STATE OF NORTH CAROLINA

COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE
SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,

REBECCA HARPER, et al.,

Plaintiffs,

v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

AFFIDAVIT OF DR. MOON DUCHIN ON REMEDIES

I, Dr. Moon Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:

1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.

2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

Background and qualifications

3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.

4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.

5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.

6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.

7. A current copy of my full CV is attached to this report.

8. I am compensated at the rate of $400 per hour.

Framework for Analysis of Remedial Districting Plans in North Carolina

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

February 18, 2022

1 Introduction

On November 4, 2021, the North Carolina General Assembly enacted three districting plans: maps of 14 U.S. Congressional districts, 50 state Senate districts, and 120 state House districts. Since then, there has been significant activity in the state court, culminating in the invalidation of those plans by the Supreme Court of North Carolina and a remand to assess and select remedial maps.

In this report, I analyze the remedial maps for Congress, state Senate, and state House proposed by the North Carolina League of Conservation Voters plaintiffs, which I will denote NCLCV-Cong, NCLCV-Sen, and NCLCV-House. These maps have excellent properties in terms of partisan balance, minority opportunity to elect candidates of choice, and the full complement of traditional districting principles that are operative in North Carolina.

Below, I will set up an analytical framework for evaluating remedial plans and will illustrate this framework by comparing the LCV plans to the now-invalidated state plans (SL-174, SL-173, SL-175). In a follow-up report, I plan to apply this analytical framework to other remedial plans submitted to the court, including the remedial plans recently enacted by the General Assembly. I have included a quantitative review of the traditional districting principles in Appendix A; below, I will focus on partisan fairness (§2-4) and on race and the Stephenson framework (§5).

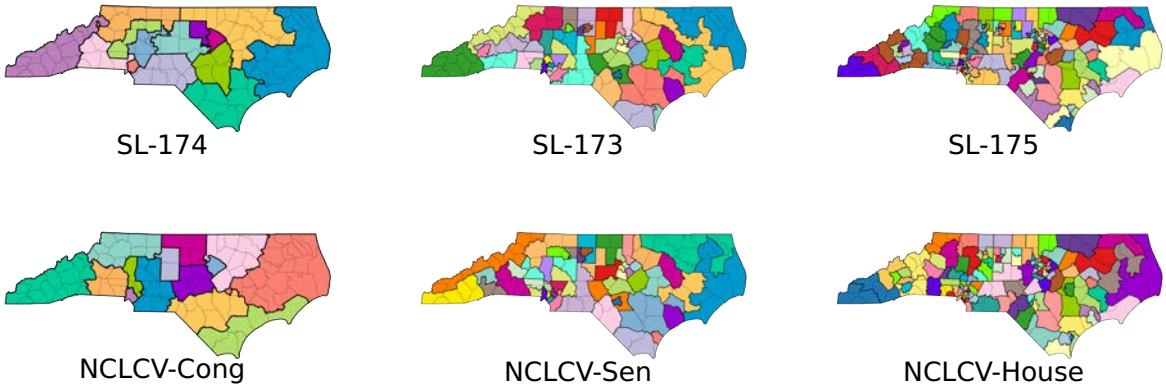


Figure 1: The six plans used to illustrate the analytical framework presented here.

2 Partisan fairness

2.1 Abstract partisan fairness

There are many notions of partisan fairness that can be found in the scholarly literature and in redistricting practitioner guides and software. Most of them are numerical, in the sense that they address *how a certain quantitative share of the vote should be translated to a quantitative share of the seats* in a state legislature or Congressional delegation. These include: disproportionality, efficiency gap (original and simplified versions), mean-median gap, partisan bias, partisan Gini, and a relatively new alternative that I am calling the Eguia county skew. All of these are discussed below in §4.

Conceptually, the numerical notions of partisan fairness all tend to agree on one central point: an electoral climate with a roughly 50-50 split in partisan preference should produce a roughly 50-50 representational split. I will call this the *Close-Votes-Close-Seats* principle. North Carolina voting has displayed a partisan split staying consistently close to even between the two major parties over the last ten years, but the plans released by the General Assembly after the 2010 census were very far from realizing the ideal of converting even voting to even representation. This time, with a 14th seat added to North Carolina's apportionment, an exactly even seat outcome is possible. In an election that is very close to even statewide, an ideally fair map would give each party, say, 6-8 Congressional seats out of 14, 23-27 Senate seats out of 50, and 55-65 House seats out of 120, with no particular lean to either side.

Importantly, Close-Votes-Close-Seats is *not* tantamount to a requirement for proportionality. Rather, it is closely related to the principle of *Majority Rule*: a party or group with more than half of the votes should be able to secure more than half of the seats. In fact, Close-Votes-Close-Seats is essentially a corollary (or byproduct) of Majority Rule. It is not practicable to design a map that *always* attains these properties, but by contrast a map that *consistently thwarts* them should be closely scrutinized and usually rejected.

Unlike proportionality, neither Close-Votes-Close-Seats nor Majority Rule has any bearing on the preferred representational outcome when one party has a significant voting advantage: these principles are silent about whether 70% vote share should secure 70% of the seats, as proportionality would dictate, or 90% of the seats, as supporters of the efficiency gap would prefer. The size of the "winner's bonus" is not at all prescribed by a Close-Votes-Close-Seats norm.

2.2 Geography and fairness

Some scholars have argued that all numerical ideals, including Close-Votes-Close-Seats, ignore the crucial *political geography*—this school of thought reminds us that the location of votes for each party, and not just the aggregate preferences, has a major impact on redistricting outcomes. In [6], my co-authors and I gave a vivid demonstration of the impacts of political geography in Massachusetts: we showed that for a ten-year span of observed voting patterns, even though Republicans tended to get over one-third of the statewide vote, it was impossible to draw a single Congressional district with a Republican majority. That is, the geography of Massachusetts Republicans locked them out of Congressional representation. It is therefore not reasonable to charge the Massachusetts legislature with gerrymandering for having produced maps which yielded all-Democratic delegations; they could not have done otherwise.

In North Carolina, this is not the case. The NC-LCV plans demonstrate that it is possible to produce maps that give the two major parties a roughly equal opportunity to elect their candidates. These plans are just examples among many thousands of plausible maps that convert voter preferences to far more even representation by party. In Congressional redistricting, present-day North Carolina geography is easily conducive to a seat share squarely in line with the vote share. In Senate and House plans, even following the strict detail of the Whole County Provisions, there are likewise many alternatives converting nearly even voting patterns to nearly even representation, across a large set of recent elections.

The clear conclusion is that the political geography of North Carolina today does not obstruct the selection of a map that treats Democratic and Republican voters fairly and even-handedly.

2.3 Millions of maps – and the median mistake

My research lab has been at the forefront of the development of *ensemble analysis*: algorithmic techniques for building large collections (or "ensembles") of alternative districting plans. In fact, most of the leading teams in this space now use a variant of the "spanning tree" method we initiated for generating new plans.

The power of ensembles is that you can understand a plan in comparison to alternatives that are drawn on the same political geography. If they are done well, the plans will be made using the same rules and criteria that were applied by the primary line-drawers.

