## SUPREME COURT OF NORTH CAROLINA

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NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,
REBECCA HARPER, et al.,
Plaintiffs, COMMON CAUSE,

Plaintiff-Intervenor,
From Wake County
No. 21 CVS 015426
No. 21 CVS 500085

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC., ET AL.'S EMERGENCY APPLICATION FOR STAY PENDING APPEAL, PETITION FOR WRIT OF SUPERSEDEAS, MANDAMUS, AND/OR PROHIBITION, ALTERNATIVE PETITION FOR WRIT OF CERTIORARI, MOTION TO SUSPEND APPELLATE RULES, AND MOTION FOR PRELIMINARY INJUNCTION

## TO THE HONORABLE SUPREME COURT OF NORTH CAROLINA:

The NCLCV Plaintiffs hereby seek an emergency stay pending appeal of the trial court's order approving the North Carolina General Assembly's newly enacted redistricting map for North Carolina Senate, as well as other relief. ${ }^{1}$

Three weeks ago, this Court invalidated the redistricting plans the General Assembly enacted in November 2021 (the "2021 Enacted Plans") as "extreme partisan gerrymanders" and held that, under our Constitution, lawful remedial plans must give "voters of all political parties substantially equal opportunity to translate votes into seats." Harper v. Hall, 2022-NCSC17, 【163. Earlier today, the trial court correctly rejected the General Assembly's congressional plan because it did not satisfy this standard, and adopted its own map that complies with this standard. It also accepted the General Assembly's remedial House plan, which passed with an overwhelming bipartisan majority.

The trial court erred, however, in approving the Legislative Defendants' Senate plan (the "Legislative Senate Plan"), which passed on a party-line vote and fails to satisfy the standard this Court set. Order on Remedial Plans, N.C. League of Conservation Voters, Inc., v. Hall, 21 CVS 015426, 21 CVS 500085
${ }^{1}$ Given the deadline, the NCLCV Plaintiffs were unable to prepare an accurate table of contents and authorities to accompany this filing.
(Sup. Ct. Feb. 23, 2022) (the " 23 February Order"). Plaintiffs North Carolina League of Conservation Voters et al. have filed a notice of appeal from that decision. App. 169. Via this application, the NCLCV Plaintiffs seek emergency relief pursuant to this Court's 4 February Order, which provided for emergency applications for a stay pending appeal to be filed on 23 February 2022. Order © 9, Harper v. Hall, No. 413PA21 (N.C. Feb. 4, 2022) ("Harper Order"). A stay of the decision to approve the Legislative Senate Plan is necessary to preserve the voting rights of millions of North Carolinians and to ensure orderly election administration in this State.

Dr. Bernard Grofman, upon whom the Special Masters relied to redraw the General Assembly's congressional plan, found in his report accompanying the Special Masters' findings that the Legislative Senate Plan shared similar properties with the invalid congressional plan in that it was "very lopsidedly Republican." App. 131. Dr. Grofman concluded that "[b]ecause they all point in the same direction, the political effects statistical indicators of partisan gerrymandering argue for the conclusion that this NC Senate map should be viewed as a pro-Republican gerrymander. While, overall, the dilutive effects of this map do not appear quite as severe as in the congressional map they are still quite substantial." App. 132. Hence, as even the person who drew the remedial congressional plan found, the Legislative Senate Plan does not
provide an adequate remedy for the Legislative Defendants' pro-Republican gerrymandering.

In this Application, the NCLCV Plaintiffs seek three forms of relief.
First, the NCLCV Plaintiffs seek a stay of the order approving the Legislative Senate Plan. This Court held that districting plans are unlawful if "voters supporting one political party have their votes systematically devalued by having less opportunity to elect representatives to seats, compared to an equal number of voters of the favored party." Harper Op. ๆ| 162. A "meaningful ... skew," the Court held, is lawful only if it "necessarily results from North Carolina's political geography." Id. © 163.

The trial court, however, approved a party-line Senate plan that preserves roughly half of the pro-Republican bias this Court found unconstitutional. And it did so even though it had before it alternative remedial plans (from the NCLCV Plaintiffs and the Harper Plaintiffs) that all but eliminate this bias while complying as well or better with North Carolina's traditional neutral districting principles. As Dr. Grofman found, comparing the levels of partisan bias in the Legislative Senate Plan to the level of partisan bias in the Harper and NCLCV Senate Maps, "we see that each of these two bias measures is at least twice as high in the legislative map as in the alternatives and, even when we look at differences in absolute value rather than ratios, it is still clear that the legislatively proposed ... map is much more
extreme with respect to partisan bias than either of the alternatives." App. 131. Elections that are half free are not "free," N.C. Const. art I, § 10, and laws that treat citizens half-equally do not ensure "equal protection," id. art. I, § 19.

Second, the NCLCV Plaintiffs respectfully suggest that the Court immediately order (via a preliminary injunction or a writ of supersedeas, mandamus, and/or prohibition) the adoption of the NCLCV Senate Map. The NCLCV Plaintiffs attached that map to their original complaint filed 16 November 2022; it has been tested via discovery and four days of trial; and the parties addressed this map in their liability-stage briefing and argument before this Court. As this scrutiny revealed, the NCLCV Senate Map complies with and upholds each of North Carolina's constitutional requirements and traditional redistricting principles, including protecting each voter's right to substantially equal voting power and legislative representation regardless of party, race, or region. The analysis of the NCLCV Senate Map by the assistants to the Special Masters likewise shows how well the NCLCV Senate Map performs on the various metrics. Indeed, it appears from their reports that the NCLCV Senate Map scores better on every possible metric of partisan fairness than does the Legislative Senate Plan.

Third, to the extent this Court requires more time to examine proposed remedial maps, the NCLCV Plaintiffs respectfully submit that the Court
(again, via a preliminary injunction or a writ of supersedeas, mandamus, and/or prohibition) stay the candidate-filing period that is slated to open tomorrow, on 24 February 2022. In no event should candidate filing proceed under a Senate map that, as shown below, fails to satisfy this Court's standards.

Finally, once this Court establishes a lawful Senate map, the Court should further decree that under Article II, Sections 3 and 5 of the North Carolina Constitution, the districts for the House and Senate are established and shall remain unaltered until the return of another decennial population taken by order of Congress. ${ }^{2}$
${ }^{2}$ In particular, the North Carolina Constitution provides that "[w]hen established, the [House and] [S]enate districts and the apportionment of [Representatives and] Senators shall remain unaltered until the return of another decennial census of population taken by order of Congress." N.C. Const. art. II, §§ 3(4), 5(4). Accordingly, the plain and unambiguous language of Sections 3(4) and 5(4) prohibits the General Assembly from engaging in middecade redistricting to change legislative district boundaries once established. See Granville Cty. Comm'rs v. Ballard, 69 N.C. 18, 20-21 (1873).

Because the NCLCV Plaintiffs have not contested the validity of the Remedial House Plan (S.L. 2022-4, also known as House Bill 980, enacted February 17, 2022) and the trial court approved the Remedial House Plan in its February 23 Order, this Court should decree that under Article II, Section $5(4)$ of the North Carolina Constitution, the representative districts for the House hereby are established and shall remain unaltered until the return of another decennial population taken by order of Census.

This Court should enter the same decree as to the Senate districts under Article II, Section 3(4) once this appeal is resolved and a constitutionally compliant Senate map is adopted. While N.C.G.S. § 120-2.4(a)(1) purports to limit the effectiveness of any court-ordered remedial map "for ... the next

## BACKGROUND

On 4 February 2022, this Court held that the North Carolina General Assembly's 2021 Enacted Plans for the U.S. Congress, North Carolina Senate, and North Carolina House were "unconstitutional beyond a reasonable doubt under the free elections clause, the equal protection clause, the free speech clause, and the freedom of assembly clause of the North Carolina Constitution." Harper Order 『 3. The Court ruled that the 2021 Enacted Plans were "unlawful partisan gerrymanders," id. © 1, that violated the "fundamental right to vote," id. ब 4. That fundamental right, the Court explained, "includes the right to enjoy 'substantially equal voting power and substantially equal legislative representation,"" id. ब 4 (quoting Stephenson v. Bartlett, 355 N.C. 354, 382, 562 S.E.2d 377, 396 (2002) (Stephenson I)). The Court therefore "enjoin[ed] the use of these maps in any future elections, commencing with the upcoming candidate filing period scheduled to commence on 24 February 2022 for elections in 2022, including primaries scheduled to take place on 17 May 2022." Id.

[^0]The Court gave the General Assembly "the opportunity to submit new ... districting plans that satisfy all provisions of the North Carolina Constitution," consistent with N.C.G.S. § 120-2.4(a). Harper Op. © 9. Any such plan, the Court emphasized, had to "give the voters of all political parties substantially equal opportunity to translate votes into seats across the plan." Id. $\mathbb{\text { I }} 6$.

The Court remanded the matter to the trial court for remedial proceedings. The Court directed the General Assembly to submit any new plans for review to the trial court by 5:00 p.m. on 18 February 2022 and directed all parties to the proceeding and intervenors to submit any proposed remedial plans by the same deadline. Id. © 9. This Court ordered that comments on the proposed remedial plans be submitted 72 hours later, by 5:00 p.m. on 21 February 2022 and that the trial court "approve or adopt compliant congressional and state legislative districting plans" by 12:00 noon on 23 February 2022. Id. Finally, the Court ordered that any emergency application for a stay pending appeal be filed by 5:00 p.m. on 23 February 2022. Id. On 14 February 2022, this Court issued a full opinion.

On remand, the trial court appointed three Special Masters to "[a]ssist th[e] Court in reviewing any Proposed Remedial Plans enacted and submitted by the General Assembly or otherwise submitted to the Court by a party" as well as to "[a]ssist th[e] Court in fulfilling the Supreme Court's directive to this Court to develop remedial maps based upon the findings in this Court's

January 11, 2022 Judgment should the General Assembly fail to enact Proposed Remedial Plans compliant with the Supreme Court's Order within the time allowed." Order Appointing Special Masters 3-4, NCLCV v. Hall, No. 21 CVS 015426 (N.C. Super. Ct. Feb. 16, 2022); see Order on Submission of Remedial Plans for Court Review 3, NCLCV v. Hall, No. 21 CVS 015426 (N.C. Super. Ct. Feb. 8, 2022). The trial court set forth detailed instructions stipulating what the General Assembly and parties were required to submit in connection with their proposed remedial plans. Order on Submission of Remedial Plans for Court Review 3-5; Order Appointing Special Masters 5-7.

On 18 February 2022, the General Assembly and the other parties, including the NCLCV Plaintiffs, submitted proposed remedial maps and accompanying written submissions to the trial court. The maps submitted by the NCLCV Plaintiffs were identical to those they had submitted with their Complaint on 16 November 2021, litigated through discovery and trial, and addressed in their briefing and argument before this Court.

On 21 February 2022, all parties submitted comments on the proposed remedial maps. The Governor and the Attorney General also sought leave to submit an amicus brief. That brief explained that this Court's "guarantee [of] 'substantially equal voting power"' means that plans "must allow voters the equal ability to translate votes into legislative seats." Amicus Br. of Governor Cooper and Att'y Gen. Stein 2, NCLCVv. Hall, No. 21 CVS 015426 (N.C. Super.

Ct. Feb. 21, 2022) (Gov./AG Br.), https://www.nccourts.gov/locations/wake-county/cases-of-public-interest\#remedial. But the Remedial Congressional and Senate Plans, the brief explained, do not "provide voters with substantially equal voting power." Id. at 12. Thus, the brief concluded, the court should not approve the plans. Id.

On 23 February 2022, the Special Masters issued a report concerning the proposed remedial plans. Pursuant to the report's recommendations, the trial court rejected the congressional plan enacted by the General Assembly. In lieu of accepting the remedial congressional plans proposed by the Plaintiffs, the Special Masters and their assistants modified the General Assembly's plan. The court, however, accepted the Legislative Senate Plan. The court also denied leave to file the amicus brief from the Governor and the Attorney General. In addition to the report filed by the Special Masters, each of the Special Masters' assistants also filed individual reports with their findings.

## EMERGENCY APPLICATION FOR STAY PENDING APPEAL

Pursuant to Rules 2, 8(a), and 23 of the North Carolina Rules of Appellate Procedure, the NCLCV Plaintiffs respectfully apply for an emergency stay pending appeal of the trial court's 23 February 2022 decision approving the Legislative Senate Plan.

In its 4 February 2022 Order, this Court authorized the parties to this proceeding to file an emergency application for a stay pending appeal of the
trial court's 23 February Order by 5:00 p.m. today, 23 February. Under the North Carolina Rules of Appellate Procedure, a party may request such a stay from an appellate court in the first instance "when extraordinary circumstances make it impracticable to obtain a stay by deposit of security or by application to the trial court for a stay order." N.C. R. App. P. 8(a); see N.C. R. App. P. 23(a)(1). Here, the Court's timetable-which leaves only five hours between the deadline for the trial court's decision and the deadline for any emergency application for a stay pending appeal-"make[s] it impracticable" to first "obtain a stay ... by application to the trial court." Moreover, the NCLCV Plaintiffs understand this Court's Orders of 8 December 2021 and 4 February 2022 to contemplate that emergency stay applications would be filed in this Court in the first instance.

A stay pending appeal is warranted when the appellant would otherwise suffer irreparable harm and has some likelihood of success on the merits. See Meares v. Town of Beaufort, 193 N.C. App. 49, 63-64, 667 S.E.2d 244, 254 (2008); Abbott v. Town of Highlands, 52 N.C. App. 69, 79, 277 S.E.2d 820, 827 (1981); 1-23 North Carolina Appellate Practice and Procedure § 23.04 (2018).

Pursuant to these standards, a stay is warranted to protect the voting rights of millions of North Carolinas and to ensure that the 2022 elections do not violate this Court's ruling and the North Carolina Constitution by
proceeding under an unlawful Senate plan that deprives North Carolina citizens of their fundamental right to substantially equal voting power.

## I. A Stay Is Warranted to Avoid Irreparable Harm.

Absent a stay, the prejudice that looms is immense. This Court held that "our constitution's Declaration of Rights guarantees the equal power of each person's voice in our government through voting in elections that matter." Harper Op. © 1. Under the Legislative Senate Plan the trial court approved, however, voices are not equally powerful. As detailed below, this plan perpetuates the severe partisan bias in the plan the Court already invalidated-providing only half a remedy and failing to cure the constitutional violations this Court found. As a result, millions of North Carolinians will find their fundamental right to vote for North Carolina Senators drained of all meaning. "Courts routinely deem restrictions on fundamental voting rights irreparable injury." Holmes v. Moore, 270 N.C. App. 7, 35, 840 S.E.2d 244, 266 (2020). Indeed, "discriminatory voting procedures in particular are the kind of serious violation of the Constitution ... for which courts have granted immediate relief." Id.

Nor does any countervailing harm counsel against granting such relief. Entering a stay here might (or might not) ultimately necessitate a delay in the primary elections currently scheduled for 17 May 2022. But the Court has already delayed those primaries once, and the General Assembly has already
passed legislation that would have delayed them further. Although the Governor vetoed that legislation, its passage confirms that a further delay will not undermine election administration or prejudice the Legislative Defendants. Indeed, at the time the General Assembly was considering this legislation, the North Carolina State Board of Elections confirmed that primaries could be held on June 7. ${ }^{3}$

## II. The NCLCV Plaintiffs Are Likely to Succeed in Showing that the Legislative Senate Plan Fails This Court's Unambiguous Standards.

When this Court struck down the plans that the General Assembly had enacted in November 2021, it set a clear standard for remedial plans to satisfy: Plans must treat voters of both political parties fairly, so long as North Carolina's unique political geography permits doing so. The Legislative Senate Plan fails that test. It is not fair, or close to fair. Instead, this plan-passed over unanimous Democratic opposition-is again severely biased to favor Republicans. Several of the special assistants retained to assist the Special Masters reached similar conclusions. The trial court's decision to nonetheless

[^1]approve the Legislative Senate Plan was erroneous．The NCLCV Plaintiffs are likely to succeed in their challenge to the Legislative Senate Plan．

## A．This Court Required that Redistricting Plans Give＂Voters of All Political Parties Substantially Equal Opportunity to Translate Votes into Seats．＂

This Court grounded its 4 February Order and 14 February Opinion on a clear principle．A＂system of fair elections is foundational to self－ government．＂Harper Op．ब 3 （quoting Comm．to Elect Dan Forest v．Emps． Pol．Action Comm．， 376 N．C．558，2021－NCSC－6，『 86 （Newby，C．J．，concurring in the result））．And under the North Carolina Constitution，redistricting plans must＂give ．．．voters of all political parties substantially equal opportunity to translate votes into seats．＂Id．ब 163．In sum，maps must create＂a level playing field for all voters．＂Id．【 164．That was no loose language，or offhand remark．It was the Court＇s carefully considered statement of its holding，as also reflected in its 4 February Order．Harper Order 9 6；see Harper Op．【！ $\mid$ 160－163，179－180．

This Court located that core principle－a principle of partisan fairness－ in the constitutional provisions the Plaintiffs invoked：＂When the legislature denies to certain voters ．．．substantially equal voting power，＂the Court explained，＂elections are not free and do not serve to effectively ascertain the will of the people，＂in violation of the Free Elections Clause．Harper Op．ๆ 140. Likewise，when redistricting plans do not treat voters of both parties
evenhandedly, they violate the Equal Protection Clause's guarantees of "substantially equal voting power," "substantially equal legislative representation," and substantially equal "representational influence." Id. © 148 (quoting Stephenson I, 355 N.C. at 377, 382, 562 S.E.2d at 393, 396). Finally, when a redistricting plan "systematically diminishes or dilutes the power of votes on the basis of party affiliation," it violates the core promises of the Free Speech and Free Assembly Clauses. Id. © 157.

The Court also identified particular metrics that other courts had deployed when weighing the fairness of districting plans, including partisan symmetry, mean-median differences, efficiency gaps, and "close votes, close seats" analysis. Id. © 180. The Court, however, was careful to emphasize that it was identifying only "possible" metrics. Id. ब 164; accord id. 【 165 (Dr. Duchin's "close votes, close seats" measure "could be considered"); id. © 166 (mean-median difference "could be a threshold"); id. © 167 (recounting what other "courts have found" with respect to the efficiency gap). Those metrics could serve to inform, but could never replace, the core principle this Court announced: Plans must "give ... voters of all political parties substantially equal opportunity to translate votes into seats," and any "meaningful ... skew" can be constitutional only if it "necessarily results from North Carolina's
unique political geography." Id. 『 163 (emphasis added). If a map meets that standard, it is presumptively constitutional. And if not, not.

As the proposed amicus brief of the Governor and the Attorney General explained below, this standard invites a "narrow tailoring" inquiry: "[I]f a court is presented with two plans that both satisfy neutral criteria but diverge in how well they allow voters to translate votes into seats, then the plan that performed less well could not [satisfy this Court's standard]." Gov./AG Br. 67. Hence, if "an alternative plan imposes meaningfully lesser burdens on the rights of voters than an enacted plan while still complying with neutral redistricting criteria, the enacted plan would fail narrow tailoring." Id.

## B. The Legislative Senate Plan Does Not Give "Voters of All Political Parties Substantially Equal Opportunity to Translate Votes into Seats."

The Legislative Senate Plan fails to comply with the principle this Court established-that districting plans must "give ... voters of all political parties substantially equal opportunity to translate votes into seats." Harper Op. © 163. The trial court had before it multiple alternative plans that all but eliminate the substantial bias in the Remedial Senate Plan, while complying as well or better with traditional districting criteria. By nonetheless approving the Legislative Senate Plan, the trial court erred.

The analysis of several of the Special Masters' assistants shows that the Legislative Senate Plan fails to satisfy this Court's standards. Dr. Grofman
found that the Legislative Senate Plan was "very lopsidedly Republican," creating " 24 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 17 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 9 competitive districts." App. 131. Dr. Grofman noted that "Democrats would have to win nine of the nine competitive seats to win a majority in the Senate." App. 131. Dr. Grofman noted that "when we compare these levels of partisan bias to the level of partisan bias in the Harper and NCLCV [senate] maps we see that each of these two bias measures is at least twice as high in the legislative map as in the alternatives and, even when we look at differences in absolute value rather than ratios, it is still clear that the legislatively proposed ... map is much more extreme with respect to partisan bias than either of the alternatives." App. 131. Hence, Dr. Grofman concluded: "Because they all point in the same direction, the political effects statistical indicators of partisan gerrymandering argue for the conclusion that this NC Senate map should be viewed as a pro-Republican gerrymander. While, overall, the dilutive effects of this map do not appear quite as severe as in the congressional map they are still ... quite substantial." App. 132.

Dr. Wang likewise concluded that the "Legislative Defendants' plans favor Republicans in [all] six metrics evaluated: seat partisan asymmetry, mean-median difference, partisan bias, lopsided wins, declination angle, and
efficiency gap." App. 134. He found that the seat partisan asymmetry was 2.1 seats in the Legislative Senate Plan while the "NCLCV plaintiffs' plans show ... mixed or no advantage for [either party in] the Senate plan." App. 134. Dr. Wang concluded that "[i]n no case did the Legislative Defendants' remedial map come closer to partisan symmetry than the plaintiffs' alternative(s)." Id.

Those conclusions accord with the evidence the NCLCV Plaintiffs (and other Plaintiffs) presented below. Indeed, to see that the Legislative Senate Plan fails to satisfy the core principle this Court established, no complicated calculations or statistics are required. The NCLCV Plaintiffs' expert, Dr. Duchin, has overlaid the Legislative Senate Plan (along with the equivalent NCLCV and Harper maps) on all 52 contested partisan statewide general elections since 2012, which provide a rich dataset that identifies how plans would perform under historical election patterns. See App. 32. A map that treated the parties evenhandedly would yield closely divided outcomes in neartied elections and would treat narrow Democratic victories the same as narrow Republican victories, without favoring one party over the other.

The Legislative Senate Plan does not treat the parties evenhandedly:

- Near ties. In the 9 near-tied statewide elections (of 52 total) decided by less than $1 \%$, Republican candidates carry 27 or 28 of the 50 Senate districts in 8 elections; Democratic candidates typically carry 22 or 23 districts, and reach 25 wins-a bare tie-just once. (All information in these bullet points comes from App. 32.)
- 1\% to $\mathbf{3 \%}$ victories. In the elections in which Republican statewide candidates won by $1 \%$ to $3 \%$, Republican candidates carry an average of more than 28 districts and carry a supermajority of 30 districts in several elections.

In the elections in which Democratic statewide candidates won by 1\% to $3 \%$, Democratic candidates sometimes do not win even a majority of districts and average only 25 . In total, Republican candidates carry a majority of districts in 7 elections in which Democratic candidates received a majority of votes (or $44 \%$ of the total). App. 32.

- $\mathbf{3 \%}$ to $\mathbf{6 \%}$ victories. In the 9 elections that Republican statewide candidates won by $3 \%$ to $6 \%$, Republican candidates carry a 30 -plus district supermajority 6 times out of 9 .

In the elections that Democratic statewide candidates won by $3 \%$ to $6 \%$, Democratic candidates never carry a majority of Senate districts, averaging only 24 districts.

Table 1 summarizes the same information—and vividly underscores the skew in the Legislative Senate Plan.

Table 1: Senate Seats Across Remedial Plans For Close Elections


Source: Data derived from App. 32. Supermajority outcomes are defined as any election in which a party wins $60 \%$ or more seats.

Figure 1 further illustrates this lack of symmetry and again shows that the General Assembly retained about half of the partisan skew of the nowinvalidated Senate plan. The invalidated plan would have given Republicans an average of 8 more seats than Democrats for a similar vote share in a typically competitive election environment. The Legislate Senate Plan would give Republicans a 4- or 5-seat advantage. By contrast, the Plaintiffs' proposed maps shrink that figure to about 1.5 seats. The residual pro-Republican skew
in the Plaintiffs' maps is likely due to the interaction of political geography and the Whole County Provisions, as interpreted by this Court. (Yellow shading in the below figures reflect plans with an especially large partisan bias.)

Figure 1: Republican and Democratic Seat Shares for Elections Within 6 Points in the Enacted Senate Plan and Proposed Remedial Plans


Source: App. 38.
According to Dr. Duchin, the Legislative Senate Plan has a partisan bias score of $-4.4 \%$, a mean-median difference of $-2.0 \%$, and an efficiency gap of -
4.5\%. App. 38. Notably, while the Special Masters' assistants calculated different figures for some of these metrics, two of them agreed that the meanmedian difference exceeded $1 \%$ : Dr. Jarvis calculated it as $-1.40 \%$, and Dr. McGhee calculated it as $-2.20 \%$. App. 103, 166.

The NCLCV Senate Map again underscores that, while the Whole County Provision may produce some modest pro-Republican bias, the Legislative Senate Plan's bias vastly exceeds what political geography "necessarily" yields. Harper Op. ब 163. On all the metrics just described, the NCLCV Senate Map's scores improve dramatically on those of the Legislative Senate Plan: The NCLCV Senate Map's partisan bias is just $-1.5 \%$, its meanmedian difference is just $-0.9 \%$, and its efficiency gap is just $-2.0 \%$. App. 38 . Moreover, in the 9 near-tied elections decided by less than 1\%, Democratic candidates average between 24 and 25 seats in the NCLCV Senate Map-just as one would expect. When Republican statewide candidates prevail by $1 \%$ to $3 \%$, Republican candidates win an average of 27 Senate seats-and when Democratic statewide candidates prevail by $1 \%$ to $3 \%$, they receive the same 27 seats on average. And so on. That is the type of fair, symmetric plan the Legislative Defendants could have drawn, but chose not to draw.

A stark illustration of the skew comes from the results the Legislative Senate Plan yields across all 33 statewide elections decided by 4 points or less. As Dr. Duchin explains, in an evenhanded map, close elections will generally
translate into close seat shares, and any departures will not systematically advantage one party. See Harper Op. ब 165 ("Under th[e] [close votes, close seats] method, ...a plan which persistently resulted in the same level of partisan advantage to one party when the vote was closer than $52 \%$, could be considered presumptively unconstitutional.").

In those close elections, one would expect a fair map to yield something like 23 to 27 seats for each party and to minimize the extent to which departures favor one party or another. App. 31. While the Whole County Provisions produce some departures from that standard in both the NCLCV Senate Map and the Legislative Senate Plan, the Legislative Senate Plan does so in nine additional elections for a total of 23 additional Republican seats (i.e., seats above 27). Id. More than that, the NCLCV Senate Map never translates a close election within 4 points into a Republican supermajority; the Legislative Senate Plan does so five times (or $22 \%$ of the 23 elections that Republican statewide candidates win by less than 4 points). Id.

The Legislative Senate Plan yields these skewed results, moreover, while also traversing county lines seven times more than the NCLCV Senate Map (96 versus 89). The two plans' Polsby-Popper scores are nearly identical (0.38 for the Legislative Senate Plan and 0.37 for the NCLCV Map), and the NCLCV Senate Map splits fewer municipalities (with the Legislative Senate Plan splitting 65 municipalities and 52 involving population, and the NCLCV

Senate Map splitting 51 and 41, respectively). Compare N.C. Gen. Assembly, StatPack, SL 2022-2 at 76 (provided with Legislative Defendants' remedial submission), with Ex. N to NCLCV Remedial Submission at 10-26, 77. ${ }^{4}$ So again, the Legislative Senate Plan's poor results on partisan-fairness metrics are not driven by political geography. They are due to partisan gerrymandering. And again, these results violate the Supreme Court's mandate that plans must give all voters "substantially equal opportunity to translate votes into seats." Harper Op. © 163.

## C. The Trial Court Erred in Approving Plans that Fail to Comply with This Court's Standards.

The trial court's decision to approve the Legislative Senate Plan was based on several legal and factual errors. Indeed, the Special Masters and the trial court often departed from the conclusions of the expert assistants that the Special Masters had retained to analyze the Legislative Senate Plan.

First, the trial court ground its decision almost exclusively on the view that this Court's opinion established "statistical ranges" for constitutional redistricting plans. 23 Feb. Order FOF $\mathbb{T}$ 42. Because the trial court believed the Legislative Remedial Plan fell within these ranges-a "mean-median difference of $1 \%$ or less" and an "efficiency gap less than $7 \%$ "-the trial court

[^2]concluded that the plan was constitutional. Id. But even if the trial court had correctly calculated those metrics-and as explained below, the trial court often departed from the conclusions of the Special Masters' assistants—this Court specifically said it was not "identify[ing] precise mathematical thresholds which conclusively demonstrate or disprove the existence of an unconstitutional partisan gerrymander." Harper Op. | 163. The trial court simply overlooked all the exhaustive analysis the NCLCV Plaintiffs (and other Plaintiffs) provided—summarized above—showing that the Legislative Senate Plan failed this Court's core command to "give ... voters of all political parties substantially equal opportunity to translate votes into seats." Id.

Indeed, the reports from the Special Masters' assistants underscore why this Court wisely eschewed reliance on specific numerical thresholds. As to the mean-median metric, for example, Dr. Grofman explained that while it "is a very useful and easy to calculate tool," it "may be easier to manipulate by mapmakers than some other measures." App. 119. As to the Legislative Senate Plan in particular, Dr. Grofman explained that "while the median district again looks a lot like the statewide average, but again with a slight Republican edge, the median is only one district and we must look at the overall map." App. 131. And as to the Legislative Senate Plan, Dr. Grofman concluded, the " $4.07 \%$ seats bias still suggest a substantial pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win
a majority of the seats, even though it is one percentage point or so lower than the comparable statistic in the congressional map, while the $2.00 \%$ vote bias suggests that only a win by considerably more than $50 \%$ of the statewide vote can yield the Democrats a majority of the seats." Id. (emphasis original).

Second, the trial court stated that it found "that to the extent there remains a partisan skew in the [Legislative] Senate Plan, that skew is explained by the political geography of North Carolina." 23 Feb. Order FOF ब 43. This Court's opinion, however, did not ask whether political geography contributed in some way to a plan's partisan bias. It held that "meaningful" partisan bias is permissible only if it "necessarily results from North Carolina's unique political geography." Harper Op. ब 163 (emphasis added). The trial court did not find that North Carolina's political geography necessarily yielded the skew in the Legislative Senate Plan. Indeed, the trial court could not have made that finding given the undisputed record evidence that the NCLCV Senate Map (and the Harper Plaintiffs' plan) treats voters of both parties fairly while also respecting traditional districting principles.

Third, the trial court declined to accord weight to Plaintiffs' Senate maps because it viewed "Plaintiffs' arguments as tantamount to urging this Court to adopt a proportional representation standard." 23 Feb. Order COL © 2. That, however, misunderstands Plaintiffs’ arguments. The NCLCV Plaintiffs do not argue that if Democratic candidates win (say) $53 \%$ of the
statewide vote, Democratic candidates should obtain $53 \%$ of seats. Our argument is about symmetry: If Democratic candidates get the same vote share as Republican candidates, they should receive roughly the same seat share. For example, if the General Assembly draws a Senate map that usually gives Republican candidates a veto-proof supermajority with $51.5 \%$ to $53 \%$ of the vote, then the same plan should not often condemn Democratic candidates that win the same share of the vote minority status, as the Legislative Senate Plan often does (as shown above). That is not proportional representation. It is a faithful application of the core principle this Court announced-that lawful plans must "give ... voters of all political parties substantially equal opportunity to translate votes into seats." Harper Op. © 163.

Fourth, even if the trial court were correct that this Court had established presumptive thresholds based on particular metrics (which it was not), two assistants to the Special Masters-Dr. Jarvis and Dr. McGheecalculated mean-median differences for the Legislative Senate Plan that exceed 1\%. App. 103, 166. So if this Court had established such a threshold, this analysis would show that the Legislative Senate Plan falls on the wrong side of it. Cf. Harper Op. © 166. Neither the trial court nor the Special Masters acknowledged or addressed these analyses. The failure to do so was clear error.

Finally, the Special Masters recommended approving the Legislative Senate Plan largely because they "recommend[ed] to the trial court that it give
appropriate deference to the General Assembly," based on the view that the Plaintiffs had not "overcome the presumption of constitutionality." App. 75. But this is a remedial proceeding in which the General Assembly's redistricting plans have already been invalidated as extreme partisan gerrymanders. And this Court expressly identified what facts would entitle the General Assembly to a presumption of constitutionality: If "there is a significant likelihood that the districting plan will give the voters of all political parties substantially equal opportunity to translate votes into seats ..., then the plan is presumptively constitutional." Harper Op. If 163. Because the Legislative Senate Plan fails to satisfy that standard—as shown above-no presumption of constitutionality applies.

To the contrary, Plaintiffs are entitled to a remedial plan "that will fully correct past wrongs" and provide all voters genuinely equal opportunity. N.C. State Conf. of NAACP v. McCrory, 831 F.3d 204, 240 (4th Cir. 2016). As the Fourth Circuit explained in its 2016 invalidation of the General Assembly's voter-identification law, "the remedy for an unconstitutional law must completely cure the harm wrought by the prior law." Id. It was not enough that the General Assembly had amended the voter-identification law, following a successful lawsuit, to institute a partial cure-in the form of an exception for voters who declared they faced a "reasonable impediment" to obtaining identification-because the record did not show that the exception "fully
cure[d]" the constitutional harm. Id. The Court was obligated to "ensure that [the challenged] provisions do not impose any lingering burden on ... voters." $I d$. And because the exception "f[ell] short of the remedy that the Supreme Court has consistently applied in cases of this nature," it was inadequate. Id.; cf. Dayton Bd. of Ed. v. Brinkman, 433 U.S. 406, 423-24 (1977) (reinforcing the court's "duty to remedy fully those constitutional violations it finds" in the context of school-desegregation cases, in light of the "paramount importance of the constitutional rights being enforced").

The same is true here. As the Covington court explained, "courts must 'provid[e] remedies fully adequate to redress the constitutional violations which have been adjudicated and must be rectified." Covington v. North Carolina, No. 1:15CV399, 2017 WL 44840, at *2 (M.D.N.C. Jan. 4, 2017), rev’d on other grounds 137 S. Ct. 1624 (2017) (quoting White v. Weiser, 412 U.S. 783, 797 (1973)). Millions of North Carolina voters are entitled to more than a half cure for the violations of their fundamental voting rights.

This Court ordered a simple remedy for the General Assembly's unlawful partisan gerrymandering: Draw fair maps. The General Assembly could have done so. After all, for three months, the state legislatures had access to the NCLCV Maps, which are indisputably fair. But instead, on repeated partyline votes, the General Assembly enacted plan for Congress and state Senate
that were severely biased. The trial court properly rejected the congressional plan. But the trial court approved the General Assembly's Senate plan based on several legal errors, and in contradiction of the conclusions of the Special Masters' own assistants. Now, the NCLCV Plaintiffs ask the Court to enforce its mandate, ensure that the unconstitutional Legislative Senate Plan does not remain in effect, and adopt a fair Senate map that genuinely treat all voters fairly and equally, regardless of party, region, or race. North Carolinians are counting on this Court to keep its promise of fair maps, free elections, and a government that truly reflects the will of the people.

WHEREFORE, for the reasons set forth above, the NCLCV Plaintiffs respectfully request that this Court grant an emergency application to stay the 23 February Order pending appeal, insofar as the 23 February Order approved the Legislative Senate Plan.

# MOTION FOR PRELIMINARY INJUNCTION AND/OR PETITION FOR WRIT OF SUPERSEDEAS, MANDAMUS, AND/OR <br> PROHIBITION TO ADOPT A LAWFUL REMEDIAL SENATE MAP, AND/OR STAY THE CANDIDATE-FILING PERIOD 

If the Court stays the decision below, it has two options. First, it could immediately order an alternative map for Senate into effect-and the NCLCV Plaintiffs have proposed their NCLCV Senate Map for this purpose. That approach would avoid the need for any further delay to the election schedule. Second, if the Court requires more time to examine the proposed remedial
maps or to address other related issues, the Court could withhold issuance of its mandate and (via a preliminary injunction or a writ of supersedeas, mandamus, and/or prohibition) stay the candidate-filing period for the primary election that was originally scheduled to open tomorrow, on 24 February 2022, pending appellate review of the 23 February Order. The NCLCV Plaintiffs respectfully seek those alternative forms of relief pursuant to Article IV, §§ 1 and 12(1) of the North Carolina Constitution, N.C.G.S. § 7A-32(b), and Rules 2, 8, 22, 23, 32, and 37 of the North Carolina Rules of Appellate Procedure.

## REASONS WHY THE WRIT SHOULD ISSUE

## I. The Court Could Immediately Order the NCLCV Senate Map into Effect.

If the Court wishes to bring this proceeding to a prompt close, it could order the NCLCV Senate map into effect. This map is uniquely suitable as remedial maps: Because the NCLCV Plaintiffs attached this map to their complaint filed on 16 November 2021, it has been tested via discovery and examination at trial, and the parties and this Court addressed this map in their briefs and oral arguments at the liability phase.

Moreover, the NCLCV Plaintiffs developed their maps-leveraging the power of high-performance computing-to target the same standard that this Court ultimately adopted. As the evidence at trial showed, the algorithm used to produce the NCLCV Maps sought to optimize multiple traditional districting
criteria, including "population balance, contiguity, respect for counties, [and] geographic compactness," as well as "minority electoral opportunity" and "partisan fairness." Rule 9 Ex. 11,231; see T4 pp 818:8-819:1. As to the last, the algorithm pursued "symmetry"-meaning that "no one gets discriminated against based on their political viewpoint or their partisan affiliation." T5 pp 806:6, 808:5-8. That is the precise principle that this Court ultimately endorsed.

The evidence below showed that the NCLCV Senate Map is extraordinarily fair to both political parties-while also excelling on traditional districting criteria. The NCLCV Plaintiffs have already summarized much of that evidence above. In short, the NCLCV Senate Map eliminates the proRepublican bias in the General Assembly's now-invalidated plan to the maximum extent that the Whole County Provisions and North Carolina's political geography permit.

The NCLCV Senate Map is attached hereto at Appendix pages App. 25; the affidavits of Dr. Moon Duchin analyzing that map are attached at Appendix pages App. 6-48. The NCLCV Plaintiffs provided additional details to the trial court in their 18 February 2022 submission. To avoid burdening this Court with too much paper, the NCLCV Plaintiffs have refrained from attaching the full 18 February 2022 filing to this submission, but it is available at https://www.nccourts.gov/locations/wake-county/cases-of-public-interest.

The NCLCV Plaintiffs also stand ready to immediately provide this Court with copies of the full filing upon request.

## II. Alternatively, the Court Should Stay the Candidate-Filing Period.

If the Court concludes that further remedial proceedings are necessary, it should retain jurisdiction and stay the candidate-filing period currently scheduled to open tomorrow, 24 February 2022. The Court has already stayed the candidate-filing period once. Order at 1, Harper v. Hall, No. 413P21 (N.C. Dec. 8, 2021). If necessary, the Court should do so again, to avoid the waste and inconvenience of candidates filing under unlawful maps.

To the extent such a stay would also require delaying the primary election further, that relief would be appropriate. This Court has already delayed the primary election once, and the General Assembly has enacted legislation that would have delayed it further. Although the Governor vetoed that legislation, its passage confirms that a further delay will not undermine election administration or prejudice the Legislative Defendants.

## MOTION TO SUSPEND APPELLATE RULES AND FOR ALTERNATIVE RELIEF

To the extent deemed necessary to provide any of the relief sought in this filing or exercise jurisdiction over this appeal, the NCLCV Plaintiffs respectfully move to suspend the appellate rules pursuant to Rules 2 and 37(a) of the North Carolina Rules of Appellate Procedure and, in the alternative,
petition the Court for a writ of certiorari pursuant to Rule 21 or for discretionary review prior to determination by the Court of Appeals pursuant to N.C.G.S. § 7A-31(b).

## CONCLUSION

For the reasons set forth above, the NCLCV Plaintiffs ask this Court to stay the 23 February Order pending appeal insofar as it approved the Legislative Senate Plan. The NCLCV Plaintiffs also respectfully urge this Court to grant a preliminary injunction and/or issue a writ of supersedeas and/or prohibition and/or mandamus to order the adoption of the NCLCV Senate Map and/or to stay the candidate-filing period pending this Court's review of the 23 February Order. To the extent deemed necessary, the Court should also consider suspending the appellate rules, issuing a writ of certiorari, and granting discretionary review prior to determination by the Court of Appeals. Finally, the Court should further decree that under Article II, Sections 5(4) of the Constitution, the districts for the House are hereby established and shall remain unaltered until the return of another decennial population taken by order of Congress. This Court should enter the same decree as to the Senate districts under Article II, Section 3(4) once this appeal is resolved and a constitutionally compliant Senate map is adopted.

Respectfully submitted this 23rd day of February, 2022.

## ROBINSON, BRADSHAW \& HINSON, P.A.

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## VERIFICATION

I, Stephen D. Feldman, as counsel for Plaintiff-Applicants, verify that the facts stated in the attached Emergency Application, including any facts incorporated by reference in the Emergency Application, are true to the best of my knowledge, information, and belief. Pursuant to Appellate Rules 21, 22, and 23 , I also hereby certify that the documents attached to this filing are believed to be true and correct copies of the pleadings and other documents from or associated with the file in Wake County Superior Court pertaining to this action, including documents that were served or submitted for consideration as contemplated by Appellate Rule 11.


Wake County, North Carolina
Sworn to and subscribed before me this 23rd day of February 2022.


Notary Public Signature
Kevin Richeson

Printed Name

My commission expires:
$01 / 28 / 2026$

## CERTIFICATE OF SERVICE

Pursuant to Rule 26 of the North Carolina Rules of Appellate Procedure, I hereby certify that the foregoing document has been filed with the Clerk of the North Carolina Supreme Court by electronic submission. I further certify that a copy of this document has been duly served upon the following counsel of record by email:

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This the 23rd day of February, 2022.

