ADVISORY REPORT TO SPECIAL MASTERS ON PROPOSED REMEDIAL REDISTRICTING PLANS FOR NORTH CAROLINA

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1. Overview

I was asked to perform an ensemble analysis of eight proposed remedial plans: two (Congressional and Senate) from the Harper plaintiffs, and three each (Congressional, House, and Senate) from NCLCV and the legislative defendants.

Ensemble analysis consists first of constructing a large number of possible alternative plans (the ensemble). The plans are generated without using any partisan information, but in accordance with accepted criteria for redistricting in the state, including approximately equal population per district, contiguity of districts, relative compactness of districts, few boundary traversals, and so

forth. Historical election data is then used to compare election results under the proposed plans with elections results under the ensemble.

I was asked to include the following well-known metrics in my ensemble analysis: mean–median (MM), efficiency gap (EG), partisan bias (PB), and declination (D). All of these have the property that a more negative score is supposed to represent more benefit to Republicans and a more positive score is supposed to represent more benefit to Democrats. Scores closer to zero are generally expected to be less indicative of a partisan gerrymander.

But the range of possible scores also varies widely from state to state because of widely varying political geography from state to state, varying criteria for redistricting, and varying results from different elections. Although one might make a philosophical argument for why scores for a given metric that lie outside a given range should be considered evidence of a partisan gerrymander, this is an unreasonable standard if all or most of the possible scores lie outside that range. Indeed, in some cases it may not even be possible for the scores in a given state under a given set of redistricting rules to lie in that prescribed range.

Ensembles provide important context for interpreting these scores by helping to identify a typical range of score values as well as identifying outliers.

1.1. **Ensembles.** The best way to do an ensemble analysis is to generate an ensemble with a distribution of plans that specifically reflects the redistricting criteria for case under consideration. But doing that properly takes much more time than is available. Because of this I chose to use ensembles previously generated by Professor Jonathan Mattingly and his collaborators at Duke University [1]. These were generated using well-accepted MCMC methods.

My analysis is conditioned on the assumption that these ensembles are somewhat representative of the distribution of possible plans reflecting established law and intent of the court. In the case of the Congressional ensemble, I am more confident of this for the Congressional ensemble than in the case of the House and Senate ensembles. I discuss this in more detail in Sections 2.2, 3.1, and 4.

1.2. **Election Data.** For all three types of plans (congressional, senate, and house) I used historical results from the following 11 elections: the 2016 Attorney General (G16AG), Presidential (G16PR), Lieutenant Governor (G16LG) and Governor (G16AG), as well as the 2020 Attorney General (G20AG), Presidential (G20PR), Lieutenant Governor (G20LG), 2020 Governor (G16AG), Treasurer (G20TR), US Senate (G20USS), and Secretary of State (G20SST). To calculate the vote shares and other scores for the proposed plans, I used 2016 and 2020 precinct-level election results from the Voting and Election Science Team (VEST) and prorated the data to 2020 census blocks.

1.3. **Racial Considerations.** It is important to note that I have not considered racial factors or the VRA in this analysis. Incorporating those considerations may lead to other conclusions than those I have drawn here.

2. Congressional Plans Analysis

Using data from the 11 different historical elections mentioned above, I evaluated three congressional plans: one each from the Harper plaintiffs (Harper), NCLCV, and the legislative defendants (LD). I also analyzed the number of seats that would have been won under these various vote counts and the margins of victory in the most contested districts.

2.1. **Summary of Congressional Analysis.** My analysis below shows that, by all the measures I used, the LD plan favors Republicans more than the other two plans do, the NCLCV plan favors Democrats more than the other two plans, and the Harper plan lies somewhere between them. Both the LD and Harper plans are fairly typical in the ensemble distributions for all the measures I considered. The NCLCV plan, however, shows up as a significant outlier for the seat margins

for competitive seats (see Section 2.4) as well as for the mean–median and partisan bias scores. Taken together these give some evidence of partisan gerrymandering in the NCLCV congressional plan.

These conclusions do not take VRA racial considerations into account.

2.2. **Ensemble.** For analyzing the congressional plans I used the ensemble [2] (sometimes denoted the *Duke* congressional ensemble in this report). According to my reading of [4], this ensemble is generated using well-accepted Markov chain Monte Carlo methods (a parallel tempering framework using a proposal from the Multiscale Forest RECOM algorithm). Under this method specific parameters for the distribution to enforce certain requirements and to encourage certain properties of the plans chosen. According to [4] the resulting plans split no more than 14 counties and split no county into more than two districts. All districts are required to consist of one contiguous region. The deviation of the total population in any district is within 1% of the ideal district population. Districts traverse counties as few times as possible, and plans with a higher Polsby–Popper score (more compact) are more likely to be selected. This model was tuned to give similar Polsby–Popper score to the enacted congressional plan. Some have argued that tuning for a specific range of Polsby–Popper scores might skew the distribution somewhat, but in my own (unpublished) research I have explicitly checked for correlation between Polsby–Popper scores and metrics of partisan bias in ReCom MCMC and found none. I expect that this absence of correlation would hold in the other ReCom-based MCMC methods as well, including the method used to generate this ensemble. The ensemble has nearly 80,000 plans, and according to [4] the distribution seems well mixed has been sufficiently sampled to provide stable statistics. I cannot verify the mixing directly, but in my use of the ensemble, I saw no signs that the ensemble was not well mixed. Based on these I conclude that this ensemble is suitable to evaluate the Congressional plans.

2.3. **Distribution of Seats Across Elections.** Different plans perform differently under different elections. When a plan gives more seats to one party than most of the plans in the ensemble do, that can suggest a possible partisan gerrymander, especially when this occurs over several elections. To analyze this, I used histograms of seats won for the ensemble for each race, collected in Figure 1.

These histograms show that while the LD plan consistently favors Republicans and the the Harper plan consistently favors Democrats, in both cases the number of seats they give in most races is fairly typical of the ensemble distribution. The NCLCV plan also consistently favors Democrats, usually much more so than the Harper plan, and in one case (G20PR) more so than 99.7% of the ensemble, making it a significant outlier in that election.

An alternative view of the same data collected into one diagram, with histograms replaced by violin plots, is shown in Figure 2.

2.4. **Rank-Ordered Violin Plots.** The number of seats won by a plan in an election does not indicate how close the election would be. A plan that gives Democrats 51% of the vote share in their winning districts is very different from one that gives them 70% of the vote share in those districts. To analyze this effect for the proposed plans I used rank-ordered violin plots; see Figures 3 and 4. In a rank-ordered violin plot for a given election, all the congressional districts for each plan are ordered left-to-right by their Democratic vote share in the election. The numbers on the horizontal axis represent the position of the district in rank ordering (not the name given to the district in the plan). The vote share for the plans in the ensemble is represented by the gray violin-shaped distributions in each distribution, and the vote share for each plan is indicated by the corresponding colored bar.

Figure 3 shows a rank-ordered violin plot for the election G20LG, which reveals that although the NCLCV plan gives one more seat (District 8 in the figure) to the Democrats than the Harper plan, that extra seat comes by a very fine margin, with the NCLCV plan just over 50% and the Harper plan just under 50% in that district. None of the plans is a far outlier compared to the

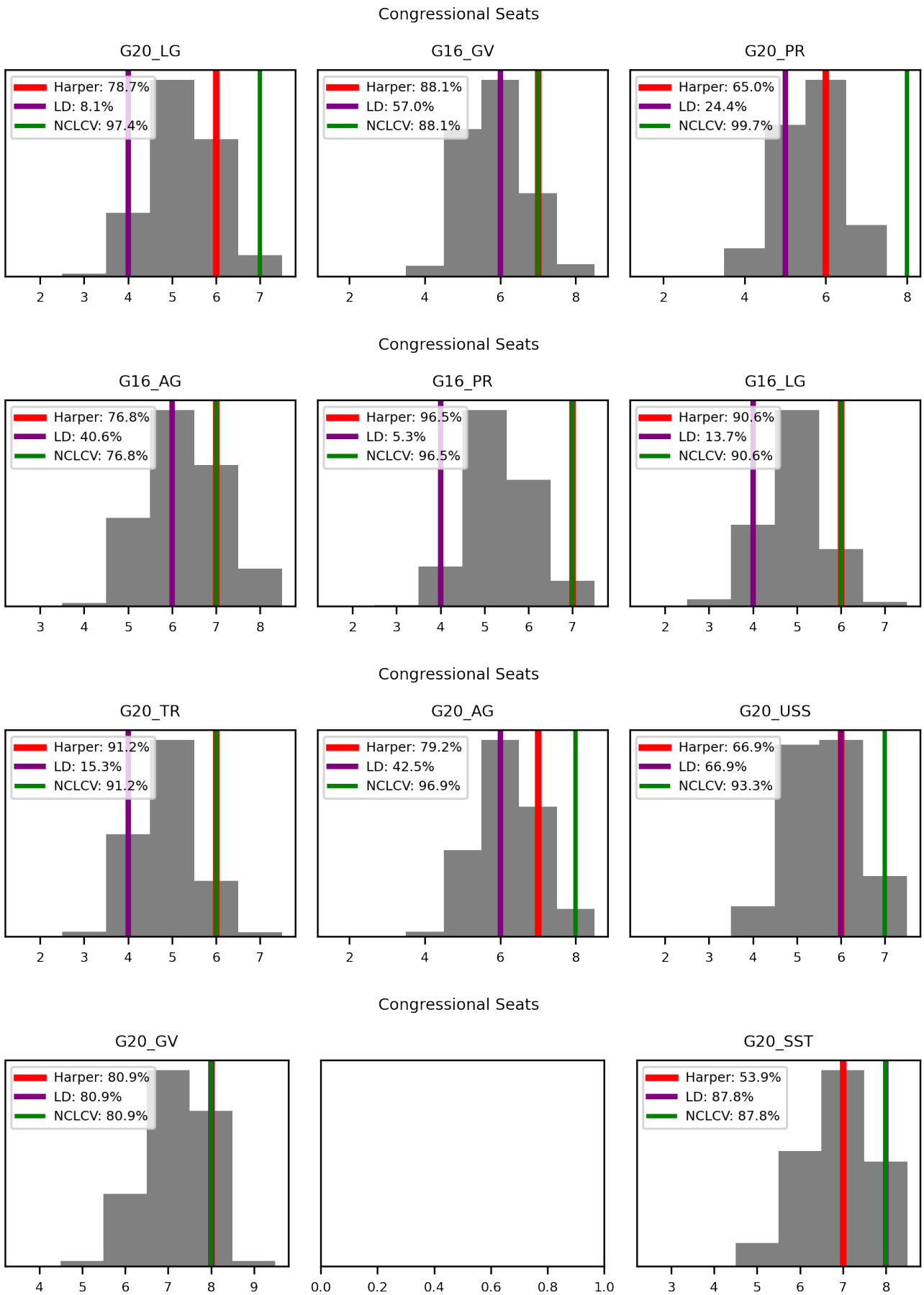

FIGURE 1. Histograms of congressional seats won in all 11 elections for the ensemble plans (gray). The proposed plans are indicated as colored vertical lines.

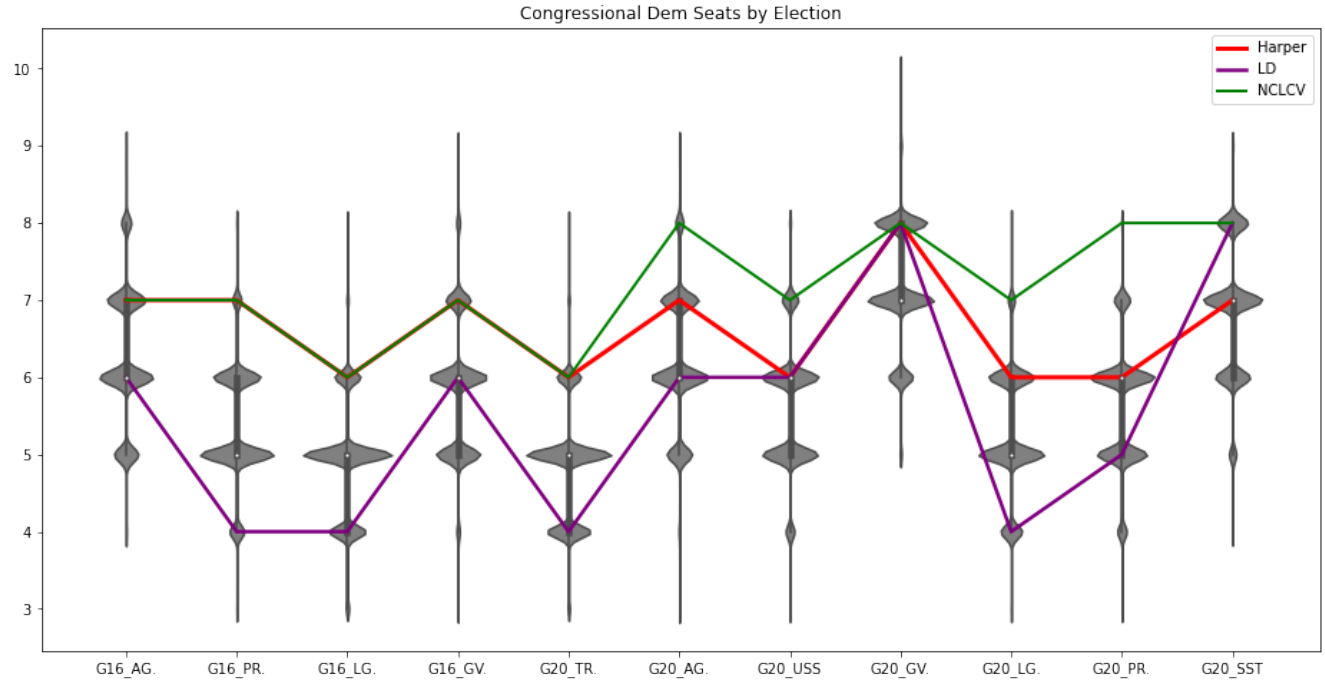

FIGURE 2. Congressional seats won across elections: shows the number of seats won (vertical axis) by Democratic candidates under each plan (colored lines) over the 11 elections (horizontal axis). The ensemble distribution of seats won for each election is indicated with the gray "violins," with wider gray regions around a point indicating more ensemble plans with the indicated number of seats won, and narrower regions indicating fewer ensemble plans with the indicated number of seats won.

ensemble in this district. This suggests that the difference in the number of seats between NCLCV and Harper in this election is not significant.