Furthermore, the large ensembles can be scored on various metrics—like the number of seats that they would give to each party, or their partisan bias score, or anything else—and this typically gives a visually appealing bell curve, with the various plans and other benchmarks falling at various places along the curve.

To illustrate this I will start with an example from an amicus brief to the U.S. Supreme Court in *Rucho v. Common Cause* for which I was one of the amici. In this case (Figure 2), neutral is good—the General Assembly's plans look like outliers in a way that suggests *packing*, while the Judges' plan has a more typical partisan makeup.

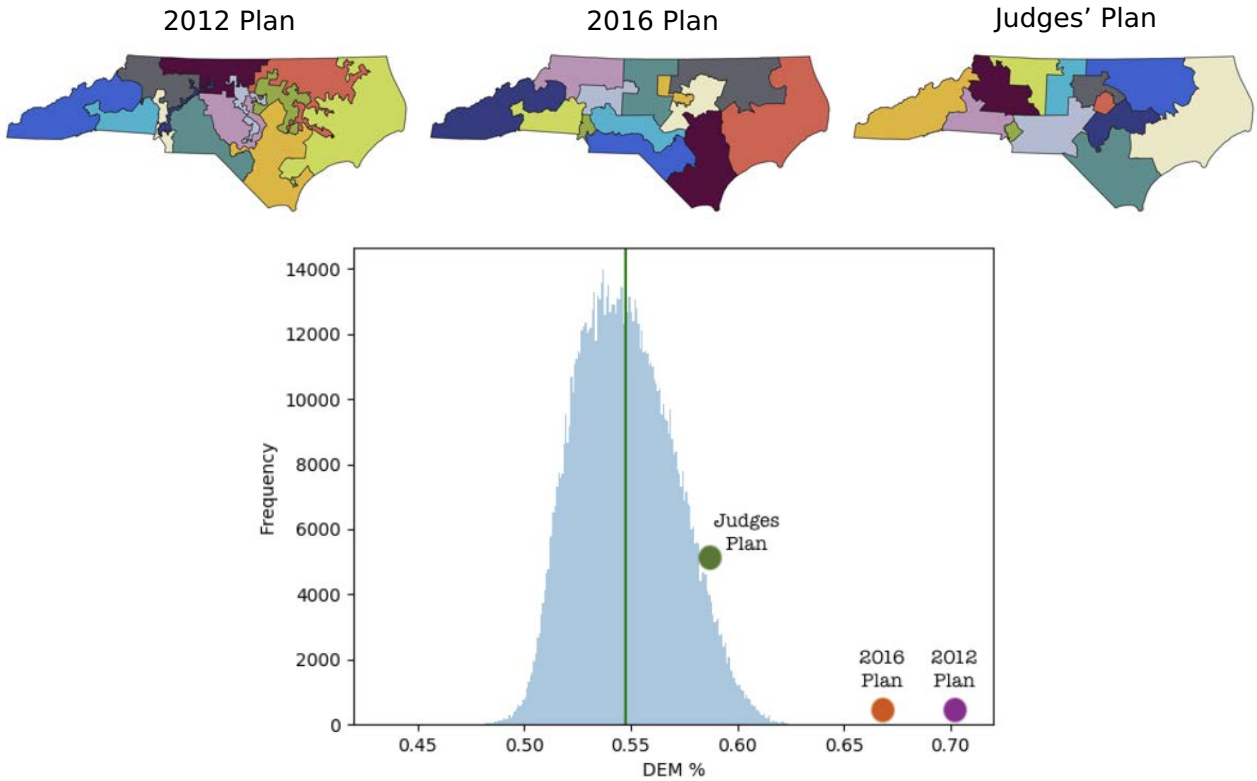


Figure 2: A hisgotram plot reproduced from the Mathematicians' Brief to the U.S. Supreme Court in *Rucho v. Common Cause*. This bell curve, based on hundreds of thousands of districting plans drawn with no partisan data, shows that the North Carolina General Assembly plans from 2012 and 2016 *pack* Democrats into the third most Democratic district. By contrast, the demonstration plan drawn by a bipartisan panel of retired judges does not.

But typical is not always best. For instance, if we had a bell curve of compactness scores, a good map wouldn't be right in the middle—it would be all the way on the extreme that represents the most compact plans. On a bell curve of county splits, we should prefer a plan that splits fewer counties to a typical map. And likewise, we do not seek a plan with typical fairness, but rather a plan that is especially fair.

To drive this home, consider the following figure, reproduced and extended from my earlier rebuttal report in the current litigation in North Carolina. In it, I have constructed a statewide ensemble of North Carolina state House plans by assembling the 50,000 plans generated by defendants' expert Michael Barber in each county cluster to make an astronomical number of maps overall.[1] The bell curve measures something that Barber calls "Democratic-leaning seats."

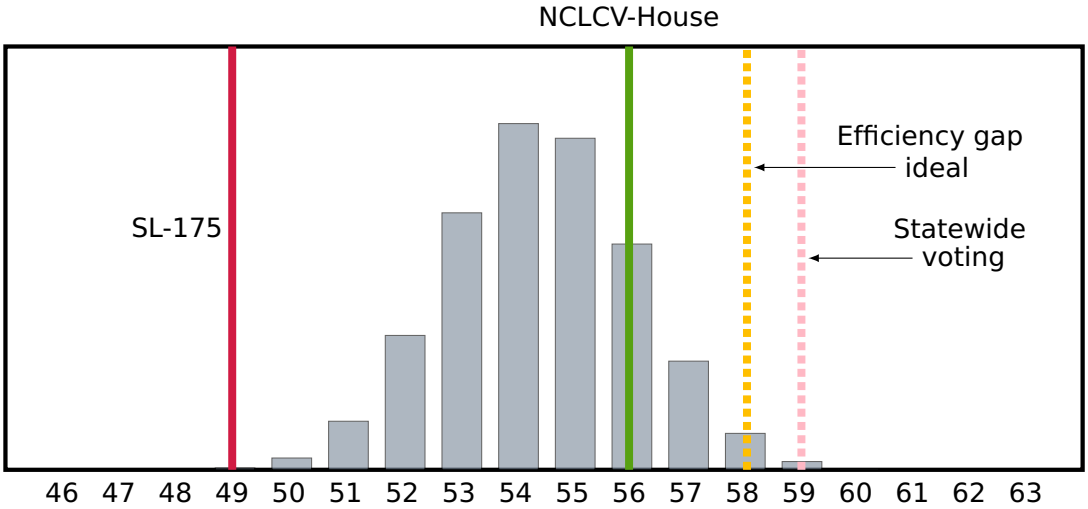


Figure 3: "Democratic-leaning seats" in Dr. Barber's House district ensemble. (Figure extended from earlier response report.) This is a case in which the fairest map might not be in the middle of the bell curve.

Dr. Barber's outputs are consistent with the often-observed fact that North Carolina, like many states, exhibits some partisan skew in blindly drawn plans, just as a function of political geography. But it would be a mistake to assume that typical is fair. The North Carolina Supreme Court has told us that we should prefer a plan in which "there is a significant likelihood that [the plan] will give the voters of all political parties substantially equal opportunity to translate votes into seats across the plan." If it is possible, while strictly following the rules and priorities in the legal framework for North Carolina redistricting, then we should certainly prefer a fair plan to a typical plan.

