Exhibit 16

STATE OF NEW HAMPSHIRE

HILLSBOROUGH, SS.
SOUTHERN DISTRICT

SUPERIOR COURT

Docket No. 226-2022-CV-00181

MILES BROWN,
ELIZABETH CROOKER,
CHRISTINE FAJARDO,
KENT HACKMANN,
BILL HAY,
PRESCOTT HERZOG,
PALANA HUNT-HAWKINS,
MATT MOOSHIAN,
THERESA NORELLI,
NATALIE QUEVEDO, and
JAMES WARD,

v.

DAVID M. SCANLAN,
in his official capacity as the New Hampshire Secretary of State

AFFIDAVIT OF DR. WESLEY PEGDEN

I, Wesley Pegden, being duly sworn, state as follows:

1. I am over the age of 18, have personal knowledge of the facts stated in this affidavit, and can competently testify to their truth.

2. Attached to this affidavit is an analysis of the recently enacted redistricting plans for the New Hampshire State Senate and New Hampshire Executive Council, as well as a current version of my curriculum vitae. I reserve the right to supplement the attached analysis in light of additional facts, testimony, and/or materials that may come to light.

Dated: 05/09/2022

Wesley Pegden

Wesley Pegden

State of Texas

County of Travis

This instrument was acknowledged before me this 9th day of May, 2022, by Wesley Pegden.

NOTARY PUBLIC STATE OF TEXAS

Jennifer Jo Thomas

ID NUMBER
133341819
COMMISSION EXPIRES
September 20, 2025

Jennifer Jo Thomas

Notary Public

Notary Public, State of Texas

My commission expires: 09/20/2025

Notarized online using audio-video communication

An analysis of New Hampshire's legislative districtings: Expert Report

Wesley Pegden

April 28, 2022

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1 Qualifications

I am an associate professor in the department of Mathematical Sciences at Carnegie Mellon University, where I have been a member of the faculty since 2013. I received my Ph.D. in Mathematics from Rutgers University in 2010 under the supervision of József Beck, and I am an expert on stochastic processes and discrete probability. My research has been funded by the National Science Foundation and the Sloan Foundation. A current CV with a list of publications is attached as Exhibit A. A list of my publications with links to online manuscripts is also available at my website at `http://math.cmu.edu/~wes`.

I am an expert on the use of Markov Chains for the rigorous analysis of gerrymandering, and have published papers[1] developing techniques for this application in *Proceedings of the National Academy of Sciences* and *Statistics and Public Policy*, hereafter referred to by [CFP] and [CFMP], respectively.

I testified as an expert witness in the *League of Women Voters of Pennsylvania v. Commonwealth of Pennsylvania* case in which the 2011 Congressional districting was found to be an unconstitutional partisan gerrymander, the 2018 *Common Cause v. Lewis* case in North Carolina which resulted in the North Carolina legislative districtings being found to be unconstitutional partisan gerrymanders, and the 2022 *Harper v. Hall* case in North Carolina Congressional and state legislative districtings were found to be unconstitutional partisan gerrymanders. I previously served as a member of the bipartisan Pennsylvania Redistricting Reform Commission under appointment by the governor. I am being compensated at a rate of $325 per hour for my work on the current case.

2 Summary of Analysis and Opinions

I was asked to analyze whether New Hampshire's Senate and Executive Council plans enacted in 2022 were drawn in a way which made extreme use of partisan considerations.

To conduct my analysis, I take a enacted plan as a starting point and make a sequence of many small random changes to the district boundaries. This methodology is intended to detect whether the district lines were carefully drawn to optimize partisan considerations; in particular, if the plans in question were not intentionally drawn to maximize partisan advantage, then making random changes should not significantly decrease the plan's partisan bias.

Specifically, my method begins with the enacted plan and uses a Markov Chain—a sequence of random changes—to generate billions of comparison districtings against which I compare the enacted plans. These comparison districtings are generated by making a sequence of small random changes to the enacted plans themselves, and preserve districting criteria such as population deviation, compactness, and limits on the splitting of counties and cities, among other criteria (a complete list is given in Section 4.3.1).

The analysis I conduct of the enacted plan using this data has two levels. The first level of my analysis consists simply of comparing the partisan properties of the enacted plans to the large sets of comparison maps produced by my Markov Chain, and I report how unusual the enacted plans are with respect to their partisan properties, against this comparison set. **Quantitatively, for the enacted Senate and Executive Council plans, I find that they each have a greater partisan bias than** 99.99% **of the billions of districtings produced by my algorithm, respectively.**

The next level of my analysis uses the mathematical theorems I have developed with my co-authors in [CFP] and [CFMP] to translate the results of the above comparison into a statement about how the enacted plans compare against *all* other possible districtings of New Hampshire satisfying the districting criteria I consider in this report. In other words, the theorem that I use in the second level analysis allows me to compare the enacted plan against not only the billions of plans that my simulations produce through making small random changes, but also against all other possible districtings of New Hampshire satisfying the districting criteria I consider.

