. "Memorandum of Amici Curiae Common Cause and Virginia New Majority Regarding Proposed Remedial Plans." U.S. District Court for the Eastern District of Virginia (3:13-cv-678).

Hodge, Stephen W. 2013. "Assessments of the Redistricting Simulations of Professors Chen and Rodden" (November). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

King, Gary, Bernard Grofman, Andrew Gelman, and Jonathan Katz. 2005. "Amicus Brief in Jackson v. Perry Submitted on Behalf of Neither Party, in the Supreme Court." (No. 05–276).

Jackman, Simon. 2015. "Assessing the Current Wisconsin State Legislative Districting Plan" (July). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

Johnson v. DeGrandy. 1994. 512 U.S. 997.

League of United Latin American Citizens v. Perry. 2006. 548 U.S. 399.

League of Women Voters of Florida v. Detzner. 2015. Supreme Court of Florida (SC14-1905).

Martin, David O. 2016. "Iowa's Redistricting Model Deserves Bipartisan Support." *Wisconsin State Journal* (May 7).

Magleby, Daniel B. and Daniel B. Mosesson. Forthcoming. "Neutral Redistricting Using a Multi-Level Weighted Graph Partitioning Algorithm." *Political Analysis*.

Mayhew, David 2011. *Partisan Balance: Why Political Parties Don't Kill the U.S. Constitutional System*. Princeton, NJ: Princeton University Press.

McCarty, Nolan. 2013. "Response to Chen and Rodden's 'Report on Computer Simulation of Florida Congressional Districting Plans'" (No date stated). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

McCarty, Nolan. 2014. "Response to Jowei Chen and Jonathan Rodden's February, 2014 Report" (April 28). Available at Justin Levitt's *All about Redistricting* website, <http://redistricting.lls.edu/>.

McDonald, Michael D. and Robin E. Best. 2015. "Unfair Partisan Gerrymandering in Politics and Law: A Diagnostic Applied to Six Cases." *Election Law Journal* 14: 312–30.

McGann, Anthony J., Charles Anthony Smith, Michael Latner, and Alex J. Keena. 2015. "A Discernable and Manageable Standard for Partisan Gerrymandering." *Election Law Journal* 14: 295–311.

McGann, Anthony J., Charles Antony Smith, Michael Latner, and Alex J. Keena. 2016. *Gerrymandering in America: The House of Representatives, the Supreme Court, and the Future of Popular Sovereignty*. New York: Cambridge University Press.

McGhee, Eric. 2014. "Measuring Partisan Bias in Single-Member District Electoral Systems." *Legislative Studies Quarterly* 55: 68–69.

Nagel, Stuart. 1965. "Simplified Bipartisan Computer Redistricting." *Stanford Law Review* 17: 863–69.

Owen, Guillermo and Bernard Grofman. 1988. "Optimal Partisan Gerrymandering." *Political Geography Quarterly* 7(1): 5–22.

Page v. Virginia State Board of Elections. 2014. 3:13-cv-678 (E.D. Va).

Quantum GIS Development Team. 2016. *Quantum GIS Geographic Information System*. Open Source Geospatial Foundation Project. <http://qgis.osgeo.org>.

Romo v. Detzner 2014. Final Judgement, CA-412 (2nd Jud. Cir. Fla.).

Stephanopoulos, Nicholas and Eric McGhee. 2015. "Partisan Gerrymandering and the Efficiency Gap." *University of Chicago Law Review* 82: 831–900.

Stevens, John Paul. 2014. *Six Amendments: How and Why We Should Change the Constitution*. New York: NY: Little, Brown.

Thornburg v. Gingles. 1986. 478 U.S. 30.

Toobin, Jeffrey. 2003. "The Great Election Grab: When Does Gerrymandering Become a Threat to Democracy?" *New Yorker* (December 8): 63–80.

Vickrey, William. 1961. "On the Prevention of Gerrymandering." *Political Science Quarterly* 76: 105–10.

Voinovich v. Quilter. 1993. 507 U.S. 146.

Wang, Samuel S.-H. 2016. "Three Tests for Practical Evaluation of Partisan Gerrymandering." *Stanford Law Review* 68: 1263–1321.

Wang, Samuel S.-H. 2015. "Brief of Amicus Curiae Samuel S. Wang, Ph.D. in Support of Appellees." U.S. Supreme Court, *Harris v. Arizona Independent Redistricting Commission* (14-232).

Whitford v. Gill. 2016. 15-cv-421-bbc, 2016 U.S. Dist. LEXIS 160811 (W.D. Wis. November 21, 2016), jurisdiction postponed, 582 U.S. ___ (June 19, 2017) (No. 16–1161).

Veith v. Jubelirer. 2004. 541 U. S. 267.

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