In other words, it would be a mistake to valorize the median, or the "top of the hill," when better maps can be made that are completely consistent with the rules and political geography of North Carolina today.

[1] In my report, I note a host of problems with his methodology, but I set that aside here to make a conceptual point about interpreting ensembles.

3 A framework for partisan fairness in North Carolina

3.1 Overlaying elections and plans

We can examine how well the Close-Votes-Close-Seats norm is upheld without invoking any predictions or assumptions about future voting behavior by using a standard technique in election analysis: pairing proposed plans with actual recent elections. This method works by overlaying (or superimposing) the districting plans on a series of observed voting patterns from the recent past; this lets us take advantage of the rich dataset of real electoral outcomes in North Carolina in the last ten years to avoid speculative or predictive modeling about voting trends in the future.[2]

The overlay method works best when there is a large set of statewide elections to apply, which is certainly true in North Carolina. Of the 52 statewide party-ID general elections from the last cycle, 29 are elections for Council of State (ten offices elected three times, with the Attorney General race uncontested in 2012), three are presidential races, three are for U.S. Senate, and 17 are judicial races since mid-decade, when those became partisan contests. See Table 1 for more detail on the election dataset.

3.2 Partisanship outcomes

North Carolina is a very "purple" state. In 38 out of the 52 contests in our dataset, the statewide partisan outcome is within a 6-point margin: 47-53 or closer.

To understand how plans perform under different electoral conditions, we will overlay the plans with voting patterns from individual elections in the past Census cycle. We can make a striking observation by laying our six plans over the vote patterns, shown in Table 1. Examining the performance of a plan as the electoral conditions vary shows many things: its responsiveness (giving sufficient variation in the seats as the votes change), its tendency to avoid anti-majoritarian outcomes, and so on. The NC-LCV Congressional plan (NCLCV-Cong) is far more faithful to the vote share, far more responsive, and tends to award more seats to the party with more votes—usually upholding both basic small-d-democratic principles of Majority Rules and Close-Votes-Close-Seats, which were violated by the invalidated plans. The same patterns are visible at the Senate and House level.

By every measure considered above that corresponds to a clear legal or good-government redistricting goal or value, the NC-LCV plans have excellent performance. This demonstrates that it is possible, without any cost to the redistricting principles in play, to select maps that are fair to the voters of North Carolina.

Below, the outcomes of overlaying the plans on the elections will be presented in a series of tables and figures. First, Table 1 overviews the overlays with numbers, repeated from my earlier reports (which included the backup data).[3]

Then, Figure 4 offers a visualization to depict the same big picture of votes versus seats with the full 52-election dataset. The diagonals show various lines of *responsiveness* that pivot around the central point of fairness: half of the votes securing half of the seats.

[2] Many authors have used this technique of overlaying "exogenous" statewide elections rather than using statistical regressions and other modeling to manipulate "endogenous" districted elections. For instance this can be found in peer-reviewed work and expert reports of scholar-practitioners such as Bernard Grofman and Steven Ansolabehere.

[3] Codes for reading Table 1: AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; LTG = Lieutenant Governor; PRS = President; SEN = Senator; SOS = Secretary of State; SUP = Superintendent of Public Instruction; TRS = Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), JS* are elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals. Where there was more than one judicial candidate from a given party on the ballot, they were combined for this analysis. The two-digit suffix designates the election year.

Do close votes translate to close seats?

The table records the number of districts in each plan with a Democratic win. This shows that the enacted maps systematically violate the principles of Close-Votes-Close-Seats and Majority Rule.

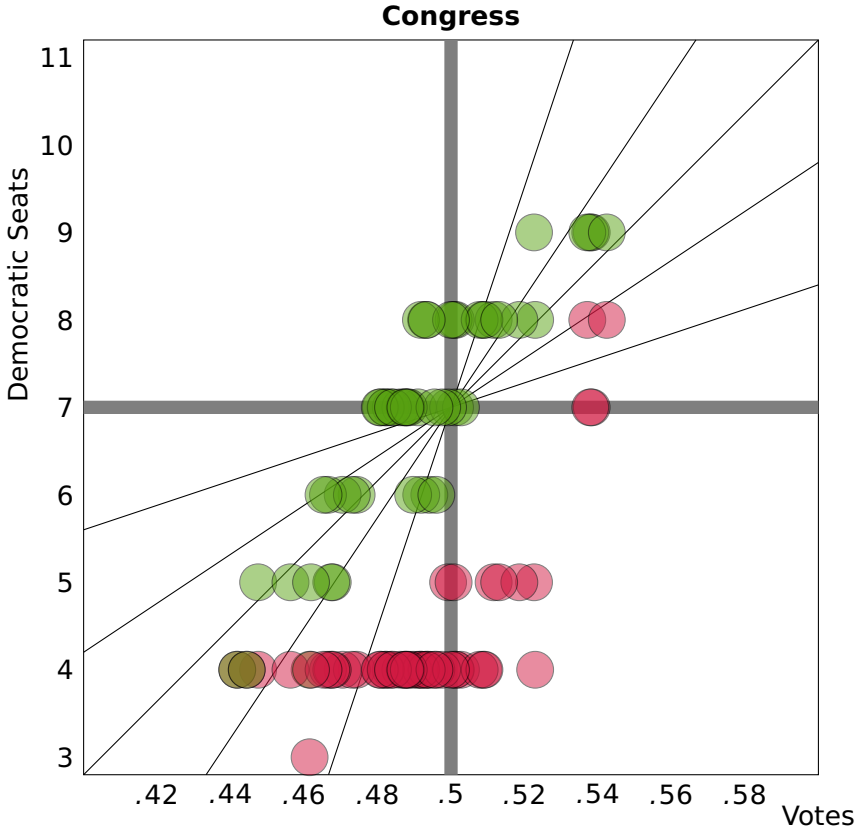
	D Vote Share	SL-174	NCLCV-Cong	SL-173	NCLCV-Sen	SL-175	NCLCV-House
GOV12	0.4418	4	4	16	18	41	44
AGC16	0.4444	4	4	17	17	40	42
LAC16	0.4475	4	5	18	20	42	45
JHU16	0.4563	4	5	18	19	42	49
AGC20	0.4615	3	4	17	19	40	51
JZA16	0.4619	4	5	19	21	43	50
JDI16	0.4653	4	6	19	21	44	53
LTG16	0.4665	4	6	19	21	44	54
LAC12	0.4674	4	5	20	20	44	51
AGC12	0.4678	4	5	18	18	43	50
SEN16	0.4705	4	6	19	21	43	55
TRS16	0.4730	4	6	19	21	45	53
TRS20	0.4743	4	6	17	20	45	51
JA620	0.4806	4	7	17	21	46	55
PRS16	0.4809	4	7	19	22	48	56
JA420	0.4822	4	7	17	22	47	56
INC20	0.4823	4	7	18	23	47	56
LTG20	0.4836	4	7	18	21	46	55
JA720	0.4842	4	7	17	22	48	56
SUP20	0.4862	4	7	19	23	49	56
JA520	0.4874	4	7	18	22	49	57
JA218	0.4876	4	7	18	22	45	55
JS420	0.4879	4	7	19	24	49	56
J1320	0.4885	4	7	19	23	49	56
PRS12	0.4897	4	6	20	21	46	55
SEN20	0.4910	4	7	20	24	48	56
LAC20	0.4918	4	8	21	25	51	58
SEN14	0.4919	4	6	20	22	46	52
PRS20	0.4932	4	8	20	25	50	60
JS220	0.4934	4	8	21	24	51	59
SUP16	0.4941	4	6	22	23	49	57
JS118	0.4955	4	7	20	25	50	58
INC16	0.4960	4	6	22	22	50	57
JST16	0.4976	4	7	21	23	50	58
LTG12	0.4992	5	7	22	22	50	58
JS120	0.5000	4	8	22	27	52	60
AUD16	0.5007	5	8	22	23	51	56
GOV16	0.5011	4	7	20	27	50	58
ATG20	0.5013	4	8	21	25	51	58
ATG16	0.5027	4	7	20	23	50	57
JA118	0.5078	4	8	22	26	51	58
AUD20	0.5088	4	8	24	28	54	61
JA318	0.5091	4	8	21	26	52	59
SOS20	0.5116	5	8	24	28	53	62
JGE16	0.5131	5	8	22	25	52	59
INC12	0.5186	5	8	22	22	55	61
SOS16	0.5226	5	9	24	24	57	62
GOV20	0.5229	4	8	23	27	58	63
AUD12	0.5371	8	9	27	28	61	65
SOS12	0.5379	7	9	26	26	59	63
TRS12	0.5383	7	9	25	24	59	65
SUP12	0.5424	8	9	28	28	61	66