[1]

- M. Chikina, A. Frieze, W. Pegden. Assessing significance in a Markov Chain without mixing, in *Proceedings of the National Academy of Sciences* **114** (2017) 2860–2864
- M. Chikina, A. Frieze, J. Mattingly, W. Pegden. Separating effect from significance in Markov chain tests, in *Statistics and Public Policy* **7** (2020) 101–114.

Consider the following: when I make a sequence of small random changes to an enacted plan as described above, this can be viewed as a test of whether the partisan bias in the current districting is fragile, in the sense that it evaporates when the boundary lines of the district are perturbed. As discussed in Section C, our theorems in [CFP] and [CFMP] establish that it is mathematically impossible for the political geography of a state to cause such a result. That is: while political geography might conceivably interact with districting criteria to create a situation where typical districtings of a state are biased in favor of one party, it is mathematically impossible for the political geography of a state to interact with districting criteria to create a situation where typical districtings of a state appear to be *optimized for partisan bias*, in the sense that their bias is fragile and evaporates when small random changes are made. This allows us to rigorously demonstrate that a districting is optimized for partisanship, and is an outlier among *all* possible districtings of a state satisfying the criteria I consider, with respect to this property.

Quantitatively, my second-level analysis establishes that the 2022 Senate and Executive Council plans are more optimized for partisanship than more than 99.9% **of all possible Senate and Executive Council districtings satisfying the districting criteria I account for in my analysis, respectively.** Thus the chance of drawing districtings that are as optimized with respect to their partisan properties as the current House and Senate districtings of New Hampshire *without* using partisan considerations is exceedingly small. **Qualitatively, I conclude based on my analysis that the 2022 New Hampshire Senate and Executive Council plans were drawn with an overarching intent to maximize political advantage for Republican candidates**.

3 Topic of Expert Report

The question motivating my analysis in this case is: "*How significant a role did partisanship play in the drawing of the 2022 Senate and Executive Council maps of New Hampshire?*"

My analysis approaches this question in a rigorous and quantifiable way. In short, I identify how much of an outlier the present districting lines are, with respect to how carefully they are drawn to line up with partisan goals. *A priori*, it is possible that political geography might conceivably interact with districting criteria to bias typical districtings for one party or another. But my analysis provides a rigorous quantifiable answer to the question of the extent to which partisanship was used in the districting process, whose validity *does not depend on the political geography of New Hampshire*.

4 Quantifying intentional and excessive use of partisanship

My approach begins with a simple idea: I make small random changes to the district boundaries of enacted plans (while maintaining districting criteria) and study the effect this has on the partisan bias of the map. More specifically:

- I begin from the enacted plan I am evaluating, and then repeatedly:
  1. Randomly select a geographical unit (a township or ward) on the boundary of two districts, and check: if I change which district this geographic unit belongs to, will the resulting districting still satisfy the districting criteria laid out in Section 4.3.1? If so, I make the change.
  2. Using historical voting data as a proxy for partisan voting patterns, evaluate the partisanship of the districting resulting from the previous step.
- These two steps are repeated many times, resulting in a sequence of districtings, each produced by a small random change to the districting preceding it, with the enacted map I am evaluating as the starting point for the sequence.

This procedure is implemented as a computer algorithm which carries out millions or billions of the above steps for a districting map.

4.1 First level analysis

The first level of my analysis simply uses the above procedure to generate a large set of comparison districtings against which one can compare the enacted plan. For example, for the New Hampshire Senate districting, I conducted 32 runs of the above procedure. A "run" in this context consists of a single consecutive sequence of small random changes to the enacted plan, producing a set of comparison districtings. For example, for the Senate districting, each run consisted of carrying out Steps 1 and 2 in the procedure above $2^{25} \approx 34$ million times. As discussed in later sections, these comparison maps adhere to districting criteria in ways that constrain them to be similar in several respects to the enacted map being evaluated. For example, the comparison districtings will preserve the same counties and cities preserved by the enacted plan.