Electronically Submitted John R. Wester
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## SUPREME COURT OF NORTH CAROLINA

************************************************

NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,

Plaintiffs,
REBECCA HARPER, et al.,
Plaintiffs,
COMMON CAUSE,
Plaintiff-Intervenor,

From Wake County
No. 21 CVS 015426
No. 21 CVS 500085
v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.
************************************************
APPENDIX TO NORTH CAROLINA LEAGUE OF CONSERVATION VOTERS, INC., ET AL.'S EMERGENCY APPLICATION FOR STAY PENDING APPEAL, PETITION FOR WRIT OF SUPERSEDEAS, MANDAMUS, AND/OR PROHIBITION, ALTERNATIVE PETITION FOR WRIT OF CERTIORARI, MOTION TO SUSPEND APPELLATE RULES, AND MOTION FOR PRELIMINARY INJUNCTION

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STATE OF NORTH CAROLINA
COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085

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NORTH CAROLINA LEAGUE OF CONSERVATION
VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,
    Plaintiffs,
REBECCA HARPER, et al.,
    Plaintiffs,
v.
REPRESENTATIVE DESTIN HALL, in his official capacity
as Chair of the House Standing Committee on Redistricting, et
al.,

I, Dr. Moon Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:
1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.
2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

\section*{Background and qualifications}
3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.
4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.
5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.
6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013-2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.
7. A current copy of my full CV is attached to this report.
8. I am compensated at the rate of \(\$ 400\) per hour.

\title{
Framework for Analysis of Remedial Districting Plans in North Carolina
}

\author{
Moon Duchin \\ Professor of Mathematics, Tufts University \\ Senior Fellow, Tisch College of Civic Life
}

February 18, 2022

\section*{1 Introduction}

On November 4, 2021, the North Carolina General Assembly enacted three districting plans: maps of 14 U.S. Congressional districts, 50 state Senate districts, and 120 state House districts. Since then, there has been significant activity in the state court, culminating in the invalidation of those plans by the Supreme Court of North Carolina and a remand to assess and select remedial maps.

In this report, I analyze the remedial maps for Congress, state Senate, and state House proposed by the North Carolina League of Conservation Voters plaintiffs, which I will denote NCLCV-Cong, NCLCV-Sen, and NCLCV-House. These maps have excellent properties in terms of partisan balance, minority opportunity to elect candidates of choice, and the full complement of traditional districting principles that are operative in North Carolina.

Below, I will set up an analytical framework for evaluating remedial plans and will illustrate this framework by comparing the LCV plans to the now-invalidated state plans (SL-174, SL-173, SL-175). In a follow-up report, I plan to apply this analytical framework to other remedial plans submitted to the court, including the remedial plans recently enacted by the General Assembly. I have included a quantitative review of the traditional districting principles in Appendix A; below, I will focus on partisan fairness ( \(\$ 2-4\) ) and on race and the Stephenson framework ( \(\$ 5\) ).


Figure 1: The six plans used to illustrate the analytical framework presented here.

\section*{2 Partisan fairness}

\subsection*{2.1 Abstract partisan fairness}

There are many notions of partisan fairness that can be found in the scholarly literature and in redistricting practitioner guides and software. Most of them are numerical, in the sense that they address how a certain quantitative share of the vote should be translated to a quantitative share of the seats in a state legislature or Congressional delegation. These include: disproportionality, efficiency gap (original and simplified versions), mean-median gap, partisan bias, partisan Gini, and a relatively new alternative that I am calling the Eguia county skew. All of these are discussed below in §4.

Conceptually, the numerical notions of partisan fairness all tend to agree on one central point: an electoral climate with a roughly \(50-50\) split in partisan preference should produce a roughly 50-50 representational split. I will call this the Close-Votes-Close-Seats principle. North Carolina voting has displayed a partisan split staying consistently close to even between the two major parties over the last ten years, but the plans released by the General Assembly after the 2010 census were very far from realizing the ideal of converting even voting to even representation. This time, with a 14th seat added to North Carolina's apportionment, an exactly even seat outcome is possible. In an election that is very close to even statewide, an ideally fair map would give each party, say, 6-8 Congressional seats out of 14, 23-27 Senate seats out of 50 , and \(55-65\) House seats out of 120 , with no particular lean to either side.

Importantly, Close-Votes-Close-Seats is not tantamount to a requirement for proportionality. Rather, it is closely related to the principle of Majority Rule: a party or group with more than half of the votes should be able to secure more than half of the seats. In fact, Close-Votes-Close-Seats is essentially a corollary (or byproduct) of Majority Rule. It is not practicable to design a map that always attains these properties, but by contrast a map that consistently thwarts them should be closely scrutinized and usually rejected.

Unlike proportionality, neither Close-Votes-Close-Seats nor Majority Rule has any bearing on the preferred representational outcome when one party has a significant voting advantage: these principles are silent about whether \(70 \%\) vote share should secure \(70 \%\) of the seats, as proportionality would dictate, or \(90 \%\) of the seats, as supporters of the efficiency gap would prefer. The size of the "winner's bonus" is not at all prescribed by a Close-Votes-Close-Seats norm.

\subsection*{2.2 Geography and fairness}

Some scholars have argued that all numerical ideals, including Close-Votes-Close-Seats, ignore the crucial political geography-this school of thought reminds us that the location of votes for each party, and not just the aggregate preferences, has a major impact on redistricting outcomes. In [6], my co-authors and I gave a vivid demonstration of the impacts of political geography in Massachusetts: we showed that for a ten-year span of observed voting patterns, even though Republicans tended to get over one-third of the statewide vote, it was impossible to draw a single Congressional district with a Republican majority. That is, the geography of Massachusetts Republicans locked them out of Congressional representation. It is therefore not reasonable to charge the Massachusetts legislature with gerrymandering for having produced maps which yielded all-Democratic delegations; they could not have done otherwise.

In North Carolina, this is not the case. The NC-LCV plans demonstrate that it is possible to produce maps that give the two major parties a roughly equal opportunity to elect their candidates. These plans are just examples among many thousands of plausible maps that convert voter preferences to far more even representation by party. In Congressional redistricting, present-day North Carolina geography is easily conducive to a seat share squarely in line with the vote share. In Senate and House plans, even following the strict detail of the Whole County Provisions, there are likewise many alternatives converting nearly even voting patterns to nearly even representation, across a large set of recent elections.

The clear conclusion is that the political geography of North Carolina today does not obstruct the selection of a map that treats Democratic and Republican voters fairly and evenhandedly.

\subsection*{2.3 Millions of maps - and the median mistake}

My research lab has been at the forefront of the development of ensemble analysis: algorithmic techniques for building large collections (or "ensembles") of alternative districting plans. In fact, most of the leading teams in this space now use a variant of the "spanning tree" method we initiated for generating new plans.

The power of ensembles is that you can understand a plan in comparison to alternatives that are drawn on the same political geography. If they are done well, the plans will be made using the same rules and criteria that were applied by the primary line-drawers.

Furthermore, the large ensembles can be scored on various metrics-like the number of seats that they would give to each party, or their partisan bias score, or anything else-and this typically gives a visually appealing bell curve, with the various plans and other benchmarks falling at various places along the curve.

To illustrate this I will start with an example from an amicus brief to the U.S. Supreme Court in Rucho v. Common Cause for which I was one of the amici. In this case (Figure 2), neutral is good-the General Assembly's plans look like outliers in a way that suggests packing, while the Judges' plan has a more typical partisan makeup.


Figure 2: A hisgotram plot reproduced from the Mathematicians' Brief to the U.S. Supreme Court in Rucho v. Common Cause. This bell curve, based on hundreds of thousands of districting plans drawn with no partisan data, shows that the North Carolina General Assembly plans from 2012 and 2016 pack Democrats into the third most Democratic district. By contrast, the demonstration plan drawn by a bipartisan panel of retired judges does not.

But typical is not always best. For instance, if we had a bell curve of compactness scores, a good map wouldn't be right in the middle-it would be all the way on the extreme that represents the most compact plans. On a bell curve of county splits, we should prefer a plan that splits fewer counties to a typical map. And likewise, we do not seek a plan with typical fairness, but rather a plan that is especially fair.

To drive this home, consider the following figure, reproduced and extended from my earlier rebuttal report in the current litigation in North Carolina. In it, I have constructed a statewide ensemble of North Carolina state House plans by assembling the 50,000 plans generated by defendants' expert Michael Barber in each county cluster to make an astronomical number of maps overall. \({ }^{1}\) The bell curve measures something that Barber calls "Democratic-leaning seats."

NCLCV-House


Figure 3: "Democratic-leaning seats" in Dr. Barber's House district ensemble. (Figure extended from earlier response report.) This is a case in which the fairest map might not be in the middle of the bell curve.

Dr. Barber's outputs are consistent with the often-observed fact that North Carolina, like many states, exhibits some partisan skew in blindly drawn plans, just as a function of political geography. But it would be a mistake to assume that typical is fair. The North Carolina Supreme Court has told us that we should prefer a plan in which "there is a significant likelihood that [the plan] will give the voters of all political parties substantially equal opportunity to translate votes into seats across the plan." If it is possible, while strictly following the rules and priorities in the legal framework for North Carolina redistricting, then we should certainly prefer a fair plan to a typical plan.

In other words, it would be a mistake to valorize the median, or the "top of the hill," when better maps can be made that are completely consistent with the rules and political geography of North Carolina today.

\footnotetext{
\({ }^{1}\) In my report, I note a host of problems with his methodology, but I set that aside here to make a conceptual point about interpreting ensembles.
}

\section*{3 A framework for partisan fairness in North Carolina}

\subsection*{3.1 Overlaying elections and plans}

We can examine how well the Close-Votes-Close-Seats norm is upheld without invoking any predictions or assumptions about future voting behavior by using a standard technique in election analysis: pairing proposed plans with actual recent elections. This method works by overlaying (or superimposing) the districting plans on a series of observed voting patterns from the recent past; this lets us take advantage of the rich dataset of real electoral outcomes in North Carolina in the last ten years to avoid speculative or predictive modeling about voting trends in the future. \({ }^{2}\)

The overlay method works best when there is a large set of statewide elections to apply, which is certainly true in North Carolina. Of the 52 statewide party-ID general elections from the last cycle, 29 are elections for Council of State (ten offices elected three times, with the Attorney General race uncontested in 2012), three are presidential races, three are for U.S. Senate, and 17 are judicial races since mid-decade, when those became partisan contests. See Table 1 for more detail on the election dataset.

\subsection*{3.2 Partisanship outcomes}

North Carolina is a very "purple" state. In 38 out of the 52 contests in our dataset, the statewide partisan outcome is within a 6-point margin: 47-53 or closer.

To understand how plans perform under different electoral conditions, we will overlay the plans with voting patterns from individual elections in the past Census cycle. We can make a striking observation by laying our six plans over the vote patterns, shown in Table 1. Examining the performance of a plan as the electoral conditions vary shows many things: its responsiveness (giving sufficient variation in the seats as the votes change), its tendency to avoid anti-majoritarian outcomes, and so on. The NC-LCV Congressional plan (NCLCV-Cong) is far more faithful to the vote share, far more responsive, and tends to award more seats to the party with more votes-usually upholding both basic small-d-democratic principles of Majority Rules and Close-Votes-Close-Seats, which were violated by the invalidated plans. The same patterns are visible at the Senate and House level.

By every measure considered above that corresponds to a clear legal or good-government redistricting goal or value, the NC-LCV plans have excellent performance. This demonstrates that it is possible, without any cost to the redistricting principles in play, to select maps that are fair to the voters of North Carolina.

Below, the outcomes of overlaying the plans on the elections will be presented in a series of tables and figures. First, Table 1 overviews the overlays with numbers, repeated from my earlier reports (which included the backup data). \({ }^{3}\)

Then, Figure 4 offers a visualization to depict the same big picture of votes versus seats with the full 52 -election dataset. The diagonals show various lines of responsiveness that pivot around the central point of fairness: half of the votes securing half of the seats.

\footnotetext{
\({ }^{2}\) Many authors have used this technique of overlaying "exogenous" statewide elections rather than using statistical regressions and other modeling to manipulate "endogenous" districted elections. For instance this can be found in peer-reviewed work and expert reports of scholar-practitioners such as Bernard Grofman and Steven Ansolabehere.
\({ }^{3}\) Codes for reading Table 1: AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; LTG = Lieutenant Governor; PRS = President; SEN \(=\) Senator; SOS \(=\) Secretary of State; SUP \(=\) Superintendent of Public Instruction; TRS \(=\) Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), JS* are elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals. Where there was more than one judicial candidate from a given party on the ballot, they were combined for this analysis. The two-digit suffix designates the election year.
}
- App. 13 -

Do close votes translate to close seats?
The table records the number of districts in each plan with a Democratic win. This shows that the enacted maps systematically violate the principles of Close-Votes-Close-Seats and Majority Rule.
\begin{tabular}{lccccccc} 
& & D Vote Share & SL-174 & NCLCV-Cong & SL-173 & NCLCV-Sen & SL-175
\end{tabular} NCLCV-House

Table 1: 52 general elections, sorted from lowest to highest Democratic share. Recall that Close-Votes-Close-Seats can be read as calling for the highlighted elections to have close outcomes-say, 6-8 Congressional seats, 23-27 Senate seats, and 55-65 House seats. (Key to election naming convention is provided on page 7.)

\section*{Seats vs. Votes}

Majority Rule says that outcomes should tend to fall in the Northeast and Southwest quadrants, avoiding the Southeast and Northwest. Close-Votes-Close-Seats says that points should not miss the bulls-eye near the center by systematically deviating to the North or the South. These principles are clearly upheld by the alternative plans (green) and violated by the enacted plans (maroon).

\section*{Congress}




Figure 4: On these seats-vs.-votes plots, we see the election results when overlaying the six maps on the 52 general election contests in the last decade; each colored dot is plotted as the coordinate pair (vote share, seat share).

\section*{4 Summary scores of partisan fairness}

Though a holistic approach to gauging partisan opportunity (such as I presented in the previous section) is strongly recommended as a primary matter, it can still be very useful to turn to simple scores and metrics to paint a supporting quantitative picture. It would be unreasonable to expect any individual metric of partisan fairness to perfectly distill the holistic picture in a bug-free and un-gameable way, \({ }^{4}\) but as with compactness scores, looking at a suite of metrics in combination creates a strong overall narrative.

\subsection*{4.1 Vote share/seat share metrics}

First I will explain the scores that are denominated in vote shares or seat shares, to be presented in Table 2.

Efficiency gap is the difference in "wasted" votes for the two parties, across the state, as a share of votes cast. Because the authors realized that it was sensitive to turnout effects, they later advocated for a simplified efficiency gap formula \(E G=2 V-S-\frac{1}{2}\), where \(V\) is the vote share in an election and \(S\) is the seat share. Original efficiency gap and simplified efficiency gap would be exactly equal if the districts had equal turnout; it's the simplified formula that was used, for example, in the language for the Freedom To Vote act. The authors proposed .08, later refined to .07 , as the flag for a presumptive gerrymander. \({ }^{5}\)

Partisan symmetry is a family of scores based on the principle of table-turning: if the votes for the parties were reversed, would the representation also be reversed? An asymmetric plan is one in which one party fares better with its portion of support than the other party would with the same portion. Scores in this group include the mean-median gap, the partisan bias score, and the partisan Gini. The mean-median gap literally takes the difference between the average vote share in a district and the median, or middle, district (or the average of the two middle districts when the number of districts is even). The gap is zero when the middle district looks like the state as a whole, so that half the districts are more favorable to one party and half are more favorable to the other. Partisan bias is described in the literature as measuring how much "extra" representation each party would secure in a hypothetical 50-50 election.

The last metric I am presenting in the seat share/vote share collection is a county skew metric based on economist Jon Eguia's "jurisdictional partisan advantage" [7]. Eguia built a metric based on comparing the actual representation secured by a party under a vote pattern to the representation if towns and counties played the role of districts. I have applied it here only to counties, because of the fundamental importance of counties in North Carolina redistricting in particular. A simple way to explain this Eguia-style metric is as follows: in a particular election, what percentage of North Carolinians live in counties that favored Republicans? That is the benchmark for Republican representation; if their seat share is higher, the map is tilted Republican, and if lower, the map is tilted Democratic.

For all five of these scores, zero is ideal.

\footnotetext{
\({ }^{4}\) Scholars who study the metrics, including myself, have explained blind spots and loopholes in each individual score. A map with extreme partisan advantage can sometimes be drawn with good partisan symmetry scores, for instance, by careful tuning; but it is unlikely that such a map would also have good efficiency gap scores. Likewise, there are cases where the scores are inappropriate entirely, like in small states with lopsided voting. North Carolina is not such a state. See [5] for more discussion.
\({ }^{5}\) In paragraph 167 of the North Carolina Supreme Court's recent decision in this case, it is noted that "With regard to the efficiency gap measure, courts have found "that an efficiency gap above \(7 \%\) in any districting plan's first election year will continue to favor that party for the life of the plan."" (Quoting the U.S. Supreme Court, from Whitford v. Gill).
}
\begin{tabular}{|c|ccc|ccc|}
\hline & All & NCLCV-Cong \\
Non-judicial \\
(52 contests) & (35 contests) & \begin{tabular}{c} 
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{c} 
All \\
\((52\) contests)
\end{tabular} & \begin{tabular}{c} 
SL-174 \\
Non-judicial \\
(35 contests) \()\)
\end{tabular} & \begin{tabular}{c} 
Up-ballot \\
(14 contests)
\end{tabular} \\
\hline \hline Efficiency Gap & 0.006 & 0.001 & -0.001 & -0.167 & -0.159 & -0.181 \\
Simplified EG & 0.011 & 0.005 & 0.003 & -0.17 & -0.163 & -0.186 \\
Mean-median & 0.007 & 0.006 & 0.007 & -0.047 & -0.044 & -0.045 \\
Partisan Bias & 0.036 & 0.029 & 0.031 & -0.192 & -0.184 & -0.204 \\
Eguia County Skew & -0.006 & -0.009 & -0.006 & -0.188 & -0.176 & -0.195 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{} & \multicolumn{3}{|c|}{NCLCV-Sen} & \multicolumn{3}{|c|}{SL-173} \\
\hline & All
(52 contests) & Non-judicial (35 contests) & \begin{tabular}{l}
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & \begin{tabular}{l}
Up-ballot \\
(14 contests)
\end{tabular} \\
\hline Efficiency Gap & -0.02 & -0.024 & -0.017 & -0.075 & -0.068 & -0.08 \\
\hline Simplified EG & -0.023 & -0.028 & -0.021 & -0.076 & -0.070 & -0.081 \\
\hline Mean-median & -0.009 & -0.012 & -0.009 & -0.036 & -0.036 & -0.037 \\
\hline Partisan Bias & -0.015 & -0.023 & -0.016 & -0.072 & -0.069 & -0.08 \\
\hline Eguia County Skew & -0.040 & -0.041 & -0.030 & -0.093 & -0.083 & -0.09 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{} & \multicolumn{3}{|c|}{NCLCV-House} & \multicolumn{3}{|c|}{SL-175} \\
\hline & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & \begin{tabular}{l}
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) \\
\hline Efficiency Gap & -0.02 & -0.022 & -0.017 & -0.076 & -0.075 & -0.078 \\
\hline Simplified EG & -0.014 & -0.016 & -0.012 & -0.074 & -0.074 & -0.077 \\
\hline Mean-median & -0.015 & -0.015 & -0.017 & -0.039 & -0.039 & -0.04 \\
\hline Partisan Bias & -0.018 & -0.019 & -0.018 & -0.082 & -0.082 & -0.086 \\
\hline Eguia County Skew & -0.031 & -0.030 & -0.021 & -0.091 & -0.088 & -0.086 \\
\hline
\end{tabular}

Table 2: Five simplified scores of partisan fairness, averaged over different sets of elections. These five metrics are all signed, meaning that they can take positive or negative values; positive and negative scores are intended to flag an advantage to Democrats and Republicans, respectively. \(E G\) and \(M M\) are computed as a share of votes; \(P B\) and the Eguia score are computed as a share of seats. See text for an explanation of scores.

\subsection*{4.2 Big-picture scores}

Next, Table 3 zooms out to the bigger picture of overall partisan properties. Here, we see the typical number of seats that would be won by either party under these maps, across different election sets.

Disproportionality then indicates how far that seat total is from reflecting the statewide vote, with negative signs indicating an advantage to Republicans.

Finally, partisan Gini is a summary statistic for all of the various kinds of symmetry measures in the political science literature. The "Partisan Symmetry Standard" of King and his co-authors asks that a seats-votes curve be literally symmetric about the center point, meaning that it predicts exactly the same representation for either party at any share of the vote [8]. The partisan Gini, first proposed by Bernard Grofman in 1983, takes this literally, measuring the area between the curve and its mirror image [9]. This is an unsigned metric, with zero as an ideal. (When the \(P G\) score is zero, all other symmetry scores, like mean-median and partisan bias, are necessarily zero as well.)
- App. 17 -
\begin{tabular}{|c|ccc|ccc|}
\hline & All & \begin{tabular}{c} 
NCLCV-Cong \\
Non-judicial \\
(35 contests)
\end{tabular} & \begin{tabular}{c} 
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{c} 
All \\
(52 contests)
\end{tabular} & \begin{tabular}{c} 
SL-174 \\
(30n-judicial \\
(35 contests)
\end{tabular} & \begin{tabular}{c} 
Up-ballot \\
(14 contests)
\end{tabular} \\
\hline \hline D Seats & 6.9 & 6.9 & 6.7 & 4.4 & 4.5 & 4.1 \\
R Seats & 7.1 & 7.1 & 7.3 & 9.6 & 9.5 & 9.9 \\
Disproportionality & 0.0 & 0.0 & -0.1 & -2.5 & -2.4 & -2.8 \\
Partisan Gini & 0.021 & 0.020 & 0.021 & 0.078 & 0.073 & 0.080 \\
\hline
\end{tabular}
\begin{tabular}{|c|ccc|ccc|}
\hline & All & \begin{tabular}{c} 
NCLCV-Sen \\
Non-judicial \\
(35 contests)
\end{tabular} & \begin{tabular}{c} 
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{c} 
All \\
(52 contests)
\end{tabular} & \begin{tabular}{c} 
SL-173 \\
Non-judicial \\
(35 contests)
\end{tabular} & \begin{tabular}{c} 
Up-ballot \\
(14 contests)
\end{tabular} \\
\hline \hline D Seats & 23.0 & 22.9 & 22.8 & 20.3 & 20.8 & 19.8 \\
R Seats & 27.0 & 27.1 & 27.2 & 29.7 & 29.2 & 30.2 \\
Disproportionality & -1.6 & -1.7 & -1.6 & -4.2 & -3.9 & -4.6 \\
Partisan Gini & 0.026 & 0.026 & 0.026 & 0.051 & 0.049 & 0.054 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{} & \multicolumn{3}{|c|}{NCLCV-House} & \multicolumn{3}{|c|}{SL-175} \\
\hline & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & \begin{tabular}{l}
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) \\
\hline D Seats & 56.2 & 56.3 & 55.8 & 49.0 & 49.3 & 47.9 \\
\hline R Seats & 63.8 & 63.7 & 64.2 & 71.0 & 70.7 & 72.1 \\
\hline Disproportionality & -2.7 & -2.9 & -2.8 & -10.0 & -9.8 & -10.7 \\
\hline Partisan Gini & 0.032 & 0.031 & 0.034 & 0.050 & 0.050 & 0.053 \\
\hline
\end{tabular}

Table 3: The average number of Dem seats, the disproportionality of that seat total, and a partisan symmetry metric called the Partisan Gini ( \(P G\) ) that tells you how far the districting plan is from being symmetric in the sense of King et al.

Slicing the same data another way, the pattern is clear.
\begin{tabular}{ccccc} 
& \multicolumn{2}{c}{ Up-ballot generals (14) } & \multicolumn{2}{c}{ All generals (52) } \\
& D vote share & D seat share & D vote share & D seat share \\
\hline \hline SL-174 & .4883 & .2908 & .4911 & .3118 \\
NCLCV-Cong & .4931 \\
\hline SL-173 & .4883 & .4996 & & .4931 \\
NCLCV-Sen & .4937 & .4911 & .4065 \\
\hline SL-175 & .4883 & .4557 & & .4592 \\
NCLCV-House & .4994 & .4911 & .4080 \\
& & .4649 & .4684
\end{tabular}

Table 4: Comparing overall fidelity of representation to the voting preferences of the electorate. Vote shares are computed with respect to the major-party vote total.

\section*{5 Race and the Stephenson framework}

North Carolina has a large minority of Black-identified residents. Over two million North Carolinians-2,107,526 out of \(10,439,388\) to be precise, or about \(20.2 \%\)-were identified as non-Hispanic Black-alone on the Census. Within the voting age population, the numbers shift to \(1,620,569\) out of \(8,155,099\), or about 19.9\%. Increasing numbers of Americans identify as Black in combination with other races and/or Hispanic ethnicity. Passing to this more expansive definition of Black raises the voting age population numbers to \(1,743,052\) out of \(8,155,099\), or \(21.4 \%\). Other minority groups, while their population can be substantial, are rarely concentrated enough to be in the majority in a district. \({ }^{6}\)

Minority groups' opportunity to elect candidates of choice is protected by both state and federal law. A detailed assessment of opportunity must not primarily hinge on the demographics of the districts, but must also rely on electoral history and an assessment of polarization patterns. \({ }^{7}\)

In North Carolina, the nine-step Stephenson/Dickson framework addresses how to reconcile the Whole County Provisions-which apply only to state legislative districts-with one-person, one-vote requirements and the Voting Rights Act (VRA). The first step of the framework is assessing whether the VRA requires the drawing of a certain number of effective districts. I will include an assessment of VRA requirements, and will conclude that the choice of "county clustering" that was made in the state's plans (and maintained in the NC-LCV plans) is consistent with the obligations imposed by the Stephenson framework.

\subsection*{5.1 Demonstration of majority-Black districts (Gingles 1)}

Using Markov chain techniques related to those that are often employed to build ensembles of plans, I have confirmed that it is possible to draw Senate maps with five districts that exceed \(50 \%\) Black voting age population, using the any-part-Black definition (Black alone or in combination). I was unable to find a Senate map with a sixth majority-BVAP district. Likewise, I was able to achieve 17 simultaneous majority-BVAP districts in a House plan in many distinct ways, but never 18. Figure 5 shows where in the state those Gingles districts are located.


Figure 5: These heatmaps show the VTDs that are most likely to be in majority-BVAP districts in plans that have 5 such districts in the Senate (left) or 17 such districts in the House (right).

\section*{Location of "Gingles districts" in Senate maps (adding to 5)}
- Mecklenburg: 2 districts
- Guilford: 1 district
- rural Northeast: 2 districts

\footnotetext{
\({ }^{6}\) An exception is American Indian/Native American population; respondents selecting AMIN identity make up 50.4\% of the population in the NC-LCV House district 47, for instance.
\({ }^{7}\) A detailed discussion of the inadequacy of using demographics alone as a proxy can be found in [3].
}

\section*{Location of "Gingles districts" in House maps (adding to 17)}
- Mecklenburg: 4-5 districts
- Guilford: 2-3 districts
- Cumberland: 2 districts
- Wake: 1-2 districts
- Durham: 1 district
- Forsyth: 1 district
- rural northeast: 4-6 districts

\subsection*{5.2 Polarization (Gingles 2-3)}

I have used industry-leading techniques called ecological inference \({ }^{8}\) to study the racial polarization patterns in North Carolina general and primary elections from the last decade, and I have corroborated those findings with ecological regression. The results indicate a consistent pattern of polarization in statewide general elections, such that White voters are estimated to support the Republican candidate at a rate of over \(61 \%\) in every general election, and Black voters are estimated to support the Democratic candidate at a rate of over \(94 \%\) each time. Polarization is present in many Democratic primary elections as well, particularly in elections in which there is a Black Democratic candidate. I confirmed these polarizaton patterns in individual regions around the state as well as at the whole-state level. A full set of EI/ER outputs is available in my backup materials from earlier reports.

\subsection*{5.3 Effectiveness}

I have designated a selection of eight elections-four generals and four primaries-chosen to be particularly informative in determining whether Black voters have an opportunity to elect their candidates of choice. \({ }^{9}\) These eight contests were chosen by a combination of factors that combine to make an election particularly informative with respect to the preferences of Black voters. Namely: I prioritized elections that are more recent, that have a Black candidate on the ballot, that are clearly polarized, and that are close enough to produce variation at the district level. Recency, polarization, and the presence of a Black candidate on the ballot are recognized as contributing to higher probativity by a well-established consensus in case law around Gingles 2-3, and the fact that landslide and uncontested elections are less informative is obvious on first principles.

The electoral alignment score derived from these elections is a value from 0 to 8 . I consider a district in which the Black candidate of choice prevails in at least 6 of these 8 contests to be aligned with Black voting preferences in the state. \({ }^{10}\) Six out of eight is not an arbitrary fraction: it ensures that at least half of primary contests and at least half of general contests are electing the Black candidate of choice. Any valid attempt to replicate this analysis must also retain that property.

\footnotetext{
\({ }^{8}\) In particular, I used the \(R \times C\) multinomial Dirichlet model, as implemented in PyEI.
\({ }^{9}\) The Black-preferred candidates are: Sutton in the 2020 Superintendent primary, Smith in the 2020 Ag. Commissioner primary, Williams in the 2016 Attorney General primary, Coleman in the 2016 Lt. Governor primary; Holley in the 2020 Lt. Governor general, Cunningham in the 2020 U.S. Senate general, Coleman in the 2016 Lt. Governor general, and Blue in the 2016 Treasurer general. Of these, Sutton, Williams, Coleman, Colley, and Blue are themselves Black-identified.
\({ }^{10}\) I have used statewide ecological inference ("EI") runs to determine the candidate of choice for Black voters. I note that it is also possible to run El on smaller geographies (such as counties or county clusters) to detect regional candidates of choice rather than statewide candidates of choice; in most cases, these will be the same, but in some cases, regional effects may be meaningful and could affect these results at the margin.
}

If, in addition, at least \(25 \%\) of the voting age population is Black, then I label the district to be effective for Black voters. This \(25 \%\) threshold is emphatically not chosen as an estimate of the level of BVAP required for performance; rather, it is strictly to confirm that there is a substantial number of Black voters in the district to benefit from the electoral alignment with their preferences. \({ }^{11}\)

I note that the use of electoral history is not just cosmetic: there are House-sized districts with \(35-39 \%\) BVAP that are nonetheless not labeled effective in these lists because they fall short of the standard of inclining to the Black candidate of choice in at least six out of the eight chosen elections.

At all three levels, the NCLCV alternative maps provide substantial numbers of effective opportunity-to-elect districts for Black voters.

\section*{Effective districts for Black voters in NC-LCV plans}

CD 2, 4, 9, 11 (4 districts)
SD \(1,5,11,14,18,19,26,27,32,38,39,40\) (12 districts)
HD 2, 8, 9, 10, 23, 24, 25, 27, 31, 32, 33, 38, 39, 40, 42, \(43,44,45,48,57,58,59,60,61,63,66,71,88,92,99\), 100, 101, 102, 106, 107, 112 (36 districts)

\subsection*{5.4 Conformance with Stephenson framework}

The NC-LCV plans have a greater number of effective opportunity districts than the number that can be drawn to meet the Bartlett threshold for Gingles 1.

In the Senate, the Gingles plans have two majority-BVAP districts in Mecklenburg; the NC-LCV plan has three effective districts there (SD 38, 39, 40). The Gingles plans have one majority-BVAP district in Guilford; the NC-LCV plan has two effective districts in that cluster, with SD 27 wholly in Guilford County and SD 26 mostly in Guilford. The Gingles plans have two majority-BVAP districts in the rural Northeast, and the NC-LCV plan has three effective districts in that region (SD 1, 5, 11).

Likewise, the obligations are met in the House. Mecklenburg has 4-5 Gingles districts, and the NC-LCV plan has 8 effective districts in the county (HD 10, 88, 92, 99, 100, 101, 102, and 107). Guilford has 2-3 Gingles districts, and the NC-LCV plan has five effective districts in the county (HD 57, 58, 59, 60, and 61). Cumberland has 2 Gingles districts, and the NC-LCV plan has four effective districts there (HD 42, 43, 44, 45). Durham has 1 Gingles district, and the NC-LCV plan has two effective districts there, one wholly in Durham County (HD 31) and one partially in the county (HD 2). Forsyth has 1 Gingles district, and the NC-LCV plan has one as well (HD 71). Wake has 1-2 Gingles districts, and the NC-LCV plan has four effective districts (HD 33, 38, 39, 40). The rural Northeast has 4-6 Gingles districts, and the NC-LCV plan has seven effective districts there (HD 8, 9, 23, 24, 25, 27, and 32).

This analysis makes it clear that not only as an overall statewide matter, but on an individual regional basis, the NC-LCV maps create a larger number of districts that obtain effective opportunity for Black voters to elect candidates of choice than the numerical standard set by the Gingles-via-Bartlett threshold of \(50 \%\) BVAP. This also illustrates that this choice of county clustering did not obstruct the creation of maps with adequate numbers of effective districts.

This analysis provides an excellent illustration that effective opportunity can frequently be found without a numerical majority of the population, and that urban areas in particular can make good use of overall diversity and less extreme polarization to afford configurations that are especially favorable to minority opportunity to elect. The BVAP tables in Appendix B can be compared to the list of effective districts to underscore this point.

\footnotetext{
\({ }^{11}\) l performed a robustness check and confirmed that lowering the threshold to \(20 \%\) makes no difference at all, and raising it to \(30 \%\) would only drop the NC-LCV effective district count by a single House district and no Senate districts, leaving the analysis materially unchanged.
}

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\section*{A Traditional districting principles}

Principles that are relevant to North Carolina redistricting include the following.
- Population balance. The standard interpretation of One Person, One Vote for Congressional districts is that districts should be fine-tuned so that their total Census population deviates by no more than one person from any district to any other.
There is more latitude with legislative districts; they typically vary top-to-bottom by no more than \(10 \%\) of ideal district size. In North Carolina, the Whole County Provisions make it very explicit that \(5 \%\) deviation must be tolerated if it means preserving more counties intact.
All six plans have acceptable population balance.

\section*{Population deviation}
\begin{tabular}{ccccc} 
& Max Positive Deviation & District & Max Negative Deviation & District \\
\hline \hline SL-174 & 0 & (eight districts) & -1 & (six districts) \\
NCLCV-Cong & 0 & (eight districts) & -1 & (six districts) \\
\hline SL-173 & \(10,355(4.960 \%)\) & 5 & \(-10,434(4.997 \%)\) & 13,18 \\
NCLCV-Sen & \(10,355(4.960 \%)\) & 5 & \(-10,427(4.994 \%)\) & 15 \\
\hline SL-175 & \(4250(4.885 \%)\) & 18 & \(-4189(4.815 \%)\) & 112 \\
NCLCV-House & \(4341(4.990 \%)\) & 82 & \(-4323(4.969 \%)\) & 87
\end{tabular}

Table 5: Deviations are calculated with respect to the rounded ideal district populations of 745,671 for Congress, 208,788 for Senate, and 86,995 for House.
- Contiguity. All six plans are contiguous; for each district, it is possible to transit from any part of the district to any other part through a sequence of census blocks that share boundary segments of positive length. As is traditional in North Carolina, contiguity through water is accepted.
- Compactness. The two compactness metrics most commonly appearing in litigation are the Polsby-Popper score and the Reock score. Polsby-Popper is the name given in redistricting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula \(4 \pi A / P^{2}\). Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1 . Reock is a different measurement of how much a shape differs from a circle: it is computed as the ratio of a region's area to that of its circumcircle, defined as the smallest circle in which the region can be circumscribed. From this definition, it is clear that it too is optimized at a value of 1 , which is achieved only by circles.
These scores depend on the contours of a district and have been criticized as being too dependent on map projections or on cartographic resolution [1, 2]. Recently, some mathematicians have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built. The most commonly cited discrete score for districts is the cut edges score, which counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations. This score improves on the contour-based scores by better controlling for factors like coastline and other natural boundaries, and by focusing on the units actually available to redistricters rather than treating districts like free-form Rorschach blots.

The alternative plans are significantly more compact than the enacted plans in all three compactness metrics.

\section*{Compactness}
\begin{tabular}{cccc} 
& \begin{tabular}{c} 
block cut edges \\
(lower is better)
\end{tabular} & \begin{tabular}{c} 
average Polsby-Popper \\
(higher is better)
\end{tabular} & \begin{tabular}{c} 
average Reock \\
(higher is better)
\end{tabular} \\
\hline \hline SL-174 & 5194 & 0.303 & 0.417 \\
NCLCV-Cong & 4124 & 0.383 & 0.470 \\
\hline SL-173 & 9702 & 0.342 & 0.416 \\
NCLCV-Sen & 9249 & 0.369 & 0.428 \\
\hline SL-175 & 16,182 & 0.351 & 0.437 \\
NCLCV-House & 13,963 & 0.414 & 0.465
\end{tabular}

Table 6: Comparing compactness scores via one discrete and two contour-based metrics. These scores were computed using dissolved districts based on the census blocks that were assigned in the plans under discussion.
- Respect for political subdivisions. For legislative redistricting, North Carolina has one of the strongest requirements for county consideration of any state in the nation. In my understanding, courts have interpreted the Whole County Provisions as follows. \({ }^{12}\)
- First, if any county is divisible into a whole number of districts that will be within \(\pm 5 \%\) of ideal population, then it must be subdivided accordingly without districts crossing into other counties.
- Next, seek any contiguous grouping of two counties that is similarly divisible into a whole number of districts.
- Repeat for groupings of three, and so on, until all counties are accounted for.

Once clusters have been formed, there are more rules about respecting county lines within clusters. The legal language is again explicit: "[T]he resulting interior county lines created by any such groupings may be crossed or traversed in the creation of districts within said multi-county grouping but only to the extent necessary" to meet the \(\pm 5 \%\) population standard for districts. To address this, I have counted the country traversals in each plan, i.e., the number of times a district crosses between adjacent counties within a grouping.
Table 7 reflects the county integrity metric that is most relevant at each level: the enacted congressional plan splits 11 counties into 25 pieces while the alternative plan splits 13, but splits no county three ways. (The enacted plans unnecessarily split three counties into three pieces.) In the legislative plans, the law specifies traversals as the fundamental integrity statistic.

\footnotetext{
\({ }^{12} \mathrm{~A}\) complete set of solutions is described in detail in the white paper of Mattingly et al.-though with the important caveat that the work "does not reflect... compliance with the Voting Rights Act" [4]. Absent a VRA conflict, the 2020 Decennial Census population data dictates that the North Carolina Senate plan must be decomposed into ten singledistrict fixed clusters and seven multi-district fixed clusters (comprising 2, 2, 3, 3, 4, 6, and 6 districts, respectively). It has four more areas in which there is a choice of groupings. In all, there are sixteen different possible clusterings for Senate, each comprising 26 county clusters. The House likewise has 11 single-district fixed clusters and 22 multidistrict fixed clusters (with two to thirteen districts per cluster), together with three more areas with a choice of groupings. In all, the House has only eight acceptable clusterings, each comprising 40 county clusters. An analysis of whether the clustering used in the LCV maps is consistent with VRA principles can be found in \(\S 5\) of the current report.
}
- App. 24 -

\section*{County and municipality preservation}
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{2}{|r|}{\# county pieces} & \multicolumn{2}{|r|}{\# traversals} \\
\hline \multirow[t]{5}{*}{\[
\begin{gathered}
\text { SL-174 } \\
\text { NCLCV-Cong }
\end{gathered}
\]} & \multirow[t]{4}{*}{\[
\begin{aligned}
& 25 \\
& 26
\end{aligned}
\]} & SL-173 & 97 \\
\hline & & NCLCV-Sen & 89 \\
\hline & & SL-175 & 69 \\
\hline & & NCLCV-House & 66 \\
\hline & \# municipal pieces (considering all blocks) & \multicolumn{2}{|l|}{\# municipal pieces (considering populated blocks)} \\
\hline SL-174 & 90 & & \\
\hline NCLCV-Cong & 58 & & \\
\hline SL-173 & 152 & & \\
\hline NCLCV-Sen & 125 & & \\
\hline SL-175 & 292 & & \\
\hline NCLCV-House & 201 & & \\
\hline
\end{tabular}

Table 7: Comparing the plans' conformance to political boundaries.