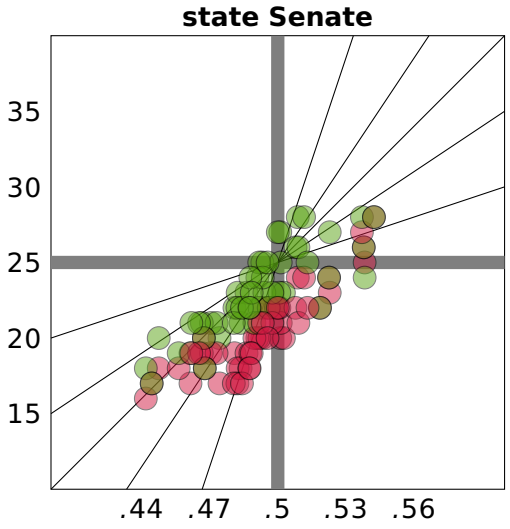
53 – 47 or closer

Table 1: 52 general elections, sorted from lowest to highest Democratic share. Recall that Close-Votes-Close-Seats can be read as calling for the highlighted elections to have close outcomes—say, 6-8 Congressional seats, 23-27 Senate seats, and 55-65 House seats. (Key to election naming convention is provided on page 7.)

Seats vs. Votes

Majority Rule says that outcomes should tend to fall in the Northeast and Southwest quadrants, avoiding the Southeast and Northwest. Close-Votes-Close-Seats says that points should not miss the bulls-eye near the center by systematically deviating to the North or the South. These principles are clearly upheld by the alternative plans (**green**) and violated by the enacted plans (**maroon**).





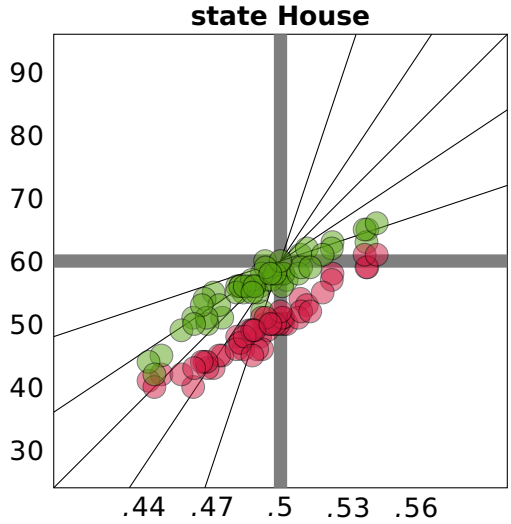


Figure 4: On these seats-vs.-votes plots, we see the election results when overlaying the six maps on the 52 general election contests in the last decade; each colored dot is plotted as the coordinate pair (vote share, seat share).

4 Summary scores of partisan fairness

Though a holistic approach to gauging partisan opportunity (such as I presented in the previous section) is strongly recommended as a primary matter, it can still be very useful to turn to simple scores and metrics to paint a supporting quantitative picture. It would be unreasonable to expect any individual metric of partisan fairness to perfectly distill the holistic picture in a bug-free and un-gameable way,[4] but as with compactness scores, looking at a suite of metrics in combination creates a strong overall narrative.

4.1 Vote share/seat share metrics

First I will explain the scores that are denominated in vote shares or seat shares, to be presented in Table 2.

Efficiency gap is the difference in "wasted" votes for the two parties, across the state, as a share of votes cast. Because the authors realized that it was sensitive to turnout effects, they later advocated for a *simplified efficiency gap* formula $EG = 2V - S - \frac{1}{2}$, where V is the vote share in an election and S is the seat share. Original efficiency gap and simplified efficiency gap would be exactly equal if the districts had equal turnout; it's the simplified formula that was used, for example, in the language for the Freedom To Vote act. The authors proposed .08, later refined to .07, as the flag for a presumptive gerrymander.[5]

Partisan symmetry is a family of scores based on the principle of table-turning: if the votes for the parties were reversed, would the representation also be reversed? An asymmetric plan is one in which one party fares better with its portion of support than the other party would with the same portion. Scores in this group include the *mean-median gap*, the *partisan bias* score, and the *partisan Gini*. The mean-median gap literally takes the difference between the average vote share in a district and the median, or middle, district (or the average of the two middle districts when the number of districts is even). The gap is zero when the middle district looks like the state as a whole, so that half the districts are more favorable to one party and half are more favorable to the other. Partisan bias is described in the literature as measuring how much "extra" representation each party would secure in a hypothetical 50-50 election.

The last metric I am presenting in the seat share/vote share collection is a county skew metric based on economist Jon Eguia's "jurisdictional partisan advantage" [7]. Eguia built a metric based on comparing the actual representation secured by a party under a vote pattern to the representation if towns and counties played the role of districts. I have applied it here only to counties, because of the fundamental importance of counties in North Carolina redistricting in particular. A simple way to explain this Eguia-style metric is as follows: in a particular election, what percentage of North Carolinians live in counties that favored Republicans? That is the benchmark for Republican representation; if their seat share is higher, the map is tilted Republican, and if lower, the map is tilted Democratic.

For all five of these scores, zero is ideal.

[4] Scholars who study the metrics, including myself, have explained blind spots and loopholes in each individual score. A map with extreme partisan advantage can sometimes be drawn with good partisan symmetry scores, for instance, by careful tuning; but it is unlikely that such a map would also have good efficiency gap scores. Likewise, there are cases where the scores are inappropriate entirely, like in small states with lopsided voting. North Carolina is not such a state. See [5] for more discussion.