However, in this election the NCLCV plan makes District 7 much more competitive (favoring the Democrats) than either the Harper or LD plans do. Although NCLCV does not actually give the seat in District 7, NCLCV gives this district a much higher Democratic vote share than either Harper or LD and, more significantly, much higher than most of the ensemble. This makes that district very close to a win for the Democrats, without actually giving the seat to them.

Taken together, Figures 4 and 3 show that in seven of the elections (G20LG, G20GV, G20AG, G20US, G20TR, G20PR, and G16GV) the NCLCV plan places the Democratic vote share in this borderline district (7) substantially higher than most of the ensemble, which either gives the seat to the Democrats or nearly gives them the seat, by pushing the Democratic vote share close to 50%. The other plans (Harper and LD) stay in a fairly typical part of the ensemble distribution across all elections. I take this as some evidence of partisan gerrymandering in the NCLCV plan, but not in the LD and Harper plans.

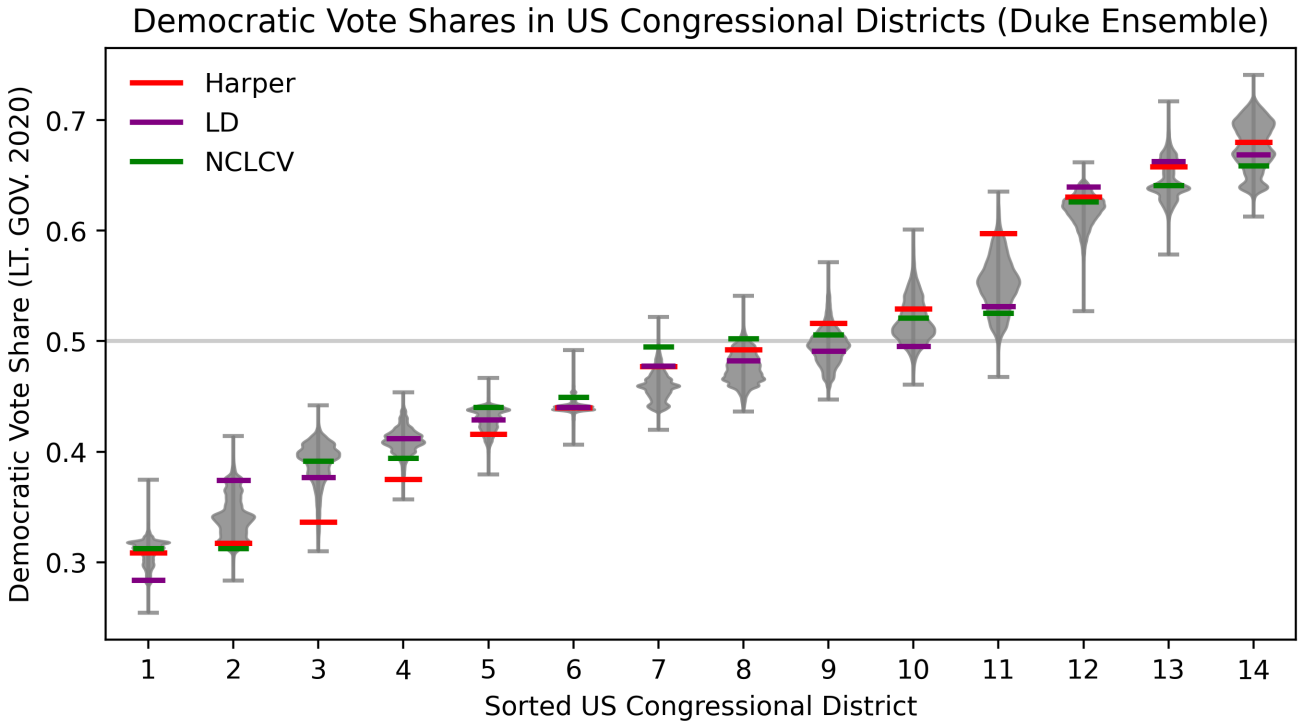

FIGURE 3. For each plan, all the congressional districts (horizontal axis) are ordered left-to-right by their Democratic vote share in the G20LG election. The numbers on the horizontal axis represent the position of the district in rank ordering (not the number given in the proposal). The vote share for the plans in the ensemble is represented by the gray violin-shaped distributions in each distribution, and the vote share for each plan is indicated by the corresponding colored bar. Points above the gray 50-percent line indicate a seat that goes to the Democrats and those below go to the Republicans.

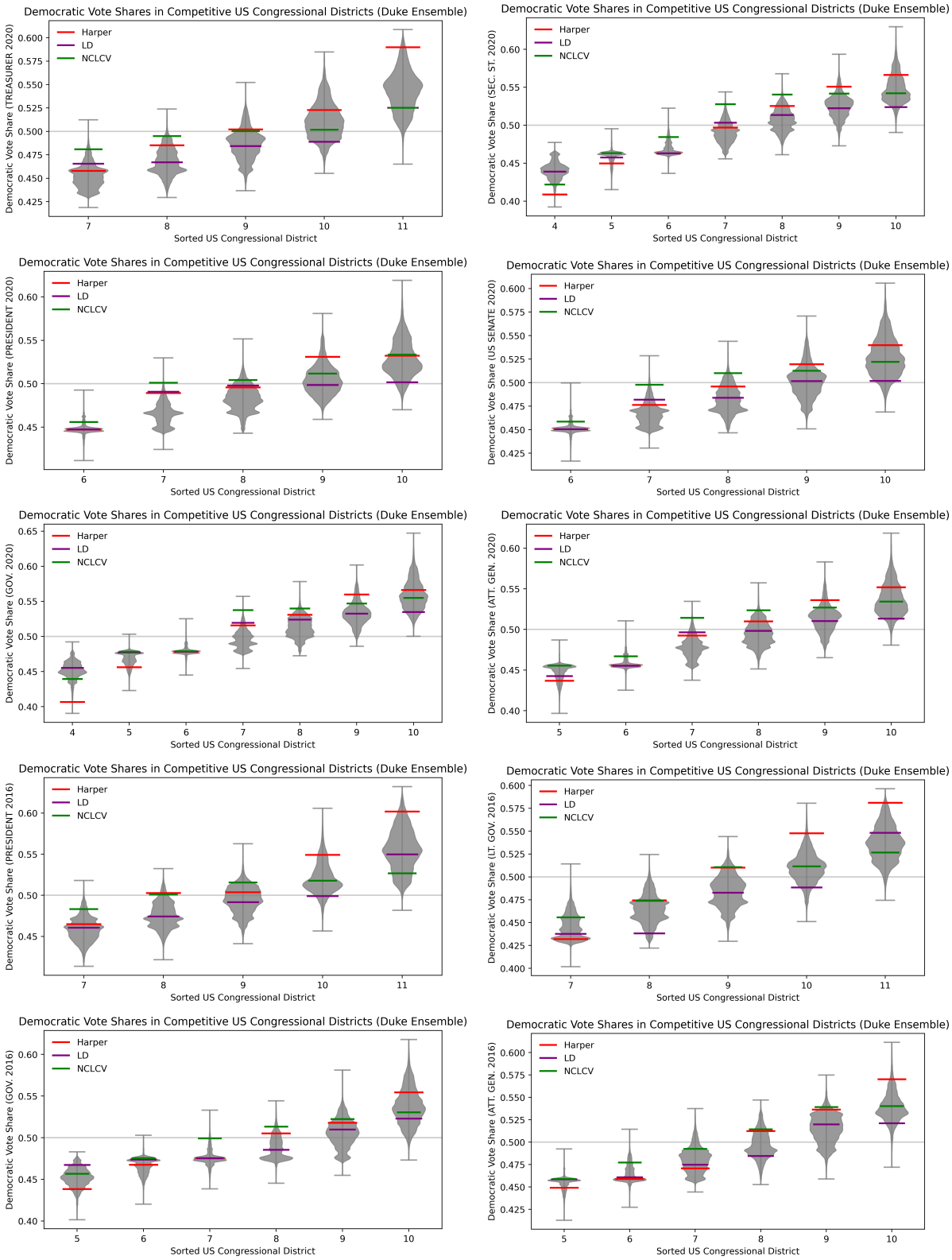

FIGURE 4. Rank-ordered congressional districts for all the elections except G20LG (shown above in Figure 3). These plots show only the most competitive districts.

2.5. **Other Metrics.** I also analyzed the plans using the mean–median score, partisan bias score, efficiency gap, and partisan declination. The first three scores are well-known and widely used. The declination is a relatively new measure proposed by Warrington. All four measures are reviewed in [5], so I will not describe them in detail here. All four of them give a single score for which a more negative score is supposed to represent more benefit to Republicans and a more positive score is supposed to represent more benefit to Democrats. Scores closer to zero are generally expected to be less indicative of a partisan gerrymander, but that depends heavily on the political geography of the state, so it is important to interpret these scores in the context of ensembles.

In the ensemble analysis below, all four scores show the LD plan favors Republicans more than the other two plans do, the NCLCV plan favors Democrats more than Harper or LD, and the Harper plan lies somewhere between them. Both the LD and Harper plans are fairly typical in the ensemble distributions for all four scores across almost all elections. The NCLCV plan, however, shows up as a significant outlier the mean–median and partisan bias scores.

2.5.1. *Mean–Median.* Table 1 shows my calculations of the mean–median scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 5 the mean–median score consistently identifies the LD plan as favoring Republicans more than the others but it is still not an outlier for the ensemble distribution. The Harper plan is is also not an outlier for the ensemble. The NCLCV plan is identified as favoring Democrats more than the others (higher scores) and is a significant outlier (greater than 99th percentile) in six of the elections.

2.5.2. *Partisan Bias.* Table 2 shows my calculations of the partisan bias scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 6, the partisan bias score also consistently identifies the LD plan as favoring Republicans more than the others but overall is more typical of the distribution than either of the other two plans. The NCLCV plan is identified as favoring Democrats more than the others and is on the very high end (over 97th percentile) of the ensemble distribution in many of the elections.

2.5.3. *Efficiency Gap.* Table 3 shows my calculations of the efficiency gap scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 7 the LD and Harper plans are mostly typical for the distribution of efficiency gap across elections. The the NCLCV plan is a significant outlier in one election (G20PR), and is somewhat high (above 90th percentile) for three other elections.

2.5.4. *Declination.* Table 4 shows my calculations of the declination scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 8 the declination only marks the NCLCV plan as a significant outlier (over 99%), but all three plans are on the outer edges (above 90% or below 10%) for some of the elections.

2.6. **Congressional Conclusion.** Both the LD and Harper plans are fairly typical in the ensemble distributions for all the measures I considered. The NCLCV plan, however, shows up as a significant outlier for the seat margins for competitive seats (see Section 2.4) as well as for the mean–median and partisan bias scores. Taken together these give evidence of partisan gerrymandering in the NCLCV congressional plan, but VRA racial considerations, which I have not considered here, might change that conclusion.