\section*{B BVAP across the districts of the NC-LCV plans}
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{ NCLCV-Cong } \\
\hline CD & B1VAP Share & APBVAP Share \\
\hline 1 & 0.289 & 0.304 \\
2 & 0.332 & 0.347 \\
3 & 0.118 & 0.131 \\
4 & 0.319 & 0.344 \\
5 & 0.226 & 0.245 \\
6 & 0.227 & 0.242 \\
7 & 0.115 & 0.128 \\
8 & 0.123 & 0.132 \\
9 & 0.277 & 0.298 \\
10 & 0.232 & 0.25 \\
11 & 0.271 & 0.289 \\
12 & 0.121 & 0.132 \\
13 & 0.114 & 0.124 \\
14 & 0.032 & 0.039 \\
\hline
\end{tabular}

Table 8: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the NC-LCV Congressional plan.
- App. 25 -
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{NCLCV-Sen} \\
\hline SD & B1VAP Share & APBVAP Share \\
\hline 1 & 0.408 & 0.423 \\
\hline 2 & 0.165 & 0.175 \\
\hline 3 & 0.253 & 0.267 \\
\hline 4 & 0.334 & 0.35 \\
\hline 5 & 0.385 & 0.403 \\
\hline 6 & 0.13 & 0.153 \\
\hline 7 & 0.125 & 0.138 \\
\hline 8 & 0.12 & 0.128 \\
\hline 9 & 0.228 & 0.239 \\
\hline 10 & 0.154 & 0.167 \\
\hline 11 & 0.352 & 0.366 \\
\hline 12 & 0.189 & 0.206 \\
\hline 13 & 0.175 & 0.188 \\
\hline 14 & 0.312 & 0.332 \\
\hline 15 & 0.136 & 0.152 \\
\hline 16 & 0.08 & 0.092 \\
\hline 17 & 0.091 & 0.104 \\
\hline 18 & 0.323 & 0.347 \\
\hline 19 & 0.439 & 0.481 \\
\hline 20 & 0.22 & 0.237 \\
\hline 21 & 0.176 & 0.195 \\
\hline 22 & 0.364 & 0.382 \\
\hline 23 & 0.155 & 0.167 \\
\hline 24 & 0.278 & 0.296 \\
\hline 25 & 0.165 & 0.178 \\
\hline 26 & 0.332 & 0.35 \\
\hline 27 & 0.297 & 0.317 \\
\hline 28 & 0.282 & 0.303 \\
\hline 29 & 0.171 & 0.18 \\
\hline 30 & 0.084 & 0.092 \\
\hline 31 & 0.122 & 0.135 \\
\hline 32 & 0.329 & 0.35 \\
\hline 33 & 0.14 & 0.149 \\
\hline 34 & 0.184 & 0.202 \\
\hline 35 & 0.105 & 0.116 \\
\hline 36 & 0.04 & 0.046 \\
\hline 37 & 0.104 & 0.115 \\
\hline 38 & 0.354 & 0.377 \\
\hline 39 & 0.4 & 0.426 \\
\hline 40 & 0.376 & 0.402 \\
\hline 41 & 0.116 & 0.131 \\
\hline 42 & 0.224 & 0.24 \\
\hline 43 & 0.181 & 0.194 \\
\hline 44 & 0.129 & 0.138 \\
\hline 45 & 0.065 & 0.074 \\
\hline 46 & 0.054 & 0.06 \\
\hline 47 & 0.028 & 0.035 \\
\hline 48 & 0.046 & 0.054 \\
\hline 49 & 0.044 & 0.052 \\
\hline 50 & 0.014 & 0.02 \\
\hline
\end{tabular}

Table 9: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the NC-LCV Senate plan.
- App. 26 -
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{NCLCV-House} \\
\hline HD & B1VAP Share & APBVAP Share \\
\hline 1 & 0.266 & 0.277 \\
\hline 2 & 0.335 & 0.351 \\
\hline 3 & 0.189 & 0.203 \\
\hline 4 & 0.219 & 0.23 \\
\hline 5 & 0.369 & 0.386 \\
\hline 6 & 0.216 & 0.24 \\
\hline 7 & 0.221 & 0.235 \\
\hline 8 & 0.333 & 0.353 \\
\hline 9 & 0.343 & 0.362 \\
\hline 10 & 0.349 & 0.37 \\
\hline 11 & 0.112 & 0.13 \\
\hline 12 & 0.373 & 0.385 \\
\hline 13 & 0.078 & 0.088 \\
\hline 14 & 0.112 & 0.134 \\
\hline 15 & 0.173 & 0.202 \\
\hline 16 & 0.106 & 0.116 \\
\hline 17 & 0.178 & 0.192 \\
\hline 18 & 0.13 & 0.144 \\
\hline 19 & 0.055 & 0.06 \\
\hline 20 & 0.04 & 0.048 \\
\hline 21 & 0.084 & 0.096 \\
\hline 22 & 0.272 & 0.285 \\
\hline 23 & 0.519 & 0.534 \\
\hline 24 & 0.371 & 0.386 \\
\hline 25 & 0.383 & 0.398 \\
\hline 26 & 0.173 & 0.189 \\
\hline 27 & 0.502 & 0.519 \\
\hline 28 & 0.158 & 0.171 \\
\hline 29 & 0.325 & 0.345 \\
\hline 30 & 0.243 & 0.26 \\
\hline 31 & 0.404 & 0.427 \\
\hline 32 & 0.42 & 0.434 \\
\hline 33 & 0.321 & 0.343 \\
\hline 34 & 0.093 & 0.104 \\
\hline 35 & 0.093 & 0.105 \\
\hline 36 & 0.058 & 0.069 \\
\hline 37 & 0.109 & 0.122 \\
\hline 38 & 0.305 & 0.324 \\
\hline 39 & 0.311 & 0.332 \\
\hline 40 & 0.316 & 0.339 \\
\hline 41 & 0.085 & 0.096 \\
\hline 42 & 0.384 & 0.415 \\
\hline 43 & 0.348 & 0.379 \\
\hline 44 & 0.365 & 0.411 \\
\hline 45 & 0.378 & 0.417 \\
\hline 46 & 0.282 & 0.295 \\
\hline 47 & 0.209 & 0.223 \\
\hline 48 & 0.346 & 0.371 \\
\hline 49 & 0.153 & 0.171 \\
\hline 50 & 0.174 & 0.185 \\
\hline 51 & 0.102 & 0.111 \\
\hline 52 & 0.199 & 0.212 \\
\hline 53 & 0.142 & 0.154 \\
\hline 54 & 0.137 & 0.149 \\
\hline 55 & 0.255 & 0.268 \\
\hline 56 & 0.096 & 0.111 \\
\hline 57 & 0.369 & 0.392 \\
\hline 58 & 0.363 & 0.386 \\
\hline 59 & 0.351 & 0.371 \\
\hline 60 & 0.286 & 0.304 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{NCLCV-House} \\
\hline HD & B1VAP Share & APBVAP Share \\
\hline 61 & 0.457 & 0.486 \\
\hline 62 & 0.115 & 0.127 \\
\hline 63 & 0.277 & 0.295 \\
\hline 64 & 0.114 & 0.126 \\
\hline 65 & 0.184 & 0.194 \\
\hline 66 & 0.31 & 0.336 \\
\hline 67 & 0.126 & 0.134 \\
\hline 68 & 0.072 & 0.081 \\
\hline 69 & 0.093 & 0.105 \\
\hline 70 & 0.065 & 0.072 \\
\hline 71 & 0.323 & 0.35 \\
\hline 72 & 0.371 & 0.393 \\
\hline 73 & 0.179 & 0.198 \\
\hline 74 & 0.108 & 0.12 \\
\hline 75 & 0.18 & 0.194 \\
\hline 76 & 0.199 & 0.21 \\
\hline 77 & 0.052 & 0.058 \\
\hline 78 & 0.081 & 0.089 \\
\hline 79 & 0.073 & 0.081 \\
\hline 80 & 0.099 & 0.108 \\
\hline 81 & 0.083 & 0.09 \\
\hline 82 & 0.183 & 0.2 \\
\hline 83 & 0.119 & 0.132 \\
\hline 84 & 0.154 & 0.166 \\
\hline 85 & 0.029 & 0.034 \\
\hline 86 & 0.057 & 0.064 \\
\hline 87 & 0.045 & 0.053 \\
\hline 88 & 0.32 & 0.341 \\
\hline 89 & 0.069 & 0.077 \\
\hline 90 & 0.032 & 0.039 \\
\hline 91 & 0.129 & 0.139 \\
\hline 92 & 0.319 & 0.345 \\
\hline 93 & 0.028 & 0.035 \\
\hline 94 & 0.049 & 0.055 \\
\hline 95 & 0.071 & 0.081 \\
\hline 96 & 0.089 & 0.1 \\
\hline 97 & 0.052 & 0.058 \\
\hline 98 & 0.075 & 0.086 \\
\hline 99 & 0.292 & 0.314 \\
\hline 100 & 0.29 & 0.316 \\
\hline 101 & 0.475 & 0.502 \\
\hline 102 & 0.302 & 0.323 \\
\hline 103 & 0.069 & 0.082 \\
\hline 104 & 0.092 & 0.103 \\
\hline 105 & 0.146 & 0.164 \\
\hline 106 & 0.451 & 0.481 \\
\hline 107 & 0.445 & 0.474 \\
\hline 108 & 0.107 & 0.116 \\
\hline 109 & 0.223 & 0.238 \\
\hline 110 & 0.169 & 0.18 \\
\hline 111 & 0.171 & 0.182 \\
\hline 112 & 0.469 & 0.493 \\
\hline 113 & 0.061 & 0.069 \\
\hline 114 & 0.035 & 0.042 \\
\hline 115 & 0.08 & 0.091 \\
\hline 116 & 0.046 & 0.055 \\
\hline 117 & 0.031 & 0.037 \\
\hline 118 & 0.011 & 0.015 \\
\hline 119 & 0.021 & 0.029 \\
\hline 120 & 0.008 & 0.013 \\
\hline
\end{tabular}

Table 10: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the NC-LCV House plan.

I declare under penalty of perjury that the foregoing is true and correct.


Sworn and subscribed before me this the 18 of February, 2022.


Notary Public


My Commission Expires:


A notary public or other officer completing this certificate verifies only the identity of the individual who signed the
document to which this certificate is attached, and not the truthfulness, accuracy, or validity of that document.
State of California, county of Alameda
Subscribed and sworn to (or affirmed) before me
\(\qquad\)

proved to me on the basis of satisfactory evidence to be the person fort who appeared before me.

- App. 28 -

STATE OF NORTH CAROLINA
COUNTY OF WAKE

IN THE GENERAL COURT OF JUSTICE SUPERIOR COURT DIVISION
21 CVS 015426, 21 CVS 500085
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NORTH CAROLINA LEAGUE OF CONSERVATION
VOTERS, INC.; HENRY M. MICHAUX, JR., et al.,
Plaintiffs,
REBECCA HARPER, et al.,
Plaintiffs,

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SECOND AFFIDAVIT OF
DR. MOON DUCHIN ON
REMEDIES
v.
REPRESENTATIVE DESTIN HALL, in his official capacity
as Chair of the House Standing Committee on Redistricting, et
al.,

I, Dr. Moon Duchin, having been duly sworn by an officer authorized to administer oaths, depose and state as follows:
1. I am over 18 years of age, legally competent to give this Affidavit, and have personal knowledge of the facts set forth in this Affidavit.
2. All of the quantitative work described in this Affidavit was performed by myself with the support of research assistants working under my direct supervision.

\section*{Background and qualifications}
3. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago as well as an A.B. in Mathematics and Women's Studies from Harvard University.
4. I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University.
5. My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections and voting. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum.
6. My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013-2018. I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018.
7. A current copy of my full CV is attached to this report.
8. I am compensated at the rate of \(\$ 400\) per hour.

\title{
Second Report on Remedial Districting Plans in North Carolina
}

\author{
Moon Duchin \\ Professor of Mathematics, Tufts University Senior Fellow, Tisch College of Civic Life
}

February 20, 2022

\section*{1 Introduction}

Below, I will execute the analytical framework for evaluating remedial plans outlined in my report of February 18. The newly-passed plans SL-3 (new Congressional), SL-2 (new Senate), and SL-4 (new House) will be compared to the earlier proposals by the Legislature, and to the plaintiffs' alternative maps.


Figure 1: The eleven plans being compared in this report.
- App. 31 -

\section*{2 Close-Votes-Close-Seats}

Below, the outcomes of overlaying the plans on the elections will be presented in a series of tables and figures. I use the full set of 52 general elections that occurred in North Carolina with a partisan ID in the last census cycle. This is a powerful tool to understand the performance of plans without the use of any vote index or counterfactuals.

First, Table 1 overviews the overlays with numbers, then Figures 2-4 illustrate the same data. \({ }^{1}\)

The seats-votes scatterplots show all 52 data points for each map: one for each election, plotted as vote share for Democrats ( \(x\) axis) against seat share for Democrats ( \(y\) axis).

The northwest and southeast quadrants of these plots are the zones where anti-majoritarian outcomes fall. In each plot, I've marked the number of these outcomes in the associated quadrant. (I have excluded the JS120 race, which was so close to a 50-50 partisan outcome that its majoritarian properties are less meaningful.)

Out of 35 elections with a Republican vote advantage, the NCLCV-Cong plan has three instances where Democrats get more seats. Out of 16 elections with a Democratic vote advantage, the LCV plans have 0,5 , and 8 anti-majoritarian outcomes favoring Republicans.

In those 16 contests, the previous generation of plans from the legislature had 12, 12, and 14 anti-majoritarian outcomes (for Congress, Senate, and House, respectively). The new remedial proposals from the Legislature have 7, 7, and 8. And the Harper plaintiffs' Congressional and Senate plans have 1 and 2. (Note that the Harper plaintiffs did not submit a House plan.)

\footnotetext{
\({ }^{1}\) Codes for reading Table 1: AGC = Agriculture Commissioner; ATG = Attorney General; AUD = Auditor; GOV = Governor; INC = Insurance Commissioner; LAC = Labor Commissioner; LTG = Lieutenant Governor; PRS = President; SEN \(=\) Senator; SOS \(=\) Secretary of State; SUP \(=\) Superintendent of Public Instruction; TRS \(=\) Treasurer. The prefix JA* refers to judicial elections to the Court of Appeals (so that, for instance, JA118 is the election to the Seat 1 on the Court of Appeals in 2018), JS* are elections to the state Supreme Court. All other J* prefixes refer to an election to replace a specific judge on the Court of Appeals. The two-digit suffix designates the election year. Where there was more than one judicial candidate from a given party on the ballot, they candidates from that party were combined for this analysis, so that there is a total Republican vote and a total Democratic vote in that contest.
}
- App. 32 -
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & D Vote Share & NCLCV-Cong & SL-174 & SL-3 & Harper-Cong & NCLCV-Sen & SL-173 & SL-2 & Harper-Sen & NCLCV-House & SL-175 & SL-4 \\
\hline GOV12 & 0.4418 & 4 & 4 & 4 & 6 & 18 & 16 & 16 & 15 & 44 & 41 & 39 \\
\hline AGC16 & 0.4444 & 4 & 4 & 4 & 6 & 17 & 17 & 17 & 16 & 42 & 40 & 40 \\
\hline LAC16 & 0.4475 & 5 & 4 & 4 & 5 & 20 & 18 & 18 & 17 & 45 & 42 & 43 \\
\hline JHU16 & 0.4563 & 5 & 4 & 4 & 6 & 19 & 18 & 19 & 17 & 49 & 42 & 44 \\
\hline AGC20 & 0.4615 & 4 & 3 & 4 & 5 & 19 & 17 & 19 & 19 & 51 & 40 & 44 \\
\hline JZA16 & 0.4619 & 5 & 4 & 4 & 6 & 21 & 19 & 20 & 18 & 50 & 43 & 46 \\
\hline JDI16 & 0.4653 & 6 & 4 & 4 & 6 & 21 & 19 & 20 & 19 & 53 & 44 & 47 \\
\hline LTG16 & 0.4665 & 6 & 4 & 4 & 6 & 21 & 19 & 20 & 21 & 54 & 44 & 47 \\
\hline LAC12 & 0.4674 & 5 & 4 & 5 & 6 & 20 & 20 & 16 & 15 & 51 & 44 & 43 \\
\hline AGC12 & 0.4678 & 5 & 4 & 5 & 6 & 18 & 18 & 16 & 16 & 50 & 43 & 42 \\
\hline SEN16 & 0.4705 & 6 & 4 & 4 & 6 & 21 & 19 & 20 & 22 & 55 & 43 & 47 \\
\hline TRS16 & 0.473 & 6 & 4 & 4 & 6 & 21 & 19 & 20 & 19 & 53 & 45 & 49 \\
\hline TRS20 & 0.4743 & 6 & 4 & 4 & 6 & 20 & 17 & 19 & 21 & 51 & 45 & 49 \\
\hline JA620 & 0.4806 & 7 & 4 & 4 & 6 & 21 & 17 & 19 & 21 & 55 & 46 & 53 \\
\hline PRS16 & 0.4809 & 7 & 4 & 4 & 7 & 22 & 19 & 21 & 23 & 56 & 48 & 52 \\
\hline JA420 & 0.4822 & 7 & 4 & 4 & 6 & 22 & 17 & 19 & 21 & 56 & 47 & 54 \\
\hline INC20 & 0.4823 & 7 & 4 & 4 & 7 & 23 & 18 & 20 & 22 & 56 & 47 & 53 \\
\hline LTG20 & 0.4836 & 7 & 4 & 4 & 6 & 21 & 18 & 21 & 21 & 55 & 46 & 54 \\
\hline JA720 & 0.4842 & 7 & 4 & 4 & 6 & 22 & 17 & 21 & 21 & 56 & 48 & 55 \\
\hline SUP20 & 0.4862 & 7 & 4 & 4 & 6 & 23 & 19 & 22 & 22 & 56 & 49 & 57 \\
\hline JA520 & 0.4874 & 7 & 4 & 4 & 6 & 22 & 18 & 21 & 21 & 57 & 49 & 57 \\
\hline JA218 & 0.4876 & 7 & 4 & 4 & 7 & 22 & 18 & 20 & 22 & 55 & 45 & 49 \\
\hline JS420 & 0.4879 & 7 & 4 & 5 & 7 & 24 & 19 & 22 & 23 & 56 & 49 & 57 \\
\hline J1320 & 0.4885 & 7 & 4 & 4 & 7 & 23 & 19 & 22 & 22 & 56 & 49 & 57 \\
\hline PRS12 & 0.4897 & 6 & 4 & 6 & 6 & 21 & 20 & 21 & 19 & 55 & 46 & 48 \\
\hline SEN20 & 0.491 & 7 & 4 & 6 & 6 & 24 & 20 & 22 & 23 & 56 & 48 & 56 \\
\hline LAC20 & 0.4918 & 8 & 4 & 5 & 7 & 25 & 21 & 23 & 23 & 58 & 51 & 56 \\
\hline SEN14 & 0.4919 & 6 & 4 & 6 & 6 & 22 & 20 & 20 & 21 & 52 & 46 & 49 \\
\hline PRS20 & 0.4932 & 8 & 4 & 5 & 6 & 25 & 20 & 22 & 22 & 60 & 50 & 59 \\
\hline JS220 & 0.4934 & 8 & 4 & 6 & 7 & 24 & 21 & 22 & 24 & 59 & 51 & 58 \\
\hline SUP16 & 0.4941 & 6 & 4 & 6 & 7 & 23 & 22 & 23 & 25 & 57 & 49 & 53 \\
\hline JS118 & 0.4955 & 7 & 4 & 5 & 7 & 25 & 20 & 22 & 23 & 58 & 50 & 54 \\
\hline INC16 & 0.496 & 6 & 4 & 5 & 7 & 22 & 22 & 22 & 25 & 57 & 50 & 53 \\
\hline JST16 & 0.4976 & 7 & 4 & 6 & 7 & 23 & 21 & 22 & 25 & 58 & 50 & 54 \\
\hline LTG12 & 0.4992 & 7 & 5 & 6 & 6 & 22 & 22 & 22 & 22 & 58 & 50 & 53 \\
\hline JS120 & 0.5 & 8 & 4 & 6 & 7 & 27 & 22 & 25 & 27 & 60 & 52 & 60 \\
\hline AUD16 & 0.5007 & 8 & 5 & 6 & 7 & 23 & 22 & 23 & 26 & 56 & 51 & 51 \\
\hline GOV16 & 0.5011 & 7 & 4 & 6 & 7 & 27 & 20 & 23 & 26 & 58 & 50 & 54 \\
\hline ATG20 & 0.5013 & 8 & 4 & 6 & 7 & 25 & 21 & 23 & 24 & 58 & 51 & 59 \\
\hline ATG16 & 0.5027 & 7 & 4 & 6 & 7 & 23 & 20 & 23 & 24 & 57 & 50 & 54 \\
\hline JA118 & 0.5078 & 8 & 4 & 7 & 7 & 26 & 22 & 24 & 25 & 58 & 51 & 59 \\
\hline AUD20 & 0.5088 & 8 & 4 & 7 & 7 & 28 & 24 & 26 & 28 & 61 & 54 & 62 \\
\hline JA318 & 0.5091 & 8 & 4 & 6 & 7 & 26 & 21 & 25 & 25 & 59 & 52 & 58 \\
\hline SOS20 & 0.5116 & 8 & 5 & 8 & 7 & 28 & 24 & 26 & 28 & 62 & 53 & 61 \\
\hline JGE16 & 0.5131 & 8 & 5 & 6 & 7 & 25 & 22 & 25 & 28 & 59 & 52 & 54 \\
\hline INC12 & 0.5186 & 8 & 5 & 6 & 6 & 22 & 22 & 22 & 25 & 61 & 55 & 57 \\
\hline SOS16 & 0.5226 & 9 & 5 & 7 & 7 & 24 & 24 & 24 & 27 & 62 & 57 & 60 \\
\hline GOV20 & 0.5229 & 8 & 4 & 8 & 8 & 27 & 23 & 25 & 27 & 63 & 58 & 64 \\
\hline AUD12 & 0.5371 & 9 & 8 & 7 & 7 & 28 & 27 & 27 & 29 & 65 & 61 & 64 \\
\hline SOS12 & 0.5379 & 9 & 7 & 8 & 7 & 26 & 26 & 25 & 29 & 63 & 59 & 62 \\
\hline TRS12 & 0.5383 & 9 & 7 & 10 & 7 & 24 & 25 & 25 & 28 & 65 & 59 & 63 \\
\hline SUP12 & 0.5424 & 9 & 8 & 9 & 8 & 28 & 28 & 28 & 31 & 66 & 61 & 64 \\
\hline
\end{tabular}

Table 1: Do close votes translate to close seats? I have identified, for each plan, the elections with a partisan margin of closer than six points, but where the outcome falls outside of the range of 6-8 Congressional seats, 23-27 Senate seats, or 55-65 House seats.
- App. 33 -


Figure 2: Congressional comparison. Top figure shows votes and seats for NCLCV-Cong (green) and the now-invalidated SL-174 (maroon); below that are SL-3 (red) and Harper-Cong (yellow). The number of anti-majoritarian outcomes for each map is noted.
- App. 34 -


Figure 3: Senate comparison. Top figure shows votes and seats for NCLCV-Sen (green) and the now-invalidated SL-173 (maroon); below that are SL-2 (red) and Harper-Sen (yellow). The number of anti-majoritarian outcomes for each map is noted.
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Figure 4: House comparison. Top figure shows votes and seats for NCLCV-House (green) and the now-invalidated SL-175 (maroon); below that is SL-4 (red). The number of anti-majoritarian outcomes for each map is noted.

\section*{3 Summary scores of partisan fairness}

\subsection*{3.1 Recap of metrics}

Recall the following metrics of partisan fairness, to be presented in Tables 2-4.
- Efficiency gap (EG) is the difference in "wasted" votes for the two parties, across the state, as a share of votes cast [10]. The authors of the paper that popularized efficiency gap (Stephanopoulos-McGhee) later advocated for a simplified efficiency gap formula \(E G=2 V-S-\frac{1}{2}\), where \(V\) is the vote share in an election and \(S\) is the seat share. Original efficiency gap and simplified efficiency gap would be exactly equal if the districts had equal turnout; it's the simplified formula that was invoked, for example, in the language for the Freedom To Vote act. The authors proposed .08, later refined to .07, as the flag for a presumptive gerrymander. \({ }^{2}\)
- Partisan symmetry is a family of scores based on the principle of table-turning: if the votes for the parties were reversed, would the representation also be reversed? An asymmetric plan is one in which one party fares better with its portion of support than the other party would with the same portion. Scores in this group include the mean-median gap \((M M)\), the partisan bias score (PB), and the partisan Gini (PG). The mean-median gap literally takes the difference between the average vote share in a district and the median, or middle, district (or the average of the two middle districts when the number of districts is even). The gap is zero when the middle district looks like the state as a whole, so that half the districts are more favorable to one party and half are more favorable to the other. Partisan bias is described in the literature as measuring how much "extra" representation each party would secure in a hypothetical 50-50 election. Finally, partisan Gini is a summary statistic for all of the various kinds of symmetry measures in the political science literature. The "Partisan Symmetry Standard" of King and his co-authors asks that a seats-votes curve be literally symmetric about the center point, meaning that it predicts exactly the same representation for either party at any share of the vote [8]. The partisan Gini, first proposed by Bernard Grofman in 1983, takes this literally, measuring the area between the curve and its mirror image [9]. This is an unsigned metric, with zero as an ideal. (When the \(P G\) score is zero, all other symmetry scores, like mean-median and partisan bias, are necessarily zero as well.)
- The metric I have called Eguia county skew (ECS) is based on economist Jon Eguia's "jurisdictional partisan advantage" [7]. Eguia built a metric based on comparing the actual representation secured by a party under a vote pattern to the representation if cities and counties played the role of districts. I have applied it here only to counties, because of the fundamental importance of counties in North Carolina redistricting in particular. A simple way to explain this Eguia-style metric is as follows: in a particular election, what percentage of North Carolinians live in counties that favored Republicans? That is the benchmark for Republican representation; if their seat share is higher, the map is tilted Republican, and if lower, the map is tilted Democratic.

\footnotetext{
\({ }^{2}\) In paragraph 167 of the North Carolina Supreme Court's recent decision in this case, it is noted that "With regard to the efficiency gap measure, courts have found "that an efficiency gap above 7\% in any districting plan's first election year will continue to favor that party for the life of the plan."" (Quoting the U.S. Supreme Court, from Whitford v. Gill).
}

From these three types, I have chosen five signed scores to present in Tables 2-3: EG, simplified \(E G, M M, P B\), and \(E C S\). For all five scores, zero is ideal.

After that, I will use a second table, Table 4 to present the seat average for each party, the size of disproportionality for each election set, and the partisan Gini PG.

In both of these tables, I will use three sets of elections: first, the full set of 52 general elections. Next, the 35 non-judicial contests. And finally, the 14 "up-ballot" contests, which are the first five to appear on the ballot: President, U.S. Senator, Governor, Lieutenant Governor, and Attorney General. (These each occurred three times in the previous cycle, except for Attorney General, which was only contested twice.)

\subsection*{3.2 Comparison of metrics}

We will see a phenomenon clearly visible in the following tour of the metrics (which was actually already apparent in Table 1 and Figures 2-4): when given a chance to re-draw maps, the Legislature produced maps that split the difference between the partisan properties of the original proposals and the properties observed in the plaintiffs' maps.

At the Congressional level, this brings the mean-median scores down substantially, but leaves all the other scores at extremely elevated levels.


Table 2: Five simplified scores of partisan fairness, averaged over different sets of elections. These five metrics are all signed, meaning that they can take positive or negative values; positive and negative scores are intended to flag an advantage to Democrats and Republicans, respectively. \(E G\) and \(M M\) are computed as a share of votes; \(P B\) and the Eguia score are computed as a share of seats. Colors are intended for ease of comparisons and are consistent within each score.

For the Senate plan, the split-the-difference approach leaves significantly inferior scores on all metrics of partisan fairness than the ones, very near zero, in the plaintiffs' maps. For the House, on the other hand, the new plan is now down to a level that is markedly better in several of the metrics.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{} & \multicolumn{3}{|c|}{NCLCV-Sen} & \multicolumn{3}{|c|}{SL-173 (old Senate plan)} \\
\hline & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & \begin{tabular}{l}
Up-ballot \\
(14 contests)
\end{tabular} & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) \\
\hline Efficiency Gap & -0.020 & -0.024 & -0.017 & -0.075 & -0.068 & -0.080 \\
\hline Simplified \(E G\) & -0.023 & -0.028 & -0.021 & -0.076 & -0.070 & -0.081 \\
\hline Mean-median & -0.009 & -0.012 & -0.009 & -0.036 & -0.036 & -0.037 \\
\hline Partisan Bias & -0.015 & -0.023 & -0.016 & -0.072 & -0.069 & -0.08 \\
\hline Eguia County Skew & -0.040 & -0.041 & -0.030 & -0.093 & -0.083 & -0.09 \\
\hline & \multicolumn{3}{|c|}{SL-2 (new Senate plan)} & \multicolumn{3}{|c|}{Harper-Sen} \\
\hline & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) \\
\hline Efficiency Gap & -0.045 & -0.048 & -0.046 & -0.022 & -0.023 & -0.029 \\
\hline Simplified \(E G\) & -0.048 & -0.051 & -0.050 & -0.027 & -0.028 & -0.034 \\
\hline Mean-median & -0.020 & -0.022 & -0.021 & -0.003 & -0.005 & -0.003 \\
\hline Partisan Bias & -0.044 & -0.045 & -0.049 & -0.013 & -0.018 & -0.002 \\
\hline Eguia County Skew & -0.065 & -0.064 & -0.059 & -0.044 & -0.041 & -0.043 \\
\hline
\end{tabular}
\begin{tabular}{|c|ccc|ccc|}
\hline & \multicolumn{3}{|c|}{ NCLCV-House } & \multicolumn{3}{c|}{ SL-175 (old House plan) } \\
& All & Non-judicial & Up-ballot & All & Non-judicial & Up-ballot \\
(52 contests) & (35 contests) & (14 contests) & (52 contests) & (35 contests) & (14 contests) \\
\hline \hline Efficiency Gap & -0.020 & -0.022 & -0.017 & -0.076 & -0.075 & -0.078 \\
Simplified EG & -0.014 & -0.016 & -0.012 & -0.074 & -0.074 & -0.077 \\
Mean-median & -0.015 & -0.015 & -0.017 & -0.039 & -0.039 & -0.04 \\
Partisan Bias & -0.018 & -0.019 & -0.018 & -0.082 & -0.082 & -0.086 \\
Eguia County Skew & -0.031 & -0.030 & -0.021 & -0.091 & -0.088 & -0.086 \\
\hline
\end{tabular}
\begin{tabular}{|c|ccc|}
\hline & \multicolumn{3}{|c|}{ SL-4 (new House plan) } \\
Non-judicial \\
Up-ballot \\
(52 contests) & (35 contests) & (14 contests)
\end{tabular}

Table 3: The same scores, now assessed for state Senate and state House maps. Across the board, the new maps from the Legislature split the difference between the invalidated plans and the LCV remedial proposals. Colors are intended for ease of comparisons and are consistent within each score.

When we turn to seats by party and the partisan Gini, the story is quite similar (Table 4).
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\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{} & \multicolumn{3}{|c|}{NCLCV-Cong} & \multicolumn{3}{|c|}{SL-174 (old Cong plan)} \\
\hline & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) \\
\hline D Seats & 6.9 & 6.9 & 6.7 & 4.4 & 4.5 & 4.1 \\
\hline R Seats & 7.1 & 7.1 & 7.3 & 9.6 & 9.5 & 9.9 \\
\hline Disproportionality & 0.0 & 0.0 & -0.1 & -2.5 & -2.4 & -2.8 \\
\hline Partisan Gini & 0.021 & 0.020 & 0.021 & 0.078 & 0.073 & 0.080 \\
\hline \multirow[t]{2}{*}{} & \multicolumn{3}{|c|}{SL-3 (new Cong plan)} & \multicolumn{3}{|c|}{Harper-Cong} \\
\hline & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) & \begin{tabular}{l}
All \\
(52 contests)
\end{tabular} & Non-judicial (35 contests) & Up-ballot (14 contests) \\
\hline D Seats & 5.4 & 5.6 & 5.4 & 6.5 & 6.5 & 6.4 \\
\hline R Seats & 8.6 & 8.4 & 8.6 & 7.5 & 7.5 & 7.6 \\
\hline Disproportionality & -1.5 & -1.3 & -1.5 & -0.4 & -0.4 & -0.4 \\
\hline Partisan Gini & 0.032 & 0.032 & 0.032 & 0.014 & 0.015 & 0.014 \\
\hline
\end{tabular}

\begin{tabular}{|c|ccc|ccc|}
\hline & \multicolumn{3}{|c|}{ NCLCV-House } & \multicolumn{2}{c|}{ SL-175 (old House plan) } \\
& All & Non-judicial & Up-ballot & All & Non-judicial & Up-ballot \\
(52 contests) & (35 contests) & (14 contests) & (52 contests) & (35 contests) & (14 contests)
\end{tabular}
\begin{tabular}{|c|ccc|}
\hline & \multicolumn{3}{|c|}{ SL-4 (new House plan) } \\
& All & Non-judicial & Up-ballot \\
& (52 contests) & (35 contests) & (14 contests) \\
\hline \hline D Seats & 53.4 & 53.2 & 52.5 \\
R Seats & 66.6 & 66.8 & 67.5 \\
Disproportionality & -5.5 & -5.9 & -6.1 \\
Partisan Gini & 0.037 & 0.037 & 0.039 \\
\hline
\end{tabular}

Table 4: Average seat totals and the distance from proportionality. The partisan Gini score measures how far the seats-votes curve is from perfect symmetry. Across the board, the "splits the difference" trend is apparent.

Finally, for another way of slicing the same data:
\begin{tabular}{ccccc} 
& \multicolumn{2}{c}{ Up-ballot generals (14) } & \multicolumn{2}{c}{ All generals (52) } \\
& D vote share & D seat share & D vote share & D seat share \\
\hline \hline NCLCV-Cong & & .4796 & & .4931 \\
SL-174 (old Cong plan) & .4883 & .2908 & .4911 & .3118 \\
SL-3 (new Cong plan) & & .3857 & .4857 \\
Harper-Cong & .4571 & & .4643 \\
\hline NCLCV-Sen & & .4557 & & .4592 \\
SL-173 (old Sen plan) & .4883 & .3957 & .4911 & .4065 \\
SL-2 (new Sen plan) & & .4280 & & .4340 \\
Harper-Sen & & .4420 & & .4560 \\
\hline NCLCV-House & & .4649 & & .4684 \\
SL-175 (old House plan) & .4883 & .3994 & .4911 & .4080 \\
SL-4 (new House plan) & & .4375 & & .4450
\end{tabular}

Table 5: Comparing overall fidelity of representation to the voting preferences of the electorate. As from every other point of view, the new plans from the Legislature split the difference from their original proposal to the LCV plans, which score better on all metrics of partisan fairness.

\section*{4 Comparison to Barber report}

I have described the scores on a range of metrics that result from overlaying eleven plans with 52 elections, and l've also presented several more selective subsets of the elections, to make it clear the that findings are robust.

Dr. Michael Barber filed a report on February 18 in which he obtains systematically less severe bias indicators for the Legislature's new proposed maps.

For instance, consider the reported efficiency gaps.
\begin{tabular}{cccc} 
& \begin{tabular}{c} 
Barber method \\
(12 elections)
\end{tabular} & \begin{tabular}{c} 
current method \\
(Barber elections) \\
current method \\
(14 "up-ballot")
\end{tabular} \\
SL-174 (old) & -.195 & -.195 & -.181 \\
SL-3 (new) & -.053 & -.093 & -.088 \\
\hline SL-173 (old) & -.080 & -.078 & -.080 \\
SL-2 (new) & -.040 & -.036 & -.046 \\
\hline SL-175 (old) & -.072 & -.079 & -.078 \\
SL-4 (new) & -.008 & -.024 & -.039
\end{tabular}

Table 6: Efficiency gap in each election using the wasted votes method (which is described above as the "original" \(E G\) ).

I have made a serious attempt at replication in the very limited time available and have not been able to figure out how Dr. Barber arrives at his numbers, exactly. My conclusion is one of two things: either the discrepancy owes to the problematic way he blends elections together, which I will describe below, or he is actually using a different method from the one he describes in his report. \({ }^{3}\)

\footnotetext{
\({ }^{3}\) For instance, there are published methods that introduce statistical corrections into the data for fractional seats, or that randomly add noise to an election index. He has not said that he is doing either of these, but it is possible that he is employing software that does this without realizing it.
}

Dr. Barber describes his election index as follows: "if a district has an index value of 0.51 , this would mean that \(51 \%\) of the votes cast for the two major parties across these 12 elections went to Democratic candidates." This means that he is adding up the votes, rather than weighting all elections equally. I will make two observations about the problems this causes.

Weighting. The first effect is to upweight higher-turnout elections. To see the effects of the up-weighting, note that ten of 12 elections are from 2020 (see Table 7 for the list), which means that he is giving over \(85 \%\) of the weight to a single election year. \({ }^{4}\) Dr. Barber indicates that he is using the same twelve elections used by Dr. Mattingly in an earlier report—but that is a selective attribution. Mattingly uses a larger set of 15 elections for his statewide analysis. Notably dropped are ATG16 and GOV16-two elections that would counteract the dominance of 2020, and that show anti-majoritarian outcomes under the SL-3 map.

Faulty averaging: practical illustration. Consider the election-by-election efficiency gaps for Barber's 12 elections.
\begin{tabular}{c|cccccc} 
& PRS20 & SEN20 & GOV20 & LTG20 & ATG20 & SOS20 \\
\hline EG & -0.1276 & -0.0532 & 0.0225 & -0.1792 & -0.0742 & 0.0457 \\
\hline D seats & 5 & 6 & 8 & 4 & 6 & 8 \\
D votes & .4932 & .4910 & .5229 & .4836 & .5013 & .5116 \\
& & & & & & \\
& TRS20 & AGC20 & AUD20 & LAC20 & PRS16 & LTG16 \\
\hline EG & -0.1602 & -0.1349 & -0.0177 & -0.1239 & -0.1693 & -0.1386 \\
\hline D seats & 4 & 4 & 7 & 5 & 4 & 4 \\
D votes & .4743 & .4615 & .5088 & .4918 & .4809 & .4665
\end{tabular}
average of these \(E G\) values: -0.09255
Barber's reported EG: -. 0529
Table 7: Election-by-election scores in Barber's elections for the original efficiency gap-the wasted-votes method that Barber describes in his report.

It is unreasonable on its face to take a set of actually observed elections that show such large efficiency gaps and propose a style of blending them that hides that effect.

Faulty averaging: abstract example. How is this happening? Most partisan scores are non-linear, meaning that if you average elections and then compute the score, this is NOT the same as reporting the average of the by-election scores.

For efficiency gap specifically, adding elections creates an unintelligible blended election from the point of view of the meaning of the metric. Is a vote wasted or not wasted? That depends on who wins the district. But a "wasted vote" is a property of the individual election, not of the composite.

Here is an illustrative example. Suppose that there have been ten elections in a twodistrict state. Nine of them had 51-49 wins for Party A in both districts. The tenth went 80-20 the other way, in favor of Party \(B\). The nine tight elections had one wasted vote for Party A and 49 for Party \(B\) in each district, for an efficiency gap of \(\frac{2(1-49)}{200}\), or -48 , indicating a huge advantage to Party A. (The largest possible magnitude of the gap is . 5 , so this is a truly massive gerrymander.) The last election had \(E G=\frac{2(20-30)}{200}=-1\), also indicating advantage to Party A. Let's apply Dr. Barber's method. We sum all the elections, so that now each district

\footnotetext{
\({ }^{4}\) For instance, the total major-party cast votes in PRS20 were \(5,443,067\) (highest) while for LTG16 it was \(4,438,769\) (lowest), giving the first contest \(23 \%\) more weight. Applying that factor of 1.2 to ten elections out of twelve gives them a \(12 / 14\) share of the weight, which is about \(85.7 \%\).
}
has 484 votes for Party A and 516 votes for Party B. Now the efficiency gap is \(\frac{2(479-21)}{2000}\), or +.458 . This looks like a single tight election, and an epic gerrymander, for Party B. That is, summing the elections gives you an uninterpretable stew. It takes a situation where one party has thin-sliced its advantage to repeatedly convert narrow preferences to a \(2-0\) sweep of seats, and it obscures that pattern completely.

Let me repeat what is illustrated by this example: an application of Barber's method takes ten elections where nine had \(E G=-.48\) and the last had \(E G=-.1\) and, by averaging the contests into an election index, produces an overall \(E G\) of +.458 . It is a strange method indeed if ten negative numbers can average to a positive total.

The same flaws permeate Dr. Barber's entire analysis, because each of his partisan metric calculations draws on the same problematic election index. This implicates not only his efficiency gap scores but also his mean-median scores and his partisan symmetry scores, which are likewise based on non-linear combinations of electoral data. (That is, the median of an average is not the average of the medians, and so on.) For each of his scores, he has applied an unreasonable averaging method that makes the systematic advantage for Republicans disappear.

North Carolina provides an extraordinary opportunity to base partisan determinations on a large number of actual election patterns from the last ten years, many of which were extremely close elections. We have a chance to employ methods that take advantage of this large naturalistically observed dataset rather than those that hide its systematic properties.

\section*{5 Electoral opportunity for Black voters}

In my previous report, I explained how I constructed a determination of which districts are effective at providing Black voters with an opportunity to elect candidates of choice.

Running the same effectiveness count for the current plans, I obtain the following numbers.