In total for this districting, I conducted 32 such runs. I then show the results of these runs in a table, like this:

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99971%	9	99.999904%	17	99.999949%	25	99.999988%
2	99.99968%	10	99.99971%	18	99.999985%	26	99.9989%
3	99.99981%	11	99.999955%	19	99.99972%	27	99.99986%
4	99.99936%	12	99.99978%	20	99.99966%	28	99.99978%
5	99.999994%	13	99.99943%	21	99.99983%	29	99.99978%
6	99.99982%	14	99.999943%	22	99.99939%	30	99.999931%
7	99.99931%	15	99.999982%	23	99.99978%	31	99.99979%
8	99.99991%	16	99.999907%	24	99.999982%	32	99.99971%

For example, we see here that in the first run, 99.99971% of the comparison districtings exhibited less Republican bias than the enacted Senate districting. Moreover, in *every* run, more than 99.9989% of the comparison districtings exhibited less Republican bias than the enacted plan. In other words, fewer than 0.0011% of comparison plans exhibited as much Republican bias as the enacted plan, in *every* run.

The first level of my analysis simply reports this comparison of the enacted map to the comparison districtings produced in these runs. Even without applying the mathematical theorems we have developed in [CFP] and [CFMP], this gives strong, intuitively clear evidence that the district lines were intentionally drawn to optimize partisan advantage in the enacted plan: if the districting had not been drawn to carefully optimize its partisan bias, we would expect naturally that making small random changes to the districting would not have such a dramatic and consistent partisan effect.

4.2 Second level analysis

In the first level of my analysis, I compare enacted plans to comparison districtings produced by my algorithm (which makes random changes to the existing map while preserving districting criteria).

The next level of my analysis goes further than this, and enables a rigorous comparison to *all* alternative districtings of New Hampshire satisfying the districting criteria I consider here. It does this by comparing how "optimized for partisanship" an evaluated plan is to how "optimized for partisanship" alternative plans are.

4.2.1 Defining "optimized for partisanship"

Roughly speaking, when I say that a districting is *optimized for partisanship*, I mean that its partisan characteristics are highly sensitive to small random changes to the boundary lines.

Formally, when I say that a districting is *optimized for partisanship* in this report, I mean that there is a high probability that when I make small random changes to the districting, its partisanship will be an extreme outlier among the comparison maps produced by the small random changes.

The yardstick I use to measure this property of a given map is the *ε-fragility* of a map. Given a small threshold ε—for example, 00.0011%, for the analysis of the Senate example discussed above—I can ask: what is the probability that when I make a sequence of small random changes to the map, the map will be

in the most extreme ε fraction of maps encountered in the sequence of random changes? The probability of this occurrence is the ε-fragility of the map, and it is this probability that I use to quantify how optimized for partisanship a map appears to be.

In other words, **one districting is considered more optimized for partisanship than another if it is more likely to have its partisan bias consistently reduced when making a random sequence of small changes to its boundary lines.**

4.2.2 Comparing an enacted plan to the set of *all* alternatives

My analysis enables a rigorous comparison of the enacted New Hampshire Senate and Executive Council plans to **all possible districting plans** for these bodies that satisfy the districting criteria I consider, with respect to how optimized for partisanship the districtings are. I can report the maximum fraction of all such possible redistricting plans which could appear as optimized for partisanship as the enacted plan, in the sense of the test described above. For example, I report that the enacted Senate districting of New Hampshire is among the most optimized-for-partisanship 00.025% of **all possible Senate districtings of New Hampshire** satisfying the districting criteria I consider here, as measured by it's ε-fragility.

My method produces a rigorous p-value (statistical significance level) which precisely captures the confidence one can have in the findings of my "second level" analyses. In particular my second-level claims in this report are all valid at a statistical significance of $p = .002$. This means that the probability that I would report an incorrect number (for example, claiming that a districting is among the most optimized-for-partisanship 00.01% of all districtings, when in fact it is merely among the most 00.015% optimized-for-partisanship) is at most 00.2%.

4.2.3 Some intuition for why this is possible

It may seem remarkable that I can make a rigorous quantifiable comparison to *all* possible districtings, without actually generating all such districtings; this is the role of our theorems from [CFP] and [CFMP], which have simple proofs which have been verified by the mathematical community.

To give some nontechnical intuition for why this kind of analysis is possible, these results roughly work by showing that in a very general sense, it is not possible for an appreciable fraction of districtings of a state to appear optimized for partisanship in the sense defined in Section 4.2.1. In other words, it is *mathematically impossible* for any state, with any political geography of voting preferences and any choice of districting criteria, to have the property that a significant fraction of the possible districtings of the state satisfying the chosen districting criteria appear optimized for partisanship (as measured by their ε-fragility).

4.3 Implementation details

Here I specify the particulars of the random changes my algorithm makes to a map, my implementation of districting criteria, and my method of comparing the partisanship of a districting to that of districtings encountered on the sequence of random changes.

4.3.1 Districting criteria

All comparison maps produced by my algorithm are required to satisfy the following districting criteria:

(a) **Contiguity**: I require comparison districtings to contain only contiguous districts.