[5] In paragraph 167 of the North Carolina Supreme Court's recent decision in this case, it is noted that "With regard to the efficiency gap measure, courts have found "that an efficiency gap above 7% in any districting plan's first election year will continue to favor that party for the life of the plan."" (Quoting the U.S. Supreme Court, from *Whitford v. Gill*).

	NCLCV-Cong			SL-174		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	0.006	0.001	−0.001	−0.167	−0.159	−0.181
Simplified *EG*	0.011	0.005	0.003	−0.17	−0.163	−0.186
Mean-median	0.007	0.006	0.007	−0.047	−0.044	−0.045
Partisan Bias	0.036	0.029	0.031	−0.192	−0.184	−0.204
Eguia County Skew	−0.006	−0.009	−0.006	−0.188	−0.176	−0.195

	NCLCV-Sen			SL-173		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.02	−0.024	−0.017	−0.075	−0.068	−0.08
Simplified *EG*	−0.023	−0.028	−0.021	−0.076	−0.070	−0.081
Mean-median	−0.009	−0.012	−0.009	−0.036	−0.036	−0.037
Partisan Bias	−0.015	−0.023	−0.016	−0.072	−0.069	−0.08
Eguia County Skew	−0.040	−0.041	−0.030	−0.093	−0.083	−0.09

	NCLCV-House			SL-175		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
Efficiency Gap	−0.02	−0.022	−0.017	−0.076	−0.075	−0.078
Simplified *EG*	−0.014	−0.016	−0.012	−0.074	−0.074	−0.077
Mean-median	−0.015	−0.015	−0.017	−0.039	−0.039	−0.04
Partisan Bias	−0.018	−0.019	−0.018	−0.082	−0.082	−0.086
Eguia County Skew	−0.031	−0.030	−0.021	−0.091	−0.088	−0.086

Table 2: Five simplified scores of partisan fairness, averaged over different sets of elections. These five metrics are all *signed*, meaning that they can take positive or negative values; positive and negative scores are intended to flag an advantage to Democrats and Republicans, respectively. *EG* and *MM* are computed as a share of votes; *PB* and the Eguia score are computed as a share of seats. See text for an explanation of scores.

4.2 Big-picture scores

Next, Table 3 zooms out to the bigger picture of overall partisan properties. Here, we see the typical number of seats that would be won by either party under these maps, across different election sets.

Disproportionality then indicates how far that seat total is from reflecting the statewide vote, with negative signs indicating an advantage to Republicans.

Finally, *partisan Gini* is a summary statistic for all of the various kinds of symmetry measures in the political science literature. The "Partisan Symmetry Standard" of King and his co-authors asks that a seats-votes curve be literally symmetric about the center point, meaning that it predicts exactly the same representation for either party at any share of the vote [8]. The partisan Gini, first proposed by Bernard Grofman in 1983, takes this literally, measuring the area between the curve and its mirror image [9]. This is an unsigned metric, with zero as an ideal. (When the *PG* score is zero, all other symmetry scores, like mean-median and partisan bias, are necessarily zero as well.)

	NCLCV-Cong			SL-174		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	6.9	6.9	6.7	4.4	4.5	4.1
R Seats	7.1	7.1	7.3	9.6	9.5	9.9
Disproportionality	0.0	0.0	−0.1	−2.5	−2.4	−2.8
Partisan Gini	0.021	0.020	0.021	0.078	0.073	0.080

	NCLCV-Sen			SL-173		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	23.0	22.9	22.8	20.3	20.8	19.8
R Seats	27.0	27.1	27.2	29.7	29.2	30.2
Disproportionality	−1.6	−1.7	−1.6	−4.2	−3.9	−4.6
Partisan Gini	0.026	0.026	0.026	0.051	0.049	0.054

	NCLCV-House			SL-175		
	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)	All (52 contests)	Non-judicial (35 contests)	Up-ballot (14 contests)
D Seats	56.2	56.3	55.8	49.0	49.3	47.9
R Seats	63.8	63.7	64.2	71.0	70.7	72.1
Disproportionality	−2.7	−2.9	−2.8	−10.0	−9.8	−10.7
Partisan Gini	0.032	0.031	0.034	0.050	0.050	0.053

Table 3: The average number of Dem seats, the disproportionality of that seat total, and a partisan symmetry metric called the Partisan Gini (*PG*) that tells you how far the districting plan is from being *symmetric* in the sense of King et al.

Slicing the same data another way, the pattern is clear.

	Up-ballot generals (14)		**All generals (52)**	
	D vote share	D seat share	D vote share	D seat share
SL-174	.4883	.2908	.4911	.3118
NCLCV-Cong		.4796		.4931
SL-173	.4883	.3957	.4911	.4065
NCLCV-Sen		.4557		.4592
SL-175	.4883	.3994	.4911	.4080
NCLCV-House		.4649		.4684

Table 4: Comparing overall fidelity of representation to the voting preferences of the electorate. Vote shares are computed with respect to the major-party vote total.

5 Race and the Stephenson framework

North Carolina has a large minority of Black-identified residents. Over two million North Carolinians—2,107,526 out of 10,439,388 to be precise, or about 20.2%—were identified as non-Hispanic Black-alone on the Census. Within the voting age population, the numbers shift to 1,620,569 out of 8,155,099, or about 19.9%. Increasing numbers of Americans identify as Black in combination with other races and/or Hispanic ethnicity. Passing to this more expansive definition of Black raises the voting age population numbers to 1,743,052 out of 8,155,099, or 21.4%. Other minority groups, while their population can be substantial, are rarely concentrated enough to be in the majority in a district.[6]

Minority groups' opportunity to elect candidates of choice is protected by both state and federal law. A detailed assessment of opportunity must not primarily hinge on the demographics of the districts, but must also rely on electoral history and an assessment of polarization patterns.[7]

In North Carolina, the nine-step *Stephenson/Dickson framework* addresses how to reconcile the Whole County Provisions—which apply only to state legislative districts—with one-person, one-vote requirements and the Voting Rights Act (VRA). The first step of the framework is assessing whether the VRA requires the drawing of a certain number of effective districts. I will include an assessment of VRA requirements, and will conclude that the choice of "county clustering" that was made in the state's plans (and maintained in the NC-LCV plans) is consistent with the obligations imposed by the Stephenson framework.

5.1 Demonstration of majority-Black districts (Gingles 1)

Using Markov chain techniques related to those that are often employed to build ensembles of plans, I have confirmed that it is possible to draw Senate maps with five districts that exceed 50% Black voting age population, using the any-part-Black definition (Black alone or in combination). I was unable to find a Senate map with a sixth majority-BVAP district. Likewise, I was able to achieve 17 simultaneous majority-BVAP districts in a House plan in many distinct ways, but never 18. Figure 5 shows where in the state those Gingles districts are located.