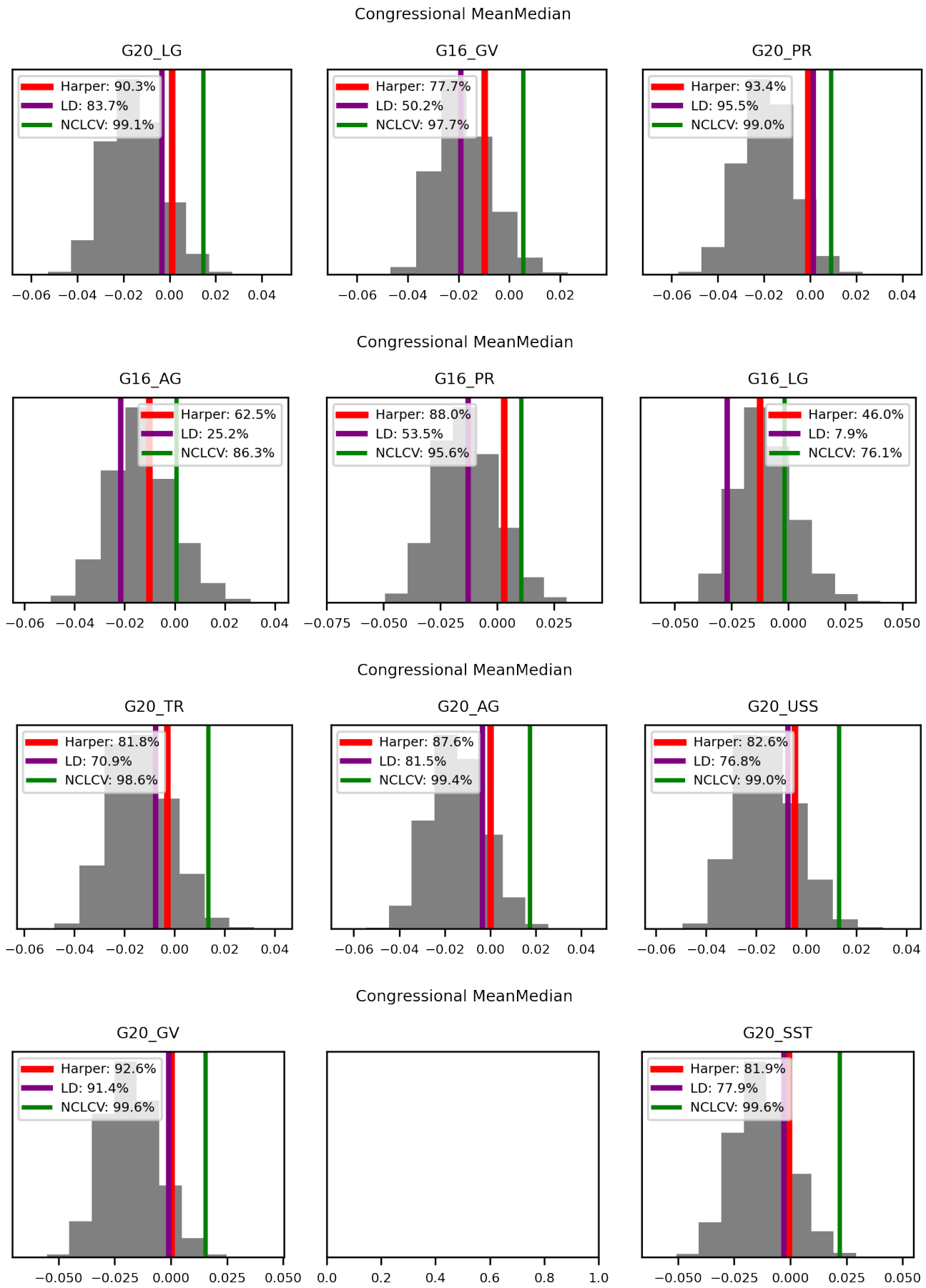

FIGURE 5. Histogram of congressional mean–median score for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

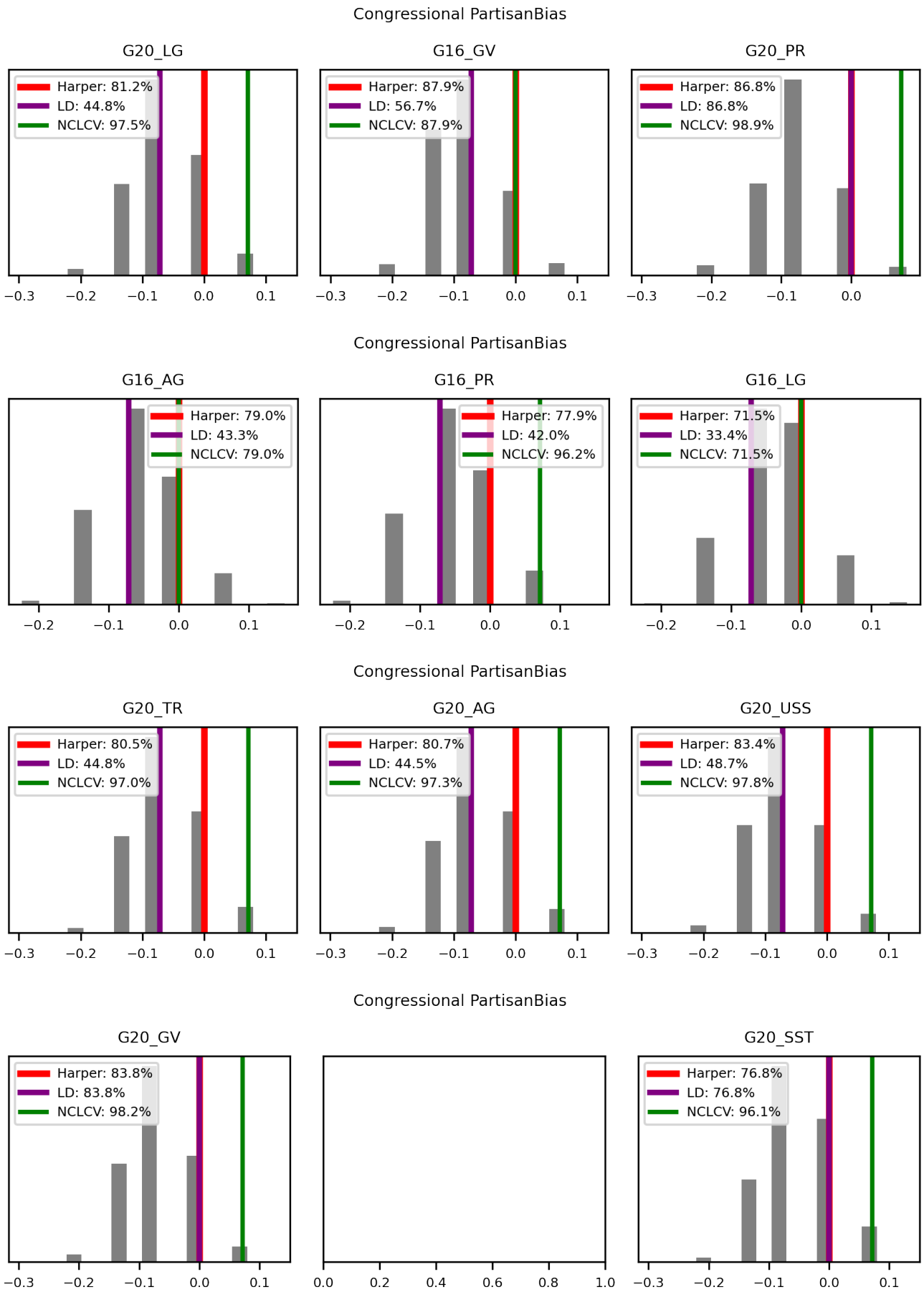

FIGURE 6. Histogram of partisan bias for all 11 elections. The numbers in the legend are the percentile in the ensemble for the corresponding plan.

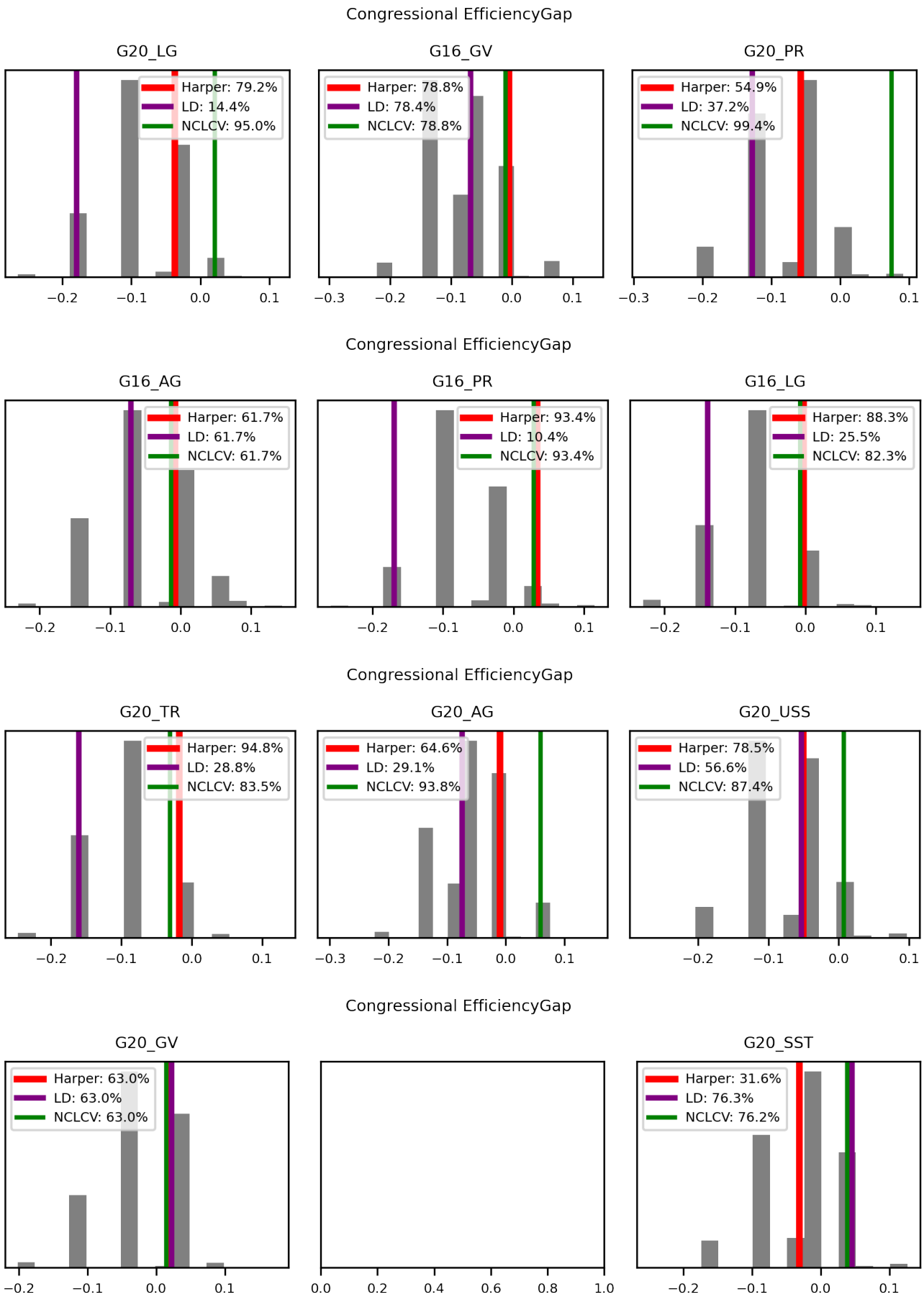

FIGURE 7. Histogram of congressional efficiency gap for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