\section*{Effective districts for Black voters}
\begin{tabular}{|c|c|c|c|c|}
\hline & NC-LCV maps & previous Leg. maps & new Leg. maps & Harper maps \\
\hline Congress & \begin{tabular}{l}
4 \\
(CD 2, 4, 9, 11)
\end{tabular} & \[
\begin{aligned}
& \hline \hline 2 \\
& (C D 2,9)
\end{aligned}
\] & \[
\begin{aligned}
& \hline 2 \\
& (C D 1,12)
\end{aligned}
\] & \begin{tabular}{l}
\[
3
\] \\
(CD 1, 6, 12)
\end{tabular} \\
\hline Senate & 12
(SD 1, 5, 11, 14, 18,
\(19,26,27,32,38,39\),
40 ) & \[
\begin{aligned}
& 8 \\
& \text { (SD 5, 11, 14, 19, 28, } \\
& 38,39,40 \text { ) }
\end{aligned}
\] & \[
\begin{aligned}
& 10 \\
& \text { (SD 3, 5, 11, 14, 19, } \\
& 27,28,38,40,41 \text { ) }
\end{aligned}
\] & ```
11
(SD 3, 5, 11, 13, 16,
19, 27, 28, 38, 40, 41)
``` \\
\hline House & 36
(HD 2, 8, 9, 10, 23, 24 , 25, 27, 31, 32, 33, 38, \(39,40,42,43,44,45\), 48, 57, 58, 59, 60, 61, \(63,66,71,88,92,99\), 100, 101, 102, 106, & 24
(HD 8, 23, 24, 25, 27, \(32,38,39,42,44\), \(48,57,58,60,66,71\), 92, 99, 100, 101, 102, 106, 107, 112) & \begin{tabular}{l}
27 \\
(HD 8, 23, 24, 25, 27 , \(31,32,33,38,39,42\), \(44,45,48,58,60,61\), 66, 71, 92, 99, 100, 101, 102, 106, 107, 112)
\end{tabular} & - \\
\hline
\end{tabular}

Table 8: The plaintiffs' plans secure additional electoral opportunity for Black voters in North Carolina.

For comparison, Black voting age population (BVAP) levels by district can be found in Appendix A .

\section*{6 Conclusion}

At a high level, the situation with the Legislature's new maps of all three types is clear throughout all of the analysis presented here: they chose maps with intermediate partisan properties between the now-invalidated original proposals and a truly even-handed map. This is quite evident in Table 4, where the number of R Congressional seats was 7.1 in the LCV maps and 9.6 in the invalidated plans; the new plans average to 8.6 . For Senate, the new plans split the difference between 27.0 and 29.7 seats, giving 28.3. And in the House, they split the difference between 56.2 seats and 49.0, giving 53.4.

I find the Legislature's new Congressional and Senate plans to be particularly problematic from a Close-Votes-Close-Seats perspective, often giving four out of 14 Congressional seats (28\%) or twenty out of 50 Senate seats (40\%) to Democrats even when Democrats poll at better than \(48 \%\) of the major-party vote. This is borne out in the partisan fairness scores, which show the new proposals splitting the difference from the now-invalidated maps to the plaintiffs' alternatives.

The plaintiffs' proposed remedial plans simply perform far better on the Close-Votes-CloseSeats norm and on the full suite of partisan fairness scores. For the scores, there are 63 opportunities to compare the plans numerically: seven metrics ( \(E G\), simplified \(E G, M M, P B\), disproportionality, and \(P G\) ) times three election sets (all, non-judicial, up-ballot) times three maps (Congress, Senate, House). The newly enacted plans improve on their predecessors all 63 times, but they likewise fall significantly short of the LCV maps all 63 times (and fall short of the Harper maps in 42 of 42 available comparisons). It is as consistent and robust of a finding as can be.

The LCV plans are also superlative on the traditional districting principles (recalling previous reports) and contain a large number of districts that provide effective electoral opportunitybut not a guarantee-for Black voters. In sum, they are an excellent choice of remedial plans for adoption by the Court.

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\section*{A BVAP across the districts of the proposed remedial plans}
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{ NCLCV-Cong } \\
\hline CD & B1VAP & APBVAP \\
\hline 1 & 0.289 & 0.304 \\
2 & 0.332 & 0.347 \\
3 & 0.118 & 0.131 \\
4 & 0.319 & 0.344 \\
5 & 0.226 & 0.245 \\
6 & 0.227 & 0.242 \\
7 & 0.115 & 0.128 \\
8 & 0.123 & 0.132 \\
9 & 0.277 & 0.298 \\
10 & 0.232 & 0.25 \\
11 & 0.271 & 0.289 \\
12 & 0.121 & 0.132 \\
13 & 0.114 & 0.124 \\
14 & 0.032 & 0.039 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{ SL-3 } \\
\hline CD & B1VAP & APBVAP \\
\hline 1 & 0.403 & 0.42 \\
2 & 0.205 & 0.224 \\
3 & 0.17 & 0.185 \\
4 & 0.249 & 0.266 \\
5 & 0.156 & 0.168 \\
6 & 0.239 & 0.257 \\
7 & 0.23 & 0.252 \\
8 & 0.176 & 0.19 \\
9 & 0.182 & 0.195 \\
10 & 0.071 & 0.079 \\
11 & 0.033 & 0.04 \\
12 & 0.317 & 0.339 \\
13 & 0.162 & 0.175 \\
14 & 0.196 & 0.211 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{ Harper-Cong } \\
\hline CD & B1VAP & APBVAP \\
\hline 1 & 0.396 & 0.412 \\
2 & 0.225 & 0.243 \\
3 & 0.173 & 0.187 \\
4 & 0.247 & 0.263 \\
5 & 0.08 & 0.089 \\
6 & 0.316 & 0.336 \\
7 & 0.166 & 0.178 \\
8 & 0.111 & 0.121 \\
9 & 0.181 & 0.197 \\
10 & 0.127 & 0.137 \\
11 & 0.032 & 0.039 \\
12 & 0.312 & 0.334 \\
13 & 0.127 & 0.141 \\
14 & 0.297 & 0.321 \\
\hline
\end{tabular}

Table 9: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the proposed remedial plans for Congress.
- App. 46 -
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{3}{|c|}{NCLCV-Sen} & \multicolumn{3}{|c|}{SL-2} & \multicolumn{3}{|c|}{Harper-Sen} \\
\hline SD & B1VAP & APBVAP & SD & B1VAP & APBVAP & SD & B1VAP & APBVAP \\
\hline 1 & 0.408 & 0.423 & 1 & 0.165 & 0.175 & 1 & 0.165 & 0.175 \\
\hline 2 & 0.165 & 0.175 & 2 & 0.253 & 0.267 & 2 & 0.253 & 0.267 \\
\hline 3 & 0.253 & 0.267 & 3 & 0.408 & 0.423 & 3 & 0.408 & 0.423 \\
\hline 4 & 0.334 & 0.35 & 4 & 0.334 & 0.35 & 4 & 0.334 & 0.35 \\
\hline 5 & 0.385 & 0.403 & 5 & 0.385 & 0.403 & 5 & 0.385 & 0.403 \\
\hline 6 & 0.13 & 0.153 & 6 & 0.13 & 0.153 & 6 & 0.13 & 0.153 \\
\hline 7 & 0.125 & 0.138 & 7 & 0.105 & 0.117 & 7 & 0.1 & 0.112 \\
\hline 8 & 0.12 & 0.128 & 8 & 0.139 & 0.148 & 8 & 0.142 & 0.152 \\
\hline 9 & 0.228 & 0.239 & 9 & 0.228 & 0.239 & 9 & 0.228 & 0.239 \\
\hline 10 & 0.154 & 0.167 & 10 & 0.154 & 0.167 & 10 & 0.154 & 0.167 \\
\hline 11 & 0.352 & 0.366 & 11 & 0.352 & 0.366 & 11 & 0.352 & 0.366 \\
\hline 12 & 0.189 & 0.206 & 12 & 0.189 & 0.206 & 12 & 0.189 & 0.206 \\
\hline 13 & 0.175 & 0.188 & 13 & 0.181 & 0.199 & 13 & 0.246 & 0.267 \\
\hline 14 & 0.312 & 0.332 & 14 & 0.406 & 0.43 & 14 & 0.115 & 0.131 \\
\hline 15 & 0.136 & 0.152 & 15 & 0.128 & 0.143 & 15 & 0.124 & 0.138 \\
\hline 16 & 0.08 & 0.092 & 16 & 0.094 & 0.107 & 16 & 0.382 & 0.405 \\
\hline 17 & 0.091 & 0.104 & 17 & 0.102 & 0.115 & 17 & 0.087 & 0.099 \\
\hline 18 & 0.323 & 0.347 & 18 & 0.215 & 0.23 & 18 & 0.169 & 0.181 \\
\hline 19 & 0.439 & 0.481 & 19 & 0.356 & 0.392 & 19 & 0.363 & 0.397 \\
\hline 20 & 0.22 & 0.237 & 20 & 0.256 & 0.273 & 20 & 0.39 & 0.41 \\
\hline 21 & 0.176 & 0.195 & 21 & 0.259 & 0.284 & 21 & 0.252 & 0.278 \\
\hline 22 & 0.364 & 0.382 & 22 & 0.326 & 0.344 & 22 & 0.195 & 0.211 \\
\hline 23 & 0.155 & 0.167 & 23 & 0.154 & 0.167 & 23 & 0.154 & 0.167 \\
\hline 24 & 0.278 & 0.296 & 24 & 0.278 & 0.296 & 24 & 0.278 & 0.296 \\
\hline 25 & 0.165 & 0.178 & 25 & 0.165 & 0.179 & 25 & 0.17 & 0.184 \\
\hline 26 & 0.332 & 0.35 & 26 & 0.207 & 0.221 & 26 & 0.283 & 0.3 \\
\hline 27 & 0.297 & 0.317 & 27 & 0.272 & 0.29 & 27 & 0.249 & 0.266 \\
\hline 28 & 0.282 & 0.303 & 28 & 0.43 & 0.456 & 28 & 0.376 & 0.399 \\
\hline 29 & 0.171 & 0.18 & 29 & 0.169 & 0.178 & 29 & 0.169 & 0.178 \\
\hline 30 & 0.084 & 0.092 & 30 & 0.084 & 0.092 & 30 & 0.084 & 0.092 \\
\hline 31 & 0.122 & 0.135 & 31 & 0.207 & 0.222 & 31 & 0.222 & 0.239 \\
\hline 32 & 0.329 & 0.35 & 32 & 0.234 & 0.252 & 32 & 0.224 & 0.24 \\
\hline 33 & 0.14 & 0.149 & 33 & 0.14 & 0.149 & 33 & 0.14 & 0.149 \\
\hline 34 & 0.184 & 0.202 & 34 & 0.184 & 0.201 & 34 & 0.184 & 0.201 \\
\hline 35 & 0.105 & 0.116 & 35 & 0.106 & 0.117 & 35 & 0.1 & 0.112 \\
\hline 36 & 0.04 & 0.046 & 36 & 0.039 & 0.045 & 36 & 0.04 & 0.046 \\
\hline 37 & 0.104 & 0.115 & 37 & 0.104 & 0.114 & 37 & 0.105 & 0.116 \\
\hline 38 & 0.354 & 0.377 & 38 & 0.411 & 0.437 & 38 & 0.422 & 0.448 \\
\hline 39 & 0.4 & 0.426 & 39 & 0.212 & 0.231 & 39 & 0.203 & 0.223 \\
\hline 40 & 0.376 & 0.402 & 40 & 0.361 & 0.387 & 40 & 0.341 & 0.365 \\
\hline 41 & 0.116 & 0.131 & 41 & 0.374 & 0.396 & 41 & 0.371 & 0.394 \\
\hline 42 & 0.224 & 0.24 & 42 & 0.11 & 0.125 & 42 & 0.127 & 0.143 \\
\hline 43 & 0.181 & 0.194 & 43 & 0.173 & 0.186 & 43 & 0.179 & 0.192 \\
\hline 44 & 0.129 & 0.138 & 44 & 0.123 & 0.131 & 44 & 0.129 & 0.138 \\
\hline 45 & 0.065 & 0.074 & 45 & 0.066 & 0.076 & 45 & 0.067 & 0.076 \\
\hline 46 & 0.054 & 0.06 & 46 & 0.042 & 0.049 & 46 & 0.056 & 0.063 \\
\hline 47 & 0.028 & 0.035 & 47 & 0.028 & 0.034 & 47 & 0.029 & 0.035 \\
\hline 48 & 0.046 & 0.054 & 48 & 0.048 & 0.055 & 48 & 0.044 & 0.051 \\
\hline 49 & 0.044 & 0.052 & 49 & 0.063 & 0.072 & 49 & 0.046 & 0.054 \\
\hline 50 & 0.014 & 0.02 & 50 & 0.014 & 0.02 & 50 & 0.014 & 0.02 \\
\hline
\end{tabular}

Table 10: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the proposed remedial plans for state Senate.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{3}{|c|}{NCLCV-House} & \multicolumn{3}{|c|}{NCLCV-House} & \multicolumn{3}{|c|}{SL-4} & \multicolumn{3}{|c|}{SL-4} \\
\hline HD & B1VAP & APBVAP & HD & B1VAP & APBVAP & HD & B1VAP & APBVAP & HD & B1VAP & APBVAP \\
\hline 1 & 0.266 & 0.277 & 61 & 0.457 & 0.486 & 1 & 0.172 & 0.182 & 61 & 0.465 & 0.493 \\
\hline 2 & 0.335 & 0.351 & 62 & 0.115 & 0.127 & 2 & 0.292 & 0.307 & 62 & 0.152 & 0.166 \\
\hline 3 & 0.189 & 0.203 & 63 & 0.277 & 0.295 & 3 & 0.188 & 0.202 & 63 & 0.264 & 0.282 \\
\hline 4 & 0.219 & 0.23 & 64 & 0.114 & 0.126 & 4 & 0.244 & 0.255 & 64 & 0.128 & 0.141 \\
\hline 5 & 0.369 & 0.386 & 65 & 0.184 & 0.194 & 5 & 0.369 & 0.386 & 65 & 0.184 & 0.194 \\
\hline 6 & 0.216 & 0.24 & 66 & 0.31 & 0.336 & 6 & 0.222 & 0.246 & 66 & 0.309 & 0.335 \\
\hline 7 & 0.221 & 0.235 & 67 & 0.126 & 0.134 & 7 & 0.221 & 0.235 & 67 & 0.126 & 0.134 \\
\hline 8 & 0.333 & 0.353 & 68 & 0.072 & 0.081 & 8 & 0.361 & 0.381 & 68 & 0.082 & 0.093 \\
\hline 9 & 0.343 & 0.362 & 69 & 0.093 & 0.105 & 9 & 0.313 & 0.332 & 69 & 0.095 & 0.106 \\
\hline 10 & 0.349 & 0.37 & 70 & 0.065 & 0.072 & 10 & 0.323 & 0.344 & 70 & 0.066 & 0.074 \\
\hline 11 & 0.112 & 0.13 & 71 & 0.323 & 0.35 & 11 & 0.121 & 0.136 & 71 & 0.322 & 0.348 \\
\hline 12 & 0.373 & 0.385 & 72 & 0.371 & 0.393 & 12 & 0.373 & 0.385 & 72 & 0.383 & 0.404 \\
\hline 13 & 0.078 & 0.088 & 73 & 0.179 & 0.198 & 13 & 0.079 & 0.088 & 73 & 0.217 & 0.239 \\
\hline 14 & 0.112 & 0.134 & 74 & 0.108 & 0.12 & 14 & 0.121 & 0.144 & 74 & 0.118 & 0.13 \\
\hline 15 & 0.173 & 0.202 & 75 & 0.18 & 0.194 & 15 & 0.164 & 0.191 & 75 & 0.189 & 0.205 \\
\hline 16 & 0.106 & 0.116 & 76 & 0.199 & 0.21 & 16 & 0.107 & 0.117 & 76 & 0.199 & 0.21 \\
\hline 17 & 0.178 & 0.192 & 77 & 0.052 & 0.058 & 17 & 0.099 & 0.107 & 77 & 0.052 & 0.058 \\
\hline 18 & 0.13 & 0.144 & 78 & 0.081 & 0.089 & 18 & 0.188 & 0.203 & 78 & 0.052 & 0.058 \\
\hline 19 & 0.055 & 0.06 & 79 & 0.073 & 0.081 & 19 & 0.047 & 0.054 & 79 & 0.165 & 0.174 \\
\hline 20 & 0.04 & 0.048 & 80 & 0.099 & 0.108 & 20 & 0.07 & 0.081 & 80 & 0.09 & 0.098 \\
\hline 21 & 0.084 & 0.096 & 81 & 0.083 & 0.09 & 21 & 0.085 & 0.096 & 81 & 0.092 & 0.1 \\
\hline 22 & 0.272 & 0.285 & 82 & 0.183 & 0.2 & 22 & 0.272 & 0.285 & 82 & 0.191 & 0.209 \\
\hline 23 & 0.519 & 0.534 & 83 & 0.119 & 0.132 & 23 & 0.519 & 0.534 & 83 & 0.079 & 0.088 \\
\hline 24 & 0.371 & 0.386 & 84 & 0.154 & 0.166 & 24 & 0.369 & 0.385 & 84 & 0.155 & 0.167 \\
\hline 25 & 0.383 & 0.398 & 85 & 0.029 & 0.034 & 25 & 0.385 & 0.4 & 85 & 0.03 & 0.034 \\
\hline 26 & 0.173 & 0.189 & 86 & 0.057 & 0.064 & 26 & 0.165 & 0.181 & 86 & 0.057 & 0.064 \\
\hline 27 & 0.502 & 0.519 & 87 & 0.045 & 0.053 & 27 & 0.502 & 0.518 & 87 & 0.045 & 0.052 \\
\hline 28 & 0.158 & 0.171 & 88 & 0.32 & 0.341 & 28 & 0.158 & 0.17 & 88 & 0.228 & 0.247 \\
\hline 29 & 0.325 & 0.345 & 89 & 0.069 & 0.077 & 29 & 0.29 & 0.31 & 89 & 0.063 & 0.07 \\
\hline 30 & 0.243 & 0.26 & 90 & 0.032 & 0.039 & 30 & 0.288 & 0.307 & 90 & 0.032 & 0.038 \\
\hline 31 & 0.404 & 0.427 & 91 & 0.129 & 0.139 & 31 & 0.434 & 0.456 & 91 & 0.104 & 0.112 \\
\hline 32 & 0.42 & 0.434 & 92 & 0.319 & 0.345 & 32 & 0.419 & 0.434 & 92 & 0.318 & 0.344 \\
\hline 33 & 0.321 & 0.343 & 93 & 0.028 & 0.035 & 33 & 0.32 & 0.34 & 93 & 0.028 & 0.035 \\
\hline 34 & 0.093 & 0.104 & 94 & 0.049 & 0.055 & 34 & 0.105 & 0.117 & 94 & 0.049 & 0.055 \\
\hline 35 & 0.093 & 0.105 & 95 & 0.071 & 0.081 & 35 & 0.17 & 0.187 & 95 & 0.071 & 0.081 \\
\hline 36 & 0.058 & 0.069 & 96 & 0.089 & 0.1 & 36 & 0.073 & 0.086 & 96 & 0.092 & 0.105 \\
\hline 37 & 0.109 & 0.122 & 97 & 0.052 & 0.058 & 37 & 0.111 & 0.124 & 97 & 0.052 & 0.058 \\
\hline 38 & 0.305 & 0.324 & 98 & 0.075 & 0.086 & 38 & 0.416 & 0.439 & 98 & 0.074 & 0.085 \\
\hline 39 & 0.311 & 0.332 & 99 & 0.292 & 0.314 & 39 & 0.314 & 0.336 & 99 & 0.459 & 0.488 \\
\hline 40 & 0.316 & 0.339 & 100 & 0.29 & 0.316 & 40 & 0.097 & 0.11 & 100 & 0.334 & 0.36 \\
\hline 41 & 0.085 & 0.096 & 101 & 0.475 & 0.502 & 41 & 0.07 & 0.083 & 101 & 0.506 & 0.534 \\
\hline 42 & 0.384 & 0.415 & 102 & 0.302 & 0.323 & 42 & 0.376 & 0.42 & 102 & 0.309 & 0.33 \\
\hline 43 & 0.348 & 0.379 & 103 & 0.069 & 0.082 & 43 & 0.342 & 0.369 & 103 & 0.087 & 0.1 \\
\hline 44 & 0.365 & 0.411 & 104 & 0.092 & 0.103 & 44 & 0.4 & 0.438 & 104 & 0.086 & 0.098 \\
\hline 45 & 0.378 & 0.417 & 105 & 0.146 & 0.164 & 45 & 0.354 & 0.392 & 105 & 0.126 & 0.141 \\
\hline 46 & 0.282 & 0.295 & 106 & 0.451 & 0.481 & 46 & 0.251 & 0.264 & 106 & 0.351 & 0.376 \\
\hline 47 & 0.209 & 0.223 & 107 & 0.445 & 0.474 & 47 & 0.241 & 0.256 & 107 & 0.562 & 0.592 \\
\hline 48 & 0.346 & 0.371 & 108 & 0.107 & 0.116 & 48 & 0.346 & 0.371 & 108 & 0.137 & 0.147 \\
\hline 49 & 0.153 & 0.171 & 109 & 0.223 & 0.238 & 49 & 0.142 & 0.16 & 109 & 0.178 & 0.191 \\
\hline 50 & 0.174 & 0.185 & 110 & 0.169 & 0.18 & 50 & 0.174 & 0.185 & 110 & 0.187 & 0.198 \\
\hline 51 & 0.102 & 0.111 & 111 & 0.171 & 0.182 & 51 & 0.154 & 0.167 & 111 & 0.157 & 0.167 \\
\hline 52 & 0.199 & 0.212 & 112 & 0.469 & 0.493 & 52 & 0.218 & 0.231 & 112 & 0.308 & 0.331 \\
\hline 53 & 0.142 & 0.154 & 113 & 0.061 & 0.069 & 53 & 0.147 & 0.16 & 113 & 0.065 & 0.073 \\
\hline 54 & 0.137 & 0.149 & 114 & 0.035 & 0.042 & 54 & 0.106 & 0.116 & 114 & 0.077 & 0.086 \\
\hline 55 & 0.255 & 0.268 & 115 & 0.08 & 0.091 & 55 & 0.248 & 0.261 & 115 & 0.051 & 0.06 \\
\hline 56 & 0.096 & 0.111 & 116 & 0.046 & 0.055 & 56 & 0.094 & 0.109 & 116 & 0.033 & 0.04 \\
\hline 57 & 0.369 & 0.392 & 117 & 0.031 & 0.037 & 57 & 0.233 & 0.251 & 117 & 0.03 & 0.036 \\
\hline 58 & 0.363 & 0.386 & 118 & 0.011 & 0.015 & 58 & 0.456 & 0.484 & 118 & 0.011 & 0.015 \\
\hline 59 & 0.351 & 0.371 & 119 & 0.021 & 0.029 & 59 & 0.306 & 0.325 & 119 & 0.021 & 0.029 \\
\hline 60 & 0.286 & 0.304 & 120 & 0.008 & 0.013 & 60 & 0.328 & 0.347 & 120 & 0.008 & 0.013 \\
\hline
\end{tabular}

Table 11: Non-Hispanic Black alone (B1) and any-part-Black (APB) voting age population in the proposed remedial plans for the state House.

I declare under penalty of perjury that the foregoing is true and correct.

Executed:


Sworn and subscribed before me
this the 20 of February, 2022.


Notary Public


My Commission Expires:


State of California, county of Ala med q
Subscribed and sworn to (or affirmed) before me
\(\qquad\)
by. \(\qquad\) Moon Duchin
proved to me on the basis of satisfactory evidence to be the person ss) who appeared before me.


STATE OF NORTH CAROLINA
COUNTY OF WAKE
NORTH CAROLINA LEAGUE, OF CONSERVATION VOTERS, INC., et al.,

Plaintiffs,
COMMON CAUSE,
Plaintiff-Intervenor,
v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

STATE OF NORTH CAROLINA

COUNTY OF WAKE
REBECCA HARPER, et al., Plaintiffs,
v.

REPRESENTATIVE DESTIN HALL, in his official capacity as Chair of the House Standing Committee on Redistricting, et al.,

Defendants.

IN THE GENERAL COURT OF JUSTICE SUPERIOR COURT DIVISION FILE NO. 21 CVS 015426

IN THE GENERAL COURT OF JUSTICE SUPERIOR COURT DIVISION

FILE NO. 21 CVS 500085
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THIS MATTER comes before the undersigned three-judge panel pursuant to the February 4, 2022, Order of the Supreme Court of North Carolina ("Supreme Court Remedial Order) for review of Remedial Redistricting Plans to apportion the state legislative and congressional districts within North Carolina (hereinafter collectively referred to as the "Remedial Plans") enacted by the North Carolina General Assembly on February 17, 2022. 2022 N.C. Sess. Laws. 2 (also known as Senate Bill 744 and hereafter referred to as "Remedial Senate Plan"); 2022 N.C. Sess. Laws. 4 (also known as House Bill 980 and hereafter referred to as "Remedial House Plan"); 2022 N.C. Sess. Laws. 3 (also known as Senate Bill 745 and hereafter referred to as "Remedial Congressional Plan").

The Remedial Plans were enacted following entry of the Supreme Court Remedial Order. This Court entered a Judgment on January 11, 2022, wherein the Court upheld the constitutionality of the 2021 Enacted State Legislative and Congressional redistricting plans (hereinafter "Enacted Plans"). Thereafter, Harper Plaintiffs, North Carolina League of Conservation Voters Plaintiffs, and PlaintiffIntervenor Common Cause (hereinafter collectively referred to as "Plaintiffs") appealed this Court's Judgment directly to the Supreme Court of North Carolina. On February 4, 2022, the Supreme Court of North Carolina entered its Remedial Order, with opinion to follow, adopting in full this Court's findings of fact in the January 11, 2022, Judgment; however, the Supreme Court concluded that the Enacted Plans are unconstitutional under N.C. Const., art. I, \(\S \S 10,12,14\), and 19 and remanded the action to this Court for remedial proceedings. On February 14, 2022, the Supreme

Court filed its full opinion in this action. Harper v. Hall, 2022-NCSC-17 (Feb. 14, 2022).

Pursuant to the Supreme Court Remedial Order and full opinion, and after reviewing all remedial and alternative plans submitted to this Court, as well as additional documents, materials, and information pertaining to the submitted plans, including the report of this Court's appointed Special Masters and comments received from the parties, this Court sets out the following:

\section*{FINDINGS OF FACT}

\section*{I. Summary of Requirements for Remedial Process}
1. The Supreme Court's Order required the submission to this Court of remedial state legislative and congressional redistricting plans that "satisfy all provisions of the North Carolina Constitution"; both the General Assembly, and any parties to this action who chose to submit proposed remedial plans for this Court's consideration, were required to submit such plans, and additional information, on or before February 18, 2022, at 5:00 p.m.
2. The Supreme Court's Order also provided for a comment period in which parties to these consolidated cases were permitted to file and submit to this Court comments on any plans submitted for this Court's consideration by February 21, 2022 at 5:00 p.m.
3. The Supreme Court's Order also mandated that this Court must approve or adopt constitutionally compliant remedial plans by noon on February 23, 2022.
4. This Court subsequently entered an order on February 8, 2022, providing initial guidance on the remedial phase of the litigation before this Court,
requiring written submissions containing the information the Supreme Court set forth in its Order pertaining to redistricting plans in general and the ordered Remedial Plans specifically. The written submissions were required to provide an explanation of the data and other considerations the mapmaker relied upon to create any submitted proposed remedial plan and to determine that the proposed remedial plan was constitutional-i.e., compliant with the Supreme Court Remedial Order. The full opinion of the Supreme Court, Harper v. Hall, 2022-NCSC-17, thereafter provided further guidance for the Remedial Plans.
5. On February 16, 2022, this Court entered an Order appointing three former jurists of our State appellate and trial courts-Robert F. Orr, Robert H. Edmunds, Jr., and Thomas W. Ross-to serve as Special Masters for the purposes of: 1) assisting this Court in reviewing any Proposed Remedial Plans enacted and submitted by the General Assembly or otherwise submitted to the Court by a party in these consolidated cases; and, 2) assisting this Court in fulfilling the Supreme Court's directive to this Court to develop remedial plans based upon the findings in this Court's January 11, 2022, Judgment should the General Assembly fail to enact and submit Proposed Remedial Plans compliant with the Supreme Court's Order within the time allowed. This Appointment Order also required the submission of additional information, data, and materials for review by the Court, the parties, and the Special Masters.
6. The Appointment Order further provided that the Special Masters were authorized to hire assistants and advisors reasonably necessary to complete their
work. Pursuant to this authorization, the Special Masters hired the following advisors to assist in evaluating the Remedial Plans:
a. Bernard Grofman: PhD in political science from the University of Chicago, and currently the Jack W. Peltason Endowed Chair and Distinguished Professor at the University of California, Irvine, School of Social Sciences;
b. Tyler Jarvis: PhD in mathematics from Princeton University, and currently a Professor at Brigham Young University's College of Physical and Mathematical Sciences;
c. Eric McGhee: PhD in political science from the University of California, Berkeley, and currently a Senior Fellow at Public Policy Institute of California, a non-partisan, non-profit think tank; and,
d. Samuel Wang: PhD in Neurosciences from Stanford University, and currently a Professor of neuroscience at Princeton University and Director of the Electoral Innovation Lab.
7. The Court finds that these advisors were reasonably necessary to facilitate the work of the Special Masters to provide this Court with an analysis of the Remedial Plans. \({ }^{1}\)

\section*{II. The General Assembly's Remedial Plans as a Whole}
8. Pursuant to the Supreme Court's directive, the General Assembly enacted Remedial Plans and, through the Legislative Defendants, timely submitted the Remedial Plans to this Court on February 18, 2022.

\footnotetext{
\({ }^{1}\) On February 20, 2022, counsel for Harper Plaintiffs submitted a notice of communications wherein the Court was informed that Dr. Wang and Dr. Jarvis had contacted some of Harper Plaintiffs' retained experts by email regarding their algorithms and analysis models. Legislative Defendants subsequently filed a motion to disqualify Dr. Wang and Dr. Jarvis from assisting the Special Masters. The Special Masters have provided additional review of the issues presented in this motion, as noted in the Report attached to this Order, and the Court will address the Motion in a separate order that will be filed contemporaneously herewith.
}

\section*{A. Participants in the General Assembly's Drawing of Remedial Plans}
9. The House participants involved in the drawing of the Remedial Plans consisted of twenty-one Republican members and one Democratic member, with five Republican staff members and two Democratic staff members.
10. The Senate participants involved in the drawing of the Remedial Plans consisted of four Republican members and five Democratic members, with four Republican staff members and one Democratic staff member.
11. The General Assembly members were also supported by fifteen Legislative Analysis and Bill Drafting Division staff members, as well as four Information Systems Division staff members.
12. Legislative Defendants, through counsel, also relied for limited purposes on their experts and non-testifying experts in this case, including Clark Bensen and Sean Trende for statistical analysis, Dr. Jeffrey Lewis to conduct a Racially Polarized Voting Analysis for both the 2021 and the 2022 districts, and Dr. Michael Barber for statistical analyses of the Remedial Plans and other BVAP-related information.

\section*{B. The General Assembly's Remedial Criteria for Drawing the Remedial Plans}
13. The General Assembly's Remedial Criteria governing the remedial map drawing process were those neutral and traditional redistricting criteria adopted by the Joint Redistricting Committees on August 12, 2021, (received into evidence at trial as exhibit LDTX15) unless the criteria conflicted with the Supreme Court Remedial Order and full opinion.
14. Although expressly forbidden by the previously-used August 2021 Criteria, the General Assembly as part of its Remedial Criteria intentionally used partisan election data as directed by the Supreme Court's Remedial Order. The General Assembly did so by loading such data into Maptitude, the map drawing software utilized by the General Assembly in creating districting plans. The elections used by the General Assembly to evaluate the projected partisan effects of district lines were as follows: Lt. Gov 2016, President 2016, Commissioner of Agriculture 2020, Treasurer 2020, Lt. Gov. 2020, US Senate 2020, Commissioner of Labor 2020, President 2020, Attorney General 2020, Auditor 2020, Secretary of State 2020, and Governor 2020.
15. The Court finds that the General Assembly's use of partisan data in this manner comported with the Supreme Court Remedial Order.

\section*{C. The General Assembly's Racially Polarized Voting Analysis}
16. Paragraph 8 of the Supreme Court Remedial Order required the General Assembly to "assess whether, using current election and population data, racially polarized voting is legally sufficient in any area of the state such that Section 2 of the Voting Rights Act requires the drawing of a district to avoid diluting the voting strength of African-American voters."
17. The General Assembly conducted an abbreviated racially polarized voting ("RPV") analysis to determine whether racially polarized voting is legally sufficient in any area of the state such that Section 2 of the Voting Rights Act requires the drawing of a district to avoid diluting the voting strength of African American voters during the remedial process. Legislative Defendants' expert Dr. Jeffery B.

Lewis ran an analysis and concluded that all three Remedial Plans provide African Americans with proportional opportunity to elect their candidates of choice.
18. The Court finds that the General Assembly satisfied the directive in the Supreme Court Remedial Order to determine whether the drawing of a district in an area of the state is required to comply with Section 2 of the Voting Rights Act.

\section*{D. Plaintiffs' Objections and Comments to the Plans}
19. Pursuant to the Supreme Court's directive, Plaintiffs timely submitted comments on and objections to the Remedial Plans on February 21, 2022.
20. NCLCV Plaintiffs object to the Remedial Senate and Congressional Plans. NCLCV Plaintiffs do not specifically object to the Remedial House Plan but instead request the Court conduct its own analysis of the Remedial House Plan.
21. Harper Plaintiffs object to the Remedial Congressional Plan and Remedial Senate Plan. Harper Plaintiffs do not object to the Remedial House Plan.
22. Plaintiff Common Cause objects to all three Remedial Plans in general and specifically contends the Remedial Senate and House Plans must be redrawn for Senate District 4 and House District 10.

\section*{E. Report of Special Masters}
23. Pursuant to this Court's Appointment Order, the Special Masters prepared a Report containing their analysis and submitted that Report to this Court for its consideration. The Report is attached to this Order as an exhibit and has been filed with the Court.
24. The Special Masters, and their advisors, conducted an analysis of the Remedial Plans using a variety of metrics to determine whether the submitted maps
meet the requirements of the North Carolina Constitution as set out by the Supreme Court of North Carolina in its Remedial Order and full opinion.
25. The Special Masters' findings demonstrate that the Remedial House and Senate Plans meet the requirements of the Supreme Court's Remedial Order and full opinion.
26. The Special Masters' findings demonstrate that the Remedial Congressional Plan does not meet the requirements of the Supreme Court's Remedial Order and full opinion.
27. This Court adopts in full the findings of the Special Masters and sets out additional specific findings on the Remedial Plans' compliance with the Supreme Court Remedial Order below.

\section*{III. Remedial Congressional Plan}

\section*{A. The General Assembly's Starting Point and Subsequently Proposed Amendments}
28. In determining the base map for the Congressional Districts in the Remedial Congressional Plan that was eventually enacted, the Senate started from scratch.
29. There was a House Draft of a remedial congressional plan that was never voted on and therefore never considered by a committee or the full General Assembly.
30. Senator Clark offered one amendment to the Remedial Congressional Plan, a statewide plan, that was tabled.
31. The Remedial Congressional Plans passed the Senate by a vote of 25-19. The "aye" votes in the Senate were solely by members of the Republican party, while the "no" votes in the Senate were solely by members of the Democratic Party. The Remedial Congressional Plan passed the House by voice vote along party lines.
B. Analysis of Partisanship Reflected in the Remedial Congressional Plan
32. The Remedial Congressional Plan reflects key differences from the 2021 Enacted Congressional Plan in the projected partisan makeup of certain districts.
a. Four congressional districts are some of the most politically competitive in the country (i.e., presidential election differences of less than 5\%): District 6, District 7, District 13, and District 14.
b. Wake and Mecklenburg Counties are only split across two districts unlike in the 2021 Enacted Congressional Plan when each county was split across three districts.
33. The Supreme Court Remedial Order stated that a combination of different methods could be used to evaluate the partisan fairness of a districting plan; of those methods, the General Assembly used the "mean-median" test and the "efficiency gap" test to analyze the partisan fairness of the Remedial Plans.
34. The Court finds, based upon the analysis performed by the Special Masters and their advisors, that the Remedial Congressional Plan is not satisfactorily within the statistical ranges set forth in the Supreme Court's full opinion. See Harper v. Hall, 2022-NCSC-17, 『[166 (mean-median difference of \(1 \%\) or less) and \(\mathbb{T}[167\) (efficiency gap less than 7\%).
35. The Court finds that the partisan skew in the Remedial Congressional Plan is not explained by the political geography of North Carolina.

\section*{IV. Remedial Senate Plan}

\section*{A. The General Assembly's Starting Point and Subsequently Proposed Amendments}
36. In determining the base map for the State Senate Districts, the Senate also started from scratch. The Senate altered two county groupings and adopted groupings for Senate Districts 1 and 2 that were preferred by Common Cause Plaintiffs. The remaining county groupings remained the same. As a result, the 13 wholly-contained single district county groupings in the Remedial Plan were kept from the Enacted Plan.
37. Alternative county groupings were proposed but not adopted.
a. The Senate considered the Democratic members' preferred alternate grouping for Forsyth County, which pairs it with Yadkin instead of Stokes County, but it was determined that the resulting districts in Alexander, Wilkes, Surry, and Stokes Counties would have been less compact. Additionally, Yadkin County is more Republican than Stokes County.
b. Alternative county groupings around Buncombe County were considered as well, but the Senate determined that any change from the chosen grouping would have resulted in districts that would have been significantly less compact.
38. The Remedial Senate Plan passed the Senate by a vote of 26-19. The "aye" votes in the Senate were solely by members of the Republican party, while the "no" votes in the Senate were solely by members of the Democratic Party. The Remedial Senate Plan passed the House by voice vote along party lines.

\section*{B. Analysis of Partisanship Reflected in the Remedial Senate Plan}
39. The process for the development of the Remedial Senate Plan began with separate maps being drawn by the Senate Democratic Caucus and the Republican Redistricting and Election Committee members, respectively. The plans were then exchanged and discussed; however, after the two groups could not come to a resolution, the plan proposed by the Republican Redistricting and Election Committee members was then put to a vote by the Senate Committee and advanced to the full chamber.
40. The Remedial Senate Plan includes ten districts that were within ten points in the 2020 presidential race.
41. The Remedial Senate Plan reflects key differences from the 2021 Enacted Senate Plan in the projected partisan makeup of districts in certain county groupings.
a. In the Cumberland-Moore County grouping, Senate District 21 is now more competitive.
b. In the Iredell-Mecklenburg County grouping, one district is more competitive.
c. In New Hanover County, the districts were made more competitive, resulting in a Senate District 7 that leans Democratic.
d. In Wake County, Senate Districts 17 and 18 are more Democratic leaning.
42. The Court finds, based upon the analysis performed by the Special Masters and their advisors, that the Remedial Senate Plan is satisfactorily within the statistical ranges set forth in the Supreme Court's full opinion. See Harper v. Hall, 2022-NCSC-17, 『166 (mean-median difference of \(1 \%\) or less) and \(\mathbb{\|} 167\) (efficiency gap less than 7\%).
43. The Court finds that to the extent there remains a partisan skew in the Remedial Senate Plan, that partisan skew is explained by the political geography of North Carolina.
C. The General Assembly's Consideration of Incumbency Protection and Traditional Neutral Districting Criteria
44. For the Remedial Senate Plan, current members of either chamber who announced retirement or their intention to seek another office were not considered as "incumbents."
45. In the Senate, incumbency was considered evenly. No Senators are double bunked unless as a result of the mandatory county groupings, and no Democratic members are double bunked with other incumbents.
46. The Court finds that the measures taken by the General Assembly for the purposes of incumbency protection in the Remedial Senate Plan were applied evenhandedly.
47. The current membership of the General Assembly was elected under a districting plan that was approved by the trial court in Common Cause v. Lewis and, as stated above, the General Assembly began anew the process of drawing district lines after choosing county groupings for the remedial state legislative districts in this case.
48. The Court finds that the measures taken by the General Assembly for the purposes of incumbency protection in the Remedial Senate Plan do not perpetuate a prior unconstitutional redistricting plan.
49. The Court finds that the measures taken by the General Assembly for the purposes of incumbency protection in the Remedial Senate Plan are consistent with the equal voting power requirements of the North Carolina Constitution.
50. The Court finds that the General Assembly did not subordinate traditional neutral districting criteria to partisan criteria or considerations in the Remedial Senate Plan.

\section*{V. Remedial House Plan}

\section*{A. The General Assembly's Starting Point and Subsequently Proposed Amendments}
51. In determining the base map for the State House Districts, the House started from scratch after keeping only the 14 districts that were the product of single district county groupings.
52. The Remedial House Plan was ultimately amended by six amendments offered by Democratic Representatives.
a. Three amendments, drawn by Representative Reives, redrew certain districts in Wake, Mecklenburg, and Buncombe, which were already Democratic leaning, to be more Democratic leaning.
b. An additional amendment, also drawn by Representative Reives, added an additional district in Cabarrus County that is more Democratic leaning.
c. An amendment offered by Representative Meyer swapped two precincts in Orange County in order to keep Carrboro whole.
d. An amendment offered by Representative Hawkins adjusted district lines in Durham County in order to better follow educational district lines.
53. The Remedial House Plan passed the House by a vote of \(115-5\) and was passed by the Senate by a vote of 41-3. The "aye" votes in the House and Senate were by members of both political parties. The "no" votes in the House and Senate were solely by members of the Democratic Party.

\section*{B. Analysis of Partisanship Reflected in the Remedial House Plan}
54. The Remedial House Plan reflects key differences from the 2021 Enacted House Plan in the projected partisan makeup of districts in certain county groupings.
a. Buncombe County, which consisted of 1 Republican and 2 Democratic districts in the Enacted Plan, consists of 3 Democratic districts in the Remedial House Plan.
b. Pitt County, which consisted of 1 Republican and 1 Democratic district in the Enacted Plan, consists of 2 Democratic districts in the Remedial House Plan.
c. Guilford County now consists of 6 Democratic leaning districts.
d. Cumberland County now consists of 3 Democratic districts and 1 competitive district.
e. Mecklenburg and Wake Counties now consist of 13 Democratic leaning districts each.
f. New Hanover, Cabarrus, and Robeson Counties now contain an additional competitive district each.
55. The Court finds, based upon and confirmed by the analysis of the Special Masters and their advisors, that the Remedial House Plans are satisfactorily within the statistical ranges set forth in the Supreme Court's full opinion. See Harper v. Hall, 2022-NCSC-17, 166 (mean-median difference of \(1 \%\) or less) and \(\mathbb{\|} 167\) (efficiency gap less than 7\%).
56. The Court finds that to the extent there remains a partisan skew in the Remedial House Plan, that partisan skew is explained by the political geography of North Carolina.

\section*{C. The General Assembly's Consideration of Incumbency Protection and Traditional Neutral Districting Criteria}
57. For the Remedial House Plan, current members of either chamber who announced retirement or their intention to seek another office were not considered as "incumbents."
58. In the House, incumbency was considered evenly. The only discretionary double bunking in the Remedial House Plan pairs two Republican members. There was no discretionary double bunking of Democratic members. The few double bunked members are double bunked solely as a result of the mandatory county groupings.
59. The Court finds that the measures taken by the General Assembly for the purposes of incumbency protection in the Remedial House Plan were applied evenhandedly.
60. The current membership of the General Assembly was elected under a districting plan that was approved by the trial court in Common Cause v. Lewis and, as stated above, the General Assembly began anew the process of drawing district lines after choosing county groupings for the remedial state legislative districts in this case.
61. The Court finds that the measures taken by the General Assembly for the purposes of incumbency protection in the Remedial House Plan do not perpetuate a prior unconstitutional redistricting plan.
62. The Court finds that the measures taken by the General Assembly for the purposes of incumbency protection in the Remedial House Plan are consistent with the equal voting power requirements of the North Carolina Constitution.
63. The Court finds that the General Assembly did not subordinate traditional neutral districting criteria to partisan criteria or considerations in the Remedial House Plan.

\section*{VI. Plaintiffs' Alternative Remedial Plans}
64. The following alternative remedial plans for the Court's consideration were submitted by NCLCV Plaintiffs, Harper Plaintiffs, and Plaintiff-Intervenor Common Cause on February 18, 2022 (hereinafter referred to as "NCLCV Alternative Plans"; "Harper Alternative Plans"; "Common Cause Alternative Plans"; or collectively, "Alternative Plans").
65. Although Plaintiffs submitted Alternative Plans, because the Court is satisfied with the Remedial House and Senate Plans, the Court did not need to consider an alternative plan for adoption.
66. Furthermore, the Court, in following N.C.G.S. § 120-2.4(a1), has chosen to order the use of an interim districting plan for the 2022 North Carolina Congressional election that differs from the Remedial Congressional Plan to the extent necessary to remedy the defects identified by the Court.

\section*{VII. Special Masters' Interim Congressional Plan}
67. As part of their Report, the Special Masters have developed a recommended congressional plan ("Interim Congressional Plan") for this Court to consider due to their findings, which the Court has adopted, that the Remedial Congressional Plan does not satisfy the requirements of the Supreme Court Remedial Order and full opinion.
68. The Court finds that the Interim Congressional Plan recommended by the Special Masters was developed in an appropriate fashion \({ }^{2}\), is consistent with N.C.G.S. § 120-2.4(a1), and is consistent with the North Carolina Constitution and the Supreme Court's full opinion.

Based upon the foregoing findings of fact, the Court makes the following:

\section*{CONCLUSIONS OF LAW}
1. In Harper v. Hall, 2022-NCSC-17, the Supreme Court stated:

We do not believe it prudent or necessary to, at this time, identify an exhaustive set of metrics or precise mathematical thresholds which conclusively demonstrate or disprove the existence of an unconstitutional partisan gerrymander. Cf. Reynolds v. Sims, 377 U.S. 533,578 (1964) ("What is marginally permissible in one [case] may be unsatisfactory in another, depending on the particular circumstances of the case. Developing a body of doctrine on a case-by-case basis appears to us to provide the most satisfactory means of arriving at detailed constitutional requirements in the area of . . . apportionment."). As in Reynolds, "[l]ower courts can and assuredly will work out more concrete and specific standards for evaluating state legislative apportionment schemes in the context of actual litigation." Id. However, as the trial court's findings of fact indicate, there are multiple reliable ways of demonstrating the existence of an unconstitutional partisan gerrymander. In particular, mean-median difference analysis; efficiency gap analysis; close-votes, close-seats analysis; and partisan symmetry analysis may be useful in assessing whether the mapmaker adhered to traditional neutral districting criteria and whether a meaningful partisan skew necessarily results from North Carolina's unique political geography. If some combination of these metrics demonstrates there is a significant likelihood that the districting plan will give the voters of all political parties substantially equal opportunity to translate votes into seats across the plan, then the plan is presumptively constitutional.

Id. at 『163.

\footnotetext{