(b) **Compact districts**: I require comparison districtings to be at least as compact as the enacted plan being evaluated, up to an error of 5%. I impose this condition both on the Polsby-Popper score of individual districts as well as on the districting-wide district-average of the ratio of the perimeter squared to the area (Polsby-Popper reciprocal), as well as to the total perimeter of all districts in the plan. In the robustness checks section of the report, I show the insensitivity of my analysis to the particular threshold chosen.

(c) **Country traversals**: I require comparison districts to not contain more county traversals than the enacted plan.

(d) **Municipality preservation:** There are at most as many city splits as in the enacted plan.

(e) **Incumbency protection:** In my analysis of the enacted Senate plan, I performed an alternative analysis where I implement an incumbency protection requirement. In these runs, any incumbent who, in the enacted plan, is not paired with any other incumbent must remain unpaired in the comparison districtings. Because I did not have access to home addresses of Executive Council incumbents, I did not perform this analysis for the enacted Executive Council plan.

(f) **Population deviation**: When evaluating the enacted Executive Council districting, I impose a 1% population deviation-from-ideal requirement on my comparison maps, while for Senate maps, I impose a 5% constraint. (The enacted Executive Council and Senate districtings exhibit population deviations of roughly 0.9% and 4.9%, respectively, from the ideal district size.) In the robustness checks section of the report, I show the insensitivity of my analysis to the particular thresholds chosen.

4.3.2 A conservative application of the criteria

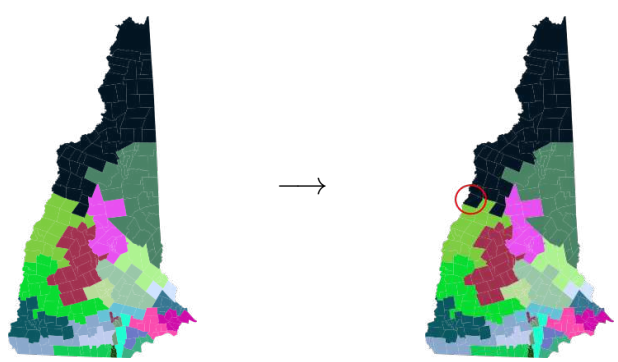
It is important to note that my analysis is designed to avoid second-guessing the mapmakers' choices in how they implemented the districting criteria. In particular, while it is reasonable to ask whether the mapmakers could have drawn districtings which adhered better to nonpartisan criteria (more compact, preserving more cities, *etc*), my approach is different, and much more conservative.

In particular, my analysis asks the question: even if we accept that the mapmakers have made appropriate choices with respect to nonpartisan criteria such as compactness, population deviation, city preservation, incumbency protection, and so on, does their plan nevertheless stand out with respect to its *partisan* qualities?

Note that, for example, I choose my compactness threshold within 5% of value of the enacted map. And with respect to incumbents, I do not try to protect as many incumbents as are protected in the enacted map, but exactly the same incumbents as protected by the mapmakers. With respect to city preservation, I am not trying to answer the question: "if the mapmakers had tried to preserve more cities, would this have resulted in a more favorable districting for Democrats?" Instead, I am asking, among all alternative districtings of New Hampshire with the same *nonpartisan* characteristics as the enacted map—their compactness constraint, how many cities they must preserve, etc.—whether the enacted plan is an extreme outlier with respect to the extent to which it is optimized for partisanship.

5 Random Changes

As described earlier, my method involves making small random changes to a map. For example, depicted here is a small random change made to the enacted Senate districting:



The geographical units used for these small random changes in this districting are towns, townships, and city wards. At each step of the sequence of random changes for the senate districting of New Hampshire, I move a randomly selected geographical unit at the boundary of two districts from one of those districts to the other (unless it would violate the constraints laid out in Section 4.3.1). For the Executive Council plan, which does not divide any cities, my algorithm operates at the township and whole-municipality level.

In each run, my chain generates comparison maps from a given enacted plan by making millions or billions of these small changes to the enacted plan, while preserving districting criteria in specific ways chosen by the mapmakers, as discussed in Section 4.3.2.

These random changes can be either be made one-at-a-time or with several steps made simultaneously; the latter allows comparison maps to be generated when any single move would lead to a violation of the constraints laid out in Section 4.3.1 (e.g., because population would become too imbalanced), but combinations of moves can be found which would preserve all these criteria.

Technical details of my implementation of these multi-moves are found in Appendix B.

5.0.1 Voting Data

To evaluate the partisanship of a given districting, I make use of the following historical statewide elections:

- 2012 Governor election
- 2012 Presidential election
- 2014 Governor election
- 2016 Governor election
- 2016 Presidential election
- 2018 Governor election
- 2020 Governor election
- 2020 Presidential election

For my main analyses, I use a historical election index which is a turnout-weighted average of these 8 historical elections. The two party vote share of this election index is 49% Democrat, 51% Republican.