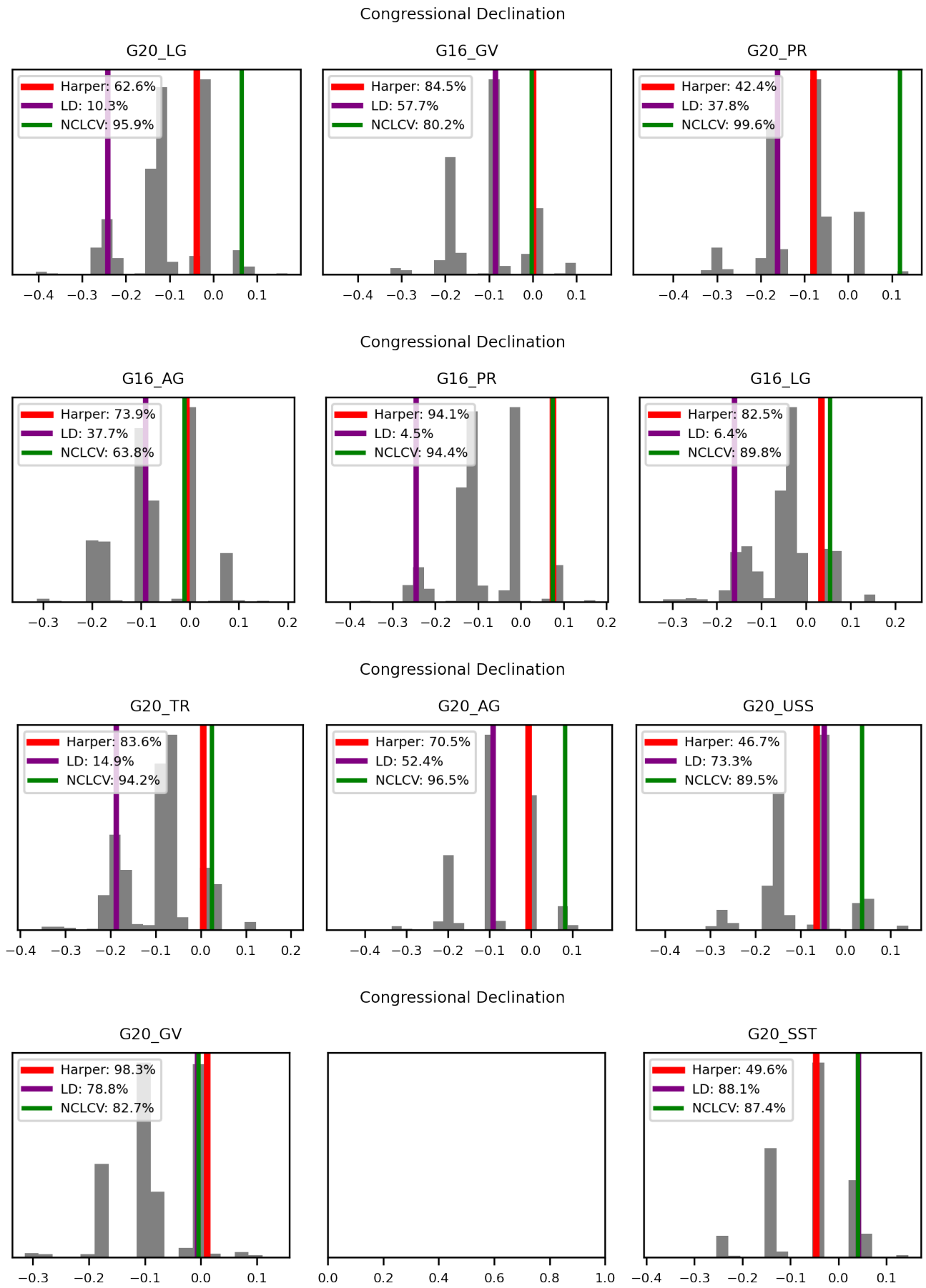

FIGURE 8. Histogram of congressional partisan declination for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

Congressional Mean–Median

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−0.1	0.1	0.9	−6.4
G20USS	−0.5	−0.7	1.3	−5.7
G20GOV	0.0	−0.1	1.5	−5.7
G20LTG	0.1	−0.3	1.5	−6.2
G20ATG	−0.0	−0.3	1.7	−6.2
G20TRE	−0.3	−0.7	1.3	−5.5
G20SOS	−0.1	−0.3	2.2	−6.1
G16PRE	0.3	−1.3	1.1	−5.3
G16GOV	−1.0	−1.9	0.6	−4.1
G16LTG	−1.3	−2.7	−0.2	−4.4
G16ATG	−1.0	−2.2	0.1	−3.8
Average	−0.3	−0.9	1.1	−5.4

TABLE 1. Mean–median scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 5)—not as isolated numbers.

Congressional Partisan Bias

Proposed_Plan	Harper	LD	NCLCV	
G20PRE	0.0	0.0	7.1	−21.4
G20USS	0.0	−7.1	7.1	−21.4
G20GOV	0.0	0.0	7.1	−21.4
G20LTG	0.0	−7.1	7.1	−21.4
G20ATG	0.0	−7.1	7.1	−21.4
G20TRE	0.0	−7.1	7.1	−21.4
G20SOS	0.0	0.0	7.1	−21.4
G16PRE	0.0	−7.1	7.1	−21.4
G16GOV	0.0	−7.1	0.0	−21.4
G16LTG	0.0	−7.1	0.0	−21.4
G16ATG	0.0	−7.1	0.0	−21.4
Average	0.0	−5.2	5.2	−21.4

TABLE 2. Partisan bias scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 6)—not as isolated numbers.

Congressional Efficiency Gap

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−5.8	−12.8	7.5	−20.1
G20USS	−5.1	−5.3	0.7	−19.5
G20GOV	1.7	2.2	1.5	−26.0
G20LTG	−3.7	−17.9	2.1	−18.1
G20ATG	−1.1	−7.4	5.9	−21.6
G20TRE	−1.8	−16.0	−3.1	−16.2
G20SOS	−3.1	4.6	3.9	−17.4
G16PRE	3.3	−16.9	2.9	−17.2
G16GOV	−0.5	−6.8	−1.0	−21.0
G16LTG	−0.3	−13.9	−0.7	−14.1
G16ATG	−0.8	−7.1	−1.3	−21.3
Average	−1.6	−8.8	1.7	−19.3

TABLE 3. Efficiency gap scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 7)—not as isolated numbers.

Congressional Declination

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−8.0	−16.2	11.8	−32.0
G20USS	−6.4	−4.7	3.7	−29.7
G20GOV	1.1	−0.6	−0.4	−41.4
G20LTG	−3.9	−24.1	6.6	−27.7
G20ATG	−0.6	−9.1	8.3	−33.8
G20TRE	0.5	−18.7	2.4	−22.3
G20SOS	−4.7	4.2	4.1	−24.6
G16PRE	7.4	−24.5	7.5	−28.3
G16GOV	−0.0	−8.5	−0.3	−32.4
G16LTG	3.3	−16.1	5.3	−19.9
G16ATG	−0.8	−9.1	−1.1	−32.6
Average	−1.1	−11.6	4.4	−29.5

TABLE 4. Partisan declination scores listed as percentages (times 100) for the proposed Congressional plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 8)—not as isolated numbers.

3. Senate Plan Analysis

I received three proposed Senate plans (LD, Harper, and NCLCV) to evaluate. I used the same methods to evaluate these plans as I did for the Congressional plans, but with a different ensemble.

3.1. **Senate Ensembles.** For analyzing the senate plans I used Dr. Mattingly's ensemble [3]. It was generated with the same method as the Congressional plan. According to my reading of [4] the resulting plans comply with the county clustering rules of *Stephenson*, maintain a population balance that deviates by no more than 5%, They are also designed to produce contiguous districts that are relatively compact and to reduce the number of counties split. This ensemble does not explicitly preserve municipalities, except as a secondary consequence of other parameter settings. This is important because municipality splits are known to have a significant interaction with partisan vote shares and measures of partisan symmetry. According to [4] the distribution seems well mixed, but I cannot verify the mixing directly.

3.2. **Seats Won.** The histograms of seats won in Figure 9 show Harper and NCLCV both are mostly typical of the ensemble, while LD is often a significant outlier in favor of the Republicans.

3.3. **Rank-Ordered Violin Plots.** As with seats won the rank-ordered violin plots show Harper and NCLCV are both mostly typical of the ensemble, while LD is often deviates in favor of the Republicans; see Figure 10.

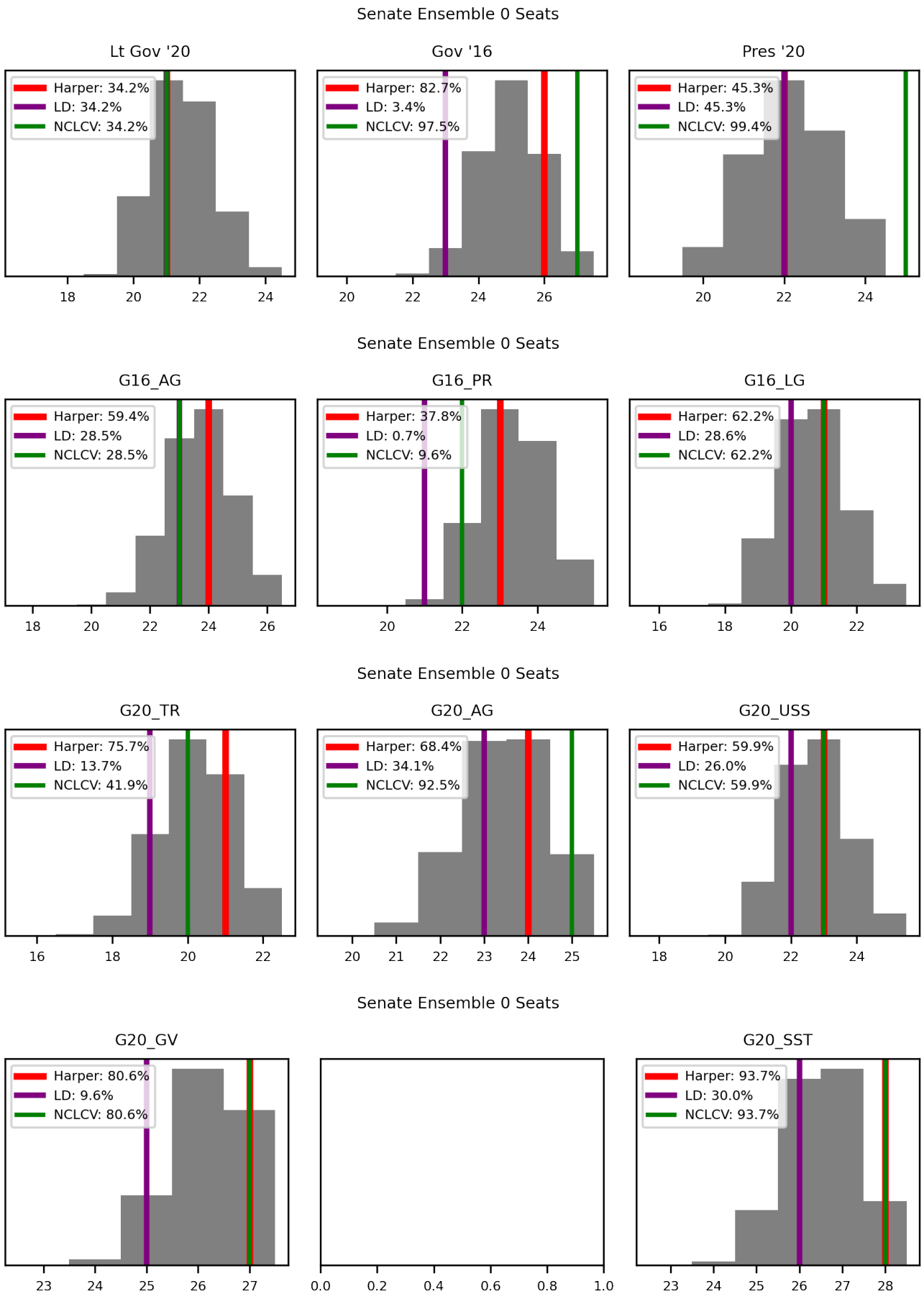

FIGURE 9. Histograms of congressional seats won in all 11 elections for the ensemble plans. The percentages in the legend represent percentile of the corresponding score in the ensemble.