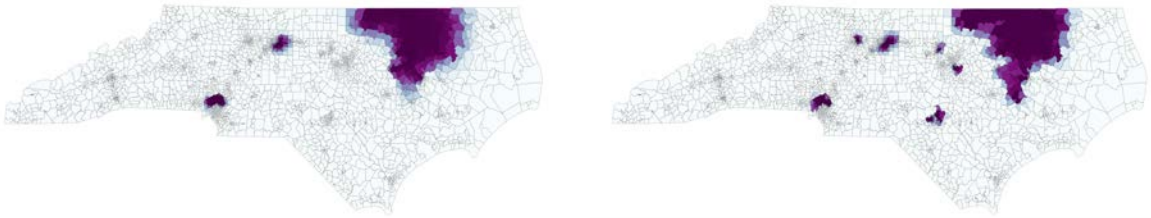


Figure 5: These heatmaps show the VTDs that are most likely to be in majority-BVAP districts in plans that have 5 such districts in the Senate (left) or 17 such districts in the House (right).

Location of "Gingles districts" in Senate maps (adding to 5)

- Mecklenburg: 2 districts
- Guilford: 1 district
- rural Northeast: 2 districts

[6] An exception is American Indian/Native American population; respondents selecting AMIN identity make up 50.4% of the population in the NC-LCV House district 47, for instance.

[7] A detailed discussion of the inadequacy of using demographics alone as a proxy can be found in [3].

Location of "Gingles districts" in House maps (adding to 17)

- Mecklenburg: 4-5 districts
- Guilford: 2-3 districts
- Cumberland: 2 districts
- Wake: 1-2 districts
- Durham: 1 district
- Forsyth: 1 district
- rural northeast: 4-6 districts

5.2 Polarization (Gingles 2-3)

I have used industry-leading techniques called *ecological inference*[8] to study the racial polarization patterns in North Carolina general and primary elections from the last decade, and I have corroborated those findings with ecological regression. The results indicate a consistent pattern of polarization in statewide general elections, such that White voters are estimated to support the Republican candidate at a rate of over 61% in every general election, and Black voters are estimated to support the Democratic candidate at a rate of over 94% each time. Polarization is present in many Democratic primary elections as well, particularly in elections in which there is a Black Democratic candidate. I confirmed these polarizaton patterns in individual regions around the state as well as at the whole-state level. A full set of EI/ER outputs is available in my backup materials from earlier reports.

5.3 Effectiveness

I have designated a selection of eight elections—four generals and four primaries—chosen to be particularly informative in determining whether Black voters have an opportunity to elect their candidates of choice.[9] These eight contests were chosen by a combination of factors that combine to make an election particularly informative with respect to the preferences of Black voters. Namely: I prioritized elections that are more recent, that have a Black candidate on the ballot, that are clearly polarized, and that are close enough to produce variation at the district level. Recency, polarization, and the presence of a Black candidate on the ballot are recognized as contributing to higher probativity by a well-established consensus in case law around Gingles 2-3, and the fact that landslide and uncontested elections are less informative is obvious on first principles.

The electoral alignment score derived from these elections is a value from 0 to 8. I consider a district in which the Black candidate of choice prevails in at least 6 of these 8 contests to be aligned with Black voting preferences in the state.[10] Six out of eight is not an arbitrary fraction: it ensures that at least half of primary contests *and* at least half of general contests are electing the Black candidate of choice. Any valid attempt to replicate this analysis must also retain that property.

[8] In particular, I used the $R \times C$ multinomial Dirichlet model, as implemented in PyEI.

[9] The Black-preferred candidates are: Sutton in the 2020 Superintendent primary, Smith in the 2020 Ag. Commissioner primary, Williams in the 2016 Attorney General primary, Coleman in the 2016 Lt. Governor primary; Holley in the 2020 Lt. Governor general, Cunningham in the 2020 U.S. Senate general, Coleman in the 2016 Lt. Governor general, and Blue in the 2016 Treasurer general. Of these, Sutton, Williams, Coleman, Colley, and Blue are themselves Black-identified.

[10] I have used statewide ecological inference ("EI") runs to determine the candidate of choice for Black voters. I note that it is also possible to run EI on smaller geographies (such as counties or county clusters) to detect regional candidates of choice rather than statewide candidates of choice; in most cases, these will be the same, but in some cases, regional effects may be meaningful and could affect these results at the margin.

If, in addition, at least 25% of the voting age population is Black, then I label the district to be effective for Black voters. This 25% threshold is emphatically not chosen as an estimate of the level of BVAP required for performance; rather, it is strictly to confirm that there is a substantial number of Black voters in the district to benefit from the electoral alignment with their preferences.[11]

I note that the use of electoral history is not just cosmetic: there are House-sized districts with 35-39% BVAP that are nonetheless not labeled effective in these lists because they fall short of the standard of inclining to the Black candidate of choice in at least six out of the eight chosen elections.

At all three levels, the NCLCV alternative maps provide substantial numbers of effective opportunity-to-elect districts for Black voters.

Effective districts for Black voters in NC-LCV plans
CD 2, 4, 9, 11 (4 districts)
SD 1, 5, 11, 14, 18, 19, 26, 27, 32, 38, 39, 40 (12 districts)
HD 2, 8, 9, 10, 23, 24, 25, 27, 31, 32, 33, 38, 39, 40, 42, 43, 44, 45, 48, 57, 58, 59, 60, 61, 63, 66, 71, 88, 92, 99, 100, 101, 102, 106, 107, 112 (36 districts)

5.4 Conformance with Stephenson framework

The NC-LCV plans have a greater number of effective opportunity districts than the number that can be drawn to meet the Bartlett threshold for Gingles 1.

In the Senate, the Gingles plans have two majority-BVAP districts in **Mecklenburg**; the NC-LCV plan has three effective districts there (SD 38, 39, 40). The Gingles plans have one majority-BVAP district in **Guilford**; the NC-LCV plan has two effective districts in that cluster, with SD 27 wholly in Guilford County and SD 26 mostly in Guilford. The Gingles plans have two majority-BVAP districts in the **rural Northeast**, and the NC-LCV plan has three effective districts in that region (SD 1, 5, 11).

Likewise, the obligations are met in the House. **Mecklenburg** has 4-5 Gingles districts, and the NC-LCV plan has 8 effective districts in the county (HD 10, 88, 92, 99, 100, 101, 102, and 107). **Guilford** has 2-3 Gingles districts, and the NC-LCV plan has five effective districts in the county (HD 57, 58, 59, 60, and 61). **Cumberland** has 2 Gingles districts, and the NC-LCV plan has four effective districts there (HD 42, 43, 44, 45). **Durham** has 1 Gingles district, and the NC-LCV plan has two effective districts there, one wholly in Durham County (HD 31) and one partially in the county (HD 2). **Forsyth** has 1 Gingles district, and the NC-LCV plan has one as well (HD 71). **Wake** has 1-2 Gingles districts, and the NC-LCV plan has four effective districts (HD 33, 38, 39, 40). The **rural Northeast** has 4-6 Gingles districts, and the NC-LCV plan has seven effective districts there (HD 8, 9, 23, 24, 25, 27, and 32).