5.0.2 The *seats expected* metric for comparing districtings

As described in Section 4.2.1, my definition of optimized for partisanship involves comparing the partisanship of an enacted plan to the partisanship of comparison districtings produced from it by a sequence of random changes. Here I describe the *seats expected* metric of partisanship I use for this comparison throughout this report. In short, the seats expected metric for the districting is **the average number of seats Democrats would expect to win in the districting**, based on a uniform swing model applied to the historical election index I use (described in Section 5.0.1).

The *uniform swing* is a simple model frequently used to make predictions about the number of seats a party might win in an election, based on partisan voting data. Suppose, for example, that given data from the last New Hampshire Senate election, we would like to predict how many seats Democrats will win in an upcoming Senate election (with the same districting), assuming that at a statewide level, we expect them to outperform by 1.5 percentage points their results from the last election.

A uniform swing would simply add 1.5 percentage points to Democrat performance in every district in data from the last election, and then evaluate how many seats would be won with these shifted voting outcomes.

When I am evaluating the partisanship of a comparison districting (to compare it to the enacted plan), I am interested in the number of seats we expect Democrats might win in the districting, given unknown shifts in partisan support. In particular, the metric I use is:

> How many seats, on average, would Democrats win in the given districting, if a random uniform swing is applied to the historical election index?

As an example, let us consider the 2022 Executive Council plan, using the 2016 Presidential election as a proxy for partisan voting patterns. Note that this election has a 50.2% Democrat 2-party vote share. Using these results as a direct proxy for future voting patterns, the enacted map would produce a 1:4 split

of Democrat:Republican seats in the executive council. If the Democrat vote share was increased by 1.26% in every district, the split would change to 2:3, and if it was increased by 1.78%, the split would rise to 3:2.

The random choice of my uniform swing is made from a normal distribution whose standard deviation is 4 percentage points[2]. Figure 1 visualizes the probabilities that this distribution assigns to the various seat splits which would arise from the enacted Exectutive Council map under uniform swings of the 2016 Presidential election:

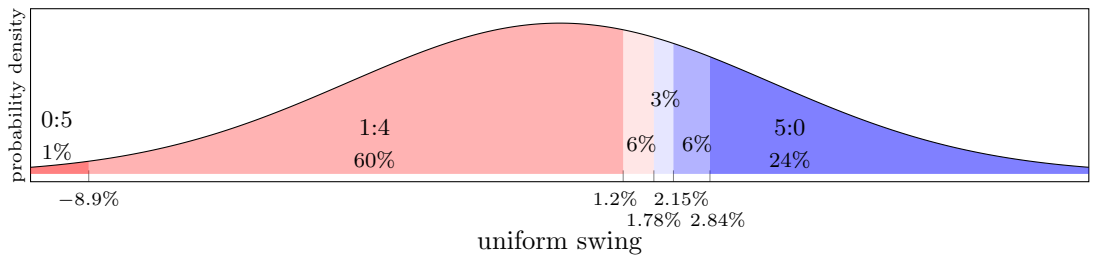


Figure 1: A normally distributed uniform swing applied to the enacted Executive Council districting.

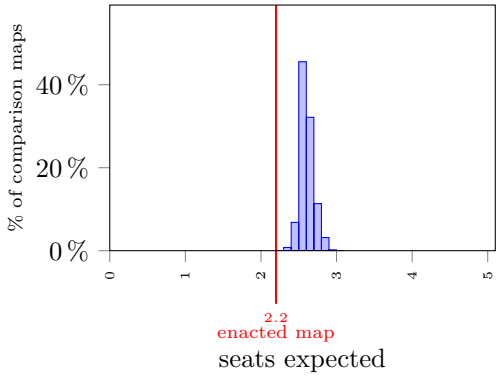
In particular, we can list the probability of any number of Democratic:Republican seat split for the enacted Executive Council plan according to this uniform swing model using the 2016 Presidential race:

0:5	1:4	2:3	3:4	4:1	5:0
1%	60%	6%	3%	6%	24%

The weighted average of these Democratseat outcomes is then computed as

$$.01 \times 0 + .60 \times 1 + .06 \times 2 + .03 \times 3 + .06 \times 4 + .24 \times 5 \approx 2.2.$$

This "seats expected" number for the Executive Council plan under the 2016 Presidential plan shows up in our analysis on page 16, in a histogram we reproduce here for the purpose of illustration, where it is compared to distribution of seats-expected values for the set of comparison maps generated by my algorithm:



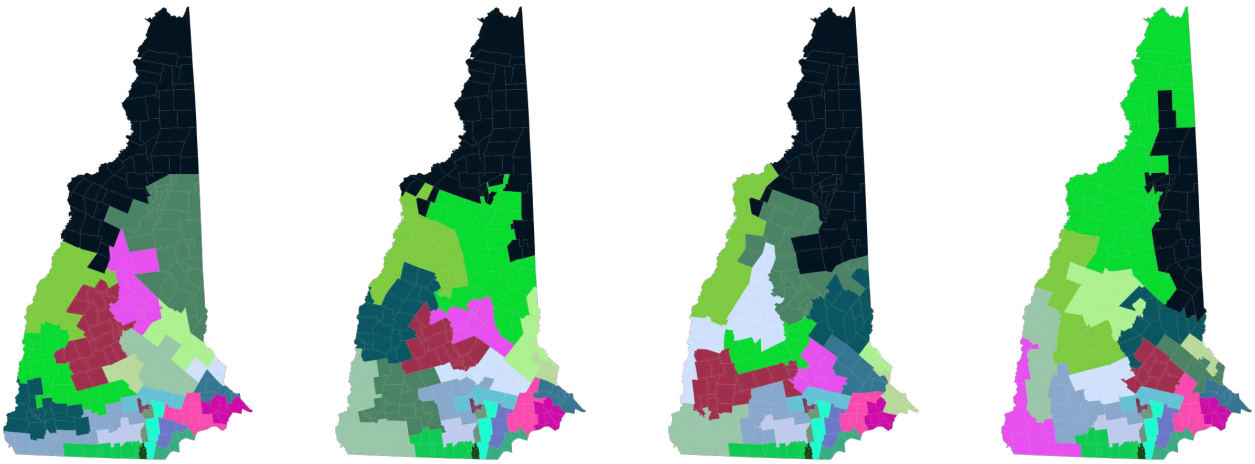
It is important to note that **my method does not evaluate the fairness of a districting by whether it produces a "small" or "large" number of seats for one party, or whether the seats-expected score calculated in this way is lower or higher than would be expected in a system of proportional representation**. Instead, this score is merely a metric used to **compare** one map to another. The only way these scores are used in my method is to evaluate which of two maps may be more advantageous to a particular political party, and when I find that a districting made extreme use of partisan consideration, it means that the enacted map is extreme outlier with respect to how optimized for partisanship it is **compared to the set of alternative comparison districtings of New Hampshire** satisfying the districting criteria I impose.

[2] For reference, the standard deviation of the historical elections I use in this report is 6.5%; it would be 3% when calculated without the 2020 Governor election.