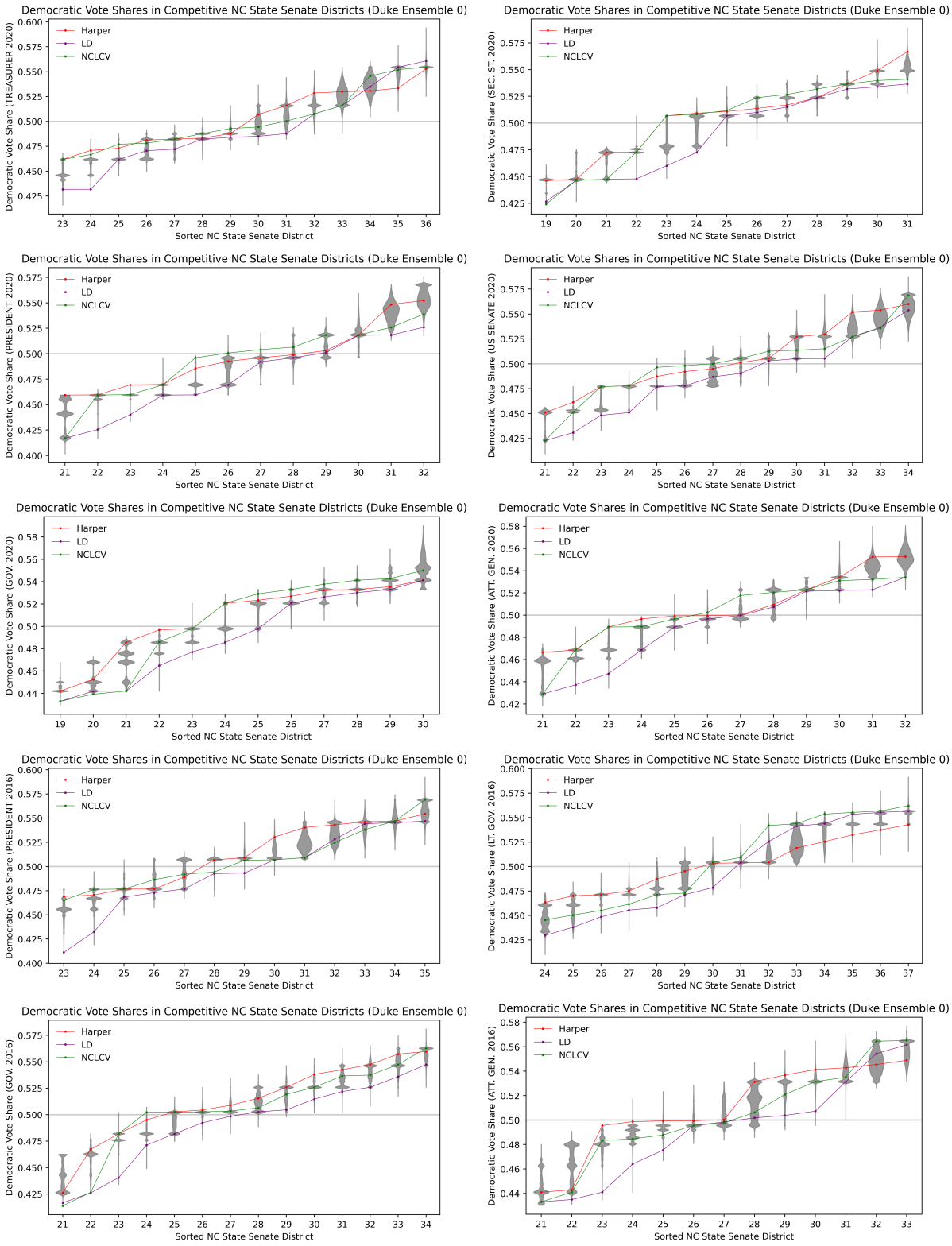

FIGURE 10. Rank-ordered senate districts for 10 of the elections (all elections but G20LG). These plots show only the most competitive districts.

3.4. **Other Metrics.**

3.4.1. *Mean–Median.* Table 5 shows my calculations of the mean–median scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 11 the mean–median score identifies the LD plan as a Republican-favoring outlier (lower than the 5th percentile) for three of the 11 elections (G20PR, G16AG, and G16LG).

The NCLCV plan is a pro-Democratic outlier (greater than 95th percentile) in four of the elections (G20PR, G20LG, G20USS, and G20GV).

The Harper plan leans toward the Democratic side of the distribution, but is not an outlier.

3.4.2. *Partisan Bias.* Table 6 shows my calculations of the partisan bias scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 12, the LD plan is a Republican-favoring outlier twice, and the Harper plan is a pro-Democratic outlier once. But the NCLCV plan stands out as a pro-Democratic outlier for partisan bias in four elections (G20PR, G20TR, G20USS, and G20GV).

3.4.3. *Efficiency Gap.* Table 7 shows my calculations of the efficiency gap scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

In Figure 13 the efficiency gap flag the NCLCV plan as a pro-Democratic outlier five times, and four of those are significant (99th percentile or greater). Harper shows up twice as Democratic outlier and LD shows up twice as a Republican outlier.

3.4.4. *Declination.* Table 8 shows my calculations of the declination scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

As shown in Figure 14 the declination marks the LD plan as a Republican outlier (below 5%) three times. The NCLCV plan shows as a Democratic outlier (over 95%) three times and Harper twice (G20GV and G20SST).

3.5. **Senate Conclusion.** The partisan symmetry scores give weak evidence of of partisan gerrymandering in the LD plan, and the seat margins in the rank-ordered violin plots give strong evidence of partisan gerrymandering in the LD plan.

The seat margins in the rank-ordered violin plots give some evidence of partisan gerrymandering in the NCLCV plan, and that is corroborated by the many outliers among the partisan symmetry scores.

These conclusions do not take VRA racial considerations into account.

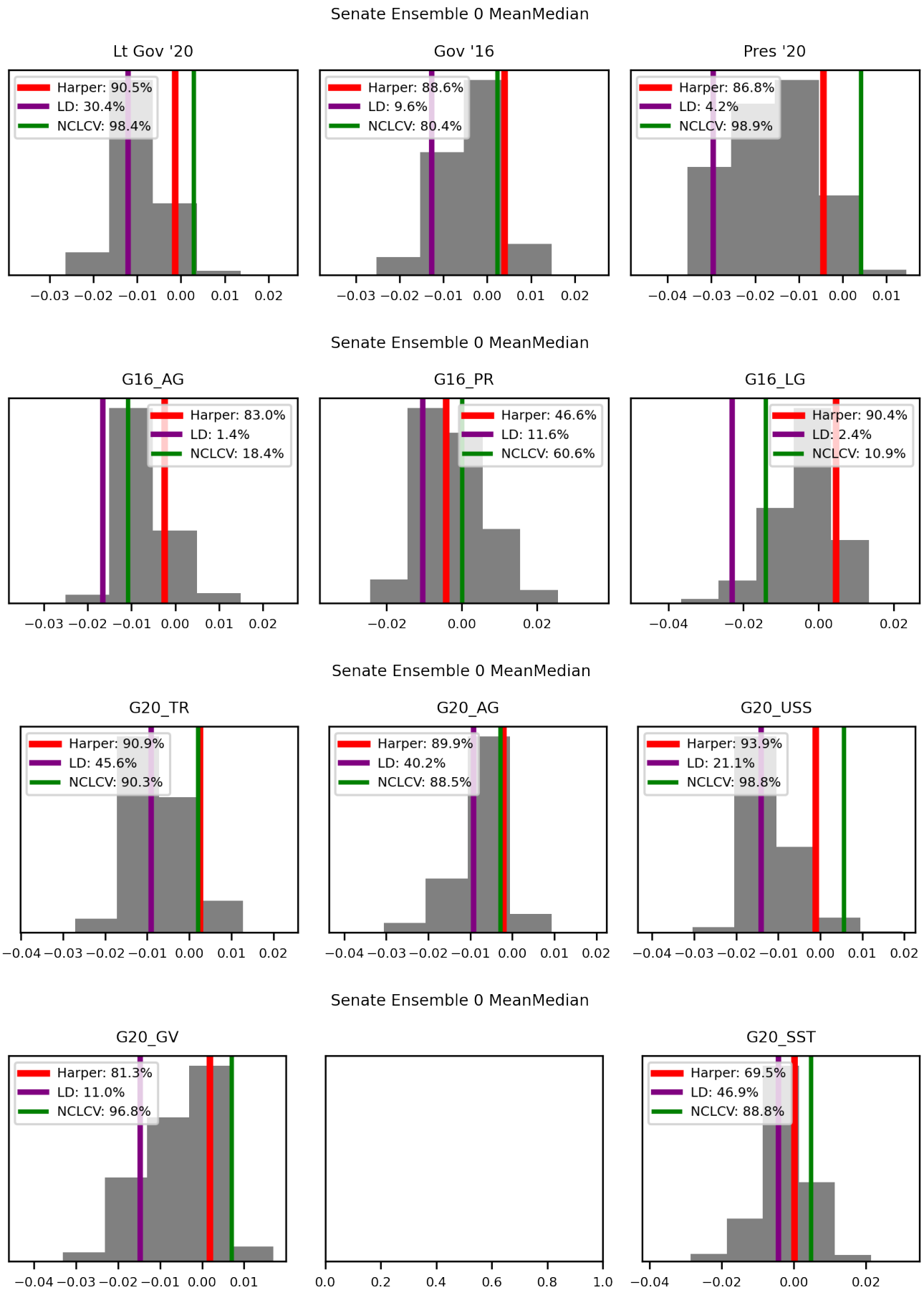

FIGURE 11. Histogram of senate ensemble mean–median score for all 11 elections.

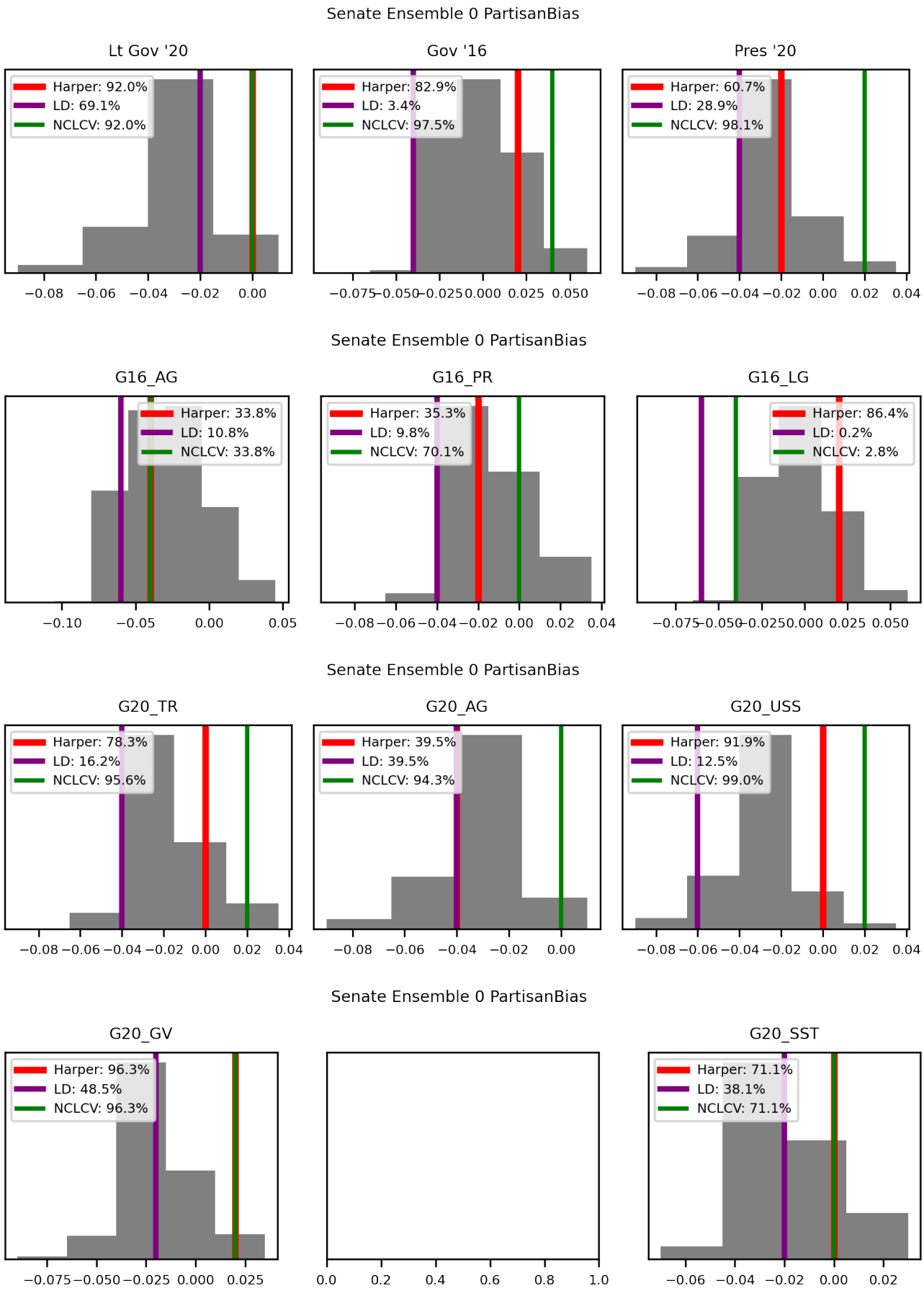

FIGURE 12. Histogram of partisan bias for all 11 elections.