\({ }^{2}\) The data files (e.g., block equivalency, shape files, population deviation results) are included in the court file with this order in native format. The equivalent of the "stat pack" has been requested from the Special Masters' advisor and will be placed in the court file and provided to the parties as soon as available.
}
2. Plaintiffs have urged upon this court that we must adopt plans that "treat voters of both political parties fairly." They argue that the "LD Congressional and Senate Plans are not fair." Further, they argue that the Supreme Court ordered "fair maps" and that " \([b]\) ecause the LD Congressional and Senate Plans are not fair maps, . . . the Court should adopt one of the fairer maps before it - such as the NCLCV Maps." We see Plaintiffs' arguments as tantamount to urging this Court to adopt a proportional representation standard, which the Supreme Court, in its order, specifically disavowed. \(I d\). at \({ }^{\text {『 }}\) [169.
3. The Court concludes that the Remedial Senate Plan satisfies the Supreme Court's standards.
4. The Court concludes that the Remedial House Plan satisfies the Supreme Court's standards.
5. Because the Court concludes that the enacted Remedial Senate and House Plans meet the Supreme Court's standards and requirements in the Supreme Court Remedial Order and full opinion, the Remedial Senate and House Plans are presumptively constitutional.
6. Furthermore, no evidence presented to the Court is sufficient to overcome this presumption for the Remedial Senate and House Plans, and those plans are therefore constitutional and will be approved.
7. The Court concludes that the Remedial Congressional Plan does not satisfy the Supreme Court's standards.
8. Plaintiffs suggest that if we conclude that a Remedial Plan passed by the General Assembly does not satisfy the Supreme Court's standards, we should simply jettison that plan and adopt one of their plans. We do not believe that our conclusion on the Remedial Congressional Plan-that it fails to satisfy the Supreme Court's standards-automatically results in the adoption of an alternate plan proposed by Plaintiffs. Given that the ultimate authority and directive is given to the Legislature to draw redistricting maps, we conclude that the appropriate remedy is to modify the Legislative Remedial Congressional Plan to bring it into compliance with the Supreme Court's order. See N.C.G.S. § 120-2.4(a1).
9. Because the Court concludes that the enacted Remedial Congressional Plan does not meet the Supreme Court's standards and requirements in the Supreme Court Remedial Order and full opinion, the Remedial Congressional Plan is not presumptively constitutional and is therefore subject to strict scrutiny.
10. The General Assembly has failed to demonstrate that their proposed Congressional map is narrowly tailored to a compelling governmental interest, and we therefore must conclude that the Remedial Congressional Map is unconstitutional.
11. The Interim Congressional Plan as proposed by the Special Masters satisfies the Supreme Court's standards and should be adopted by this Court for the 2022 North Carolina Congressional elections.

\section*{DECREE}

BASED UPON THE FOREGOING findings and conclusions, the Court here by
ORDERS the following:
1. The Remedial Senate Plan and Remedial House Plan, enacted into law by the General Assembly on February 17, 2022, are hereby APPROVED by the Court.
2. The Remedial Congressional Plan, enacted into law by the General Assembly on February 17, 2022, is hereby NOT APPROVED by the Court.
3. The Interim Congressional Plan as recommended by the Special Masters is hereby ADOPTED by the Court and approved for the 2022 North Carolina Congressional elections.
4. As the Special Masters and their retained experts may be called upon to assist this Court in this matter should the need arise in the future, the prohibition in this Court's prior order appointing the Special Masters against contacting the Special Masters or their experts remains in full force and effect.

SO ORDERED, this the 23rd day of February, 2022.

- App. 72 -


TO: Judges Shirley, Poovey, and Layton
FROM: Special Masters
DATE: February 23, 2022
SUBJECT: Special Masters' Report - Analysis and Recommendations

\section*{Introduction}

Pursuant to the trial court's "Order Appointing Special Masters" on February 16, 2022, 『ा 6 , the undersigned now file the following report with the three-judge panel in this case.

\section*{Motion for Disqualification}

In its Order Appointing the three Special Masters, the Court authorized the undersigned Special Masters (hereinafter "Special Masters") to "hire research and technical assistants and advisors reasonably necessary to facilitate [our] work." We subsequently retained Dr. Bernard Grofman, Dr. Tyler Jarvis, Dr. Eric McGhee, and Dr. Samuel Wang to assist us in satisfying our duties as Special Masters. The Curriculum Vitae for each of these individuals (hereinafter referred to as "advisors") is attached to this report. In this same Order, this Court also ordered the "parties and non-parties may not engage in any ex parte communication with the Special Masters about the subject matter of this litigation." Id.

We have been informed that Legislative Defendants have filed a motion in this case requesting that this Court disqualify Dr. Wang and Dr. Jarvis as advisors to the Special Masters and take further steps to destroy any work product completed by them and otherwise prohibit the undersigned from considering any information or materials obtained from them. We have investigated this matter and below is a detailed review of our findings.

On February 18, 2022, at 1:01 pm, Dr. Wang emailed Dr. Mattingly requesting the underlying data utilized in his analysis of the 2021 redistricting plans. On this same date at \(1: 57\) p.m., Dr. Mattingly responded, and correspondence between Dr . Wang and Dr. Mattingly continued through February 20, 2022 at 10:23 a.m.

On February 18, 2022, at 1:21 p.m., Dr. Wang emailed Dr. Pedgen, expert for Harper Plaintiffs, seeking the underlying data Dr. Pedgen utilized in his analysis of the 2021 redistricting plans. On this same date at 2:31 p.m., Dr. Pedgen responded to Dr. Wang's inquiry, directing him to use the method utilized by Dr. Mattingly, expert for Harper Plaintiffs and Plaintiff Common Cause. On February 19, 2022, at 6:59 a.m., Dr. Wang responded to Dr. Mattingly's correspondence.

On February 19, 2022, at 4:46 p.m., Dr. Jarvis contacted Dr. Mattingly to request clarification on Dr. Mattingly's analysis and underlying data. Later that day, at 8:13 p.m., Dr. Jarvis contacted Dr. Herschlag, Dr. Mattingly's colleague at Duke University, regarding Dr. Herschlag's analysis and underlying data supporting his analysis of the 2021 redistricting plans to which Dr. Herschlag responded on that same date. All email correspondence between Dr. Wang and Dr. Jarvis and the plaintiff experts Mattingly and Pegden is attached to this report and the email correspondence attached is all of the communication that occurred between the advisors and any of the experts of the parties.
The undersigned acknowledge the technical breach of this Court's mandate that no exparte communication occur between parties and non-parties with the Special Masters. The undersigned, however, respectfully recommend that the Court deny the motion for the following reasons:
- First, these communications between the advisors and Drs. Mattingly and Herschlag do not appear to have been made in bad faith and constitute the only communications between them, written or otherwise. The advisors immediately ceased contact with Drs. Mattingly and Herschlag, and have provided copies of the communications. Therefore, all parties are privy to the extent of the communications.
- Second, their communications directed at experts for Harper Plaintiffs were solely for the purpose of proceeding as quickly as possible within the abbreviated time frame allotted for the remedial process.
- Third, the Special Masters emphasize that, while the communications were in the context of the advisors' preliminary steps to evaluate the 2022 Remedial Plans, the communications sought background information pertaining to the earlier analysis of the 2021 Redistricting Plans performed by Drs. Pegden, Mattingly, and Herschlag in the merits stage of this case that was ultimately received and relied upon by the Court at trial. Additionally, as was later determined, the information sought by Dr. Wang and by Dr. Jarvis was publicly available on Dr. Hershlag's website at the time of the communications questioned herein by the Legislative Defendants.
- Finally, though the analysis provided by Drs. Wang and Jarvis was helpful and consistent with the analysis of our other expert advisors, it was not determinative of any recommendations made by the Special Masters to the court.

\section*{Review of Proposed Remedial Plans}

Pursuant to the North Carolina Supreme Court's opinion, any plan with a meanmedian difference of 1\% or less (Harper, 2022-NCSC-17 at 『 166) and an efficiency gap below 7\% (Harper, 2022-NCSC-17 at ๆ 167) should be considered presumptively constitutional. Additionally, as the Supreme Court recognized, other metrics may be instructive (Harper, 2022-NCSC-17 at 『 168). The Special Masters considered the full Order and Opinion of the North Carolina Supreme Court along with, the submissions from all of the parties as well as the reports of the advisors and reached the following conclusions:

\section*{I. Proposed Remedial House Plan}

The advisors as well as the experts of the parties ("experts") all found the efficiency gap of the proposed remedial House plan to be less than 7\%. The majority of the advisors and experts found the mean-median difference of the proposed remedial House plan to be less than 1\%. In addition to these facts, the Special Masters considered the findings of the advisors on the partisan symmetry analysis, the declination metrics, and their opinions on partisan bias and evidence of partisan gerrymandering. Considering all of this information as well as the totality of circumstances, the Special Masters conclude under the metrics identified by the North Carolina Supreme Court that the proposed remedial House plan meets the test of presumptive constitutionality. Further the Special Masters did not find substantial evidence to overcome the presumption of constitutionality and recommend to the trial court that it give appropriate deference to the General Assembly and uphold the constitutionality of the remedial House plan.

\section*{II. Proposed Remedial Senate Plan}

All of the advisors and experts found the efficiency gap of the proposed remedial Senate plan to be less than 7\%. The majority of the advisors and experts found the mean-median difference of the proposed remedial Senate plan to be less than \(1 \%\). In addition to these facts, the Special Masters considered the findings of the advisors on the partisan symmetry analysis, the declination metrics, and their opinions on partisan bias and evidence of partisan gerrymandering. Considering all of this information as well as the totality of circumstances, the Special Masters conclude under the metrics identified by the North Carolina Supreme Court the remedial Senate plan meets the test of presumptive constitutionality. Further the Special Masters did not find substantial evidence to overcome the presumption of constitutionality and recommend to the trial court that it give appropriate deference to the General Assembly and uphold the constitutionality of the remedial Senate plan.

\section*{III. Proposed Remedial Congressional Plan}

Unlike the proposed remedial House and Senate plans, there is substantial evidence from the findings of the advisors that the proposed congressional plan has an efficiency gap above \(7 \%\) and a mean-median difference of greater than \(1 \%\). The Special Masters considered this evidence along with the advisors' findings on the partisan symmetry analysis and the declination metrics. There is disagreement among the parties as to whether the proposed remedial congressional plan meets the presumptively constitutional thresholds suggested by the Supreme Court. The Special Masters, considering the reports of their advisors and the experts of the parties while giving appropriate deference to the General Assembly, are of the opinion that the proposed remedial congressional plan fails to meet the threshold of constitutionality and recommend that the Trial Court reject the proposed remedial congressional plan as being unconstitutional.

Given the recommendation that the Trial Court reject the proposed remedial congressional plan, and consistent with the instructions from the three-judge panel and the Order of the Supreme Court of North Carolina, the Special Masters have submitted a modified version of the proposed remedial congressional plan submitted by the Legislative Defendants. It is our opinion that the attached plan satisfies the requirements of the Supreme Court.

The following data files for the modified congressional plan are included with this report:
1. Block equivalency files in .CSV format for each district and the plan as a whole;
2. Environmental Systems Research Institute, Inc. (ESRI) shapefiles for each district and the plans as a whole;
3. Color maps in .PDF format of the plan as a whole;
4. Population totals and deviations for each district based on the 2020 Census P.L. 94-171 dataset; and
5. Note: due to time constraints, the functional equivalent of what the General Assembly includes in its "stat pack" is not included with this report; however, if requested we will endeavor to obtain this from Dr. Grofman.

In redrawing certain district lines, the undersigned considered all of the submitted plans and related commentary. Being mindful that the Constitution of North Carolina provides that the General Assembly has the responsibility of redistricting, we focused on the proposed remedial congressional plan submitted by the Legislative Defendants. On that basis, the Special Masters worked solely with Dr. Bernard Grofman and his assistant to amend the Legislative Defendants' plan to
enhance its consistency with the opinion of the Supreme Court of North Carolina, the Constitutions of the United States and of North Carolina, and the expressed will of the General Assembly.

Dr. Grofman prepared a preliminary exemplar map at the Special Masters' request and thereafter at the instruction of the Special Masters prepared three maps for consideration. One of these maps raised potential VRA concerns and so was discarded. A second map did not meet the 1\% threshold for mean-median difference and so was likewise discarded. The Special Masters then modified the third prepared map in order to improve the efficiency gap and mean-median difference scores as well as compactness and contiguity measures.

The following parties were involved in the process of redrawing the plans:
a. Robert F. Orr
b. Robert H. Edmunds, Jr.
c. Thomas W. Ross
d. Dr. Bernard N. Grofman
e. Zachary R. Griggy (Research Assistant to Dr. Grofman)
f. Adam H. Steele, Senior Judicial Fellow (for administrative purposes only)
g. Alison J. Rossi, Judicial Fellow (for administrative purposes only)
h. Danielle Smith, Judicial Fellow (for administrative purposes only)

Dave's Redistricting App was used in the redrawing of the plan.
The Special Masters believe the modified congressional plan recommended for adoption to the Trial Court achieves the partisan fairness and "substantially equal voting power" required by the Supreme Court of North Carolina without diluting votes under the Voting Rights Act while maintaining the number of county splits, retaining equal population, compactness, and contiguity, as well as respecting municipal boundaries. Dr. Grofman's analysis of the modified congressional plan recommended by the Special Masters indicates that the plan has an efficiency gap of \(0.63 \%\), a mean-median difference of \(0.69 \%\), seat bias of \(0.28 \%\), and vote bias of \(0.10 \%\). According to Dr. Grofman, "this is the most non-dilutive plan in partisan terms of any map that has been submitted to the Court."

Accordingly, the Special Masters recommend to the Trial Court that it order the State of North Carolina to utilize the modified congressional plan prepared by the Special Masters in the 2022 Congressional election.

This the 23rd day of February 2022.


Robert H. Edmunds, Jr.


Robert F. Orr


\section*{Acknowledgement}

We would like to thank the advisors, Dr. Grofman, Dr. Jarvis, Dr. McGee, and Dr. Wang for their analysis and advice in the extremely compressed timeframe. Additionally, we would like to thank the Judicial Fellows, Adam Steele, Alison Rossi, and Danielle Smith for their administrative support and assistance in preparing this report and for the long hours of work in bringing this matter to a conclusion.

\section*{CERTIFICATE OF SERVICE}

I hereby certify that a copy of the foregoing document was served on the persons
indicated below via electronic transmission by e-mail addressed as follows:

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Service is made upon local counsel for all attorneys who have been granted pro hac vice admission, with the same effect as if personally made on a foreign attorney within this state.

This the \(23^{\text {rd }}\) day of February 2022.


Kellie.Z.Myers@nccourts.org

\title{
ADVISORY REPORT TO SPECIAL MASTERS ON PROPOSED REMEDIAL REDISTRICTING PLANS FOR NORTH CAROLINA
}

\author{
Tyler J. Jarvis*
}

February 22, 2022
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\section*{1. Overview}

I was asked to perform an ensemble analysis of eight proposed remedial plans: two (Congressional and Senate) from the Harper plaintiffs, and three each (Congressional, House, and Senate) from NCLCV and the legislative defendants.

Ensemble analysis consists first of constructing a large number of possible alternative plans (the ensemble). The plans are generated without using any partisan information, but in accordance with accepted criteria for redistricting in the state, including approximately equal population per district, contiguity of districts, relative compactness of districts, few boundary traversals, and so
forth. Historical election data is then used to compare election results under the proposed plans with elections results under the ensemble.

I was asked to include the following well-known metrics in my ensemble analysis: mean-median (MM), efficiency gap (EG), partisan bias (PB), and declination (D). All of these have the property that a more negative score is supposed to represent more benefit to Republicans and a more positive score is supposed to represent more benefit to Democrats. Scores closer to zero are generally expected to be less indicative of a partisan gerrymander.

But the range of possible scores also varies widely from state to state because of widely varying political geography from state to state, varying criteria for redistricting, and varying results from different elections. Although one might make a philosophical argument for why scores for a given metric that lie outside a given range should be considered evidence of a partisan gerrymander, this is an unreasonable standard if all or most of the possible scores lie outside that range. Indeed, in some cases it may not even be possible for the scores in a given state under a given set of redistricting rules to lie in that prescribed range.

Ensembles provide important context for interpreting these scores by helping to identify a typical range of score values as well as identifying outliers.
1.1. Ensembles. The best way to do an ensemble analysis is to generate an ensemble with a distribution of plans that specifically reflects the redistricting criteria for case under consideration. But doing that properly takes much more time than is available. Because of this I chose to use ensembles previously generated by Professor Jonathan Mattingly and his collaborators at Duke University [1]. These were generated using well-accepted MCMC methods.

My analysis is conditioned on the assumption that these ensembles are somewhat representative of the distribution of possible plans reflecting established law and intent of the court. In the case of the Congressional ensemble, I am more confident of this for the Congressional ensemble than in the case of the House and Senate ensembles. I discuss this in more detail in Sections 2.2, 3.1, and 4.
1.2. Election Data. For all three types of plans (congressional, senate, and house) I used historical results from the following 11 elections: the 2016 Attorney General (G16AG), Presidential (G16PR), Lieutenant Governor (G16LG) and Governor (G16AG), as well as the 2020 Attorney General (G20AG), Presidential (G20PR), Lieutenant Governor (G20LG), 2020 Governor (G16AG), Treasurer (G20TR), US Senate (G20USS), and Secretary of State (G20SST). To calculate the vote shares and other scores for the proposed plans, I used 2016 and 2020 precinct-level election results from the Voting and Election Science Team (VEST) and prorated the data to 2020 census blocks.
1.3. Racial Considerations. It is important to note that I have not considered racial factors or the VRA in this analysis. Incorporating those considerations may lead to other conclusions than those I have drawn here.

\section*{2. Congressional Plans Analysis}

Using data from the 11 different historical elections mentioned above, I evaluated three congressional plans: one each from the Harper plaintiffs (Harper), NCLCV, and the legislative defendants (LD). I also analyzed the number of seats that would have been won under these various vote counts and the margins of victory in the most contested districts.
2.1. Summary of Congressional Analysis. My analysis below shows that, by all the measures I used, the LD plan favors Republicans more than the other two plans do, the NCLCV plan favors Democrats more than the other two plans, and the Harper plan lies somewhere between them. Both the LD and Harper plans are fairly typical in the ensemble distributions for all the measures I considered. The NCLCV plan, however, shows up as a significant outlier for the seat margins
for competitive seats (see Section 2.4) as well as for the mean-median and partisan bias scores. Taken together these give some evidence of partisan gerrymandering in the NCLCV congressional plan.

These conclusions do not take VRA racial considerations into account.
2.2. Ensemble. For analyzing the congressional plans I used the ensemble [2] (sometimes denoted the Duke congressional ensemble in this report). According to my reading of [4], this ensemble is generated using well-accepted Markov chain Monte Carlo methods (a parallel tempering framework using a proposal from the Multiscale Forest RECOM algorithm). Under this method specific parameters for the distribution to enforce certain requirements and to encourage certain properties of the plans chosen. According to [4] the resulting plans split no more than 14 counties and split no county into more than two districts. All districts are required to consist of one contiguous region. The deviation of the total population in any district is within \(1 \%\) of the ideal district population. Districts traverse counties as few times as possible, and plans with a higher Polsby-Popper score (more compact) are more likely to be selected. This model was tuned to give similar PolsbyPopper score to the enacted congressional plan. Some have argued that tuning for a specific range of Polsby-Popper scores might skew the distribution somewhat, but in my own (unpublished) research I have explicitly checked for correlation between Polsby-Popper scores and metrics of partisan bias in ReCom MCMC and found none. I expect that this absence of correlation would hold in the other ReCom-based MCMC methods as well, including the method used to generate this ensemble. The ensemble has nearly 80,000 plans, and according to [4] the distribution seems well mixed has been sufficiently sampled to provide stable statistics. I cannot verify the mixing directly, but in my use of the ensemble, I saw no signs that the ensemble was not well mixed. Based on these I conclude that this ensemble is suitable to evaluate the Congressional plans.
2.3. Distribution of Seats Across Elections. Different plans perform differently under different elections. When a plan gives more seats to one party than most of the plans in the ensemble do, that can suggest a possible partisan gerrymander, especially when this occurs over several elections. To analyze this, I used histograms of seats won for the ensemble for each race, collected in Figure 1.

These histograms show that while the LD plan consistently favors Republicans and the the Harper plan consistently favors Democrats, in both cases the number of seats they give in most races is fairly typical of the ensemble distribution. The NCLCV plan also consistently favors Democrats, usually much more so than the Harper plan, and in one case (G20PR) more so than \(99.7 \%\) of the ensemble, making it a significant outlier in that election.

An alternative view of the same data collected into one diagram, with histograms replaced by violin plots, is shown in Figure 2.
2.4. Rank-Ordered Violin Plots. The number of seats won by a plan in an election does not indicate how close the election would be. A plan that gives Democrats \(51 \%\) of the vote share in their winning districts is very different from one that gives them \(70 \%\) of the vote share in those districts. To analyze this effect for the proposed plans I used rank-ordered violin plots; see Figures 3 and 4. In a rank-ordered violin plot for a given election, all the congressional districts for each plan are ordered left-to-right by their Democratic vote share in the election. The numbers on the horizontal axis represent the position of the district in rank ordering (not the name given to the district in the plan). The vote share for the plans in the ensemble is represented by the gray violin-shaped distributions in each distribution, and the vote share for each plan is indicated by the corresponding colored bar.

Figure 3 shows a rank-ordered violin plot for the election G20LG, which reveals that although the NCLCV plan gives one more seat (District 8 in the figure) to the Democrats than the Harper plan, that extra seat comes by a very fine margin, with the NCLCV plan just over \(50 \%\) and the Harper plan just under \(50 \%\) in that district. None of the plans is a far outlier compared to the

Congressional Seats


Congressional Seats




Congressional Seats


Figure 1. Histograms of congressional seats won in all 11 elections for the ensemble plans (gray). The proposed plans are indicated as colored vertical lines.


Figure 2. Congressional seats won across elections: shows the number of seats won (vertical axis) by Democratic candidates under each plan (colored lines) over the 11 elections (horizontal axis). The ensemble distribution of seats won for each election is indicated with the gray "violins," with wider gray regions around a point indicating more ensemble plans with the indicated number of seats won, and narrower regions indicating fewer ensemble plans with the indicated number of seats won.
ensemble in this district. This suggests that the difference in the number of seats between NCLCV and Harper in this election is not significant.

However, in this election the NCLCV plan makes District 7 much more competitive (favoring the Democrats) than either the Harper or LD plans do. Although NCLCV does not actually give the seat in District 7, NCLCV gives this district a much higher Democratic vote share than either Harper or LD and, more significantly, much higher than most of the ensemble. This makes that district very close to a win for the Democrats, without actually giving the seat to them.

Taken together, Figures 4 and 3 show that in seven of the elections (G20LG, G20GV, G20AG, G20US, G20TR, G20PR, and G16GV) the NCLCV plan places the Democratic vote share in this borderline district (7) substantially higher than most of the ensemble, which either gives the seat to the Democrats or nearly gives them the seat, by pushing the Democratic vote share close to \(50 \%\). The other plans (Harper and LD) stay in a fairly typical part of the ensemble distribution across all elections. I take this as some evidence of partisan gerrymandering in the NCLCV plan, but not in the LD and Harper plans.

Democratic Vote Shares in US Congressional Districts (Duke Ensemble)


Figure 3. For each plan, all the congressional districts (horizontal axis) are ordered left-to-right by their Democratic vote share in the G20LG election. The numbers on the horizontal axis represent the position of the district in rank ordering (not the number given in the proposal). The vote share for the plans in the ensemble is represented by the gray violin-shaped distributions in each distribution, and the vote share for each plan is indicated by the corresponding colored bar. Points above the gray 50-percent line indicate a seat that goes to the Democrats and those below go to the Republicans.


Democratic Vote Shares in Competitive US Congressional Districts (Duke Ensemb









Figure 4. Rank-ordered congressional districts for all the elections except G20LG (shown above in Figure 3). These plots show only the most competitive districts.
2.5. Other Metrics. I also analyzed the plans using the mean-median score, partisan bias score, efficiency gap, and partisan declination. The first three scores are well-known and widely used. The declination is a relatively new measure proposed by Warrington. All four measures are reviewed in [5], so I will not describe them in detail here. All four of them give a single score for which a more negative score is supposed to represent more benefit to Republicans and a more positive score is supposed to represent more benefit to Democrats. Scores closer to zero are generally expected to be less indicative of a partisan gerrymander, but that depends heavily on the political geography of the state, so it is important to interpret these scores in the context of ensembles.

In the ensemble analysis below, all four scores show the LD plan favors Republicans more than the other two plans do, the NCLCV plan favors Democrats more than Harper or LD, and the Harper plan lies somewhere between them. Both the LD and Harper plans are fairly typical in the ensemble distributions for all four scores across almost all elections. The NCLCV plan, however, shows up as a significant outlier the mean-median and partisan bias scores.
2.5.1. Mean-Median. Table 1 shows my calculations of the mean-median scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 5 the mean-median score consistently identifies the LD plan as favoring Republicans more than the others but it is still not an outlier for the ensemble distribution. The Harper plan is is also not an outlier for the ensemble. The NCLCV plan is identified as favoring Democrats more than the others (higher scores) and is a significant outlier (greater than 99th percentile) in six of the elections.
2.5.2. Partisan Bias. Table 2 shows my calculations of the partisan bias scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 6, the partisan bias score also consistently identifies the LD plan as favoring Republicans more than the others but overall is more typical of the distribution than either of the other two plans. The NCLCV plan is identified as favoring Democrats more than the others and is on the very high end (over 97th percentile) of the ensemble distribution in many of the elections.
2.5.3. Efficiency Gap. Table 3 shows my calculations of the efficiency gap scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 7 the LD and Harper plans are mostly typical for the distribution of efficiency gap across elections. The the NCLCV plan is a significant outlier in one election (G20PR), and is somewhat high (above 90th percentile) for three other elections.
2.5.4. Declination. Table 4 shows my calculations of the declination scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 8 the declination only marks the NCLCV plan as a significant outlier (over \(99 \%\) ), but all three plans are on the outer edges (above \(90 \%\) or below \(10 \%\) ) for some of the elections.
2.6. Congressional Conclusion. Both the LD and Harper plans are fairly typical in the ensemble distributions for all the measures I considered. The NCLCV plan, however, shows up as a significant outlier for the seat margins for competitive seats (see Section 2.4) as well as for the mean-median and partisan bias scores. Taken together these give evidence of partisan gerrymandering in the NCLCV congressional plan, but VRA racial considerations, which I have not considered here, might change that conclusion.

Congressional MeanMedian


Congressional MeanMedian




Congressional MeanMedian


Figure 5. Histogram of congressional mean-median score for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

Congressional PartisanBias


Congressional PartisanBias


Figure 6. Histogram of partisan bias for all 11 elections. The numbers in the legend are the percentile in the ensemble for the corresponding plan.

Congressional EfficiencyGap


Congressional EfficiencyGap




Congressional EfficiencyGap




Congressional EfficiencyGap




Figure 7. Histogram of congressional efficiency gap for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

Congressional Declination


Congressional Declination


Figure 8. Histogram of congressional partisan declination for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.
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\section*{Congressional Mean-Median}
\begin{tabular}{lllll} 
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -0.1 & 0.1 & 0.9 & -6.4 \\
G20USS & -0.5 & -0.7 & 1.3 & -5.7 \\
G20GOV & 0.0 & -0.1 & 1.5 & -5.7 \\
G20LTG & 0.1 & -0.3 & 1.5 & -6.2 \\
G20ATG & -0.0 & -0.3 & 1.7 & -6.2 \\
G20TRE & -0.3 & -0.7 & 1.3 & -5.5 \\
G20SOS & -0.1 & -0.3 & 2.2 & -6.1 \\
G16PRE & 0.3 & -1.3 & 1.1 & -5.3 \\
G16GOV & -1.0 & -1.9 & 0.6 & -4.1 \\
G16LTG & -1.3 & -2.7 & -0.2 & -4.4 \\
G16ATG & -1.0 & -2.2 & 0.1 & -3.8 \\
\hline Average & -0.3 & -0.9 & 1.1 & -5.4
\end{tabular}

Table 1. Mean-median scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 5) - not as isolated numbers.
\begin{tabular}{lllll}
\multicolumn{5}{c}{ Congressional } \\
\multicolumn{4}{c}{ Partisan Bias } \\
Proposed_Plan & Harper & LD & NCLCV & \\
\hline G20PRE & 0.0 & 0.0 & 7.1 & -21.4 \\
G20USS & 0.0 & -7.1 & 7.1 & -21.4 \\
G20GOV & 0.0 & 0.0 & 7.1 & -21.4 \\
G20LTG & 0.0 & -7.1 & 7.1 & -21.4 \\
G20ATG & 0.0 & -7.1 & 7.1 & -21.4 \\
G20TRE & 0.0 & -7.1 & 7.1 & -21.4 \\
G20SOS & 0.0 & 0.0 & 7.1 & -21.4 \\
G16PRE & 0.0 & -7.1 & 7.1 & -21.4 \\
G16GOV & 0.0 & -7.1 & 0.0 & -21.4 \\
G16LTG & 0.0 & -7.1 & 0.0 & -21.4 \\
G16ATG & 0.0 & -7.1 & 0.0 & -21.4 \\
\hline Average & 0.0 & -5.2 & 5.2 & -21.4
\end{tabular}

Table 2. Partisan bias scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 6) - not as isolated numbers.

Congressional Efficiency Gap
\begin{tabular}{lllll} 
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -5.8 & -12.8 & 7.5 & -20.1 \\
G20USS & -5.1 & -5.3 & 0.7 & -19.5 \\
G20GOV & 1.7 & 2.2 & 1.5 & -26.0 \\
G20LTG & -3.7 & -17.9 & 2.1 & -18.1 \\
G20ATG & -1.1 & -7.4 & 5.9 & -21.6 \\
G20TRE & -1.8 & -16.0 & -3.1 & -16.2 \\
G20SOS & -3.1 & 4.6 & 3.9 & -17.4 \\
G16PRE & 3.3 & -16.9 & 2.9 & -17.2 \\
G16GOV & -0.5 & -6.8 & -1.0 & -21.0 \\
G16LTG & -0.3 & -13.9 & -0.7 & -14.1 \\
G16ATG & -0.8 & -7.1 & -1.3 & -21.3 \\
\hline Average & -1.6 & -8.8 & 1.7 & -19.3
\end{tabular}

Table 3. Efficiency gap scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 7) - not as isolated numbers.

Congressional Declination
\begin{tabular}{lllll} 
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -8.0 & -16.2 & 11.8 & -32.0 \\
G20USS & -6.4 & -4.7 & 3.7 & -29.7 \\
G20GOV & 1.1 & -0.6 & -0.4 & -41.4 \\
G20LTG & -3.9 & -24.1 & 6.6 & -27.7 \\
G20ATG & -0.6 & -9.1 & 8.3 & -33.8 \\
G20TRE & 0.5 & -18.7 & 2.4 & -22.3 \\
G20SOS & -4.7 & 4.2 & 4.1 & -24.6 \\
G16PRE & 7.4 & -24.5 & 7.5 & -28.3 \\
G16GOV & -0.0 & -8.5 & -0.3 & -32.4 \\
G16LTG & 3.3 & -16.1 & 5.3 & -19.9 \\
G16ATG & -0.8 & -9.1 & -1.1 & -32.6 \\
\hline Average & -1.1 & -11.6 & 4.4 & -29.5
\end{tabular}

Table 4. Partisan declination scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 8) - not as isolated numbers.

\section*{3. Senate Plan Analysis}

I received three proposed Senate plans (LD, Harper, and NCLCV) to evaluate. I used the same methods to evaluate these plans as I did for the Congressional plans, but with a different ensemble.
3.1. Senate Ensembles. For analyzing the senate plans I used Dr. Mattingly's ensemble [3]. It was generated with the same method as the Congressional plan. According to my reading of [4] the resulting plans comply with the county clustering rules of Stephenson, maintain a population balance that deviates by no more than \(5 \%\), They are also designed to produce contiguous districts that are relatively compact and to reduce the number of counties split. This ensemble does not explicitly preserve municipalities, except as a secondary consequence of other parameter settings. This is important because municipality splits are known to have a significant interaction with partisan vote shares and measures of partisan symmetry. According to [4] the distribution seems well mixed, but I cannot verify the mixing directly.
3.2. Seats Won. The histograms of seats won in Figure 9 show Harper and NCLCV both are mostly typical of the ensemble, while LD is often a significant outlier in favor of the Republicans.
3.3. Rank-Ordered Violin Plots. As with seats won the rank-ordered violin plots show Harper and NCLCV are both mostly typical of the ensemble, while LD is often deviates in favor of the Republicans; see Figure 10.

Senate Ensemble 0 Seats


Senate Ensemble 0 Seats


Figure 9. Histograms of congressional seats won in all 11 elections for the ensemble plans. The percentages in the legend represent percentile of the corresponding score in the ensemble.


Figure 10. Rank-ordered senate districts for 10 of the elections (all elections but G20LG). These plots show only the most competitive districts.

\subsection*{3.4. Other Metrics.}
3.4.1. Mean-Median. Table 5 shows my calculations of the mean-median scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 11 the mean-median score identifies the LD plan as a Republican-favoring outlier (lower than the 5th percentile) for three of the 11 elections (G20PR, G16AG, and G16LG).

The NCLCV plan is a pro-Democratic outlier (greater than 95th percentile) in four of the elections (G20PR, G20LG, G20USS, and G20GV).

The Harper plan leans toward the Democratic side of the distribution, but is not an outlier.
3.4.2. Partisan Bias. Table 6 shows my calculations of the partisan bias scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 12, the LD plan is a Republican-favoring outlier twice, and the Harper plan is a pro-Democratic outlier once. But the NCLCV plan stands out as a pro-Democratic outlier for partisan bias in four elections (G20PR, G20TR, G20USS, and G20GV).
3.4.3. Efficiency Gap. Table 7 shows my calculations of the efficiency gap scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

In Figure 13 the efficiency gap flag the NCLCV plan as a pro-Democratic outlier five times, and four of those are significant (99th percentile or greater). Harper shows up twice as Democratic outlier and LD shows up twice as a Republican outlier.
3.4.4. Declination. Table 8 shows my calculations of the declination scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

As shown in Figure 14 the declination marks the LD plan as a Republican outlier (below 5\%) three times. The NCLCV plan shows as a Democratic outlier (over 95\%) three times and Harper twice (G20GV and G20SST).
3.5. Senate Conclusion. The partisan symmetry scores give weak evidence of of partisan gerrymandering in the LD plan, and the seat margins in the rank-ordered violin plots give strong evidence of partisan gerrymandering in the LD plan.

The seat margins in the rank-ordered violin plots give some evidence of partisan gerrymandering in the NCLCV plan, and that is corroborated by the many outliers among the partisan symmetry scores.

These conclusions do not take VRA racial considerations into account.

Senate Ensemble 0 MeanMedian

Lt Gov '20


Gov '16


Senate Ensemble 0 MeanMedian

G16_PR


Pres '20



Senate Ensemble 0 MeanMedian




Senate Ensemble 0 MeanMedian



Figure 11. Histogram of senate ensemble mean-median score for all 11 elections.

Senate Ensemble 0 PartisanBias

Lt Gov '20


Gov '16


Pres '20


Senate Ensemble 0 PartisanBias
G16_AG


G16_LG


Senate Ensemble 0 PartisanBias

G20_TR


G20_AG


G20_USS


Senate Ensemble 0 PartisanBias

G20_SST


Figure 12. Histogram of partisan bias for all 11 elections.

Senate Ensemble 0 EfficiencyGap


Senate Ensemble 0 EfficiencyGap



Figure 13. Histogram of senate ensemble efficiency gap for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

Senate Ensemble 0 Declination


Senate Ensemble 0 Declination

G16_AG


G16_PR


Senate Ensemble 0 Declination


Figure 14. Histogram of senate ensemble partisan declination for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.
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\section*{Senate Mean-Median}
\begin{tabular}{lllll} 
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -0.4 & -3.0 & 0.4 & -3.8 \\
G20USS & -0.1 & -1.4 & 0.6 & -4.0 \\
G20GOV & 0.2 & -1.5 & 0.7 & -4.5 \\
G20LTG & -0.1 & -1.2 & 0.3 & -3.7 \\
G20ATG & -0.2 & -0.9 & -0.3 & -3.9 \\
G20TRE & 0.2 & -0.9 & 0.2 & -3.3 \\
G20SOS & 0.0 & -0.4 & 0.5 & -3.7 \\
G16PRE & -0.4 & -1.0 & 0.0 & -2.0 \\
G16GOV & 0.4 & -1.3 & 0.2 & -3.1 \\
G16LTG & 0.5 & -2.3 & -1.4 & -4.1 \\
G16ATG & -0.3 & -1.7 & -1.1 & -3.2 \\
\hline Average & -0.0 & -1.4 & 0.0 & -3.6
\end{tabular}

Table 5. Mean-median scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 11) - not as isolated numbers.
\begin{tabular}{lllll}
\multicolumn{5}{c}{ Senate Partisan Bias } \\
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -2.0 & -4.0 & 2.0 & -8.0 \\
G20USS & 0.0 & -6.0 & 2.0 & -8.0 \\
G20GOV & 2.0 & -2.0 & 2.0 & -6.0 \\
G20LTG & 0.0 & -2.0 & 0.0 & -8.0 \\
G20ATG & -4.0 & -4.0 & 0.0 & -8.0 \\
G20TRE & 0.0 & -4.0 & 2.0 & -10.0 \\
G20SOS & 0.0 & -2.0 & 0.0 & -6.0 \\
G16PRE & -2.0 & -4.0 & 0.0 & -10.0 \\
G16GOV & 2.0 & -4.0 & 4.0 & -10.0 \\
G16LTG & 2.0 & -6.0 & -4.0 & -8.0 \\
G16ATG & -4.0 & -6.0 & -4.0 & -10.0 \\
\hline Average & -0.5 & -4.0 & 0.4 & -8.4 \\
\hline
\end{tabular}

Table 6. Partisan bias scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 12) - not as isolated numbers.
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\begin{tabular}{lllll} 
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -4.1 & -4.3 & 1.8 & -8.5 \\
G20USS & -1.6 & -3.9 & -2.1 & -8.0 \\
G20GOV & -0.8 & -4.9 & -0.6 & -8.8 \\
G20LTG & -4.2 & -4.5 & -4.7 & -10.9 \\
G20ATG & -1.7 & -3.8 & 0.2 & -8.0 \\
G20TRE & -2.3 & -6.8 & -4.9 & -11.2 \\
G20SOS & 3.5 & -0.6 & 3.7 & -4.6 \\
G16PRE & 0.1 & -4.0 & -2.1 & -8.5 \\
G16GOV & 2.4 & -3.6 & 4.4 & -10.2 \\
G16LTG & -1.3 & -3.2 & -1.3 & -5.5 \\