6 Analysis: Senate

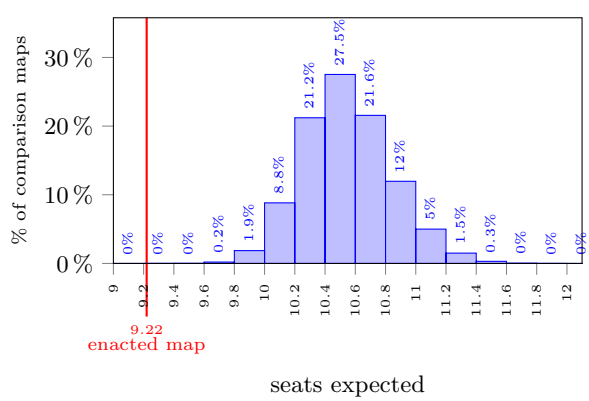
6.1 Main Analysis

Comparison map examples



Results

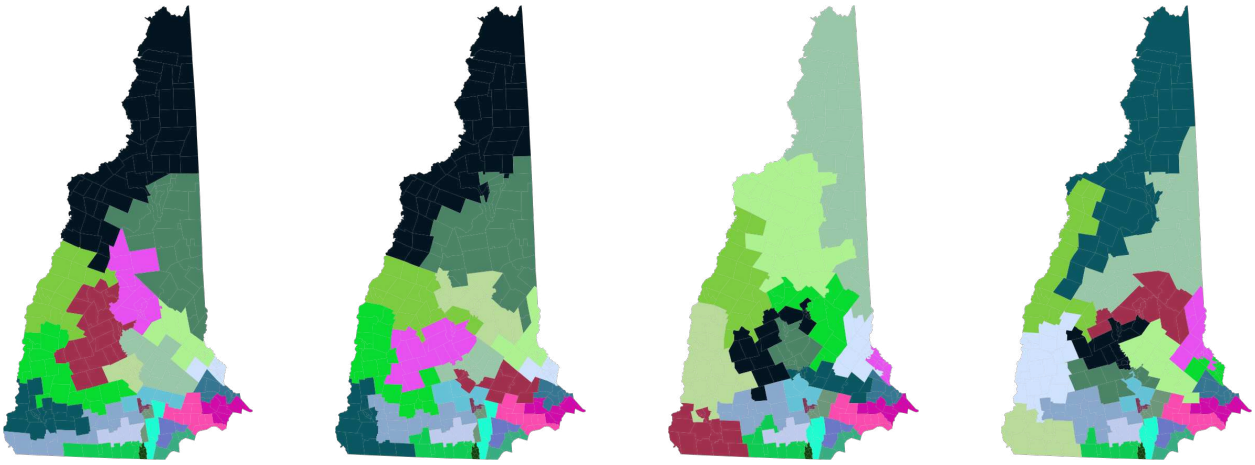
Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99971%	9	99.999904%	17	99.999949%	25	99.999988%
2	99.99968%	10	99.99971%	18	99.999985%	26	99.9989%
3	99.99981%	11	99.999955%	19	99.99972%	27	99.99986%
4	99.99936%	12	99.99978%	20	99.99966%	28	99.99978%
5	99.999994%	13	99.99943%	21	99.99983%	29	99.99978%
6	99.99982%	14	99.999943%	22	99.99939%	30	99.999931%
7	99.99931%	15	99.999982%	23	99.99978%	31	99.99979%
8	99.99991%	16	99.999907%	24	99.999982%	32	99.99971%



- **First level analysis:** In *every* run, the districting was in the most partisan 0.0011% of districtings (in other words, 99.999% were less partisan, in *every* run).
- **Second level analysis:** My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.025% of all alternative districtings of New Hampshire satisfying my districting criteria (in other words, 99.975% are less optimized-for-partisanship), measured by their ε-fragility for $\varepsilon = 0.0011\%$.

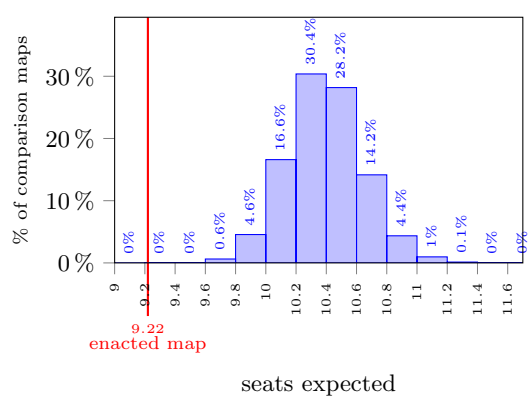
6.2 Analysis with incumbency protection

Comparison map examples



Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.9998%	9	99.999961%	17	99.999949%	25	99.999988%
2	99.99969%	10	99.99976%	18	99.999985%	26	99.99987%
3	99.99981%	11	99.999%	19	99.99973%	27	99.99953%
4	99.99955%	12	99.99985%	20	99.99963%	28	99.99978%
5	99.999994%	13	99.999934%	21	99.999967%	29	99.99982%
6	99.99965%	14	99.999961%	22	99.99962%	30	99.999931%
7	99.999949%	15	99.999982%	23	99.99965%	31	99.99978%
8	99.99989%	16	99.999907%	24	99.999982%	32	99.99961%

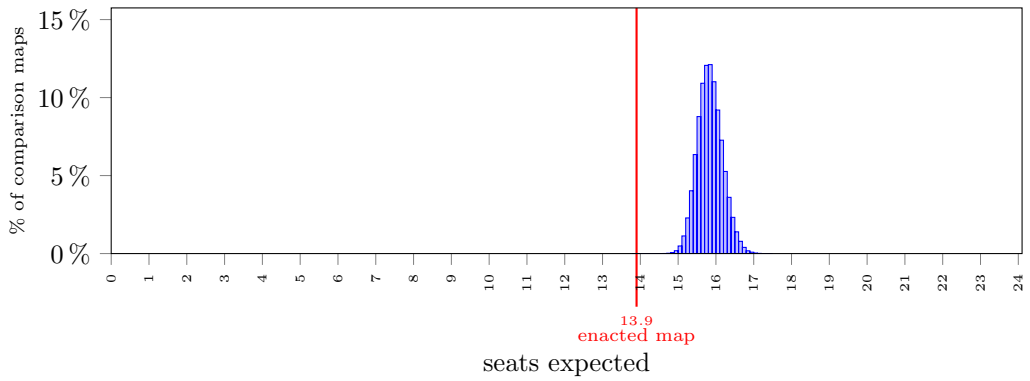


- **First level analysis:** In *every* run, the districting was in the most partisan 0.0011% of districtings (in other words, 99.999% were less partisan, in *every* run).
- **Second level analysis:** My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.025% of all alternative districtings of New Hampshire satisfying my districting criteria (in other words, 99.975% are less optimized-for-partisanship), measured by their ε-fragility for $\varepsilon = 0.0011\%$.