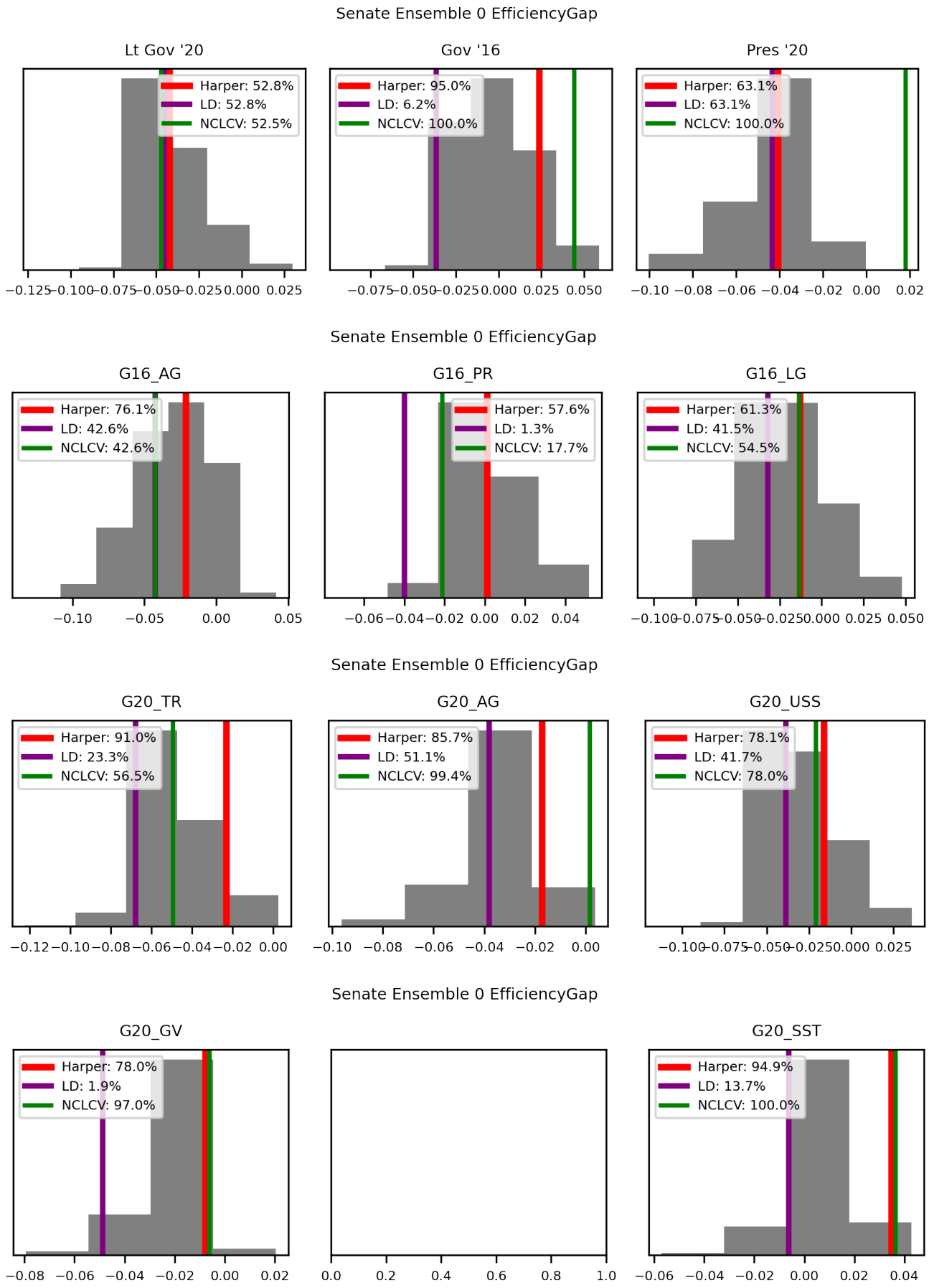

FIGURE 13. Histogram of senate ensemble efficiency gap for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

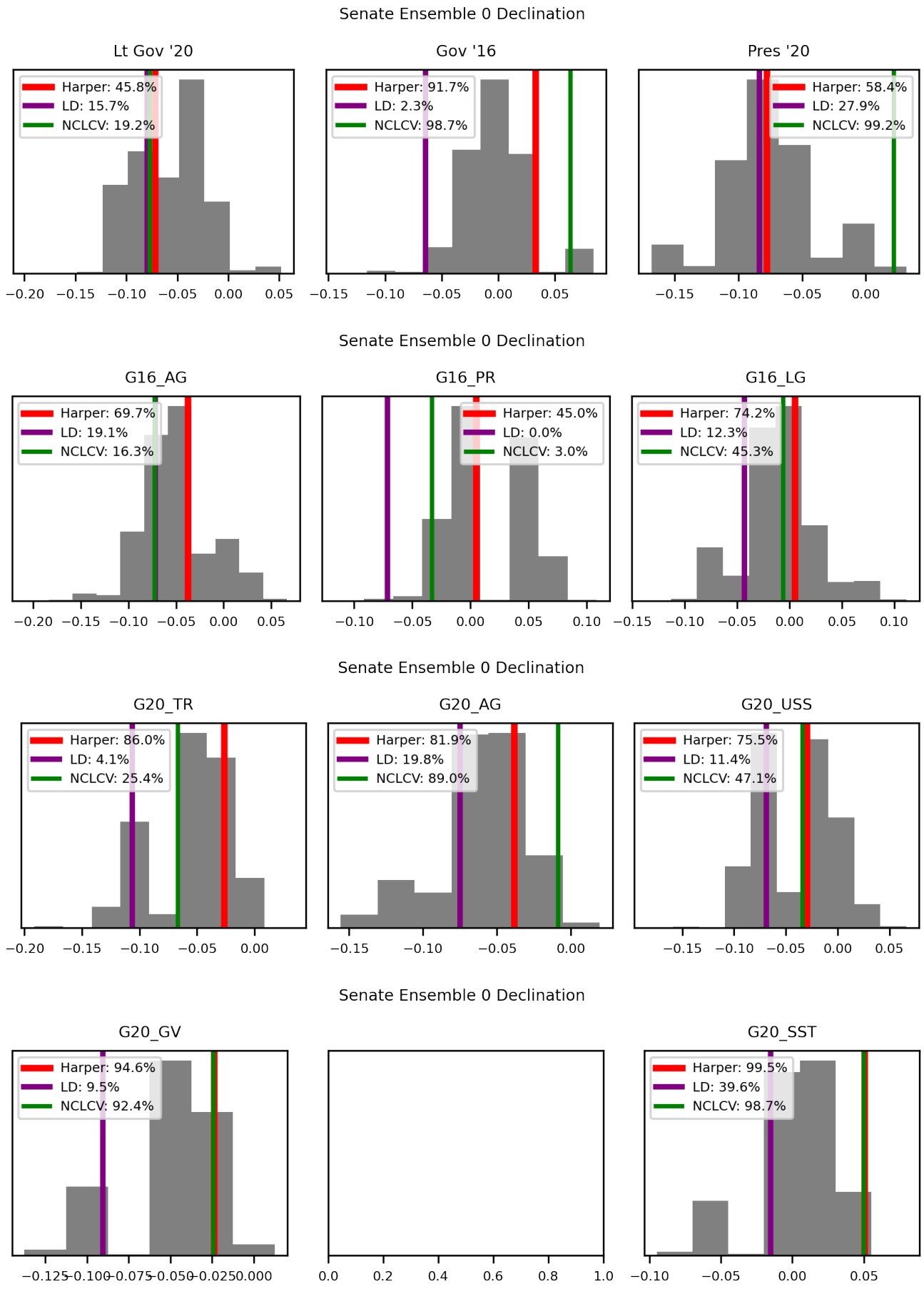

FIGURE 14. Histogram of senate ensemble partisan declination for all 11 elections. The percentages in the legend represent percentile of the corresponding score in the ensemble.

Senate Mean–Median

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−0.4	−3.0	0.4	−3.8
G20USS	−0.1	−1.4	0.6	−4.0
G20GOV	0.2	−1.5	0.7	−4.5
G20LTG	−0.1	−1.2	0.3	−3.7
G20ATG	−0.2	−0.9	−0.3	−3.9
G20TRE	0.2	−0.9	0.2	−3.3
G20SOS	0.0	−0.4	0.5	−3.7
G16PRE	−0.4	−1.0	0.0	−2.0
G16GOV	0.4	−1.3	0.2	−3.1
G16LTG	0.5	−2.3	−1.4	−4.1
G16ATG	−0.3	−1.7	−1.1	−3.2
Average	−0.0	−1.4	0.0	−3.6

TABLE 5. Mean–median scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 11)—not as isolated numbers.

Senate Partisan Bias

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−2.0	−4.0	2.0	−8.0
G20USS	0.0	−6.0	2.0	−8.0
G20GOV	2.0	−2.0	2.0	−6.0
G20LTG	0.0	−2.0	0.0	−8.0
G20ATG	−4.0	−4.0	0.0	−8.0
G20TRE	0.0	−4.0	2.0	−10.0
G20SOS	0.0	−2.0	0.0	−6.0
G16PRE	−2.0	−4.0	0.0	−10.0
G16GOV	2.0	−4.0	4.0	−10.0
G16LTG	2.0	−6.0	−4.0	−8.0
G16ATG	−4.0	−6.0	−4.0	−10.0
Average	−0.5	−4.0	0.4	−8.4

TABLE 6. Partisan bias scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 12)—not as isolated numbers.

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−4.1	−4.3	1.8	−8.5
G20USS	−1.6	−3.9	−2.1	−8.0
G20GOV	−0.8	−4.9	−0.6	−8.8
G20LTG	−4.2	−4.5	−4.7	−10.9
G20ATG	−1.7	−3.8	0.2	−8.0
G20TRE	−2.3	−6.8	−4.9	−11.2
G20SOS	3.5	−0.6	3.7	−4.6
G16PRE	0.1	−4.0	−2.1	−8.5
G16GOV	2.4	−3.6	4.4	−10.2
G16LTG	−1.3	−3.2	−1.3	−5.5
G16ATG	−2.1	−4.2	−4.2	−10.5
Average	−1.1	−4.0	−0.9	−8.6

TABLE 7. Efficiency gap scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 13)—not as isolated numbers.

Senate Declination

Proposed_Plan	Harper	LD	NCLCV	Enacted
G20PRE	−7.8	−8.4	2.2	−16.9
G20USS	−3.0	−6.9	−3.4	−15.3
G20GOV	−2.4	−9.1	−2.4	−16.2
G20LTG	−7.2	−8.0	−7.7	−20.9
G20ATG	−3.8	−7.5	−0.9	−15.3
G20TRE	−2.7	−10.6	−6.7	−20.5
G20SOS	5.1	−1.5	5.0	−8.5
G16PRE	0.5	−7.1	−3.3	−16.2
G16GOV	3.3	−6.4	6.4	−17.9
G16LTG	0.5	−4.3	−0.6	−10.0
G16ATG	−3.8	−7.2	−7.2	−18.4
Average	−1.9	−7.0	−1.7	−16.0

TABLE 8. Declination scores listed as percentages (times 100) for the proposed Senate plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 14)—not as isolated numbers.

4. House Plan Analysis

I followed the same procedures for analyzing the House plans as I did for the Senate and Congressional plans, but here I had only two plans (LD and NCLCV). I used the ensemble [**?**], whose characteristics are similar to those of the Senate ensemble used above.

4.1. **Seats Won.** Considering the number of seats won in each election, as shown in Figure 15, Both the LD and NCLCV plans appear to be mostly typical in terms of the number of seats won, except in G20PR and G16LG where NCLCV is much higher (pro Democrat) than the main distribution.

4.2. **Rank-Ordered Violin Plots.** Referring to Figure 16, which focuses only on the most competitive districts, the NCLCV plan appears to deviate much more from the ensemble than the LD plan does.

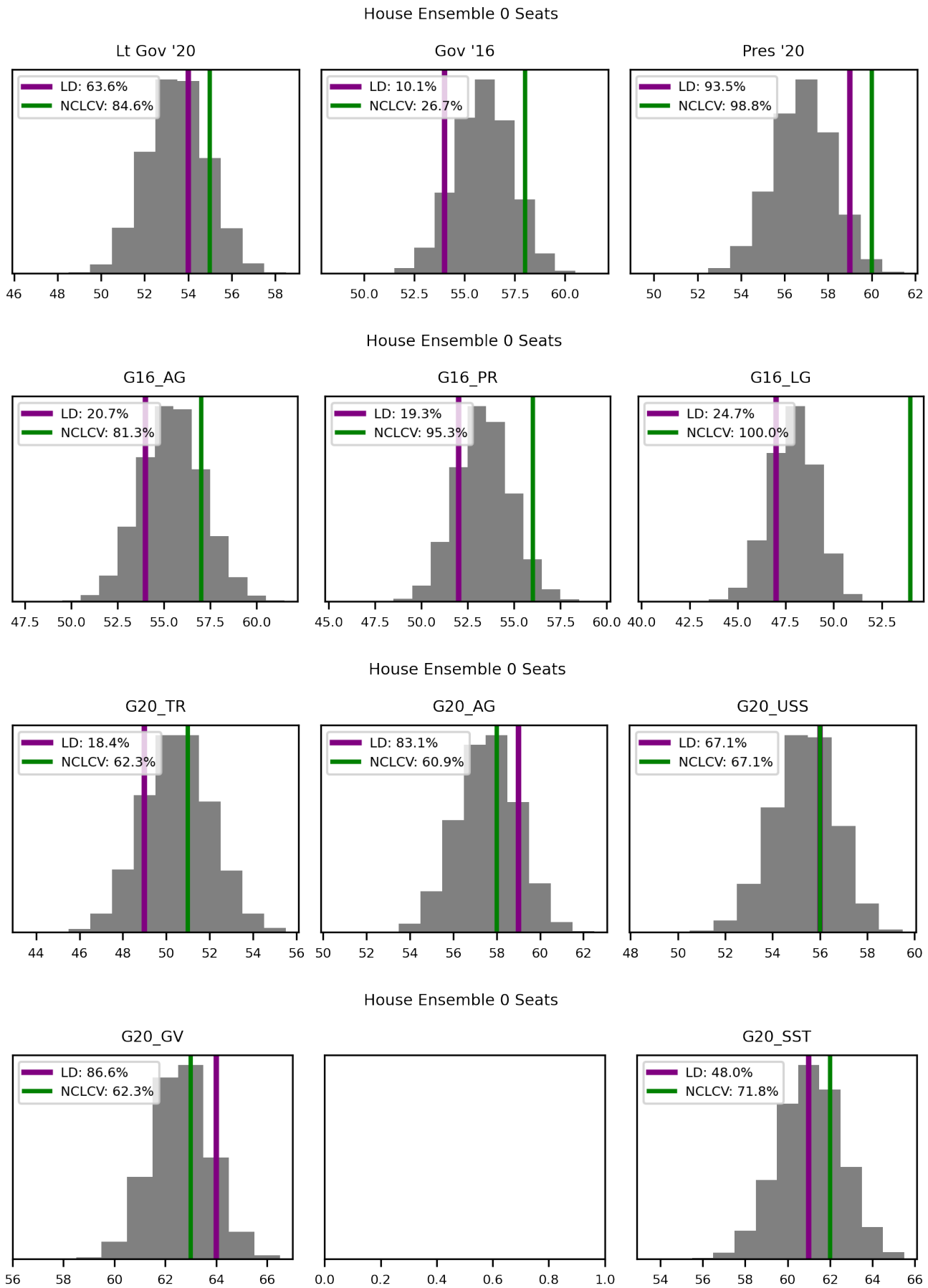

FIGURE 15. Histograms of congressional seats won in all 11 elections for Ensemble 0 plans. The percentages in the legend represent percentile of the corresponding score in the ensemble.