G16ATG & -2.1 & -4.2 & -4.2 & -10.5 \\
\hline Average & -1.1 & -4.0 & -0.9 & -8.6 \\
\hline
\end{tabular}

Table 7. Efficiency gap scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 13) - not as isolated numbers.

Senate Declination
\begin{tabular}{lllll} 
Proposed_Plan & Harper & LD & NCLCV & Enacted \\
\hline G20PRE & -7.8 & -8.4 & 2.2 & -16.9 \\
G20USS & -3.0 & -6.9 & -3.4 & -15.3 \\
G20GOV & -2.4 & -9.1 & -2.4 & -16.2 \\
G20LTG & -7.2 & -8.0 & -7.7 & -20.9 \\
G20ATG & -3.8 & -7.5 & -0.9 & -15.3 \\
G20TRE & -2.7 & -10.6 & -6.7 & -20.5 \\
G20SOS & 5.1 & -1.5 & 5.0 & -8.5 \\
G16PRE & 0.5 & -7.1 & -3.3 & -16.2 \\
G16GOV & 3.3 & -6.4 & 6.4 & -17.9 \\
G16LTG & 0.5 & -4.3 & -0.6 & -10.0 \\
G16ATG & -3.8 & -7.2 & -7.2 & -18.4 \\
\hline Average & -1.9 & -7.0 & -1.7 & -16.0 \\
\hline
\end{tabular}

Table 8. Declination scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 14) - not as isolated numbers.

\section*{4. House Plan Analysis}

I followed the same procedures for analyzing the House plans as I did for the Senate and Congressional plans, but here I had only two plans (LD and NCLCV). I used the ensemble [?], whose characteristics are similar to those of the Senate ensemble used above.
4.1. Seats Won. Considering the number of seats won in each election, as shown in Figure 15, Both the LD and NCLCV plans appear to be mostly typical in terms of the number of seats won, except in G20PR and G16LG where NCLCV is much higher (pro Democrat) than the main distribution.
4.2. Rank-Ordered Violin Plots. Referring to Figure 16, which focuses only on the most competitive districts, the NCLCV plan appears to deviate much more from the ensemble than the LD plan does.

House Ensemble 0 Seats

Lt Gov '20


Gov '16


House Ensemble 0 Seats
G16_AG


G16_PR


G16_LG


House Ensemble 0 Seats

G20_TR


G20_AG


G20_USS


House Ensemble 0 Seats



G20_SST


Figure 15. Histograms of congressional seats won in all 11 elections for Ensemble 0 plans. The percentages in the legend represent percentile of the corresponding score in the ensemble.


Figure 16. Rank-ordered house districts for 10 of the elections (all elections but G20LG) using Ensemble 0. These plots show only the most competitive districts.

\subsection*{4.3. Other Metrics.}
4.3.1. Mean-Median. Table 9 shows my calculations of the mean-median scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 17)—not as isolated numbers.

The distribution is shifted in the negative direction, so scores very close to 0 look more like outliers than large negative scores. Specifically, the NCLCV score of \(0.1 \%\) in the election G20PR is very close to zero, but it is more Democratic favoring than \(98 \%\) of all plans, so this plan is an outlier for this distribution, while the LD plan's score of \(-0.7 \%\) is more typical of the distribution.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of mean-median scores.
4.3.2. Partisan Bias. Table 10 shows my calculations of the partisan bias scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 18)—not as isolated numbers.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of partisan bias scores.
4.3.3. Efficiency Gap. Table 11 shows my calculations of the efficiency gap scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 19) - not as isolated numbers.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of efficiency gap scores.
4.3.4. Declination. Table 4 shows my calculations of the declination scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 20)—not as isolated numbers.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of declination scores.
4.4. House Conclusion. The seat margins shown in the rank-ordered violin plots of Figure 16 give evidence of partisan gerrymandering in the NCLCV plan.

These conclusions do not take VRA racial considerations into account.

House Ensemble 0 MeanMedian


House Ensemble 0 MeanMedian



House Ensemble 0 MeanMedian

G16_LG



House Ensemble 0 MeanMedian


\(-0.04-0.03-0.02-0.01 \quad 0.00 \quad 0.01\)


G20_SST


Figure 17. Histogram of house ensemble 0 mean-median score for all 11 elections.

House Ensemble 0 PartisanBias


House Ensemble 0 PartisanBias


House Ensemble 0 PartisanBias


Figure 18. Histogram of partisan bias for all 11 elections for Ensemble 0.

House Ensemble 0 EfficiencyGap


Figure 19. Histogram of house ensemble 0 efficiency gap for all 11 elections.

House Ensemble 0 Declination


House Ensemble 0 Declination




House Ensemble 0 Declination


G20_SST


Figure 20. Histogram of house ensemble 0 partisan declination for all 11 elections.
\begin{tabular}{llll}
\multicolumn{4}{c}{ House Mean-Median } \\
Proposed_Plan & LD & NCLCV & Enacted \\
\hline G20PRE & -0.7 & 0.1 & -3.0 \\
G20USS & -1.5 & -1.0 & -3.4 \\
G20GOV & -0.5 & -0.3 & -3.3 \\
G20LTG & -1.3 & -1.0 & -3.4 \\
G20ATG & -1.4 & -1.2 & -3.1 \\
G20TRE & -1.1 & -1.4 & -3.4 \\
G20SOS & -0.7 & -0.8 & -3.1 \\
G16PRE & -1.8 & -1.6 & -5.1 \\
G16GOV & -2.0 & -2.2 & -4.1 \\
G16LTG & -3.0 & -3.2 & -4.4 \\
G16ATG & -2.6 & -2.5 & -4.5 \\
\hline Average & -1.5 & -1.4 & -3.7
\end{tabular}

Table 9. Mean-median scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.
\begin{tabular}{llll}
\multicolumn{4}{c}{ House Partisan Bias } \\
Proposed_Plan & LD & NCLCV & Enacted \\
\hline G20PRE & -0.8 & 0.0 & -6.7 \\
G20USS & -2.5 & -0.8 & -7.5 \\
G20GOV & -1.7 & -0.8 & -8.3 \\
G20LTG & -1.7 & -0.8 & -7.5 \\
G20ATG & -0.8 & -1.7 & -7.5 \\
G20TRE & -1.7 & -1.7 & -7.5 \\
G20SOS & -0.8 & -1.7 & -7.5 \\
G16PRE & -4.2 & -1.7 & -9.2 \\
G16GOV & -5.0 & -2.5 & -8.3 \\
G16LTG & -5.0 & -0.8 & -8.3 \\
G16ATG & -5.8 & -2.5 & -9.2 \\
\hline Average & -2.7 & -1.4 & -8.0 \\
\hline
\end{tabular}

Table 10. Partisan bias scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.
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\section*{House Efficiency Gap}
\begin{tabular}{llll} 
Proposed_Plan & LD & NCLCV & Enacted \\
\hline G20PRE & 0.4 & 1.0 & -7.5 \\
G20USS & -1.5 & -2.8 & -8.7 \\
G20GOV & -1.4 & -2.7 & -6.3 \\
G20LTG & -2.1 & -1.8 & -8.9 \\
G20ATG & -1.2 & -2.4 & -8.6 \\
G20TRE & -4.6 & -3.2 & -8.0 \\
G20SOS & -1.9 & -1.8 & -8.9 \\
G16PRE & -3.1 & 0.1 & -5.2 \\
G16GOV & -4.7 & -1.8 & -8.2 \\
G16LTG & -4.4 & 1.0 & -7.0 \\
G16ATG & -5.5 & -3.4 & -9.0 \\
Hou EG mean & -2.7 & -1.6 & -7.8 \\
\hline
\end{tabular}

Table 11. Efficiency gap scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

\section*{House Declination}
\begin{tabular}{llll} 
Proposed_Plan & LD & NCLCV & Enacted \\
\hline G20PRE & -0.3 & 1.3 & -14.9 \\
G20USS & -3.9 & -4.0 & -16.8 \\
G20GOV & -3.6 & -5.1 & -12.7 \\
G20LTG & -4.5 & -3.0 & -18.0 \\
G20ATG & -3.2 & -4.7 & -15.7 \\
G20TRE & -8.1 & -5.1 & -15.2 \\
G20SOS & -4.1 & -2.6 & -16.3 \\
G16PRE & -6.5 & -0.2 & -11.8 \\
G16GOV & -9.4 & -3.5 & -15.7 \\
G16LTG & -8.7 & 2.4 & -14.4 \\
G16ATG & -10.2 & -6.0 & -16.4 \\
\hline Average & -5.7 & -2.8 & -15.3 \\
\hline
\end{tabular}

Table 12. Declination scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms) - not as isolated numbers.

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\section*{Preliminary Report:}

\title{
Proposed Legislative and Congressional Remedial Plans in North Carolina
}

\section*{Revised draft (please discared the older version)}

\author{
Bernard Grofman*
}

March 21, 2022
* I am Jack W. Peltason Chair of Democracy Studies and Distinguished Professor of Political Science at the University of California, Irvine. My research deals primarily with issues of representation, including minority voting rights and party competition. I am a Fellow of the American Academy of Arts and Sciences. I have an honorary Ph.D. from the University of Copenhagen for my work on the cross-national study of elections and voting rules. I am the recipient of a lifetime achievement award from the American Political Science Association for my work on elections and voting rights. I am co-author of five books with major university presses (Cambridge (4), Yale (1), and co-editor of 26 other books, (including books with Oxford (3), U. Michigan (4), and Princeton) with over 300 research articles and book chapters.. Over the past six years I have served as a special master to draw remedial maps for five different federal courts, including redrawing a Virginia congressional district and redrawing eleven districts in the Virginia House of Delegates, and preparing remedial maps s in local elections in Georgia, Virginia, and Utah. In addition I served as co-special master in the 2021 redistricting, drawing the remedial maps adopted by the Virginia State Supreme Court for that state's legislative and congressional districts. Over a 40+ year career, I have served as an expert witness or consultant in redistricting cases in nearly a dozen states I have worked as an expert for both political parties, the NAACP, MALDEF, the U.S, Department of Justice, and non-partisan redistricting authorities. My work has been cited in a dozen different U.S. Supreme Court cases, perhaps most notably in Thornburg v. Gingles 478 U.S. 30 (1986). In mid-February 2022 I was asked to serve as an expert consultant to the three Special Masters appointed to present recommendations to the North Carolina Supreme Court in the case of Harper v. Hall. North Carolina maps and block equivalency files were provided by the parties in this case; North Carolina election data was provided courtesy of the Voting and Election Science Team: https://dataverse.harvard.edu/dataverse/electionscience, disseminated by Dave's Redistricting App : https://davesredistricting.org of which I made extensive use.. I am also deeply indebted to my research assistant, Zachary Griggy, for the work he provided under my direction.

\section*{I. Introduction: Thinking About Partisan Gerrymandering.}

We can address the questions of partisan or racial gerrymandering either directly in terms of observed or expected political or racial consequences or, more indirectly, by examining features of maps (e.g., undue fragmentation of existing political subunits) that are often manipulated for partisan purposes. In this report my focus is on political consequences. \({ }^{1}\)

Another useful distinction in thinking about gerrymandering is whether the focus is to be on statewide indicators of gerrymandering or on evidence of gerrymandering at the district (or additionally, in North Carolina, county cluster) level. I believe in a holistic view of gerrymandering in which we examine both statewide effects and look in detail at evidence of manipulation at the level of districts/districts within clusters. Below I discuss both approaches. \({ }^{2}\)
(1) Using statistical metrics to directly evaluate the degree to which a map as a whole is non-dilutive in its expected partisan (or racial) consequences?

Most analyses of partisan effects of gerrymandering rely on a set of measures in the political science literature such as the mean minus median gap, or partisan bias that are applied on a jurisdiction-wide basis. These two metrics are intended to be effectively independent of the actual state-wide vote share in any given election. \({ }^{3}\) The mean-median gap builds in the value of the statewide vote average; by comparing means and medians of the partisan distribution, it is looking at one aspect of the skewness of a distribution, which is a measure of asymmetry. The partisan bias measure is evaluated in terms of what happens when both parties get a \(50 \%\) vote share, and thus checks to see if one party is advantaged when the vote share is evenly divided at

\footnotetext{
\({ }^{1}\) Since I have written extensively on racially polarized voting and racial vote dilution, if requested, I could extend my Report to analyze racial representation in the proposed maps. But, given the intense time pressure, I have limited myself here to issues involving partisan gerrymandering.
\({ }^{2}\) Courts have differed in how they approached this issue. One possible synthesis is to evaluate maps at the jurisdiction wide level but to determine remedies in particular districts or particular areas of the state where the key problems seemed to lie. In the racial context, the finding of violations and the remedies for gerrymandering (or for a violation of the Shaw v. Reno 509 U.S. 630 (1993) test for a constitutionally unlawful racially preponderant motive) have usually been localized.
\({ }^{3}\) However, ceteris paribus, both methods work best when, as in North Carolina, the state-wide two party vote share is close to fifty-fifty.
}
the statewide level. \({ }^{4}\) Note also that the mean-median gap and partisan bias are NOT tests for proportionality; they are tests for unequal treatment.

The best known metric to evaluate partisan inequities is partisan bias, one measure of which is reported for proposed NC maps in Table 1 later in the Report. \({ }^{5}\) The partisan bias metric, which focuses on what happens when the vote share is \(50 \%\), implicitly incorporates what Dr. Duchin in her first expert witness report refers to as the majoritarian principle, namely that a majority of votes should translate into a majority of seats. As the Supreme Court said in Reynolds v. Sims," to sanction minority control of state legislative bodies would appear to deny majority rights in a
\({ }^{4}\) Similarly, the difference between the value of the efficiency gap for a given plan and a value of the efficiency gap of zero can be taken to be an indicator of possible gerrymandering.
\({ }^{5}\) The partisan bias test, based on symmetry, was developed by the Princeton political scientist, Edward Tufte in 1973 and the statistical methodology for calculating it was improved by the Harvard political scientist Gary King and his co-authors in the 1980s, mostly notably in joint work with the Columbia University statistician, Andrew Gelman. A relatively non-technical introduction can be found in Bernard Grofman and Gary King. "Partisan Symmetry and the Test for Gerrymandering Claims after LULAC v. Perry." 6 Election L.J. 2 (2007). Also see Katz Jonathan N., Gary King, and Elizabeth Rosenblatt. 2019. "Theoretical Foundations and Empirical Evaluations of Partisan Fairness in District-Based Democracies." American Political Science Review. Partisan has a simple intuition but requires a somewhat complicated method to generate results. Take a situation in which Democrats typically won approximately \(53 \%\) of the statewide two-party vote. Say that with \(53 \%\) of the vote Democrats would win \(57 \%\) of the seats in some legislative or congressional election. Now, say that in a succeeding election, Democrats lost 6 percentage points in the popular vote so that they, not the Republicans had \(47 \%\) of the popular vote. If the map were perfectly symmetric, with \(53 \%\) of the vote, the Republicans also should win \(57 \%\) of the seats, as the Democrats did with this same vote share. Calculating partisan symmetry requires that a researcher estimate a 50-50 election. In our example above-the researcher begins with a \(53 \%\) vote share and then shifts the vote share, on average, a point at a time in both the Republican and Democratic direction while tracking the expected outcomes in seats won and lost. Then the relationship between vote share and seat share is calculated. If the parties move identically up and down what is called a votes-seats curve, the deviations should cancel out and you are left with a 0 deviation from symmetry, i.e., an estimated seat share of \(50 \%\) at a vote share of \(50 \%\) (i.e., vote share of \(50 \%\) at a seat share of \(50 \%\) ). If the outcome at a \(50 \%\) vote share is something other than a \(50 \%\) seat share then there is partisan bias in favor of one party or the other. While this metric can be time consuming to calculate by hand, a computer can calculate this quickly. Note that a \(53 \%\) vote share need not require a \(53 \%\) seat share for the map to be non-dilutive. Note also that we need to a test to see if the observed level of bias is statistically significant. If a large proportion of seats are competitive, then an estimated bias may not be statistically significant, since a small change in vote share in some of the competitive seats can shift seat share substantially. This metric is the only one to attract favorable mention by some Supreme Court Justices (see Grofman and
way that far surpasses any possible denial of minority rights that might otherwise be thought to result " 377 U.S. 533 at 565 (1964). \({ }^{6}\)

While the mean-median gap is a very useful and easy to calculate tool for getting a handle on the presence of partisan gerrymandering, it cannot stand as the sole statistical measure of partisan gerrymandering. Not only does it need to be informed by the results other measures, such as partisan bias, but it also can usefully be supplemented by measures which extend its basic approach beyond a single district.

Dr. Duchin in her first expert witness trial report (PX150, Figure 2, at p.7) shows data for the enacted congressional map and congressional ensembles. and looks at the set of most competitive districts (not just at one district, the median district). She examines whether the set of competitive districts are skewed in favor of one party. She refers to this approach as the "close votes, close seats" principle. Analogous analyses are performed by Dr. Chen in his trial testimony (see PX482, pp. 30-31). This approach can be thought of as a generalization of the mean-median gap, and is arguably to be preferred to it, since the mean-median gap only deals with results for a single district and thus can present a misleading picture of the partisan consequences of a map as a whole. Also, the mean-median gap may be easier to manipulate by mapmakers than some other measures, e.g., by assuring that in the particular district which is the median, the mean-median gap is not that big even though the map as a whole remains a clear partisan gerrymander. Nonetheless, largely because of its simplicity, the mean-median metric is an important one. I have used it myself in evaluating maps when appointed in 2021 by the Virginia Supreme Court as co-Special Master for Virginia congressional and legislative redistricting.

But, regardless of which measure of partisan vote dilution is being used, it is important to also consider how likely to be durable is the gerrymandering effect. As the Supreme Court of North Carolina observed in Harper v. Hall. "While partisan gerrymandering is not a new tool, modern technologies enable mapmakers to achieve extremes of imbalance that, 'with almost surgical precision,' undermine our constitutional system of government. Indeed, the programs and algorithms now available for drawing electoral districts have become so sophisticated that it is possible to implement extreme and durable partisan gerrymanders that can enable one party to effectively guarantee itself a supermajority for an entire decade, even as electoral conditions change and voter preferences shift" (slip op., p.1, footnotes omitted).

\section*{(2) Looking at evidence of partisan manipulation at the district or county cluster level}

\footnotetext{
\({ }^{6}\) The majoritarian principle is much weaker than the proportionality principle; the latter requires that a given vote share for a party translate into the identical share of legislative seats for that party. My 1985 essay, "Criteria for Districting: A Social Science Perspective." UCLA Law Review, 33(1):77-184," is among the many which discuss the importance of the majoritarian principle for democratic theory and election law
}

To look for evidence of gerrymandering at the district or county cluster requires an intensively local appraisal of how political subunits, concentrations of voters of a given party, and demographic groups are being treated (as well as of the degree to which compactness concerns were being met). This can be accomplished in two different ways.

One way is to look for evidence about intentional manipulation of boundaries at the district or county cluster level by careful use of the eyeball (and perhaps also some simple descriptive statistics) by individuals who have detailed knowledge of the state and who then provide a description of how particular pieces of geography were manipulated. Here, we can either be looking to identify areas where gerrymandering is found and to which remedies might be directed and/or we look for "patterns and practices" that are common across subunits of a kind that are indicative of gerrymandering even if we do not formally test for statistical significance \({ }^{7}\) This type of common-sense evidence can be compelling, both at the level of individual districts and for understanding an overall pattern of dilutive acts.

The second way is to make use statistical analyses for districts or county clusters is to do analyses based on ensembles in ways that closely resemble those used for statewide analyses.

For example, one useful approach to understanding the degree to which the two key tools of gerrymandering, packing and cracking, were used by mapmakers at the district level employs ensemble analysis and calculation of statistical outliers. Dr. Jowei Chen in his expert witness trial report. Dr. Chen (PX882, Figure 4, p. 25) ranked congressional districts from most Republican to least Republican in the enacted congressional map, and considered whether there was evidence of manipulation in that the districts Republicans did best in were, in general, being won by lower than expected vote margins (i.e., the map "efficiently" placed Republican voters to win without wasting Republican votes), while the districts in which the Democrats did best were, in general, being won by higher than expected vote margins (i.e., the map "inefficiently" placed Democratic voters to "pack" them and thus waste their votes), while districts that were somewhat competitive by and large showed a higher than expected Republican votes hare (those districts were "shored up" to make Republican loss unlikely).This creates an sshaped pattern in the data that is clearly visible in Figure \(4 .{ }^{8}\) This type of evidence suggests, even if it cannot prove, intentional partisan gerrymandering,
\({ }^{7}\) Descriptive statistics simply describe data and patterns in the data; inferential statistics seek to assign probability of occurrence of events relative to some null hypothesis. With ensemble analysis, the null hypothesis against which statistical significance is determined is that the plan was drawn from a set of plans like those in the ensemble.
\({ }^{8}\) Chen observes statistically significant results in 10 of 14 of the county clusters and the overall pattern is striking. Here it is important not to be misled by the fact that there were some clusters that were not statistically significant; it is the overall pattern that shows the improbability of the results. Indeed, even if there were NO clusters with statistically significant results but the directionality of manipulation was as predicted across virtually all the clusters, properly applied statistical calculations that look at multiple clusters at the same time can show the reality of statistically significant results even if no single cluster is a statistically significant outlier. \({ }^{8}\)

At the county cluster level, we can also evaluate whether there were excess city splits or county cuts within that cluster from what we would expect of plans in the ensemble in that same clusters. We should also note that we can ask if expected partisan outcomes within the cluster in terms of mean expected wins were extreme statistical outliers, or whether particular groups such as African-Americans or other minorities were either cracked or packed within the cluster in ways that signaled improper attention to race. But we must be careful not to mistake failures to find statistically significant results at the cluster level with the absence of significant (and substantively important) bias in the plan as a whole, since what is a clear overall pattern of discrimination can be missed if we look only small groupings.

But, in looking at districts or clusters, just as in looking at stateside indicia of partisan gerrymandering, we must also ask whether difference from what is predicted in an ensemble takes us toward partisan equity or away from it (see below).

\section*{II. Baselines and Thresholds in Evaluating Partisan Gerrymandering}

What is the appropriate baseline against which to judge whether some given value of a metric such as partisan bias or mean-median difference supports a claim of egregious gerrymandering?

There are two ways in which the question of appropriate baseline for statistical analyses of partisan gerrymandering effects has been addressed in the political science literature. The most obvious way to evaluate statistical metrics used to identify partisan gerrymandering effects, such as those shown in Table 1, is simply to ask questions such as: "How close is the mean-median gap to zero?" "How close to a zero level of (vote or seat) partisan bias does the plan have?, etc. As a result of my recent experiences as a special master I have come to the view that this is not just the simplest, but also the best, way to think about statistical metrics that seek to directly measure gerrymandering. But a second way in which this question has been addressed is to ask: "How does a map compare in its properties vis-a-vis various metrics to those in an ensemble of computer drawn maps constructed in a partisan blind fashion?"

Ensembles are sets of computer-generated plans based on the geographic distribution of population in the unit (usually at the level of census blocks) which may also have "built in" instructions to the computer to take into some features besides population, e.g., respecting county or other subunit borders, or avoiding pairing incumbents, or seeking to draw compact districts. \({ }^{9}\) For ensembles, for any given metric, the baseline is established by answer the question: "Is a given map a statistical outlier with respect to the ensemble, with properties that by chance alone would occur only at the tails of the ensemble distribution, e.g., with probability less than 05 (the familiar two standard deviation test for adverse impact from Griggs v. Duke Power Co, 401 US

\footnotetext{
\({ }^{9}\) In North Carolina, ensemble simulations for state legislative districts (NC House and NC Senate) introduced by experts in Harper v. Hall are programmed to take into account, the state's county clustering rule.
}

424 (1971))?" Ensemble analysis can be applied to features of maps such as splitting of counties or other subunits, or features such as compactness, but it can also be applied to measuring expected political effects of a map via the kinds of metrics used by experts in the Hall v. Harper litigation, that were subsequently referenced in the Harper v. Hall majority opinion. \({ }^{10}\) Electionrelated metrics are calculated using a distribution of recent partisan (and/or racial) voting patterns in the unit (usually with data drawn from statewide elections that is projected into census geography). with the values of these metrics and of expected partisan outcomes in the plan (or portions of the plan) are compared to those in the ensemble.

In evaluating any map in terms of political effect metrics it is important to be able to separate out the effects of so-called "natural" bias, i.e., partisan bias that arises from historical patterns of electoral geography and environmental features such as mountains or rivers, \({ }^{11}\) from partisan bias that arises from contemporaneous map-making practices, including and especially intentional gerrymandering. Using ensembles as the basis for our evaluations directly allows us to compare the bias (or other features) in any given map with the bias (or other feature) in the ensemble, since we are holding constant the electoral geography of the state and other features of the state, such as rivers or mountains.

The use of ensembles has allowed for major theoretical and empirical advances in studying redistricting and gerrymandering, and I strongly endorse their previous use in this litigation. If a map exhibits more evidence of bias or other kinds of distortions than we find in an ensemble to a statistically significant degree, I view this fact as very strong prima facie evidence of manipulation. But there are two ways to make errors based on ensemble analyses involving political election-based metrics: on the one hand, concluding that a plan is dilutive when instead it is vote-dilution reducing and, on the other hand, concluding that a plan is not dilutive because it is not an outlier in the ensemble for some parameters when, in fact, it is a carefully crafted gerrymander (Type I and Type II errors).

First, we must be careful to look at the directionality of deviation from ensemble expectation. If a map has lower (absolute) values on metrics such as partisan bias than most of the maps in the ensemble, ceteris paribus, that is something to be desired, not condemned, even if the map is outside the \(\mathbf{9 5 \%}\) confidence range of the ensemble. It is only when the map has higher values of metrics that show vote dilution than most of the maps in the ensemble that we see evidence of partisan gerrymandering that might rise to the level of unconstitutionality. Thus, even if we opt only for an ensemble based approach to evaluating vote dilution, when we do look at how far from an ensemble expectation is the observed value on some metric it is critical to

\footnotetext{
\({ }^{10}\) See, e.g., the discussion of the findings of Plaintiffs expert Dr. Jowei Chen in League of Women Voters v. Pennsylvania (J-1-2018, Supreme Court Of Pennsylvania Middle District).
}
\({ }^{11}\) Although I have used the term "natural bias" because it has become standard, I regard it as a misnomer. For example, there is nothing natural about the disproportionate presence of AfricanAmericans in areas good for cotton growing that continues to the present day, unless you think slavery is natural. And redlining and other practices have led to geographic segregation of minorities within cities.
distinguish whether the value in the map takes us in the direction of more dilution or in the direction of less dilution.

Second, even if a map is within the \(\mathbf{9 5 \%}\) confidence bounds of an ensemble on some particular metric, that does not mean that the map is NOT a partisan gerrymander. There are multiple statistical metrics to evaluate the level of partisan gerrymandering, and we need to be careful to look at multiple indicators, both at the state level and ones that are district or county-cluster specific. Also, there may be non-statistical evidence of intentional gerrymandering derived from careful analysis by knowledgeable observers of exactly where particular lines on the map have been drawn. Such evidence may lead to a conclusion of a constitutional violation even in the absence of use of ensembles or of statistical inference tests. \({ }^{12}\) Or they may be inferences of intentional gerrymandering based on the redistricting process itself or based on statements made by mapmakers.

Third, because of how ensembles are created, when we look at the political effects metrics, they may show a map to be non-dilutive even when dilution is present because the natural bias in a state favors a particular party and thus tilts the ensembles toward maps favorable to that party.

An ensemble-based standard for vote dilution takes as given the distribution of voters in the state at some low level of census geography such as the block. But because it is built on the distribution of voters, when we look at partisan behavior in past elections, we often find that the voters of one party are more concentrated than voters in the other party. In particular, Democrats (and minorities) are likely to be highly concentrated in cities. When one group has its voters more geographically concentrated than another, redistricting can create inequities, e.g., by packing Democratic voters into districts in such a fashion as to "waste" their votes.

While I can attest from my own knowledge that Dr. Duchin (PX150, p. 4) is correct that North Carolina is a jurisdiction that has a low level of so-called natural bias compared to most other states, \({ }^{13}\) a low level of natural bias is not zero bias.

Consider the ensembles created by Dr. Daniel Magleby which he uses to evaluate whether some given plan's mean-median value is (considerably) outside the \(95 \%\) confidence range generated by the ensemble (see PX 1483). For Congress, Magleby finds the mean-median value in his ensemble to be around \(1 \%\) more Republican than the statewide average (see Figure 5 in his first Report). A similar \(1 \%\) pro-Republican bias is found for the Senate (see Figure 4 in his first

\footnotetext{
\({ }^{12}\) Much of the litigation involving claims involving racial gerrymandering or race as a preponderant motive illustrates this point.
}
\({ }^{13}\) The existence of what has been called "natural bias," has led some commentators to claim that whatever bias is found in a given plan is due to geography, not intent to discriminate. However, as Dr. Duchin correctly points out, the level of natural bias in North Carolina in no way prevents the production of "maps that give the two major parties a roughly equal opportunity to elect their candidates" (PX150, p. 4).

Report), while the pro-Republican bias in the House for the mean-median ensemble is between \(2 \%\) and 3\%.(see Figure 2 in his first Report)

Further evidence of a pro-Republican "natural bias" obtain from simulations that focus on the expected number of seats a party will be expected to get if the partisan vote share is at the historical recent average. Dr. Magleby has done analyses of this kind (see PX1483), but so have other plaintiffs' experts. For example, with a projected \(50.8 \%\) Republican vote share, while the 10-4 projected vote outcome in the 2022 enacted legislative congressional map is a clear statistical outlier, Dr. Chen finds that a modal congressional outcome in his simulation would have an expected 9 Republican and 5 Democratic seats for the U.S. House (see Report of Dr. Chen PX882, Figure 7, p. 33). Dr. Mageleby's simulation (Figure 6 in his first Report) is similar, with about 8-9 Republican seats.

In sum, so-called "natural bias" tilts the ensembles for the North Carolina upper and lower chambers and for the U.S. House of Representatives somewhat in a pro-Republican direction. \({ }^{14}\)

Resting analyses of partisan bias solely on outlier analysis in ensembles creates a two-sided risk. On the one hand, plans that are highly dilutive might be accepted if the only analysis of equal treatment is an ensemble-based comparison. Indeed, if we judge partisan outcomes only by whether they closely match the mean results in an ensemble, we might conclude that, in North Carolina, for both branches of the legislature and for Congress, only at least a somewhat pro-Republican gerrymander is non-dilutive. \({ }^{15}\) On the other hand, any attempt to move toward a truly unbiased map might require moving away from the level of bias that is created by geography, i.e., outside the middle zone of the ensembles, and thus be attacked as a gerrymandering outlier. Such perverse results would, in my view, fly in the face of the North Carolina Supreme Court's assertion that "We hold that our constitution's Declaration of Rights guarantees the equal power of each person's voice in our government through voting in elections that matter" (slip op. p.1).

\footnotetext{
\({ }^{14}\) As best I can judge all the ensembles created by plaintiffs' experts show an expected proRepublican tilt in partisan effects measures such as mean-median difference.
\({ }^{15}\) The ensemble analyses conducted by Plaintiffs experts in Harper v. Hall concluded that the enacted maps to be partisan gerrymanders in that these maps were so egregiously gerrymandered that, on multiple indicators, they fell very far outside the ensemble-based expectations of the amount of expected pro-Republican bias even though the computer-generated ensembles were themselves exhibiting a pro-Republican bias (see above). The ensembles-based conclusions that these maps were egregiously gerrymandered in favor of Republicans, combined with the other evidence of intent and examination of how gerrymandering was done in particular areas of the state, combined with the evidence that the extreme level of pro-Republican bias in these plans would continue throughout the decade under realistic scenarios of future changes in statewide vote, thus locking in a permanent Republican majority in both houses of the legislature and in the state's congressional delegation, made it apparent that the plans should have been struck down as unconstitutional once partisan vote dilution was held to be justiciable under North Carolina state law.
}

Can we specify some threshold value of a metric such as partisan bias or mean-median difference as being required to supports a claim of egregious gerrymandering that rises to the level of unconstitutionality?

Both the zero baseline approach and the ensemble-based approach still leave open the question of the point at which the accumulated evidence of gerrymandering leads to a conclusion that this gerrymandering rises to a level of unconstitutionality. But there is one question on which I think there would be widespread agreement, namely that a legislative map does not have to be the "best possible map." The mere fact that a better map on multiple criteria exists does not require a court to choose that map over a map that is adopted through legal channels and due process. The Court's role as mapmaker only begins after the challenged map has been found to be unconstitutional and the legislature has forfeited any right to continue to prepare alternative maps. Moreover, if we think about criteria for demonstrating unconstitutional partisan gerrymandering, there probably also would be agreement that (a) the mere fact that the value of on some metric is a statistical outlier is not enough to show a violation, rather there must be evidence of the substantive importance of the discrepancy, \({ }^{16}\) and (b) before a finding of a constitutional violation, it would be important to demonstrate that the political effects of a plan are likely to be non-ephemeral.

However, while it might be seen as desirable for courts to clearly set a threshold for what differences from zero for any given metric are de minimis with respect to a claim of unconstitutional partisan gerrymandering, there are two reasons to reject such an approach at this time. First, state courts are only recently come to grips with partisan gerrymandering claims brought under state law. There simply has not been time enough for a body of jurisprudence to emerge. Rather, as the Court Opinion in Harper v. Hall suggested, courts should strike down egregious examples of partisan gerrymandering. Only in later cases will courts be in a position to set clear "safe harbor" thresholds if they eventually determine, as the U.S. Supreme Court did in the "one person, one vote" cases, that numerical de minimis standards were appropriate. \({ }^{17}\)
\({ }^{16}\) In the context of redistricting, this would translate as a finding that the consequences of the statistically significant disparate impact involved an expected seat share change of, say, at least one district (though that number might vary with the size of the legislature). For example, in League of Women Voters v. Pennsylvania (slip op. p. 128) the Pennsylvania Supreme Court favorably cites to Dr. Jowei Chen's finding that " while his simulated plans [the ensemble] created a range of up to 10 safe Republican districts... , the 2011 [enacted] Plan creates 13 safe Republican districts."
\({ }^{17}\) There are multiple statistical measures of malapportionment such as total deviation, defined as the sum of the deviation from ideal in the largest district plus the deviation from ideal in the smallest district, and average deviation, among others measures (see e.g., Cervas, Jonathan R., and Bernard Grofman. 2021. Legal, political science and economics approaches to measuring malapportionment: The U.S. House, the Senate, and the Electoral College 1790-2010. Social Sciences Quarterly. 101(6): 2238-2256), but, after a while, the Supreme Court largely settled on total population deviation as the key metric for OPOV. In the OPOV cases, after dealing with "horribles," The US. Supreme Court eventually adopted a \(10 \%\) total population deviation safe

Second, ascertaining the level of gerrymandering in a map is harder than ascertaining the degree of malapportionment in a map. Not only are some of the statistical tools, such as ensembles, much more complicated than simple arithmetic but, perhaps even more importantly, there are multiple (but related) metrics and multiple factors to consider, all of which require careful parsing in terms of forging an overall assessment. Thus, I see the early phases of state court partisan gerrymandering litigation employing a "totality of the circumstances approach," even though also relying on the various specific statistical indicators the Harper v. Hall opinion highlighted. \({ }^{18}\)

\section*{III. Preliminary Evaluations from a Political Science Perspective of the New Legislatively-Drawn Maps for Congress, the NC Senate, and the NC House}

Below is a table showing, for each of the five proposed plans and for the three previously enacted maps, a variety of metrics: projections of how many Democratic and Republican leaning seats would be expected and how many districts would be competitive (from \(45 \%\) to \(55 \%\) ) and also, among the competitive seats, what is the relative balance of Democratic and Republican vote shares; the mean-median gap; two standard measures of partisan bias based on symmetry in a seats-votes curve (one based on how much above a \(50 \%\) vote share the party with diluted votes would need to win a majority of seats, the other based on the seat share a minority party would get if it won \(50 \%\) of the vote); the efficiency gap; and a composite measure of compactness that incorporates Polsby-Popper and Reock scores. The calculations are provided from a program, Dave's Redistricting App, which can calculate the standard electionbased indices of partisan gerrymandering. The political data reflect major statewide races 20162020. The metrics used give a historical baseline of \(49.4 \%\) Democratic two party vote and 50.8\% Republican two-party vote. \({ }^{19}\)
harbor for legislative districts - at least absent evidence of discrepancies lacking a legitimate state purpose, but required population deviation as close as practicable to zero for congressional maps. Having read the OPOV cases and gone back to read key academic commentary both just before and just after Baker v. Carr, I think it fair to say that nobody could have predicted the final OPOV standards chosen .