This analysis makes it clear that not only as an overall statewide matter, but on an individual regional basis, the NC-LCV maps create a larger number of districts that obtain effective opportunity for Black voters to elect candidates of choice than the numerical standard set by the Gingles-via-Bartlett threshold of 50% BVAP. This also illustrates that this choice of county clustering did not obstruct the creation of maps with adequate numbers of effective districts.

This analysis provides an excellent illustration that effective opportunity can frequently be found without a numerical majority of the population, and that urban areas in particular can make good use of overall diversity and less extreme polarization to afford configurations that are especially favorable to minority opportunity to elect. The BVAP tables in Appendix B can be compared to the list of effective districts to underscore this point.

[11] I performed a robustness check and confirmed that lowering the threshold to 20% makes no difference at all, and raising it to 30% would only drop the NC-LCV effective district count by a single House district and no Senate districts, leaving the analysis materially unchanged.

References

[1] Assaf Bar-Natan, Elle Najt, and Zachary Schutzmann, *The gerrymandering jumble: Map projections permute districts' compactness scores*. Cartography and Geographic Information Science, Volume 47, Issue 4, 2020, 321–335.

[2] Richard Barnes and Justin Solomon, *Gerrymandering and Compactness: Implementation Flexibility and Abuse.* Political Analysis, Volume 29, Issue 4, October 2021, 448–466.

[3] Amariah Becker, Moon Duchin, Dara Gold, and Sam Hirsch, *Computational redistricting and the Voting Rights Act.* Election Law Journal. Volume 20, Number 4, 2021. `https://www.liebertpub.com/doi/epdf/10.1089/elj.2020.0704`

[4] Christopher Cooper, Blake Esselstyn, Gregory Herschlag, Jonathan Mattingly, and Rebecca Tippett, *NC General Assembly County Clusterings from the 2020 Census.* `https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf`

[5] Daryl DeFord, Natasha Dhamankar, Moon Duchin, Varun Gupta, Mackenzie McPike, Gabe Schoenbach, Ki Wan Sim, *Implementing partisan symmetry: Problems and paradoxes*. Political Analysis, to appear 2022.
Preprint available at `https://arxiv.org/pdf/2008.06930.pdf`.

[6] Moon Duchin, Taissa Gladkova, Eugene Henninger-Voss, Heather Newman, and Hannah Wheelen, *Locating the Representational Baseline: Republicans in Massachusetts.* Election Law Journal, Volume 18, Number 4, 2019, 388–401.
`https://www.liebertpub.com/doi/pdf/10.1089/elj.2018.0537`

[7] Jon X. Eguia, *A Measure of Partisan Advantage in Redistricting*. Election Law Journal, published online October 8, 2021. `https://doi.org/10.1089/elj.2020.0691`

[8] J.N. Katz, G. King, E. Rosenblatt, *Theoretical Foundations and Empirical Evaluations of Partisan Fairness in District-Based Democracies.* American Political Science Review, Volume 114, Issue 1, 2020, 164–178.

[9] Bernard Grofman, *Measures of Bias and Proportionality in Seats-Votes Relationships*. Political Methodology, Vol. 9, No. 3 (1983), 295–327.

A Traditional districting principles

Principles that are relevant to North Carolina redistricting include the following.

- **Population balance.** The standard interpretation of *One Person, One Vote* for Congressional districts is that districts should be fine-tuned so that their total Census population deviates by no more than one person from any district to any other.

  There is more latitude with legislative districts; they typically vary top-to-bottom by no more than 10% of ideal district size. In North Carolina, the Whole County Provisions make it very explicit that 5% deviation must be tolerated if it means preserving more counties intact.

  All six plans have acceptable population balance.

Population deviation

	Max Positive Deviation	District	Max Negative Deviation	District
SL-174	0	(eight districts)	−1	(six districts)
NCLCV-Cong	0	(eight districts)	−1	(six districts)
SL-173	10,355 (4.960%)	5	−10,434 (4.997%)	13,18
NCLCV-Sen	10,355 (4.960%)	5	−10,427 (4.994%)	15
SL-175	4250 (4.885%)	18	−4189 (4.815%)	112
NCLCV-House	4341 (4.990%)	82	−4323 (4.969%)	87

Table 5: Deviations are calculated with respect to the rounded ideal district populations of 745,671 for Congress, 208,788 for Senate, and 86,995 for House.

- **Contiguity.** All six plans are contiguous; for each district, it is possible to transit from any part of the district to any other part through a sequence of census blocks that share boundary segments of positive length. As is traditional in North Carolina, contiguity through water is accepted.

- **Compactness.** The two compactness metrics most commonly appearing in litigation are the *Polsby-Popper score* and the *Reock score*. Polsby-Popper is the name given in redistricting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula $4\pi A/P^2$. Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1. Reock is a different measurement of how much a shape differs from a circle: it is computed as the ratio of a region's area to that of its circumcircle, defined as the smallest circle in which the region can be circumscribed. From this definition, it is clear that it too is optimized at a value of 1, which is achieved only by circles.

  These scores depend on the contours of a district and have been criticized as being too dependent on map projections or on cartographic resolution [1, 2]. Recently, some mathematicians have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built. The most commonly cited discrete score for districts is the *cut edges score*, which counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations. This score improves on the contour-based scores by better controlling for factors like coastline and other natural boundaries, and by focusing on the units actually available to redistricters rather than treating districts like free-form Rorschach blots.

The alternative plans are significantly more compact than the enacted plans in all three compactness metrics.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
SL-174	5194	0.303	0.417
NCLCV-Cong	4124	0.383	0.470
SL-173	9702	0.342	0.416
NCLCV-Sen	9249	0.369	0.428
SL-175	16,182	0.351	0.437
NCLCV-House	13,963	0.414	0.465

Table 6: Comparing compactness scores via one discrete and two contour-based metrics. These scores were computed using dissolved districts based on the census blocks that were assigned in the plans under discussion.

- **Respect for political subdivisions.** For legislative redistricting, North Carolina has one of the strongest requirements for county consideration of any state in the nation. In my understanding, courts have interpreted the Whole County Provisions as follows.[12]
  - First, if any county is divisible into a whole number of districts that will be within ±5% of ideal population, then it must be subdivided accordingly without districts crossing into other counties.
  - Next, seek any contiguous grouping of two counties that is similarly divisible into a whole number of districts.
  - Repeat for groupings of three, and so on, until all counties are accounted for.

  Once clusters have been formed, there are more rules about respecting county lines within clusters. The legal language is again explicit: "[T]he resulting interior county lines created by any such groupings may be crossed or traversed in the creation of districts within said multi-county grouping but only to the extent necessary" to meet the ±5% population standard for districts. To address this, I have counted the *county traversals* in each plan, i.e., the number of times a district crosses between adjacent counties within a grouping.

  Table 7 reflects the county integrity metric that is most relevant at each level: the enacted congressional plan splits 11 counties into 25 pieces while the alternative plan splits 13, but splits no county three ways. (The enacted plans unnecessarily split three counties into three pieces.) In the legislative plans, the law specifies traversals as the fundamental integrity statistic.