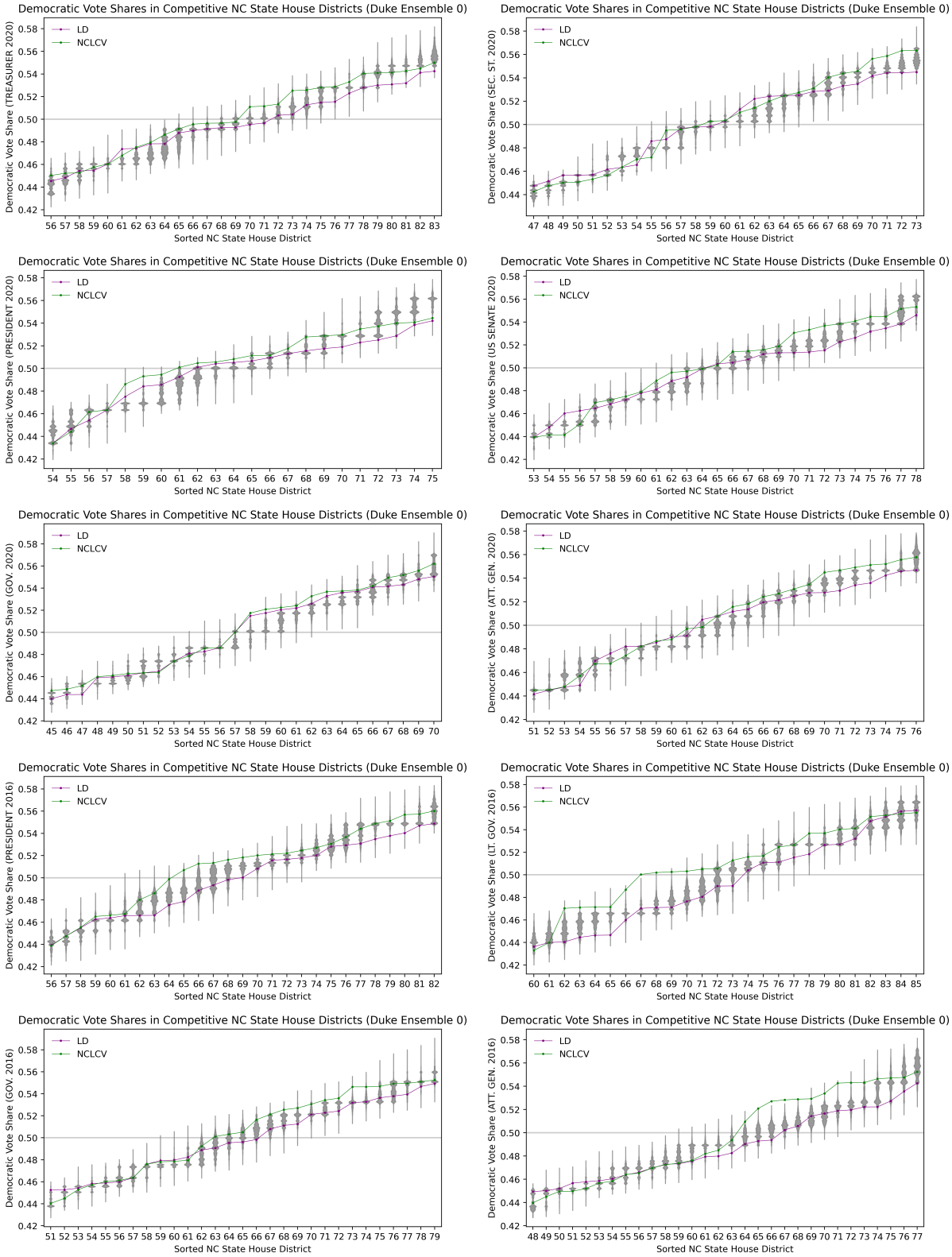

FIGURE 16. Rank-ordered house districts for 10 of the elections (all elections but G20LG) using Ensemble 0. These plots show only the most competitive districts.

4.3. **Other Metrics.**

4.3.1. *Mean–Median.* Table 9 shows my calculations of the mean–median scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 17)—not as isolated numbers.

The distribution is shifted in the negative direction, so scores very close to 0 look more like outliers than large negative scores. Specifically, the NCLCV score of 0.1% in the election G20PR is very close to zero, but it is more Democratic favoring than 98% of all plans, so this plan is an outlier for this distribution, while the LD plan's score of −0.7% is more typical of the distribution.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of mean–median scores.

4.3.2. *Partisan Bias.* Table 10 shows my calculations of the partisan bias scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 18)—not as isolated numbers.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of partisan bias scores.

4.3.3. *Efficiency Gap.* Table 11 shows my calculations of the efficiency gap scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 19)—not as isolated numbers.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of efficiency gap scores.

4.3.4. *Declination.* Table 4 shows my calculations of the declination scores of the three plans in the different races. These scores should be interpreted in light of the full distribution of scores (histograms in Figure 20)—not as isolated numbers.

Although there are occasional outliers, taken as a whole, neither proposed plan looks to me like a partisan gerrymander with respect to the distribution of declination scores.

4.4. **House Conclusion.** The seat margins shown in the rank-ordered violin plots of Figure 16 give evidence of partisan gerrymandering in the NCLCV plan.

These conclusions do not take VRA racial considerations into account.

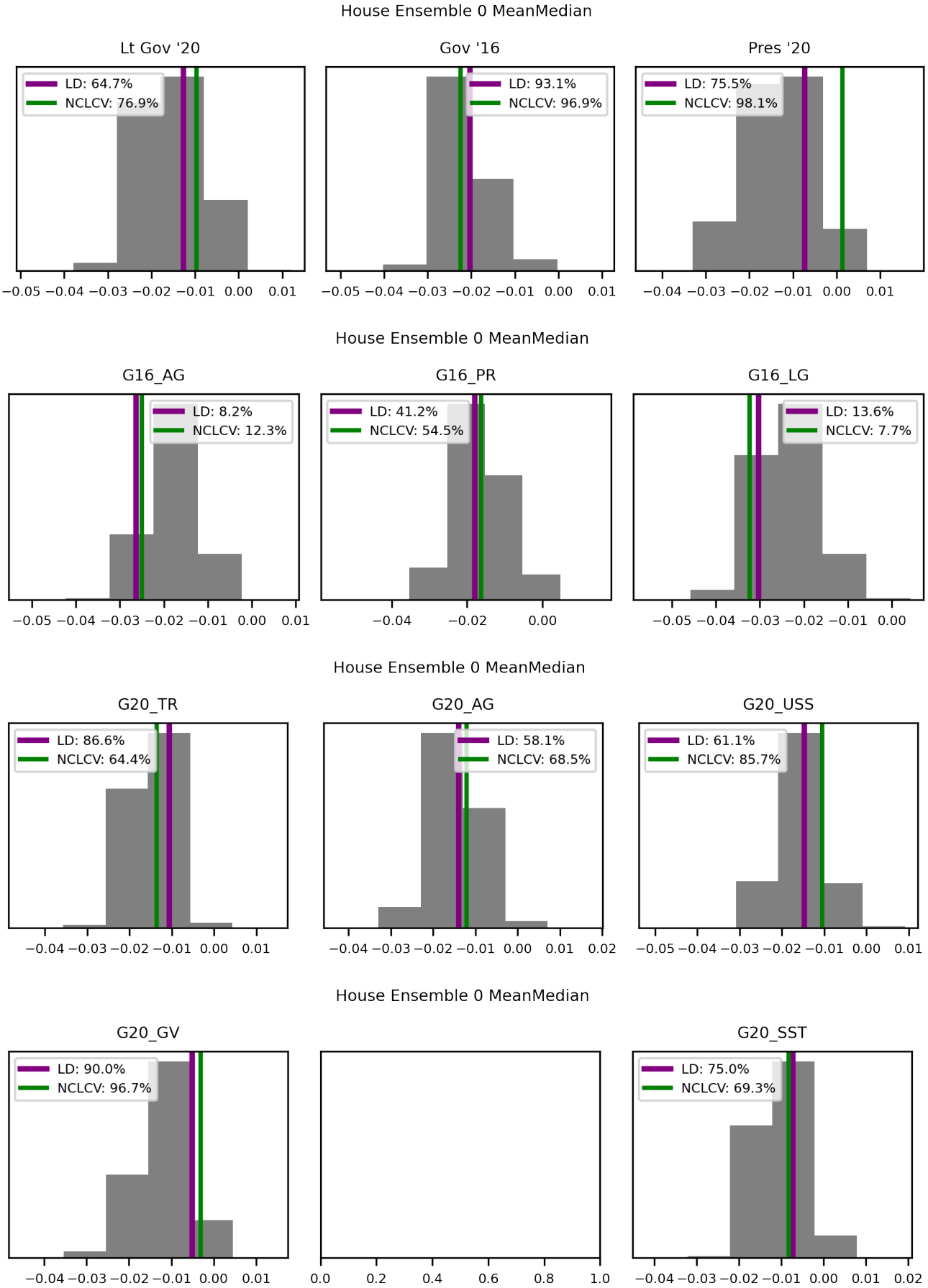

FIGURE 17. Histogram of house ensemble 0 mean–median score for all 11 elections.