\({ }^{18}\) Brnovich v. Democratic National Committee 594 U.S.__ (2021) makes it clear that, in federal jurisprudence, in the context of Section 2 of the Voting Rights Act, a finding of disparate impact is not sufficient, standing alone, to prove a Section 2 violation, since other factors need to be taken into account, the U.S. Supreme Court also asserted " \(\$ 2\) does not transfer the States' authority to set non-discriminatory voting rules to the federal courts." This observation is doubly relevant, in my view, to the present litigation. On the one hand, the Supreme Court recognized the power of the states to set non-discriminatory voting rules. On the other hand, the Supreme Court recognized that no single metric may be enough to prove (or disprove) a constitutional violation, and that contextual analysis is needed.
\({ }^{19}\) There is no dispute among experts that, in Dr. Duchin's words, "North Carolina voting has displayed a partisan split staying consistently close to even between the two major
- App. 127 -
<<Table 1 about here. See below>>
parties over the last ten years." (PX150, p.4).
- App. 128 -

TABLE 1: Plan Comparisons on Multiple Metrics
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Plan Name & Map Type & \# of Districts & Rep Districts & Dem Districts & Competive Districts & \begin{tabular}{l}
Mean- \\
Median Dist
\end{tabular} & Votes Bias & Seats Bias & Efficiency Gap & Compactness \\
\hline Overturned Congress Plan & Congress & 14 & 8 & 3 & 3 (2R, 1D) & 5.78\% & 3.68\% & 16.86\% & 17.32\% & 51 \\
\hline Legislature Congress Plan & Congress & 14 & 6 & 3 & 5 (2R, 3D) & 0.66\% & 1.27\% & 5.27\% & 6.37\% & 45 \\
\hline Harper et al. Congress Plan & Congress & 14 & 6 & 4 & 4 (1R, 3 D) & 0.05\% & 0.32\% & 0.93\% & 1.50\% & 66 \\
\hline LCV et al. Congress Plan & Congress & 14 & 5 & 3 & 6 (1R, 5D) & -1.66\% & -0.10\% & -0.36\% & 0.67\% & 74 \\
\hline Overturned Senate Plan & Senate & 50 & 24 & 17 & \(9(5 R, 4 D)\) & 3.66\% & 3.31\% & 7.22\% & 7.14\% & 61 \\
\hline Legislature Senate Plan & Senate & 50 & 24 & 17 & \(9(4 R, 5 D)\) & 0.77\% & 2.02\% & 4.07\% & 4.24\% & 69 \\
\hline Harper et al. Senate Plan & Senate & 50 & 21 & 19 & 10 (7R, 3D) & -0.08\% & 0.48\% & 1.07\% & 1.21\% & 63 \\
\hline LCV et al. Senate Plan & Senate & 50 & 22 & 17 & 11 (4R, 7D) & -0.07\% & 0.72\% & 1.56\% & 1.67\% & 69 \\
\hline Overturned House Plan & House & 120 & 56 & 40 & 24 (14R, 10D) & 3.61\% & 2.94\% & 6.77\% & 6.71\% & 65 \\
\hline Legislature House Plan & House & 120 & 54 & 43 & 23 (9R,14D) & 0.89\% & 1.29\% & 2.70\% & 2.72\% & 72 \\
\hline LCV et al. House Plan & House & 120 & 55 & 44 & 21 (7R, 14D) & 1.11\% & 0.91\% & 1.69\% & 1.58\% & 81 \\
\hline
\end{tabular}

Because lack of constitutionality must be established before any consideration can be given to choosing an alternative map, here I will limit myself to political science perspectives on the constitutionality of each of the legislature's proposed maps. I will not discuss the question of which alternative map should be adopted by the court if the map proposed by the legislature is found to be unconstitutional, except to note that the maps proposed by one or more plaintiffs would seem to be ones that the Court could adopt (perhaps as is, perhaps with very minor modifications) if the corresponding legislative map was struck down. However, while I will not discuss which alternative map is best, since that issue is premature, I will use the alternative maps to show how much closer to zero values on the various metrics it would have been possible to come.

My discussion will be limited to the data presented in Table 1, which reports only metrics calculated at the statewide level. \({ }^{20}\) I recognize that the information in this table is not the only relevant material. Thus, my conclusions might be changed upon exposure to expert witness testimony about the various plans. In particular, I am not able to incorporate into my conclusions finding about the maps in terms of the spatial configurations of individual districts or county clusters and how those might have been manipulated For these reasons, I have labeled my Report a Preliminary Report.

Before I turn to the three specific maps proposed by the legislature I should note that, on virtually all statistical metrics, the new plans are significant improvements from the old plans. But the plans previously rejected by the Court were such egregious gerrymanders that the standard of doing better is a very low bar. I would also note that perusal of Figure 4 in 22.2.21 NCLV Plaintiffs' Remedial Comments (at p. 18) suggests that the new proposed congressional map has the most pro-Republican bias of the three proposed maps, and the State House map has the least pro-Republican bias. This is generally consistent with my own findings. Thus, a legal decision about which proposed maps are constitutional/unconstitutional need not be the same for all three maps.

\section*{Congress}

There are several key facts about the congressional map proposed by the legislature.
First and foremost, in a state that is in recent history one that is nearly evenly divided, it creates a distribution of voting strength across districts that is very lopsidedly Republican: 6 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 3 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 5 competitive districts. A sports analogy may be helpful here. Imagine a playoff series of 14 games of which a majority ( 9 of 14) have already been played, with five games still to go. The team that has won only 3 of the 9 games would need to win all five of the remaining

\footnotetext{
\({ }^{20}\) I believe the data presented in Table 1 to be a faithful representation of what is found App for the various metrics in Dave's Redistricting, but I recognize that there is always the possibility of error in converting shape files from one GIS program to another and always the possibility of typographical error in my entering data into this Report.
}
games in order to win the series, and it would need to win four of the five just to get a tie. If the teams were evenly matched in the remaining games of the series the likelihood of winning all five is under \(5 \% .{ }^{21}\) Of course, we need to examine much more closely the expected degree of competition in the districts that DRA labels competitive districts in the proposed congressional map. While there is an apparent Democratic 3-2 advantage in the competitive seats, a close look at the data shows that in 2 of the 3 competitive seats showing a mean Democratic edge that edge is razor thin, and smaller than the still narrow pro-Republican edge in the two Republican leaning competitive districts, while the \(3^{\text {rd }}\) district labeled as competitive has a substantial Democratic edge and is a very heavily African-American district Looking at vote margins more closely, we might thus view this map as \(\left\{6 \mathrm{R}, 4 \mathrm{D}, 4\right.\) very competitive\}. \({ }^{22}\) But even so, Democrats would still have to win four of the four competitive seats to win a majority in the delegation.

Second, while the results in the median district look a lot like the statewide average, but with a slight Republican edge, the median is only one district and we must look at the overall map. Here the \(5.27 \%\) seats bias suggest a substantial pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win a majority of the seats, and the \(1.27 \%\) vote bias suggests that only a win by more than \(50 \%\) of the statewide vote can yield the Democrats a majority of the seats. When we compare these levels of partisan bias to the level of partisan bias in the Harper and NCLCV maps we see that each of these two bias measures is multiple times higher in the legislative map than in the alternatives and, even when we look at differences in absolute value rather than ratios, it is still clear that the legislatively proposed congressional map is much more extreme with respect to partisan bias.

Third, the level of compactness of the districts in the previous map was a statistical outlier relative to the ensembles (Chen Expert Report PX482, pp. 17-19) and since the DRA compactness score the new congressional map proposed by the legislature is even lower, my expectation is that, with respect to district compactness the new map will also be a clear statistical outlier. However, unlike its predecessor (Chen Expert Report PX482, Figure 1, p. 14), doing a visual check, the new congressional map does not appear to split any counties in more than two pieces.

Fourth, there has been a substantial drop in the efficiency gap in the new map as compared to the congressional map found to be unconstitutional. But the efficiency gap is not directly a test for

\footnotetext{
\({ }^{21}\) Of course, this is an improved situation for the Democrats compared to the enacted congressional map, since that map ( 8 Rep, 3 Dem, 3 competitive) in effect said that the outcome was foreordained before the last three games were played. Barring a political tsunami, that map locked in a permanent Republican majority, and it was shown in the expert witness testimony to make a 10R-4D outcome very likely. Of course, that map was also one of the handful of most blatant and egregious partisan gerrymanders in the nation.
\({ }^{22}\) Note that to do this exactly we would need to look election by election to see how often Democrats won, since the mean vote share averaged across elections can lead to misleading conclusions because of variation in Democratic performance. See discussion of essentially this point in Dr. Duchin's Rebuttal Expert Witness Report (PX234).
}
bias; rather it measures, roughly speaking, how far from a responsiveness level of 2 a map implements. As Dr. Duchin has argued in her previous work, in a view that I share, high values of the efficiency gap are a sign that something may be seriously wrong and signal a need to investigate carefully. However, in my view, low values of the efficiency gap, are not a proof that there is no vote dilution. By offering a map with an efficiency gap of \(6.37 \%\) for their congressional map, i.e., one with an efficiency gap below 7, the legislative map drawers have apparently sought to draw a congressional map that just narrowly pass a supposed threshold test for partisan gerrymandering (see Memorandum on Remedial Process 4876-1419-931, at p. 7). And the efficiency gap is still a result in a pro-Republican direction.

Because they all point in the same direction, the political effects statistical indicators of partisan gerrymandering strongly suggest the conclusion that this congressional map should be viewed as a pro-Republican gerrymander, but whether these gerrymandering effect rises to the level of a constitutional violation must, of course, be left to legal determination. On the other hand, if I am correct that the compactness of the districts is at a level to show proof of severe outlier status, that in and of itself may be sufficient reason to reject the plan. But of course, that again is entirely a legal question up to the Court to resolve.

\section*{NC Senate}

My analysis and conclusions for the legislatively proposed NC Senate map are similar to those for legislatively proposed congressional map. In a state that is in recent history one that is nearly evenly divided, this map, too, creates a distribution of voting strength across districts that is very lopsidedly Republican: 24 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 17 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 5 competitive districts. Democrats would have to win nine of the nine competitive seats to win a majority in the Senate.

Second, while the median district again looks a lot like the statewide average, but again with a slight Republican edge, the median is only one district and we must look at the overall map. Here the \(4.07 \%\) seats bias still suggest a substantial pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win a majority of the seats, even though it is one percentage point or so lower than the comparable statistic in the congressional map, while the \(2.00 \%\) vote bias suggests that only a win by considerably more than \(50 \%\) of the statewide vote can yield the Democrats a majority of the seats. Indeed, on this metric the new NC Senate map is more extreme by nearly a percentage point than the new NC House map. When we compare these levels of partisan bias to the level of partisan bias in the Harper and NCLCV maps we see that each of these two bias measures is at least twice as high in the legislative map as in the alternatives and, even when we look at differences in absolute value rather than ratios, it is still clear that the legislatively proposed congressional map is much more extreme with respect to partisan bias than either of the alternatives.

Third, the compactness level in the Senate map is comparable or higher than that in the alternative Senate maps.

Fourth, there has been a substantial drop in the efficiency gap in the new map as compared to the congressional map found to be unconstitutional. But it remains in a pro-Republican direction.

Because they all point in the same direction, the political effects statistical indicators of partisan gerrymandering argue for the conclusion that this NC Senate map should be viewed as a proRepublican gerrymander. While, overall, the dilutive effects of this map do not appear quite as severe as in the congressional map they are still still quite substantial. However, I have not had time to analyze how the map may have been manipulated at the level of individual districts in terms of things like city cuts or county transversals. Of course, whether the clear indicators of partisan gerrymandering effects identified in Table 1 and my discussion rise to the level of a constitutional violation requires determination by this Court.

\section*{NC State House}

My analysis for the legislatively proposed NC House map uses the same approach as for the previously considered maps. In a state that is in recent history one that is nearly evenly divided, this map, too, creates a distribution of voting strength across districts that is very lopsidedly Republican: 54 Republican leaning districts that, based on averaged recent data will, barring a political tsunami, elect Republicans; 43 Democratic leaning districts that will, barring a political tsunami, elect Democrats; and 23 competitive districts. In the House, however, unlike the other maps, the Democrats do not have to win all of the competitive seats to win a majority in the House. Moreover, unlike the other two proposed maps, when we look at the proposed NC House map we see that the competitive seats are substantially Democrat in directionality (9R, 14D). This map is genuinely far more competitive than either of the other two legislatively proposed maps even though (see below) it remains tilted in a pro-Republican direction.

Second, while the median district again looks a lot like the statewide average, but again with a slight Republican edge, the \(2.70 \%\) seats bias still suggest a substantial pro-Republican bias in terms of the likelihood that a majority of the voters will be able to win a majority of the seats. But the value on this metric is one which is more than one percentage point lower than the comparable statistic in the Senate map, and the \(1.29 \%\) vote bias in this map is again almost one percentage point lower than the 2.00 value of this metric for the Senate. But arguably quit important in judging the constitutionality of this map in the full context are the facts that: (a) the Harper plaintiffs have not chosen to offer an alternative NC House map but are apparently content to see the legislative map implemented by the Court, (b) the map was passed by a clear bipartisan consensus in the legislature, including members of the legislature who belong to particular minority communities, and (c) that while it still is further from being non-dilutive than the NCLCV House map alternative, it is far closer to Plaintiffs' map than it is to the rejected enacted NC House map.

Third, the compactness level in the Senate map is high relative to the other maps in Table 1, even though the NCLCV House map alternative has an even higher score.

Fourth, there has been a substantial drop in the efficiency gap in the new map as compared to the NC House map found to be unconstitutional. It is at the low level of 2.72 even though it remains in a pro-Republican direction.

I have not had time to analyze how this map may have been manipulated at the level of individual districts in terms of things like city cuts or county transversals or racial fragmentaion. But of the three legislatively proposed maps, for the reasons given above, this is the one that I would feel most comfortable with seeing ordered by the Court. Looking at the totality of the circumstances insofar as these are presently known to me, and recognizing that this map is still not ideal (nor need it be), this legislatively proposed NC House map simply lacks the same clear indicia of egregious bias found in the previously rejected maps and still found, but to a lesser extent than in the rejected maps, in the legislatively proposed maps for Congress and for the NC Senate that I discuss above.

\author{
Evaluation of Remedial Plans \\ Prof. Sam Wang, Princeton University \\ February 21, 2022
}

Summary: This report evaluates the likely performance and partisan fairness of remedial plans for North Carolina Congressional, state Senate, and state House maps in the cases of Harper v. Hall and NCLCV v. Hall. Remedial plans were submitted by the North Carolina General Assembly ("Legislative Defendants"). Harper plaintiffs offered two remedial maps, Congressional and state Senate. The NCLCV plaintiffs also offered a set of three remedial maps. This report finds that all three of the Legislative Defendants' plans favor Republicans in six metrics evaluated: seat partisan asymmetry, mean-median difference, partisan bias, lopsided wins, declination angle, and efficiency gap. The seat partisan asymmetry in 1.7 seats in the Congressional plan, 2.1 seats in the Senate plan, and 7.2 seats in the House plan. The Harper plaintiffs' plans show mixed or no advantage for either party. The NCLCV plaintiffs' plans show a Democratic advantage for the Congressional plan, mixed or no advantage for the Senate plan, and a Republican advantage for the House plan. In no case did the Legislative Defendants' remedial map come closer to partisan symmetry than the plaintiffs' alternative(s).

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\begin{abstract}
About Prof. Wang: Sam Wang is a professor at Princeton University, appointed in neuroscience with affiliation with the Program in Law and Public Affairs. He directs the Electoral Innovation Lab, a policy and research group which uses statistics, science, and law to analyze election systems, and in which capacity he soversee the Princeton Gerrymandering Project (gerrymander.princeton.edu), which provides non-partisan analysis of redistricting plans and reforms. He has published extensively on the subject of redistricting. In particular, he has written in the Stanford Law Review and the Election Law Journal on the subject of practical tests for detecting partisan gerrymandering. In these articles he has analyzed the mean-median difference and introduced a new measure, the lopsided-wins test. These measures fall into a broad category of tests of partisan symmetry, a topic on which he has been cited in two U.S. Supreme Court decisions.
\end{abstract}

\section*{I. INTRODUCTION}

This report analyzes the remedial plans offered by parties in the North Carolina redistricting cases Harper v. Hall and NCLCV v. Hall. Those cases found that North Carolina's new Congressional, state House, and state Senate redistricting plans were illegal partisan gerrymanders in violation of the state constitution. The state Supreme Court has instructed the General Assembly to provide remedial maps for all three plans. The General Assembly provided these remedial maps on Friday, February 18, 2022, two passed on a partisan vote (Congressional and Senate) and one passed on a bipartisan vote (House). At that time the NCLCV plaintiffs also offered a set of three remedial maps. Harper plaintiffs offered two remedial maps, Congressional and state Senate.

I have analyzed these plans to determine their likely partisan performance. I apply statistical measures of partisan fairness to determine the amount of partisan favor that these maps show to either Republicans or Democrats.

Before applying the many tests for partisan fairness, I will briefly review the rationale and interpretations of the various tests.

\section*{II. MEASURING PARTISAN FAIRNESS}

The broad majority of metrics used by the court to evaluate partisan fairness address the question of whether voters, counted in total within the state, are represented fairly given a particular arrangement of Congressional or legislative districts. These metrics are calculated based on voter behavior in recent elections. Some of the metrics allow a variety of likely future scenarios to be explored empirically.

Because the relationship between voting and representation is complex, it is useful to evaluate multiple metrics. The use of multiple metrics helps guard against the possibility that a particular metric may vary by chance. The use of multiple metrics also guards against the possibility that redistricters might cater to one specific metric, to create the appearance of compliance while maintaining a hidden partisan advantage. I will therefore consider a variety of metrics together, in their totality.

\section*{A. Partisan seat asymmetry}

An important concept is whether the two parties would have commensurate outcomes if their vote shares were exchanged. The general concept of partisan symmetry has old roots \({ }^{1}\). A particularly simple measure is to ask how many seats each party would win if it attained the same statewide share of the vote; in this report I refer to the difference between the two seat counts as partisan seat asymmetry. Partisan seat asymmetry can also be calculated for a variety of likely swings in voter behavior; in this case, the average amount of asymmetry serves as a straightforward measure of partisan advantage over a range of plausible scenarios.

Another method for evaluating the fairness in the number of seats, given a total statewide vote, is the efficiency gap. The efficiency gap measures how far a pattern of outcomes deviates from expectations for a particular statewide vote, and is therefore a way of quantifying partisan advantage (though not necessarily a bright-line test) \({ }^{2}\). It has been proposed that an efficiency gap of 7 percentage points be used as a threshold to define undue advantage. However, it must be noted that the efficiency gap can jump in value when a single close race is won or lost. Therefore it is helpful to average the efficiency gap across a range of scenarios.

\section*{B. The mean-median difference}

The mean-median difference is a long-standing measure of what statisticians call skewness \({ }^{3}\). Applied to a district plan, the mean-median difference provides one way of testing whether an unusual pattern of districts is found above or below the statewide average. Such an unusual pattern is one way that an artful redistricting plan can build systematic advantage for one party. The mean-median difference can often help detect undue partisan advantage in a closely divided state such as North Carolina \({ }^{4}\).

\section*{C. Tests of voter packing}

When one side's voters are packed into a few districts to reduce their opportunities to elect representatives, they will be present in unusually large numbers in those districts. A direct way to measure packing is to compare the average vote share of Democratic-leaning and Republican-leaning districts. The party with the larger average win is potentially packed by its opponents in order to dilute

\footnotetext{
\({ }^{1}\) Gill v. Whitford, 138 S. Ct. 1916, 1933 (2018).(citing Brief of Heather K. Gerken et al. as Amici Curiae Supporting Appellees at 27.
\({ }^{2}\) Eric McGhee, Symposium: The efficiency gap is a measure, not a test. SCOTUSblog, August 11, 2017. https://www.scotusblog.com/2017/08/symposium-efficiency-gap-measure-not-test/ (last visited on February 21, 2022).
\({ }^{3}\) David P. Doane \& Lori E. Seward, Measuring Skewness: A Forgotten Statistic?, J. STAT. EDUC., July 2011, at 9-10; Karl Pearson, Contributions to the Mathematical Theory of Evolution-1: Skew Variation in Homogeneous Material, PHIL.TRANSACTIONS ROYAL SOC'Y, 1895, at 343, 374-76.
\({ }^{4}\) Samuel S.-H. Wang, Three Tests for Practical Evaluation of Partisan Gerrymandering, 68 STAN. L. REV. 1263, 1263-1321 (2016); Michael D. McDonald \& Robin E. Best, Unfair Partisan Gerrymanders in Politics and Law1: A Diagnostic Applied to Six Cases, 14 ELECTION L.J. 312, 312 (2015).
}
voting power. The concept of comparing averages dates to the foundations of statistics \({ }^{5}\), and when applied to redistricting such a comparison is called the "lopsided-wins test".

A more recent measure of packing is the declination, a measure that can be read from a graph visually. Declination takes advantage of the fact that a pattern of packing induces an elbow-like shape in the graph. The amount of bend in the elbow defines the declination. Declination also makes use of the number of districts won by each party. the larger the declination, the more voters are packed into a small number of districts.

\section*{III. ELECTION DATA AND ANALYSIS METHODS}

I estimated the likely performance of Congressional, Senate, and House maps in two ways. First, I evaluated vote totals in the proposed districts using ten statewide elections from 2014 to 2020. Second, I allowed the vote totals to vary above and below an average of these elections, as a means of evaluating a range of future scenarios that may arise in the coming decade. After these two steps, I then evaluated a variety of measures of partisan symmetry.

I used datasets for the following elections:
- President: 2016, 2020
- Senate: 2014, 2016, 2020
- Governor: 2016, 2020
- Lieutenant Governor: 2016, 2020
- Attorney General: 2016, 2020

In these elections, the two-party vote share ranged between \(46.7 \%\) and \(52.3 \%\) for Democrats, and between \(47.7 \%\) and \(53.3 \%\) for Republicans.

In addition, I used a composite ("2016-2020 Composite") that is averaged with equal weights from three components: (1) the average of President 2016 and 2020, (2) the average of Senate 2016 and 2020, and (3) the average of Governor and Attorney General 2020. In the 2016-2020 Composite, the two-party vote share was \(49.0 \%\) for Democrats and \(51.0 \%\) for Republicans.

\section*{IV. EVALUATION OF CONGRESSIONAL REMEDIAL PLANS}

\section*{A. Legislative Defendants' Plan}

As an example of how the analysis is done, Exhibit 1 shows calculations for the Legislative Defendants' plan in district-by-district form, using the 2016-2020 Composite. The plan is also evaluated according to the 10 individual election datasets (Exhibit 2).

\footnotetext{
\({ }^{5}\) Rigorous methods for comparing averages were first developed for controlling the quality of ingredients in the production of Guinness beer. The "Student \(t\) test" was devised by a master brewer, William Sealy Gossett, working pseudonymously to protect the trade secret. S.L. Zabell, "On Student's 1908 Article 'The Probable Error of a Mean'", 103 JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION 1.
}

For all 10 election datasets evaluated, the projected outcome for the Legislative Defendants' map was always between 4 and 8 Democratic seats, and between 6 and 10 Republican seats. The average outcome for the 10 election datasets was 5.3 Democratic seats and 8.7 Republican seats.

Exhibit 1: A fairness "dashboard" for the Legislative Defendants' remedial Congressional plan. Pink shading indicates Republican advantage, and blue shading indicates Democratic advantage.
LEGISLATIVE DEFENDANTS' CONGRESSIONAL PLAN
\begin{tabular}{c|cc|c} 
District & D share & R share & Margin (\%) \\
1 & \(54.7 \%\) & \(45.3 \%\) & 9.5 \\
2 & \(64.8 \%\) & \(35.2 \%\) & 29.7 \\
3 & \(38.7 \%\) & \(61.3 \%\) & -22.5 \\
4 & \(67.5 \%\) & \(32.5 \%\) & 35.1 \\
5 & \(44.2 \%\) & \(55.8 \%\) & -11.6 \\
6 & \(50.6 \%\) & \(49.4 \%\) & 1.2 \\
7 & \(50.6 \%\) & \(49.4 \%\) & 1.1 \\
8 & \(42.0 \%\) & \(58.0 \%\) & -16.0 \\
9 & \(38.7 \%\) & \(61.3 \%\) & -22.6 \\
10 & \(29.5 \%\) & \(70.5 \%\) & -40.9 \\
11 & \(44.9 \%\) & \(55.1 \%\) & -10.2 \\
12 & \(67.4 \%\) & \(32.6 \%\) & 34.8 \\
13 & \(48.5 \%\) & \(51.5 \%\) & -2.9 \\
14 & \(48.9 \%\) & \(51.1 \%\) & -2.2 \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline & DISTRICT RATINGS \\
6 & D favored \\
8 & R favored \\
4 & competitive within 7 points \\
\multicolumn{2}{|c|}{\begin{tabular}{c} 
PROBABILISTIC OUTCOME \\
5.8
\end{tabular}\(\quad\) Expected D wins } \\
8.2 & Expected R wins \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline & LOPSIDED WINS \\
59.3 & Average D win voteshare (\%) \\
58.1 & Average R win voteshare (\%) \\
1.2 & R advantage \\
\hline
\end{tabular}
-1.3 average margin
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|r|}{PARTISAN ASYMMETRY} \\
\hline 6.3 & D seats in 50-50 election \\
\hline 7.7 & \(R\) seats in 50-50 election \\
\hline 1.4 & \(R\) seat advantage \\
\hline 2.4 & average R seat advantage \\
\hline \multicolumn{2}{|r|}{MEAN-MEDIAN DIFFERENCE} \\
\hline -1.26\% & mean D-R (margin \%) \\
\hline -2.58\% & median D-R (margin \%) \\
\hline 0.7\% & R advantage (vote \%) \\
\hline & DECLINATION \\
\hline 11.4 & R advantage \\
\hline & EFFICIENCY GAP \\
\hline 7.4\% & R advantage (averaged) \\
\hline
\end{tabular}

Exhibit 2: Evaluation of the Legislative Defendants' remedial Congressional plan using data from ten elections.

Pink shading indicates a Republican advantage. Blue indicates a Democratic advantage.
The efficiency gap for the 2016-2020 composite is averaged over a range of possibilities within 7 points of the composite.

\section*{1. Partisan seat asymmetry}

I calculated the partisan seat asymmetry, i.e. the difference in seat breakdown that would result if the two parties traded total vote shares. I did this by creating a counterfactual in which I added a fixed percentage to the vote share in all districts, an assumption called "uniform swing.' In 9 out of 10 cases \(^{6}\), Republicans won more seats than the Democrats with the same vote share. For example, using the Governor 2020 race, Democrats win \(52.3 \%\) of the vote and get 8 out of 14 districts. In my counterfactual, if Republicans win \(52.3 \%\) of the vote, they would get 10 out of 14 districts. The difference between 10 and 8 is 2 - in other words, this plan has 2-seat partisan seat asymmetry.

Averaging across all 10 elections, the advantage was 1.7 more seats for Republicans, or \(12 \%\) of the 14 -seat Congressional delegation.

To test the robustness of this finding, I re-calculated the partisan seat asymmetry by taking the 2016-2020 composite and adding uniform swings to create scenarios in which Democrats and Republicans win an additional margin up to 7 points on top of their performance in the 2016-2020 composite. In each of these scenarios, I then calculated the partisan seat asymmetry as previously described. Averaging across these scenarios, the partisan seat asymmetry was again 1.7 seats favoring Republicans.

To summarize the partisan seat asymmetry analysis: The Legislative Defendants' remedial plan contains an average advantage of approximately 1.7 Congressional seats for Republicans, and this advantage persists across a wide range of likely scenarios that may arise.

\section*{2. Metrics of partisan fairness}

I then calculated five metrics that are used to test for partisan advantage: (a) the mean-median difference, (b) partisan bias, (c) lopsided wins, (d) the efficiency gap, and (e) the declination. I found that for all five tests, the metric showed an advantage for Republicans.

Across 10 elections, the average mean-median difference was \(1.2 \%\) favoring Republicans.

I calculated the efficiency gap for a variety of scenarios, in the same way that I calculated partisan seat asymmetry: I added uniform swings to create scenarios in which Democrats and Republicans win an additional margin up to 7 points on top of their performance in the 2016-2020 composite. Under these assumptions, the average efficiency gap was \(6.8 \%\) favoring Republicans. In six out of 10 election datasets, the efficiency gap was greater than \(7 \%\).

\section*{B. Comparisons with the Harper and NCLCV Plans}

To compare the Legislative Defendants' plan with two other Congressional plans offered by the Harper plaintiffs and the North Carolina League of Conservation Voters (NCLCV), I evaluated all three plans using the 2016-2020 Composite. This Composite has two advantages: it is close to \(50 \%\) for each party

\footnotetext{
\({ }^{6}\) The only case where there was no asymmetry was Senator 2020.
}
(favoring Republicans slightly), and it averages out effects that may be peculiar to a specific election or type of office.

The results are shown below in Exhibit 3.
Exhibit 3: Comparison of Legislative Defendants' Congressional Plan, the Harper plaintiffs' plan, and the NCLCV's plan.
\begin{tabular}{|c|c|c|c|}
\hline & \begin{tabular}{l}
Legislative \\
Defendants
\end{tabular} & Harper plaintiffs & NCLCV \\
\hline Statewide Democratic vote share (2016-2020 composite) & 49.3\% & 49.3\% & 49.3\% \\
\hline \multicolumn{4}{|l|}{Estimated performance:} \\
\hline Democratic-favored seats & 6 & 7 & 8 \\
\hline Republican-favored seats & 8 & 7 & 6 \\
\hline minimum Democratic seats & 4 & 5 & 4 \\
\hline maximum Democratic seats & 8 & 8 & 8 \\
\hline \multicolumn{4}{|l|}{Asymmetry measures (positive = Republican advantage):} \\
\hline Seat partisan asymmetry & 1.7 seats & 0.2 seats & -0.6 seats \\
\hline \multicolumn{4}{|l|}{Technical metrics:} \\
\hline Mean-median difference & 0.7\% & 0.1\% & -1.7\% \\
\hline Partisan bias & 5.2\% & 1.0\% & -0.4\% \\
\hline Lopsided wins difference & 2.4\% & -2.3\% & -9.9\% \\
\hline Declination angle & \(11.4{ }^{\circ}\) & -3.1 \({ }^{\circ}\) & -18.6 \({ }^{\circ}\) \\
\hline Efficiency gap & 7.4\% & 1.1\% & -0.5\% \\
\hline \multicolumn{4}{|c|}{Color key:} \\
\hline Democratic advantage & \multicolumn{3}{|c|}{Republican advantage} \\
\hline
\end{tabular}

First, it should be noted that the Legislative Defendants' plan has 6 Democratic-favored districts, the Harper plaintiffs' plan has 7 Democratic-favored districts, and the NCLCV plan has 8 Democratic-favored districts. However, such an estimate does not capture the full complexity of the pattern of districts as constructed in each plan. For this reason it is helpful to evaluate the other measures. The Legislative Defendants' plan shows favor to Republicans in all six measures tested. The NCLCV plan shows favor to Democrats in all six measures.

The Harper plaintiffs' plan shows no clear pattern of advantages to either party. The metrics for the Harper plain are generally close to zero, including seat partisan asymmetry of 0.2 seat, a mean-median difference of \(0.1 \%\), and an efficiency gap of \(1.1 \%\). The smallness and mixed nature of these metrics indicates that the Harper plaintiffs' plan is balanced in a way that gives special favor to neither Democrats or Republicans.

Dashboards for the Harper and NCLCV plaintiffs' plans are given in Exhbits 4 and 5.

Exhibit 4: Fairness dashboard for the Harper plaintiffs' Congressional plan.

\section*{HARPER PLAINTIFFS' CONGRESSIONAL PLAN}
\begin{tabular}{|c|c|c|c|} 
District & D share & R share & Margin (\%) \\
1 & \(54.6 \%\) & \(45.4 \%\) & 9.2 \\
2 & \(63.9 \%\) & \(36.1 \%\) & 27.9 \\
3 & \(38.5 \%\) & \(61.5 \%\) & -23.0 \\
4 & \(67.1 \%\) & \(32.9 \%\) & 34.1 \\
5 & \(34.9 \%\) & \(65.1 \%\) & -30.2 \\
6 & \(61.0 \%\) & \(39.0 \%\) & 22.0 \\
7 & \(42.7 \%\) & \(57.3 \%\) & -14.5 \\
8 & \(31.8 \%\) & \(68.2 \%\) & -36.3 \\
9 & \(48.3 \%\) & \(51.7 \%\) & -3.5 \\
10 & \(32.7 \%\) & \(67.3 \%\) & -34.6 \\
11 & \(44.9 \%\) & \(55.1 \%\) & -10.2 \\
12 & \(68.6 \%\) & \(31.4 \%\) & 37.2 \\
13 & \(52.5 \%\) & \(47.5 \%\) & 5.1 \\
14 & \(50.5 \%\) & \(49.5 \%\) & 1.0
\end{tabular}
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|r|}{DISTRICT RATINGS} \\
\hline 7 & D favored \\
\hline 7 & R favored \\
\hline 3 & competitive within 7 points \\
\hline \multicolumn{2}{|r|}{PROBABILISTIC OUTCOME} \\
\hline 6.7 & Expected D wins \\
\hline 7.3 & Expected R wins \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline \multicolumn{2}{c|}{ LOPSIDED WINS } \\
59.7 & Average D win voteshare (\%) \\
60.9 & Average \(R\) win voteshare (\%) \\
-1.1 & D advantage \\
\hline
\end{tabular}
-1.1 average margin
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|r|}{PARTISAN ASYMMETRY} \\
\hline 6.9 & D seats in 50-50 election \\
\hline 7.1 & R seats in 50-50 election \\
\hline 0.2 & \(R\) seat advantage \\
\hline 0.8 & average R seat advantage \\
\hline \multicolumn{2}{|r|}{MEAN-MEDIAN DIFFERENCE} \\
\hline -1.14\% & mean D-R (margin \%) \\
\hline -1.23\% & median D-R (margin \%) \\
\hline 0.05\% & R advantage (vote \%) \\
\hline \multicolumn{2}{|r|}{DECLINATION} \\
\hline -3.1 & D advantage \\
\hline \multicolumn{2}{|r|}{EFFICIENCY GAP} \\
\hline 1.1\% & R advantage (averaged) \\
\hline
\end{tabular}

Exhibit 5: Fairness dashboard for the NCLCV plaintiffs' Congressional plan.

\section*{NCLCV PLAINTIFFS' CONGRESSIONAL PLAN}
\begin{tabular}{|c|c|c|c|} 
District & D share & R share & Margin (\%) \\
1 & \(46.2 \%\) & \(53.8 \%\) & -7.7 \\
2 & \(51.9 \%\) & \(48.1 \%\) & 3.7 \\
3 & \(40.3 \%\) & \(59.7 \%\) & -19.3 \\
4 & \(51.8 \%\) & \(48.2 \%\) & 3.6 \\
5 & \(65.0 \%\) & \(35.0 \%\) & 29.9 \\
6 & \(64.0 \%\) & \(36.0 \%\) & 28.0 \\
7 & \(50.5 \%\) & \(49.5 \%\) & 0.9 \\
8 & \(32.3 \%\) & \(67.7 \%\) & -35.4 \\
9 & \(66.7 \%\) & \(33.3 \%\) & 33.4 \\
10 & \(52.4 \%\) & \(47.6 \%\) & 4.9 \\
11 & \(53.9 \%\) & \(46.1 \%\) & 7.7 \\
12 & \(40.4 \%\) & \(59.6 \%\) & -19.1 \\
13 & \(32.3 \%\) & \(67.7 \%\) & -35.5 \\
14 & \(44.9 \%\) & \(55.1 \%\) & -10.2 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|r|}{DISTRICT RATINGS} \\
\hline 8 & D favored \\
\hline 6 & R favored \\
\hline 4 & competitive within 7 points \\
\hline \multicolumn{2}{|r|}{PROBABILISTIC OUTCOME} \\
\hline 7.1 & Expected D wins \\
\hline 6.9 & Expected R wins \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline & LOPSIDED WINS \\
57.0 & Average D win voteshare (\%) \\
60.6 & Average R win voteshare (\%) \\
-3.6 & D advantage \\
\hline
\end{tabular}
-1.1 average margin
\begin{tabular}{|cc|}
\hline & PARTISAN ASYMMETRY \\
7.4 & D seats in \(50-50\) election \\
6.6 & R seats in \(50-50\) election \\
-0.8 & D seat advantage \\
-0.6 & average \(D\) seat advantage \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline \multicolumn{2}{|c|}{ MEAN-MEDIAN DIFFERENCE } \\
\(-1.08 \%\) & mean D-R (margin \%) \\
\(2.24 \%\) & median D-R (margin \%) \\
\(-1.66 \%\) & D advantage (vote \%) \\
\hline
\end{tabular}


\section*{V. EVALUATION OF STATE SENATE REMEDIAL PLANS}

\section*{A. The Legislative Defendants' remedial plan}

A comparison of metrics for the Legislative Defendants' remedial Senate plan, as well as the Harper plaintiffs' and NCLVL plaintiffs' proposed plans, are shown in Exhibit 6. Individual dashboards for the three plans are shown in Exhibits 7, 8, and 9.

The Legislative Defendants' plan favors 22 Democrats and 28 Republicans as scored according to the 2016-2020 election composite (Exhibit 6). The range of likely outcomes is 19 to 26 Senate seats for Democrats, and 24 to 31 Senate seats for Republicans. The seat partisan asymmetry is a 2.1 -seat difference in favor of Republicans. All of the five other metrics also favor Republicans. This plan contains 7 competitive races, as defined as margins of 7 percentage points or smaller (Exhibit 7).

\section*{A. The Harper plaintiffs' and NCLCV plaintiffs' plans}

The Harper plaintiffs' plan favors 22 Democrats and 28 Republicans. The range of likely outcomes is 21 to 28 Senate seats for Democrats, and 22 to 29 Senate seats for Republicans. The seat partisan asymmetry is a 1.3-seat difference in favor of Democrats. The five other metrics are of mixed effect, showing no clear advantage. This plan contains 7 competitive races (Exhibit 8).

The NCLCV plaintiffs' plan favors 24 Democrats and 26 Republicans. The range of likely outcomes is 19 to 28 Senate seats for Democrats, and 22 to 31 Senate seats for Republicans. The seat partisan asymmetry is a 1.3-seat difference in favor of Democrats. The five other metrics are of mixed effect, showing no clear advantage. This plan contains 9 competitive races (Exhibit 9).

Exhibit 6: Comparison of state Senate plans.
\begin{tabular}{lc} 
Legislative & \begin{tabular}{c} 
Harper \\
pefendants
\end{tabular}
\end{tabular} NCLCV

Statewide Democratic vote share (2016-2020 composite)

Democratic-favored seats
22
22
24
Republican-favored seats
28
19
26
7
28
26
minimum Democratic seats
maximum Democratic seats
competitive races (margin <7 points)
21
19
28
28
7
9

Asymmetry measures
(positive = Republican advantage):
Seat partisan asymmetry
2.1 seats \(\quad 1.3\) seats \(\quad 1.3\) seats

Technical metrics:
\begin{tabular}{|c|ccc|}
\hline Mean-median difference & \(0.8 \%\) & \(-0.1 \%\) & \(-0.1 \%\) \\
\hline Partisan bias & \(4.2 \%\) & \(1.1 \%\) & \(1.6 \%\) \\
\hline Lopsided wins difference & \(4.0 \%\) & \(3.5 \%\) & \(0.0 \%\) \\
\hline Declination angle & \(11.4^{\circ}\) & \(11.1^{\circ}\) & \(2.3^{\circ}\) \\
Efficiency gap & \(2.2 \%\) & \(-0.9 \%\) & \(-0.9 \%\) \\
\hline
\end{tabular}

Color key:
Democratic advantage

Republican advantage

Exhibit 7: Fairness dashboard for the Legislative Defendants' state Senate remedial plan.

\section*{LEGISLATIVE DEFENDANTS' SENATE PLAN}


Exhibit 8: Fairness dashboard for the Harper plaintiffs' state Senate plan.
HARPER PLAINTIFFS' SENATE PLAN


Exhibit 9: Fairness dashboard for the NCLCV plaintiffs' state Senate plan.
NCLCV PLAINTIFFS' SENATE PLAN
\begin{tabular}{|c|c|c|c|}
\hline District & D share & R share & Margin (\%) \\
\hline 1 & 53.5\% & 46.5\% & 7.1 \\
\hline 2 & 38.7\% & 61.3\% & -22.5 \\
\hline 3 & 42.2\% & 57.8\% & -15.6 \\
\hline 4 & 48.0\% & 52.0\% & -3.9 \\
\hline 5 & 57.2\% & 42.8\% & 14.4 \\
\hline 6 & 34.6\% & 65.4\% & -30.9 \\
\hline 7 & 51.9\% & 48.1\% & 3.8 \\
\hline 8 & 38.4\% & 61.6\% & -23.3 \\
\hline 9 & 40.1\% & 59.9\% & -19.8 \\
\hline 10 & 37.8\% & 62.2\% & -24.3 \\
\hline 11 & 51.2\% & 48.8\% & 2.4 \\
\hline 12 & 40.6\% & 59.4\% & -18.9 \\
\hline 13 & 51.0\% & 49.0\% & 1.9 \\
\hline 14 & 73.1\% & 26.9\% & 46.1 \\
\hline 15 & 65.3\% & 34.7\% & 30.5 \\
\hline 16 & 64.5\% & 35.5\% & 29.0 \\
\hline 17 & 52.0\% & 48.0\% & 3.9 \\
\hline 18 & 65.5\% & 34.5\% & 31.1 \\
\hline 19 & 66.5\% & 33.5\% & 32.9 \\
\hline 20 & 72.0\% & 28.0\% & 44.0 \\
\hline 21 & 39.9\% & 60.1\% & -20.2 \\
\hline 22 & 79.6\% & 20.4\% & 59.2 \\
\hline 23 & 66.1\% & 33.9\% & 32.1 \\
\hline 24 & 49.5\% & 50.5\% & -1.0 \\
\hline 25 & 40.5\% & 59.5\% & -19.1 \\
\hline 26 & 51.4\% & 48.6\% & 2.8 \\
\hline 27 & 59.0\% & 41.0\% & 17.9 \\