[12] A complete set of solutions is described in detail in the white paper of Mattingly et al.—though with the important caveat that the work "does not reflect... compliance with the Voting Rights Act" [4]. Absent a VRA conflict, the 2020 Decennial Census population data dictates that the North Carolina Senate plan must be decomposed into ten single-district fixed clusters and seven multi-district fixed clusters (comprising 2, 2, 3, 3, 4, 6, and 6 districts, respectively). It has four more areas in which there is a choice of groupings. In all, there are sixteen different possible clusterings for Senate, each comprising 26 county clusters. The House likewise has 11 single-district fixed clusters and 22 multi-district fixed clusters (with two to thirteen districts per cluster), together with three more areas with a choice of groupings. In all, the House has only eight acceptable clusterings, each comprising 40 county clusters. An analysis of whether the clustering used in the LCV maps is consistent with VRA principles can be found in §5 of the current report.

County and municipality preservation

	# county pieces
SL-174	25
NCLCV-Cong	26

	# traversals
SL-173	97
NCLCV-Sen	89
SL-175	69
NCLCV-House	66

	# municipal pieces (considering all blocks)	# municipal pieces (considering populated blocks)
SL-174	90	50
NCLCV-Cong	58	41
SL-173	152	91
NCLCV-Sen	125	100
SL-175	292	222
NCLCV-House	201	173

Table 7: Comparing the plans' conformance to political boundaries.

B BVAP across the districts of the NC-LCV plans

NCLCV-Cong		
CD	B1VAP Share	APBVAP Share
1	0.289	0.304
2	0.332	0.347
3	0.118	0.131
4	0.319	0.344
5	0.226	0.245
6	0.227	0.242
7	0.115	0.128
8	0.123	0.132
9	0.277	0.298
10	0.232	0.25
11	0.271	0.289
12	0.121	0.132
13	0.114	0.124
14	0.032	0.039

Table 8: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the NC-LCV Congressional plan.

NCLCV-Sen		
SD	B1VAP Share	APBVAP Share
1	0.408	0.423
2	0.165	0.175
3	0.253	0.267
4	0.334	0.35
5	0.385	0.403
6	0.13	0.153
7	0.125	0.138
8	0.12	0.128
9	0.228	0.239
10	0.154	0.167
11	0.352	0.366
12	0.189	0.206
13	0.175	0.188
14	0.312	0.332
15	0.136	0.152
16	0.08	0.092
17	0.091	0.104
18	0.323	0.347
19	0.439	0.481
20	0.22	0.237
21	0.176	0.195
22	0.364	0.382
23	0.155	0.167
24	0.278	0.296
25	0.165	0.178
26	0.332	0.35
27	0.297	0.317
28	0.282	0.303
29	0.171	0.18
30	0.084	0.092
31	0.122	0.135
32	0.329	0.35
33	0.14	0.149
34	0.184	0.202
35	0.105	0.116
36	0.04	0.046
37	0.104	0.115
38	0.354	0.377
39	0.4	0.426
40	0.376	0.402
41	0.116	0.131
42	0.224	0.24
43	0.181	0.194
44	0.129	0.138
45	0.065	0.074
46	0.054	0.06
47	0.028	0.035
48	0.046	0.054
49	0.044	0.052
50	0.014	0.02

Table 9: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the NC-LCV Senate plan.

NCLCV-House		
HD	B1VAP Share	APBVAP Share
1	0.266	0.277
2	0.335	0.351
3	0.189	0.203
4	0.219	0.23
5	0.369	0.386
6	0.216	0.24
7	0.221	0.235
8	0.333	0.353
9	0.343	0.362
10	0.349	0.37
11	0.112	0.13
12	0.373	0.385
13	0.078	0.088
14	0.112	0.134
15	0.173	0.202
16	0.106	0.116
17	0.178	0.192
18	0.13	0.144
19	0.055	0.06
20	0.04	0.048
21	0.084	0.096
22	0.272	0.285
23	0.519	0.534
24	0.371	0.386
25	0.383	0.398
26	0.173	0.189
27	0.502	0.519
28	0.158	0.171
29	0.325	0.345
30	0.243	0.26
31	0.404	0.427
32	0.42	0.434
33	0.321	0.343
34	0.093	0.104
35	0.093	0.105
36	0.058	0.069
37	0.109	0.122
38	0.305	0.324
39	0.311	0.332
40	0.316	0.339
41	0.085	0.096
42	0.384	0.415
43	0.348	0.379
44	0.365	0.411
45	0.378	0.417
46	0.282	0.295
47	0.209	0.223
48	0.346	0.371
49	0.153	0.171
50	0.174	0.185
51	0.102	0.111
52	0.199	0.212
53	0.142	0.154
54	0.137	0.149
55	0.255	0.268
56	0.096	0.111
57	0.369	0.392
58	0.363	0.386
59	0.351	0.371
60	0.286	0.304

NCLCV-House		
HD	B1VAP Share	APBVAP Share
61	0.457	0.486
62	0.115	0.127
63	0.277	0.295
64	0.114	0.126
65	0.184	0.194
66	0.31	0.336
67	0.126	0.134
68	0.072	0.081
69	0.093	0.105
70	0.065	0.072
71	0.323	0.35
72	0.371	0.393
73	0.179	0.198
74	0.108	0.12
75	0.18	0.194
76	0.199	0.21
77	0.052	0.058
78	0.081	0.089
79	0.073	0.081
80	0.099	0.108
81	0.083	0.09
82	0.183	0.2
83	0.119	0.132
84	0.154	0.166
85	0.029	0.034
86	0.057	0.064
87	0.045	0.053
88	0.32	0.341
89	0.069	0.077
90	0.032	0.039
91	0.129	0.139
92	0.319	0.345
93	0.028	0.035
94	0.049	0.055
95	0.071	0.081
96	0.089	0.1
97	0.052	0.058
98	0.075	0.086
99	0.292	0.314
100	0.29	0.316
101	0.475	0.502
102	0.302	0.323
103	0.069	0.082
104	0.092	0.103
105	0.146	0.164
106	0.451	0.481
107	0.445	0.474
108	0.107	0.116
109	0.223	0.238
110	0.169	0.18
111	0.171	0.182
112	0.469	0.493
113	0.061	0.069
114	0.035	0.042
115	0.08	0.091
116	0.046	0.055
117	0.031	0.037
118	0.011	0.015
119	0.021	0.029
120	0.008	0.013

Table 10: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the NC-LCV House plan.

I declare under penalty of perjury that the foregoing is true and correct.

Executed Feb 18, 2022

Prof. Moon Duchin

Sworn and subscribed before me

this the 18 of February, 2022.

Notary Public

Name: Simarjit Manhas

My Commission Expires: 02/02/2024

A notary public or other officer completing this certificate verifies only the identity of the individual who signed the document to which this certificate is attached, and not the truthfulness, accuracy, or validity of that document.

State of California, County of Alameda

Subscribed and sworn to (or affirmed) before me on this 18 day of February, 20 22, by Moon Duchin, proved to me on the basis of satisfactory evidence to be the person(s) who appeared before me.

Signature:

SIMARJIT MANHAS
COMM. #2317737
Notary Public - California
Contra Costa County
My Comm. Expires Feb. 2, 2024