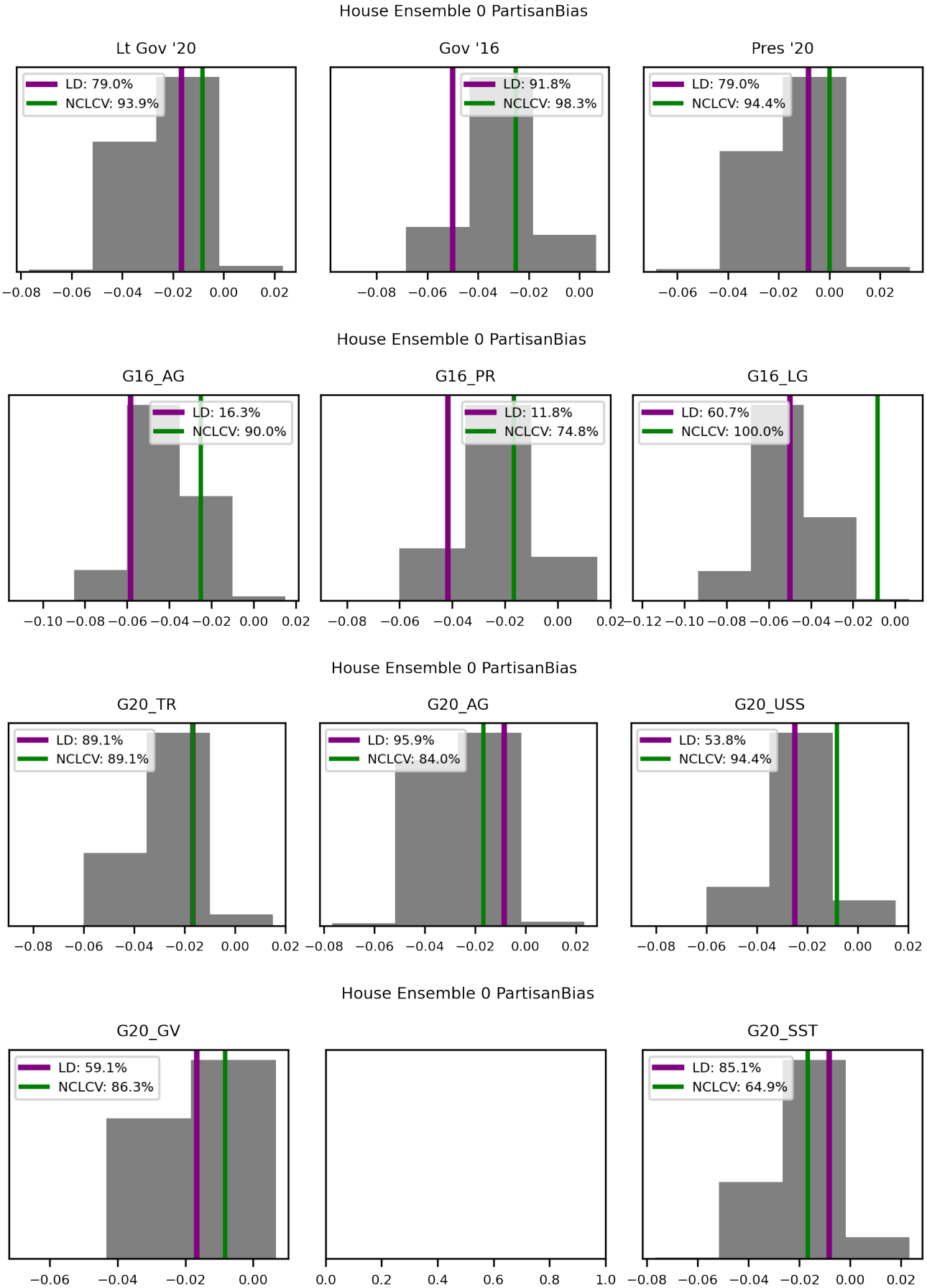

FIGURE 18. Histogram of partisan bias for all 11 elections for Ensemble 0.

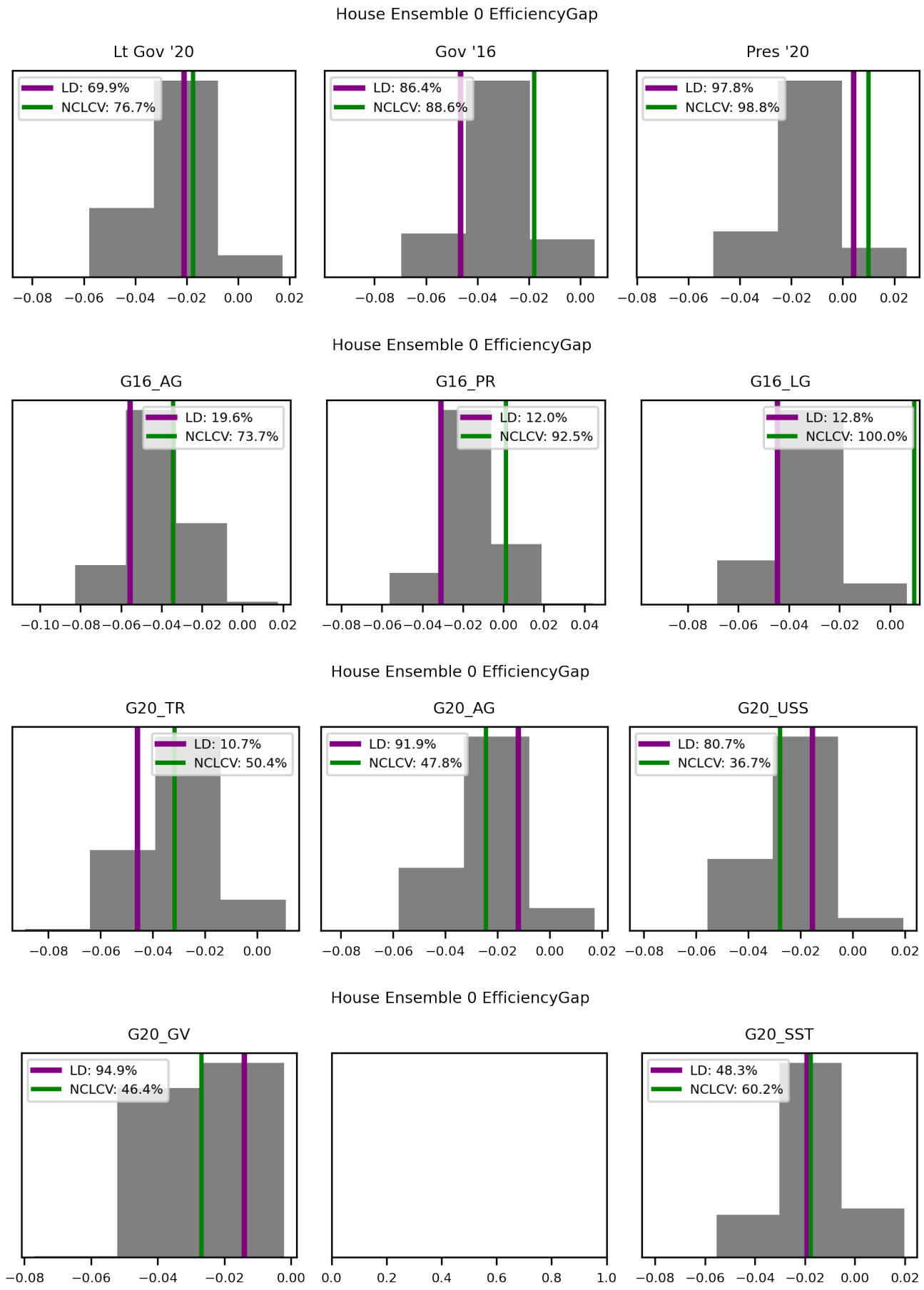

FIGURE 19. Histogram of house ensemble 0 efficiency gap for all 11 elections.

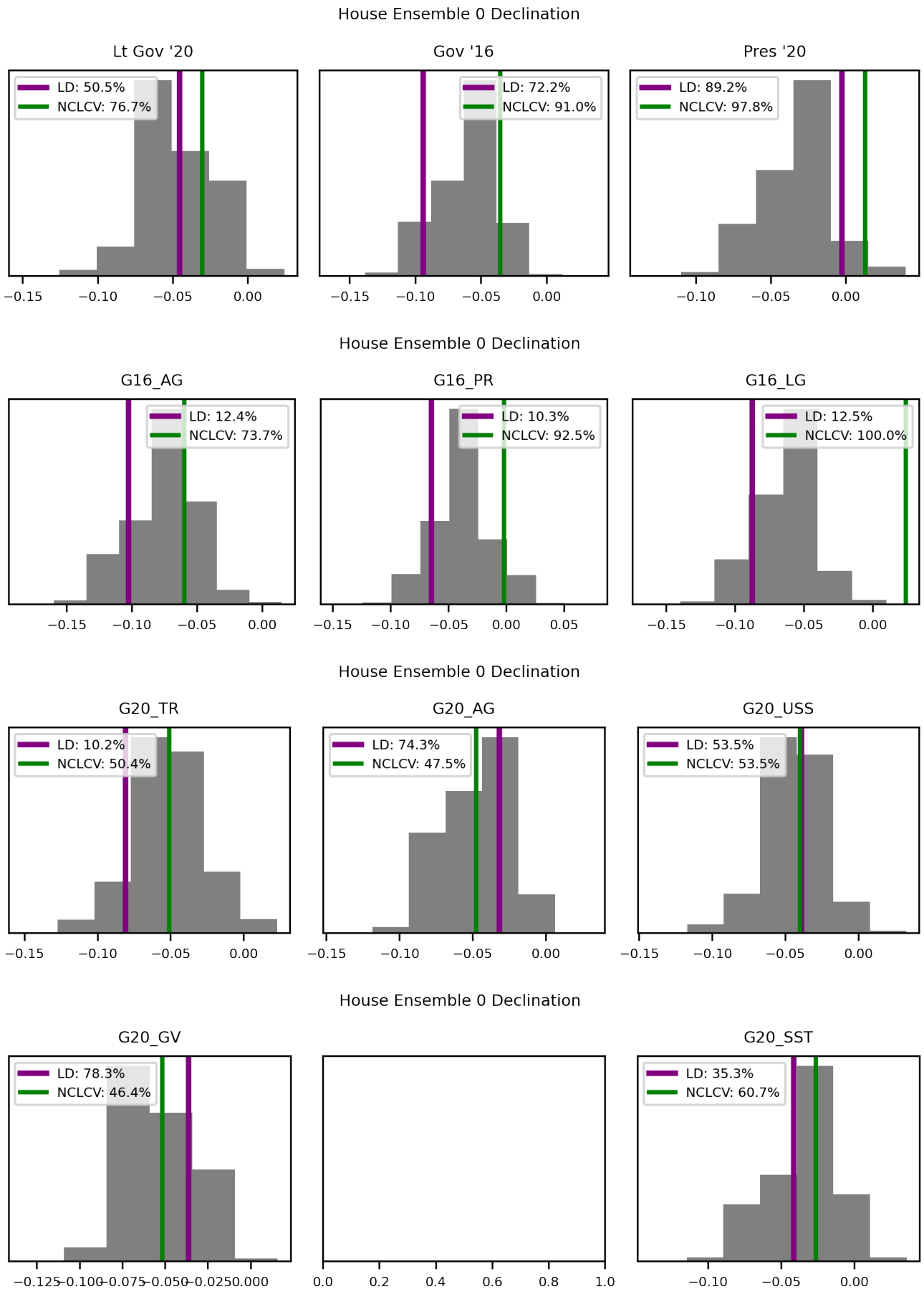

FIGURE 20. Histogram of house ensemble 0 partisan declination for all 11 elections.

House Mean–Median

Proposed_Plan	LD	NCLCV	Enacted
G20PRE	−0.7	0.1	−3.0
G20USS	−1.5	−1.0	−3.4
G20GOV	−0.5	−0.3	−3.3
G20LTG	−1.3	−1.0	−3.4
G20ATG	−1.4	−1.2	−3.1
G20TRE	−1.1	−1.4	−3.4
G20SOS	−0.7	−0.8	−3.1
G16PRE	−1.8	−1.6	−5.1
G16GOV	−2.0	−2.2	−4.1
G16LTG	−3.0	−3.2	−4.4
G16ATG	−2.6	−2.5	−4.5
Average	−1.5	−1.4	−3.7

TABLE 9. Mean–median scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

House Partisan Bias

Proposed_Plan	LD	NCLCV	Enacted
G20PRE	−0.8	0.0	−6.7
G20USS	−2.5	−0.8	−7.5
G20GOV	−1.7	−0.8	−8.3
G20LTG	−1.7	−0.8	−7.5
G20ATG	−0.8	−1.7	−7.5
G20TRE	−1.7	−1.7	−7.5
G20SOS	−0.8	−1.7	−7.5
G16PRE	−4.2	−1.7	−9.2
G16GOV	−5.0	−2.5	−8.3
G16LTG	−5.0	−0.8	−8.3
G16ATG	−5.8	−2.5	−9.2
Average	−2.7	−1.4	−8.0

TABLE 10. Partisan bias scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

House Efficiency Gap

Proposed_Plan	LD	NCLCV	Enacted
G20PRE	0.4	1.0	−7.5
G20USS	−1.5	−2.8	−8.7
G20GOV	−1.4	−2.7	−6.3
G20LTG	−2.1	−1.8	−8.9
G20ATG	−1.2	−2.4	−8.6
G20TRE	−4.6	−3.2	−8.0
G20SOS	−1.9	−1.8	−8.9
G16PRE	−3.1	0.1	−5.2
G16GOV	−4.7	−1.8	−8.2
G16LTG	−4.4	1.0	−7.0
G16ATG	−5.5	−3.4	−9.0
Hou EG mean	−2.7	−1.6	−7.8

TABLE 11. Efficiency gap scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

House Declination

Proposed_Plan	LD	NCLCV	Enacted
G20PRE	−0.3	1.3	−14.9
G20USS	−3.9	−4.0	−16.8
G20GOV	−3.6	−5.1	−12.7
G20LTG	−4.5	−3.0	−18.0
G20ATG	−3.2	−4.7	−15.7
G20TRE	−8.1	−5.1	−15.2
G20SOS	−4.1	−2.6	−16.3
G16PRE	−6.5	−0.2	−11.8
G16GOV	−9.4	−3.5	−15.7
G16LTG	−8.7	2.4	−14.4
G16ATG	−10.2	−6.0	−16.4
Average	−5.7	−2.8	−15.3

TABLE 12. Declination scores listed as percentages (times 100) for the proposed House plans across the 11 elections. These scores should be interpreted in light of the full distribution of scores (histograms)—not as isolated numbers.

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