\hline 28 & 62.6\% & 37.4\% & 25.2 \\
\hline 29 & 33.4\% & 66.6\% & -33.2 \\
\hline 30 & 27.3\% & 72.7\% & -45.4 \\
\hline 31 & 49.3\% & 50.7\% & -1.4 \\
\hline 32 & 57.6\% & 42.4\% & 15.1 \\
\hline 33 & 30.7\% & 69.3\% & -38.6 \\
\hline 34 & 44.8\% & 55.2\% & -10.3 \\
\hline 35 & 37.0\% & 63.0\% & -25.9 \\
\hline 36 & 24.1\% & 75.9\% & -51.8 \\
\hline 37 & 36.6\% & 63.4\% & -26.9 \\
\hline 38 & 66.6\% & 33.4\% & 33.1 \\
\hline 39 & 73.1\% & 26.9\% & 46.3 \\
\hline 40 & 72.5\% & 27.5\% & 45.0 \\
\hline 41 & 54.4\% & 45.6\% & 8.9 \\
\hline 42 & 68.8\% & 31.2\% & 37.6 \\
\hline 43 & 38.2\% & 61.8\% & -23.7 \\
\hline 44 & 31.1\% & 68.9\% & -37.8 \\
\hline 45 & 29.7\% & 70.3\% & -40.6 \\
\hline 46 & 28.5\% & 71.5\% & -42.9 \\
\hline 47 & 36.8\% & 63.2\% & -26.3 \\
\hline 48 & 49.7\% & 50.3\% & -0.7 \\
\hline 49 & 56.6\% & 43.4\% & 13.3 \\
\hline 50 & 36.4\% & 63.6\% & -27.2 \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline & DISTRICT RATINGS \\
24 & D favored \\
26 & R favored \\
9 & competitive within 7 points \\
\multicolumn{2}{|c|}{ PROBABILISTIC OUTCOME } \\
24.1 & Expected D wins \\
25.9 & Expected R wins \\
\hline
\end{tabular}
\begin{tabular}{|cc|}
\hline \multicolumn{2}{|c|}{ LOPSIDED WINS } \\
62.2 & Average D win voteshare (\%) \\
62.2 & Average R win voteshare (\%) \\
0.0 & R advantage \\
\hline
\end{tabular}


Exhibit 10: Comparison of state House plans.
\begin{tabular}{ccc} 
& \begin{tabular}{c} 
Legislative \\
Defendants \\
Democratic vote share (two-party)
\end{tabular} & NCLCV \\
Democratic-majority seats & \(59.3 \%\) & \(49.3 \%\) \\
Republican-majority seats & 63 & 58 \\
minimum Democratic seats & 47 & 62 \\
maximum Democratic seats & 64 & 51 \\
Seat partisan asymmetry & 7.2 & 65 \\
Mean-median difference & \(0.9 \%\) & 4.1 \\
Partisan bias & \(2.7 \%\) & \(1.1 \%\) \\
Lopsided wins difference & \(7.1 \%\) & \(3.5 \%\) \\
Declination angle (degrees) & 4.5 & 2.7 \\
Efficiency gap & \(3.0 \%\) & \(1.6 \%\) \\
\hline Color key: & & \\
Democratic advantage & Republican advantage
\end{tabular}

Exhibit 11: Fairness dashboard for the Legislative defendants' state House plan.
LEGISLATIVE DEFENDANTS' REMEDIAL HOUSE PLAN
\begin{tabular}{|c|c|c|c|c|}
\hline District & D \% & R \% & Margin (\%) & DISTRICT RATINGS \\
\hline 1 & 38.0\% & 62.0\% & -23.9 & 57 D favored \\
\hline 2 & 55.9\% & 44.1\% & 11.8 & 63 R favored \\
\hline 3 & 40.4\% & 59.6\% & -19.2 & 17 competitive within 7 points \\
\hline 4 & 40.6\% & 59.4\% & -18.7 & \\
\hline 5 & 51.1\% & 48.9\% & 2.2 & PROBABILISTIC OUTCOME \\
\hline 6 & 43.4\% & 56.6\% & -13.2 & 55.9 Expected D wins \\
\hline 7 & 43.8\% & 56.2\% & -12.4 & 64.1 Expected R wins \\
\hline 8 & 58.2\% & 41.8\% & 16.3 & \\
\hline 9 & 51.4\% & 48.6\% & 2.8 & \\
\hline 10 & 46.2\% & 53.8\% & -7.7 & LOPSIDED WINS \\
\hline 11 & 66.1\% & 33.9\% & 32.2 & 64.0 Average D win voteshare (\%) \\
\hline 12 & 47.3\% & 52.7\% & -5.4 & 63.2 Average R win voteshare (\%) \\
\hline 13 & 30.7\% & 69.3\% & -38.6 & \(0.8 \quad \mathrm{R}\) advantage (\%) \\
\hline 14 & 37.1\% & 62.9\% & -25.7 & \\
\hline 15 & 35.8\% & 64.2\% & -28.4 & PARTISAN ASYMMETRY \\
\hline 16 & 33.0\% & 67.0\% & -34.0 & 56.7 D seats in 50-50 election \\
\hline 17 & 39.1\% & 60.9\% & -21.7 & 63.3 \(\quad R\) seats in 50-50 election \\
\hline 18 & 55.1\% & 44.9\% & 10.2 & 4.0 R seat advantage \\
\hline 19 & 38.9\% & 61.1\% & -22.1 & 7.2 average \(R\) seat advantage \\
\hline 20 & 49.5\% & 50.5\% & -1.1 & \\
\hline 21 & 66.7\% & 33.3\% & 33.4 & MEAN-MEDIAN DIFFERENCE \\
\hline 22 & 42.2\% & 57.8\% & -15.7 & -0.6\% mean D-R margin (\%) \\
\hline 23 & 60.7\% & 39.3\% & 21.4 & -2.4\% median D-R margin (\%) \\
\hline 24 & 51.8\% & 48.2\% & 3.6 & 0.9\% R advantage \\
\hline 25 & 52.0\% & 48.0\% & 4.0 & \\
\hline 26 & 42.2\% & 57.8\% & -15.6 & DECLINATION ANGLE (degrees) \\
\hline 27 & 63.1\% & 36.9\% & 26.2 & 4.5 R advantage \\
\hline 28 & 33.8\% & 66.2\% & -32.4 & \\
\hline 29 & 83.8\% & 16.2\% & 67.5 & EFFICIENCY GAP \\
\hline 30 & 85.6\% & 14.4\% & 71.2 & 3.1\% \(\quad \mathrm{R}\) advantage (averaged) \\
\hline 31 & 81.2\% & 18.8\% & 62.4 & \\
\hline 32 & 57.3\% & 42.7\% & 14.6 & \\
\hline 33 & 62.8\% & 37.2\% & 25.6 & \\
\hline 34 & 62.0\% & 38.0\% & 24.1 & \\
\hline 35 & 51.1\% & 48.9\% & 2.2 & \\
\hline 36 & 56.2\% & 43.8\% & 12.4 & \\
\hline 37 & 47.3\% & 52.7\% & -5.4 & \\
\hline 38 & 85.4\% & 14.6\% & 70.9 & \\
\hline 39 & 61.2\% & 38.8\% & 22.4 & \\
\hline 40 & 54.7\% & 45.3\% & 9.4 & \\
\hline 41 & 63.1\% & 36.9\% & 26.3 & \\
\hline 42 & 71.2\% & 28.8\% & 42.3 & \\
\hline 43 & 49.6\% & 50.4\% & -0.8 & \\
\hline
\end{tabular}
- App. 151 -
\begin{tabular}{|c|c|c|c|}
\hline 44 & 62.3\% & 37.7\% & 24.6 \\
\hline 45 & 57.4\% & 42.6\% & 14.8 \\
\hline 46 & 37.9\% & 62.1\% & -24.1 \\
\hline 47 & 49.1\% & 50.9\% & -1.9 \\
\hline 48 & 54.6\% & 45.4\% & 9.2 \\
\hline 49 & 67.7\% & 32.3\% & 35.3 \\
\hline 50 & 57.5\% & 42.5\% & 15.1 \\
\hline 51 & 40.7\% & 59.3\% & -18.6 \\
\hline 52 & 43.5\% & 56.5\% & -13.0 \\
\hline 53 & 35.2\% & 64.8\% & -29.6 \\
\hline 54 & 53.8\% & 46.2\% & 7.6 \\
\hline 55 & 41.6\% & 58.4\% & -16.7 \\
\hline 56 & 85.9\% & 14.1\% & 71.8 \\
\hline 57 & 57.6\% & 42.4\% & 15.2 \\
\hline 58 & 73.0\% & 27.0\% & 46.0 \\
\hline 59 & 50.3\% & 49.7\% & 0.7 \\
\hline 60 & 62.3\% & 37.7\% & 24.6 \\
\hline 61 & 80.7\% & 19.3\% & 61.4 \\
\hline 62 & 50.3\% & 49.7\% & 0.6 \\
\hline 63 & 52.2\% & 47.8\% & 4.5 \\
\hline 64 & 40.5\% & 59.5\% & -18.9 \\
\hline 65 & 35.8\% & 64.2\% & -28.5 \\
\hline 66 & 70.7\% & 29.3\% & 41.4 \\
\hline 67 & 28.9\% & 71.1\% & -42.3 \\
\hline 68 & 38.3\% & 61.7\% & -23.5 \\
\hline 69 & 34.8\% & 65.2\% & -30.4 \\
\hline 70 & 24.6\% & 75.4\% & -50.9 \\
\hline 71 & 71.0\% & 29.0\% & 42.1 \\
\hline 72 & 75.2\% & 24.8\% & 50.5 \\
\hline 73 & 50.9\% & 49.1\% & 1.9 \\
\hline 74 & 47.5\% & 52.5\% & -4.9 \\
\hline 75 & 44.4\% & 55.6\% & -11.1 \\
\hline 76 & 39.2\% & 60.8\% & -21.6 \\
\hline 77 & 24.7\% & 75.3\% & -50.6 \\
\hline 78 & 26.3\% & 73.7\% & -47.5 \\
\hline 79 & 38.7\% & 61.3\% & -22.6 \\
\hline 80 & 25.6\% & 74.4\% & -48.8 \\
\hline 81 & 28.9\% & 71.1\% & -42.2 \\
\hline 82 & 46.2\% & 53.8\% & -7.6 \\
\hline 83 & 25.9\% & 74.1\% & -48.2 \\
\hline 84 & 33.9\% & 66.1\% & -32.2 \\
\hline 85 & 27.6\% & 72.4\% & -44.8 \\
\hline 86 & 31.5\% & 68.5\% & -37.0 \\
\hline 87 & 27.2\% & 72.8\% & -45.5 \\
\hline 88 & 69.5\% & 30.5\% & 39.0 \\
\hline 89 & 26.0\% & 74.0\% & -48.0 \\
\hline 90 & 24.9\% & 75.1\% & -50.1 \\
\hline
\end{tabular}
- App. 152 -
\begin{tabular}{|c|c|c|c|}
\hline 91 & \(30.9 \%\) & \(69.1 \%\) & -38.3 \\
\hline 92 & \(69.3 \%\) & \(30.7 \%\) & 38.6 \\
\hline 93 & \(42.9 \%\) & \(57.1 \%\) & -14.2 \\
94 & \(24.0 \%\) & \(76.0 \%\) & -52.1 \\
95 & \(34.5 \%\) & \(65.5 \%\) & -30.9 \\
96 & \(37.1 \%\) & \(62.9 \%\) & -25.7 \\
97 & \(27.5 \%\) & \(72.5 \%\) & -45.0 \\
98 & \(48.6 \%\) & \(51.4 \%\) & -2.8 \\
99 & \(84.1 \%\) & \(15.9 \%\) & 68.2 \\
100 & \(75.9 \%\) & \(24.1 \%\) & 51.8 \\
101 & \(76.6 \%\) & \(23.4 \%\) & 53.2 \\
102 & \(78.9 \%\) & \(21.1 \%\) & 57.8 \\
103 & \(51.1 \%\) & \(48.9 \%\) & 2.3 \\
104 & \(54.3 \%\) & \(45.7 \%\) & 8.6 \\
105 & \(55.9 \%\) & \(44.1 \%\) & 11.8 \\
106 & \(69.0 \%\) & \(31.0 \%\) & 38.0 \\
107 & \(82.3 \%\) & \(17.7 \%\) & 64.5 \\
\hline 108 & \(32.9 \%\) & \(67.1 \%\) & -34.3 \\
109 & \(40.8 \%\) & \(59.2 \%\) & -18.5 \\
\hline 110 & \(33.3 \%\) & \(66.7 \%\) & -33.5 \\
\hline 111 & \(31.2 \%\) & \(68.8 \%\) & -37.7 \\
\hline 112 & \(61.2 \%\) & \(38.8 \%\) & 22.5 \\
\hline 113 & \(34.3 \%\) & \(65.7 \%\) & -31.4 \\
\hline 114 & \(65.7 \%\) & \(34.3 \%\) & 31.3 \\
115 & \(55.1 \%\) & \(44.9 \%\) & 10.2 \\
116 & \(59.3 \%\) & \(40.7 \%\) & 18.6 \\
\hline 117 & \(40.4 \%\) & \(59.6 \%\) & -19.2 \\
118 & \(38.5 \%\) & \(61.5 \%\) & -22.9 \\
\hline 119 & \(43.9 \%\) & \(56.1 \%\) & -12.1 \\
120 & \(27.0 \%\) & \(73.0 \%\) & -46.1 \\
\hline & & & \\
\hline 10
\end{tabular}

Exhibit 12: Fairness dashboard for the NCLCV plaintiffs' state House plan. NCLCV PLAINTIFFS' HOUSE PLAN

- App. 154 -
\begin{tabular}{|c|c|c|c|}
\hline 44 & 72.4\% & 27.6\% & 44.9 \\
\hline 45 & 60.5\% & 39.5\% & 20.9 \\
\hline 46 & 40.0\% & 60.0\% & -20.0 \\
\hline 47 & 46.8\% & 53.2\% & -6.4 \\
\hline 48 & 54.6\% & 45.4\% & 9.2 \\
\hline 49 & 65.3\% & 34.7\% & 30.5 \\
\hline 50 & 56.7\% & 43.3\% & 13.3 \\
\hline 51 & 34.9\% & 65.1\% & -30.2 \\
\hline 52 & 41.3\% & 58.7\% & -17.3 \\
\hline 53 & 33.0\% & 67.0\% & -33.9 \\
\hline 54 & 58.0\% & 42.0\% & 16.1 \\
\hline 55 & 43.0\% & 57.0\% & -14.0 \\
\hline 56 & 85.8\% & 14.2\% & 71.6 \\
\hline 57 & 65.6\% & 34.4\% & 31.3 \\
\hline 58 & 65.8\% & 34.2\% & 31.7 \\
\hline 59 & 54.7\% & 45.3\% & 9.3 \\
\hline 60 & 58.1\% & 41.9\% & 16.1 \\
\hline 61 & 80.8\% & 19.2\% & 61.7 \\
\hline 62 & 49.0\% & 51.0\% & -2.0 \\
\hline 63 & 54.2\% & 45.8\% & 8.3 \\
\hline 64 & 39.2\% & 60.8\% & -21.5 \\
\hline 65 & 35.8\% & 64.2\% & -28.5 \\
\hline 66 & 63.6\% & 36.4\% & 27.2 \\
\hline 67 & 28.9\% & 71.1\% & -42.3 \\
\hline 68 & 36.6\% & 63.4\% & -26.7 \\
\hline 69 & 35.2\% & 64.8\% & -29.6 \\
\hline 70 & 24.3\% & 75.7\% & -51.4 \\
\hline 71 & 69.7\% & 30.3\% & 39.4 \\
\hline 72 & 74.0\% & 26.0\% & 48.1 \\
\hline 73 & 44.3\% & 55.7\% & -11.4 \\
\hline 74 & 47.4\% & 52.6\% & -5.2 \\
\hline 75 & 42.6\% & 57.4\% & -14.7 \\
\hline 76 & 39.2\% & 60.8\% & -21.6 \\
\hline 77 & 24.7\% & 75.3\% & -50.6 \\
\hline 78 & 26.5\% & 73.5\% & -47.0 \\
\hline 79 & 36.0\% & 64.0\% & -28.1 \\
\hline 80 & 28.0\% & 72.0\% & -44.1 \\
\hline 81 & 26.4\% & 73.6\% & -47.1 \\
\hline 82 & 41.7\% & 58.3\% & -16.5 \\
\hline 83 & 35.2\% & 64.8\% & -29.5 \\
\hline 84 & 33.6\% & 66.4\% & -32.8 \\
\hline 85 & 27.7\% & 72.3\% & -44.7 \\
\hline 86 & 31.5\% & 68.5\% & -37.0 \\
\hline 87 & 26.6\% & 73.4\% & -46.8 \\
\hline 88 & 75.0\% & 25.0\% & 49.9 \\
\hline 89 & 27.5\% & 72.5\% & -45.0 \\
\hline 90 & 24.8\% & 75.2\% & -50.4 \\
\hline
\end{tabular}
- App. 155 -
\begin{tabular}{|c|c|c|c|}
\hline 91 & \(35.0 \%\) & \(65.0 \%\) & -30.0 \\
\hline 92 & \(69.5 \%\) & \(30.5 \%\) & 39.0 \\
\hline 93 & \(43.0 \%\) & \(57.0 \%\) & -14.0 \\
94 & \(24.1 \%\) & \(75.9 \%\) & -51.8 \\
95 & \(34.3 \%\) & \(65.7 \%\) & -31.4 \\
96 & \(36.1 \%\) & \(63.9 \%\) & -27.9 \\
97 & \(27.5 \%\) & \(72.5 \%\) & -45.0 \\
98 & \(48.2 \%\) & \(51.8 \%\) & -3.5 \\
99 & \(59.9 \%\) & \(40.1 \%\) & 19.9 \\
100 & \(69.1 \%\) & \(30.9 \%\) & 38.2 \\
101 & \(75.0 \%\) & \(25.0 \%\) & 50.0 \\
102 & \(80.5 \%\) & \(19.5 \%\) & 61.1 \\
103 & \(50.7 \%\) & \(49.3 \%\) & 1.4 \\
104 & \(56.9 \%\) & \(43.1 \%\) & 13.7 \\
105 & \(57.1 \%\) & \(42.9 \%\) & 14.2 \\
106 & \(83.1 \%\) & \(16.9 \%\) & 66.2 \\
107 & \(76.3 \%\) & \(23.7 \%\) & 52.7 \\
\hline 108 & \(32.7 \%\) & \(67.3 \%\) & -34.6 \\
109 & \(43.2 \%\) & \(56.8 \%\) & -13.6 \\
110 & \(31.5 \%\) & \(68.5 \%\) & -37.1 \\
111 & \(32.7 \%\) & \(67.3 \%\) & -34.7 \\
\hline 112 & \(75.6 \%\) & \(24.4 \%\) & 51.2 \\
\hline 113 & \(33.1 \%\) & \(66.9 \%\) & -33.8 \\
\hline 114 & \(62.5 \%\) & \(37.5 \%\) & 25.0 \\
115 & \(61.0 \%\) & \(39.0 \%\) & 21.9 \\
116 & \(56.7 \%\) & \(43.3 \%\) & 13.5 \\
\hline 117 & \(41.1 \%\) & \(58.9 \%\) & -17.8 \\
118 & \(38.5 \%\) & \(61.5 \%\) & -22.9 \\
119 & \(43.9 \%\) & \(56.1 \%\) & -12.1 \\
120 & \(27.0 \%\) & \(73.0 \%\) & -46.1 \\
\hline & & & \\
\hline
\end{tabular}
- App. 156 -

\section*{VIII. CONCLUSIONS}

All three of the Legislative Defendants' plans favor Republicans in six metrics evaluated. The Harper plaintiffs' plans show mixed or no advantage for either party. The NCLCV plaintiffs' plans show a Democratic advantage for the Congressional plan, mixed or no advantage for the Senate plan, and a Republican advantage for the House plan. In each case, the plaintiffs' alternative(s) came closer to partisan symmetry than the Legislative Defendants' remedial maps.

To: Special Masters, North Carolina Superior Court, Wake County
From: Eric McGhee

Re: Measures of partisan fairness

Date: February 20, 2022

The Special Masters appointed by the North Carolina Superior Court of Wake County have asked me to provide my assessment of the partisan fairness of the remedial maps submitted by the parties to \(N C L C V\) and Common Cause v Hall, and Harper v Hall. I am a political scientist who studies elections, election administration, redistricting, public opinion, and legislative behavior. I am the creator of a popular measure of partisan gerrymandering called the efficiency gap, and co-creator with Nicholas Stephanopoulos of Harvard University of a legal test using the same. I am also a member of the Board of Directors of PlanScore, a nonpartisan website that scores redistricting plans on measures of partisan advantage. I have numerous published articles on the subject of partisan advantage in redistricting and am frequently consulted on the topic by policymakers and the media. I have a PhD in political science from the University of California, Berkeley.

The Special Masters have asked me to evaluate the remedial plans on four measures of partisan advantage: partisan symmetry, the mean-median difference, the efficiency gap, and the declination. In this memo I will explain the basis for each of these metrics to serve as background for the memos on the plans themselves.

All of these metrics measure some form of partisan advantage: an advantage for one party beyond what a normative concept of partisan balance would otherwise dictate. This normative baseline differs for each measure employed, meaning each captures a slightly different sense of fairness. As it happens, in a competitive state like North Carolina all the metrics tend to produce very similar results. I will describe the metrics and explain the particular sense of fairness each is meant to represent in the plainest language I can.

For those less inclined to learn the details, below is a quick summary of each measure:
- Partisan symmetry measures the excess seat share a party receives when it wins \(50 \%\) of the votes. In a fair plan it should win \(50 \%\) of the seats as well. It is measured in seat share.
- Mean-median difference measures the excess vote share required for a party to claim \(50 \%\) of the seats. In a fair plan is should only take \(50 \%\) of the vote to win half the seats. It is measured in vote share.
- Efficiency gap is the partisan balance of inefficient votes that do not contribute directly to a victory. In a fair plan the parties should have equal inefficient votes, reflecting no "packing" or "cracking." Despite using votes as its inputs it works out be a measure of seat share.
- Declination captures partisan differences in the pattern of district vote shares when plotted on a graph. In a fair plan the points will have similar patterns because they were drawn without the win/loss threshold specifically in mind. It is not measured in either vote share or seat share, so magnitudes are more difficult to compare with the other metrics.

\section*{Partisan Symmetry}

Broadly speaking, partisan symmetry is the idea that parties with equal vote shares should receive equal seat shares. It is most commonly assessed at the point when both parties have half the votes, in which case both parties should also have half the seats. If one party has more than half the seats for exactly half the votes, it has managed to claim an outright majority of the legislative power without receiving majority support from the public. This violates the fundamental principle of majority rule.

Both parties rarely have exactly half the seats, so partisan symmetry almost always imagines what the outcome might be if both parties had equal vote shares. The most typical way of calculating this counterfactual is to simulate a uniform partisan tide that shifts each district's actual vote share an equal amount, to the point where both parties end up with half the votes in the aggregate. After shifting the vote shares this way, the analyst identifies how many seats have changed hands and records the new seat share. The difference between this new seat share and 50 percent is partisan symmetry. The larger the difference between the two numbers, the more unfair the plan. This means partisan symmetry is measured in terms of seat share: the seat share above or below \(50 \%\).

As an example, suppose there are two parties: Party A and Party B. Party A has 53 percent of the vote and Party B has 47 percent, and with that outcome Party A wins 60 of 100 seats in the state legislature. Partisan symmetry requires understanding what might happen if both parties had half the votes, so Party A must lose three percent of the vote in every district-a "uniform swing"-to bring its overall vote share down from 53 percent to 50 percent (and bring Party B's vote share up three points from 47 percent to 50 percent). If Party A loses, say, four seats as a result of this counterfactual, then it will end up with 56 percent of the seats \((56 / 100)\) for 50 percent of the vote, a fairly clear violation of the principle of majority rule.

This is not the only way that partisan symmetry has been implemented. Since partisan symmetry says that
parties with equal vote shares should receive equal seat shares, it can actually be calculated for any pair of vote shares. In the example above, one might explore what happens if the party's roles were reversed, and Party A received 47 percent of the vote and Party B 53 percent. In a symmetric plan, Party A should receive 40 percent of the seats just as Party B did when it had 47 percent of the vote. If it has more than 40 percent, the plan favors Party A; if it has less than 40 percent, the plan favors Party B. This is the approach to symmetry used by plaintiffs Harper et al. and their experts.

As a practical matter this version of symmetry tends to give similar answers to the one calculated at 50 percent, but the two can diverge in any given plan. I prefer the " \(50 \%\) " version for two reasons. First, this "vote-swapping" version of symmetry necessarily requires a counterfactual twice as large as the \(50 \%\) version. In the example above, the uniform swing is 3 points for the version assessed at \(50 \%\), and 6 points ( \(53 \%\) \(47 \%\) ) for the vote-swapping version. Sometimes this makes sense, but often it can take the counterfactual into highly fanciful scenarios.

Second, the vote-swapping version has a more tenuous connection to the principle of majority rule. The \(50 \%\) version tests a normatively critical threshold: the point at which more voters support a party than support its opposition. If a party manages to acquire more power-in the form of seats in the relevant legislative body-without clearly winning more support, there is a universal sense that something is amiss.

When the parties are competitive with each other statewide, each has a vote share close to 50 percent already and the counterfactual is not very large - in fact, something close to it may have happened recently or could be expected to happen soon. This is the case with North Carolina, making partisan symmetry a useful metric for this state.

\section*{Mean-Median Difference}

The mean-median difference is just like it sounds: it is the difference between the average district vote share and the median district vote share (where half the districts have a vote share higher and half lower). The mean-median difference favors a party when its median vote share is higher than its mean vote share.

Returning to the example above, Party A has an average (mean) vote share of 53 percent. Imagine that the median vote share is 60 percent, so that Party A has a vote share higher than 60 percent in half the seats, and lower than 60 percent in the other half. The mean-median difference in this case would be -7 percent: 53 percent minus 60 percent. This would be a substantial advantage for Party A.

The mean-median difference may appear to measure something purely mathematical, but in fact it captures something very straightforward: the vote share above (below) \(50 \%\) that a party needs in order to capture
exactly half the seats. If a party needs more than 50 percent of the vote to claim half the seats, it is at a disadvantage. This means the mean-median difference is measured in terms of vote share: the vote share above or below \(50 \%\) required to win half the seats.

The mean-median difference and the \(50 \%\) version of partisan symmetry are close cousins. Partisan symmetry is the seat share above (below) \(50 \%\) that a party receives when it has half the votes; the mean-median difference is the vote share above (below) \(50 \%\) that a party needs in order to win half the seats. So the mean-median difference is a counterfactual every bit as much as partisan symmetry. Again, for a competitive state like North Carolina, the counterfactual is entirely plausible and so not an issue.

\section*{The Efficiency Gap}

Every single-member district plan has "inefficient" votes that do not contribute directly to a victory. These include votes beyond the number needed for a candidate to win, and those cast for a candidate that will definitely lose. Though these votes do not change the outcome in the race where they were actually cast, they might be enough to help a candidate of the same party win in a neighboring district instead.

The party with fewer inefficient votes than the opposition has an advantage, because its support translates more efficiently into victories. This is why a gerrymander tries to "crack" most of the opposition's supporters across many districts where victory is close but still elusive, and "pack" the remainder in a small number of districts which that party will win by large margins.

The efficiency gap captures this partisan difference in efficient votes. It is the difference between each party's total inefficient votes, divided by all the votes cast in the election. If both parties have equal inefficient votes the efficiency gap is zero and the plan is as balanced as possible.

Despite using votes as its inputs, the efficiency gap works out mathematically to be a measure of excess seat share. A balanced efficiency gap expects that for every additional one percent of the vote beyond \(50 \%\), a party should get an extra two percent of the seats. This is the "winner's bonus" so common to singlemember district electoral systems, where simple proportionality between seats and votes is extremely rare. The efficiency gap says a party can receive a higher seat share than vote share and still have a fair result, but there is a limit to how high that gap can go. This means the efficiency gap is measured in terms of seat share: the seat share above or below the ideal implied by balanced inefficient votes.

Though it comes at the problem from a very different direction, the efficiency gap also satisfies the symmetry principle. In fact, when both parties have half the votes, the efficiency gap says they should each have half the seats, and partisan symmetry and the efficiency gap are equal. For competitive states like North Carolina,
the two metrics tend to give very similar results, except that the efficiency gap is calculated at the actual election result instead of at a counterfactual.

\section*{The Declination}

Another way of thinking about a gerrymandering party is that it pays very close attention to the win/loss threshold when designing the districts. It is acutely conscious of which party will win each seat because it is trying to extract as many winning seats as possible.

The declination captures this idea through some clever geometry. Imagine plotting a plan's districts in order from lowest to highest vote share for the minority party (left to right). The vertical axis is the vote share in each district. The districts are ordered by design, so each point is always higher than all the ones to its left. In a plan drawn without regard to which party wins each seat, these points will likely just climb higher until they cross the win/loss threshold. The constellation of points above and below the win/loss threshold will look similar. Examples of such patterns are in Figure 1.


Figure 1: Fair plans are balanced around the win/loss threshold

In an unfair plan, the gerrymandering party will force as many seats on its side of the win/loss threshold as possible, leaving a few districts for the opposition to win by large margins. This is the basic logic of the efficiency gap. As an example, in Figure 2 the pattern of points on one side of the win/loss threshold is very different than on the other. \({ }^{1}\)

\footnotetext{
\({ }^{1}\) This example is an actual election from North Carolina's 2011 redistricting plan.
}


Figure 2: Unfair plans skirt the threshold in unequal ways

The declination connects the center of each party's points to a point on the win/loss threshold line. It turns out that differences between angles formed by these lines offer a way to summarize the difference between the two groups of points, and so to summarize the bias in the way the districts were drawn. However, this means the declination is measured in units other than votes or seats, making its magnitude difficult to compare to the other measures. It is best to first compare its direction to those of other measures of the same plan, and then compare its magnitude to the declination values of other plans.

Like the efficiency gap, the declination is calculated without a counterfactual, using the actual election results. The two metrics are in fact highly correlated, but the declination does not expect a particular relationship between vote share and seat share the way the efficiency gap does. The declination may best be considered a measure of intent: how much evidence is there that the pattern of election results is distorted in a way that implies an effort to win more seats for one side?

To: Special Masters, North Carolina Superior Court, Wake County
From: Eric McGhee
Re: Remedial state senate maps in NCLCV v Hall and Harper v Hall

Date: February 20, 2022

The Special Masters appointed by the North Carolina Superior Court of Wake County have asked me to provide my assessment of the partisan fairness of the remedial maps submitted by the parties to \(N C L C V \mathrm{v}\) Hall, and Harper v Hall. I am a political scientist who studies elections, election administration, redistricting, public opinion, and legislative behavior. I am the creator of a popular measure of partisan gerrymandering called the efficiency gap, and co-creator with Nicholas Stephanopoulos of Harvard University of a legal test using the same. I am also a member of the Board of Directors of PlanScore, a nonpartisan website that scores redistricting plans on measures of partisan advantage. I have numerous published articles on the subject of partisan advantage in redistricting and am frequently consulted on the topic by policymakers and the media. I have a PhD in political science from the University of California, Berkeley.

In this memo I will evaluate the remedial state senate maps against each other and against the original enacted maps that were struck down. I will use four measures of partisan advantage: partisan symmetry (PS), the mean-median difference (MMD), the efficiency gap (EG), and the declination (D). I will also offer some evidence of the competitiveness of each plan, the compactness of the districts, and how many counties have been split.

In an earlier memo (mcghee_nc_remedial_metrics.pdf) I described the logic behind each of the fairness metrics, so I direct the reader to that memo for further information and I will not cover that topic here. However, it is worth mentioning that PS and MMD have special significance for this state legislative analysis. The two metrics offer a sense of how difficult it would be for a party with majority voter support to control a majority of the seats. Majority control of North Carolina's congressional delegation does not by itself dictate majority control of the U.S. House of Representatives. But majority control of state legislative seats does decide which party organizes chambers of the state legislature. This gives the PS and MMD a clearer nexus to the fair allocation of political power for state legislature in a competitive state like North Carolina, where majority support is a live issue.

I will use PlanScore to conduct the great majority of this analysis. In an earlier memo analyzing the congressional plans (mcghee_nc_remedial_congress.pdf) I offered an explanation of PlanScore's approach
to estimating partisan outcomes, so I direct the reader to that memo for most of the details on that topic. In my analysis of the congressional plans I mentioned that PlanScore also reports EG values using presidential and U.S. Senate votes, without any statistical modeling. I prefer the model predictions, but I will note how the presidential and U.S. Senate values compare for each of the plans.

\section*{The Stephenson rule}

In North Carolina redistricting, the Stephenson rule controls how many counties may be split across multiple districts. Counties are grouped together before any lines are drawn, and boundaries are chosen within each of these county groups. In this particular litigation it appears that both sides agree on the parameters of the Stephenson constraint, so the precise groupings of counties is not at issue in this case.

The Stephenson rule does not fundamentally change the partisan fairness evaluation I will conduct here. The partisan fairness metrics used in this memo can tell us which party is advantaged by a plan, but not whether that advantage can be avoided without running afoul of other legal constraints like the Stephenson rule. That question can only be answered by identifying one or more alternative plans that do successfully avoid the bias. These plans might be submitted by other parties, or they might be generated randomly by a computer through an ensemble analysis that programs the Stephenson rule into its algorithm.

It is tempting to think that the Stephenson rule requires breaking the partisan fairness evaluation into a series of separate assessments, one for each county group. It certainly breaks up line drawing this way. But the fairness metrics are always a plan-wide consideration, because the overall effect is what matters for the allocation of political power and so for fairness. To evaluate each county group separately would be akin to deciding the winner of a basketball game by counting the number of quarters won by each team: it might say something about which team played better, but would miss the main point of the game.

\section*{Partisan fairness}

Table 1 contains a comparison of PlanScore results for the original enacted plans that were struck down and each of the proposed remedial plans. The columns headed "Open" contain predictions that simulate what might happen if no incumbents ran for reelection and every seat was open. The columns headed "Incumb." place incumbents in the seats they were drawn into and treat as open any seat where the Legislative Defendants indicated the incumbent was retiring (see footnote 11 on p. 21 of the file " 22.02 .18 -

LD Memo re Remedial Maps and Related Materials.pdf"). \({ }^{1}\) The difference between these columns in each case is the effect of incumbency on the outcome.

The PlanScore pages for these results can be found at each of the links below:

\section*{- Enacted}
- Open
- Incumbent

\section*{- Legislative Defendants}
- Open
- Incumbent

\section*{- NCLCV}
- Open
- Incumbent

\section*{- Harper}
- Open
- Incumbent

The metrics are on different scales (see the memo on metrics) so the best way to understand the values is to compare the plans to each other on the same metric, to compare the direction of the bias across different measures of the same plan, and to see how likely those directions are to persist over the life of the plan. To facilitate this last evaluation, I have added an asterisk \((*)\) to those values that are likely to favor the same party over all five elections of the plan according to the model.

The original enacted plan is the most biased of the ones considered here, with similar Republican advantages when every seat is open (EG: \(7.0 \%\); MMD: \(3.6 \%\); PS: \(7.3 \%\); D: 0.30 ) and when incumbents are running (EG: \(6.9 \%\); MMD: \(4.3 \%\); PS: \(7.5 \%\); D: 0.32 ). These advantages would all be highly likely to favor Republicans throughout the decade. The MMD and PS values suggest the Democrats would find it difficult to win a majority of the seats without an extraordinary majority of popular support.

\footnotetext{
\({ }^{1}\) A note is in order on "double bunking," where more than one incumbent has a residence in the same district. When at least one Republican and one Democrat were double bunked, I treated these incumbency effects as offsetting, making the district functionally open for the purposes of the analysis. This occurred in District 37 in the Legislative Defendants' plan and District 21 in the Harper plaintiffs' plan. When only incumbents of the same party were double bunked, I treated such districts as having one incumbent of the doubled-bunked party. Every submission had at least one doubled-bunked district of this type.
}

Table 1: Legislative Defendants' plan is fairer than enacted plans but not plaintiff submissions
\begin{tabular}{llcccccccc}
\hline \hline & \multicolumn{2}{c}{ Efficiency Gap } & \multicolumn{2}{c}{ Mean-Median Diff. } & Symmetry & Declination \\
\hline & Open & Incumb. & Open & Incumb. & Open & Incumb. & Open & Incumb. \\
\hline Enacted & \(7.0 R^{*}\) & \(6.9 R^{*}\) & \(3.6 R^{*}\) & \(4.3 R^{*}\) & \(7.3 R^{*}\) & \(7.5 R^{*}\) & \(0.30 R^{*}\) & \(0.32 R^{*}\) \\
Legislative & \(4.8 R^{*}\) & \(4.5 R^{*}\) & \(2.2 R^{*}\) & \(3.0 R^{*}\) & \(4.8 R^{*}\) & \(5.1 R^{*}\) & \(0.20 R^{*}\) & \(0.20 R^{*}\) \\
Defendants & & & & & & & & & \\
NCLCV & \(2.6 R\) & \(2.2 R\) & \(1.1 R\) & \(1.3 R\) & \(2.3 R\) & \(2.4 R\) & \(0.10 R\) & \(0.10 R\) \\
Harper & \(2.2 R\) & \(2.4 R\) & \(0.8 R\) & \(1.4 R\) & \(1.9 R\) & \(2.6 R\) & \(0.08 R\) & \(0.11 R\) \\
\hline \hline
\end{tabular}

Note: "Open" values are predictions from the PlanScore model that simulate an election where all incumbents stepped down and every seat was open. "Incumb." values assume incumbents will run in the district that contains their home residence. The districts containing the residences of the incumbents who are retiring-according to the Legislative Defendants-are treated as open in both calculations. \({ }^{*}=\) value that is more than \(50 \%\) likely to favor the same party over the course of the decade, using the uncertainty estimates from the PlanScore model.

The Legislative Defendants' remedial plan still favors Republicans when all seats are open, but somewhat less (EG: \(4.8 \%\); MMD: \(2.2 \%\); PS: \(4.8 \%\); D: 0.20 ). The EG value now clearly falls below the commonly identified threshold of \(7 \%\), though the MMD value falls well above the \(1 \%\) number cited by the Legislative Defendants (see p. 7 of their brief). The values with incumbency factored in are substantially similar (EG: 4.5\%; MMD: \(3.0 \%\); PS: \(5.1 \%\); D: 0.20 ). All the metric values for both the open seat and incumbency scenarios are more than \(50 \%\) likely to favor Republicans throughout the decade. The model-free calculations using presidential and U.S. Senate votes are very similar to the PlanScore model results for this plan.

The remaining two remedial plans in Table 1 are very similar to each other on these metrics. The values are only fractionally different within the open seats and incumbency scenarios. Like the other plans in Table 1, these also favor Republicans in all cases. However, this Republican advantage is often less than half the size of the same advantage in the Legislative Defendants' plan. Neither of the plaintiffs' submissions is more than \(50 \%\) likely to favor Republicans throughout the decade on any of the metrics. However, the MMD and PS values in the Harper plaintiffs' submission are close to \(50 \%\) in the incumbency scenario, with probabilities higher than \(40 \%\) in both cases. Once again, the model-free calculations with presidential and U.S. Senate votes are very similar to the numbers presented here.

\section*{Competition and traditional geography}

In addition to these questions of partisan fairness, it is possible to evaluate the maps in terms of competitiveness and respect for traditional geography.

A plan can favor one party but have more or fewer competitive seats. PlanScore identifies districts that are more than \(50 \%\) likely to switch party hands at least once in the five elections under the plan. As a practical matter, this works out to districts with expected two-party vote shares between about 45 and 55 percent.

For traditional geography, I look at two dimensions of the issue. The first is compactness: the extent to which the districts resemble a simple shape like a circle. I capture this concept with two different metrics: the Reock score and the Polsby-Popper score. Neither is dispositive of compactness, but they tend to capture some sense of what is meant by the concept and they are correlated with each other. I also report the total number of counties that have have been split across multiple districts, as reported by Dave's Redistricting App.

Table 2: Competition and compactness are largely similar across remedial plans
\begin{tabular}{lccccc}
\hline \hline & \multicolumn{2}{c}{ Competitive Seats } & \multicolumn{2}{c}{ Compactness } & \\
\hline & Open & Incumb. & Reock & Polsby-Popper & Split Counties \\
Enacted & 9 & 7 & 0.42 & 0.34 & 15 \\
Legislative Defendants & 10 & 7 & 0.43 & 0.38 & 15 \\
NCLCV & 11 & 9 & 0.43 & 0.37 & 15 \\
Harper & 12 & 10 & 0.41 & 0.35 & 15 \\
\hline \hline
\end{tabular}

Note: "Open" values are predictions from the PlanScore model that simulate an election where all incumbents stepped down and every seat was open. "Incumb." values assume incumbents will run in the district that contains their home residence. The districts containing the residences of the incumbents who are retiring-according to the Legislative Defendants-are treated as open in both calculations. "Competitive Seats" are those more than \(50 \%\) likely to favor the same party over the course of the decade, using the uncertainty estimates from the PlanScore model. The Reock and Polsby-Popper compactness scores both range from zero for not compact to one for maximally compact. "Split Counties" is the number of counties that have been divided into more than one district, as identified in Dave's Redistricting App.

Table 2 has the results. The enacted plan has the fewest competitive seats when all seats are open (9), followed by the Legislative Defendants' plan (10), the NCLCV plan (11), and the Harper plaintiffs' plan (12). Incumbents bring the number of competitive seats down somewhat, and there emerges a modest distinction between the number for the enacted and Legislative Defendants' plans ( 7 each), and the number
in the NCLCV (9) and Harper plaintiffs' (10) plans.

Likely reflecting the constraints of the Stephenson rule, all four plans do a reasonably good job of respecting traditional geographic principles. All four have very similar compactness on both measures considered here, and each splits 15 counties.

\section*{Conclusion}

Much like with the congressional submissions, the Legislative Defendants' remedial senate plan appears to fall in between the original enacted plan and the plaintiffs' remedial proposals on the fairness metrics. The MMD and PS metrics, which are more relevant for a state legislative plan because they connect directly to control of the chamber, suggest that in a tied election Republicans would still hold 27 or 28 seats, and that Democrats would need to win as much as 53 percent of the vote to claim 25 seats. The odds are about three to one that Republicans would maintain this advantage throughout the decade. Over the course of the last decade, Republicans managed to win 53 percent of the state senate vote once, while the most Democrats achieved was just over 50 percent.

In the plaintiffs' submissions, Republicans would win about 26 seats in a tied election, and Democrats would need about 51 percent of the vote to tie Republicans at 25 seats. The odds are about two to one or better that Republicans would lose this advantage at some point over the next decade. This suggests that there is nothing foreordained about the advantages in the Legislative Defendants' plan. The question would seem to turn instead on whether the Legislative Defendants' plan is to be preferred for other reasons.

There is far less difference between the plans in competitiveness or traditional geographic criteria. The plans are virtually identical on geography, and while the number of competitive seats is lower in the enacted and Legislative Defendants' plans, the difference is small.

STATE OF NORTH CAROLINA
IN THE GENERAL COURT OF JUSTICE
FH! [-M SUPERIOR COURT DIVISION
COUNTY OF WAKE
- \(1 . .-\omega-\omega-21\) CVS 015426,21 CVS 500085


\section*{NOTICE OF APPEAL}

TO THE HONORABLE SUPREME COURT OF NORTH CAROLINA:
Pursuant to the Supreme Court's Orders of December 8, 2021 and February 4, 2022, Plaintiffs North Carolina League of Conservation Voters, Inc., et al., in Case No. 21 CVS 015426 hereby give notice of appeal to the Supreme Court from the Order on Remedial Plans entered in this action on February 23, 2022 in Wake County Superior Court.

Dated: February 23, 2022

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\section*{CERTIFICATE OF SERVICE}

I hereby certify that the foregoing Notice of Appeal was served upon each of the parties to this action by electronic mail and by U.S. mail, first-class postage prepaid, to the following counsel:

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This 23rd day of February, 2022.
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[^0]:    general election only," that statute violates the Constitution's bans on legislative mid-decade redistricting, see N.C. Const. art. II, §§ 3(4), 5(4), and is void. The proposed amicus brief of the Governor and the Attorney General submitted below details the reasons supporting that conclusion. Amicus Br. of Governor Cooper and Att'y Gen. Stein 12-15, NCLCV v. Hall, No. 21 CVS 015426 (N.C. Super. Ct. Feb. 21, 2022), https://www.nccourts.gov/locations/wake-county/cases-of-publicinterest\#remedial.

[^1]:    ${ }^{3}$ Jordan Wilkie, Bill to Delay NC primaries Further Sharply Divides Parties, Carolina Public Press (Jan. 20, 2022), https://carolinapublicpress.org/50851 /bill-to-further-delay-nc-primaries-sharply-divides-parties; see Travis Fain \& Laura Leslie, NC Republicans Vote to Delay 2022 Primary Elections, WRAL (Jan. 19, 2022), https://www.wral.com/nc-republicans-vote-to-delay-2022-primary -elections/20086129 ("The State Board of Elections said that, if needed, any runoffs would be held Aug. 16 under the bill if any federal races require a runoff. Runoffs would be held July 26 if there aren't any federal runoffs.").

[^2]:    4 These files are available from the trial court's website at https://www.nccourts.gov/locations/wake-county/cases-of-publicinterest\#remedial.

[^3]:    Senate Clusters
    Counties

    -     - Major Interstate Highway
    $\cdots$ Major US Highway