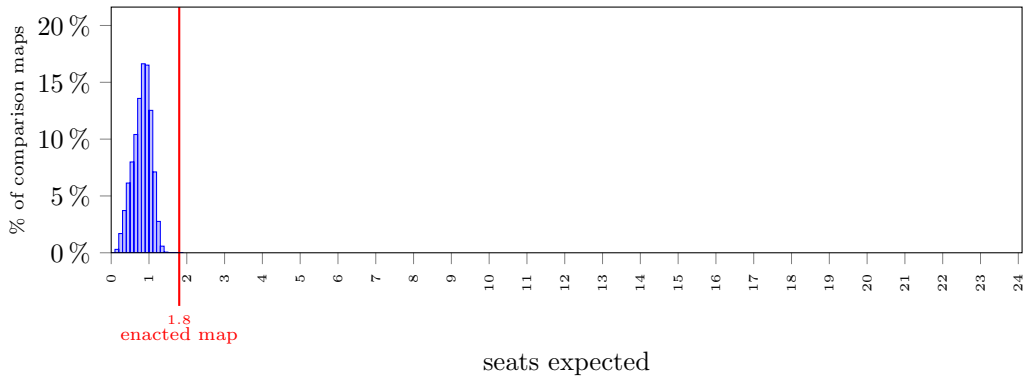
6.3 Results for individual elections

Below we show histogram results for individual elections. In each case we indicate the 2-party vote share of the Democrat in the election. We see that the map builds in structural advantages for Republicans which allows them to outperform the vast majority of comparison maps through the lens of close elections. The only election where the enacted map gives Repulicans less seats on average than a typical comparison map is the 2020 Governor election, where the Republican Chris Sununu had a 66% 2-party vote share (and where the enacted map still gives Democrats fewer than 2 out of 24 seats in expectation).

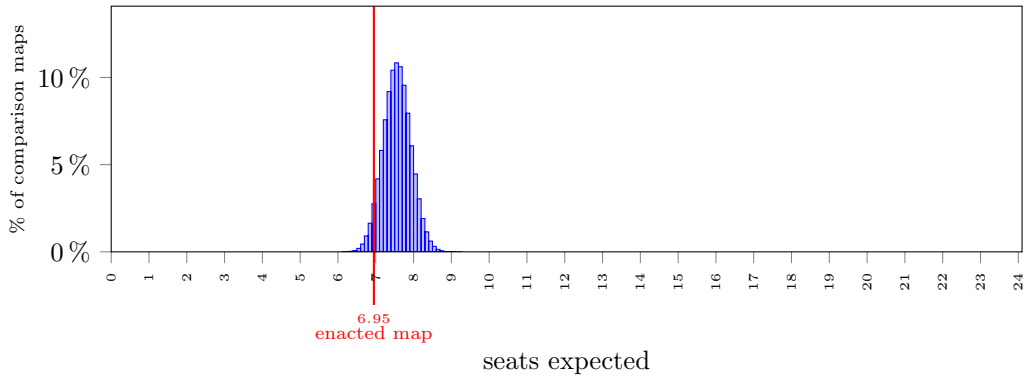
2020 Presidential Election, 53.7% **TPVS for Democrats**



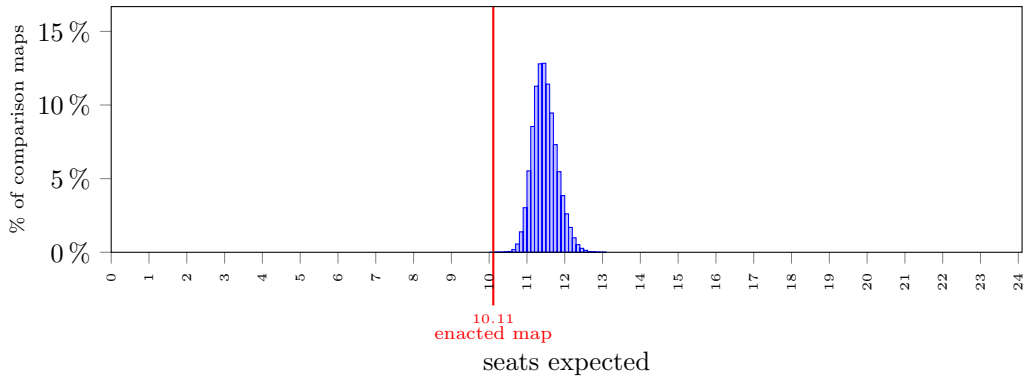
2020 Governor Election, 33.9% **TPVS for Democrats**



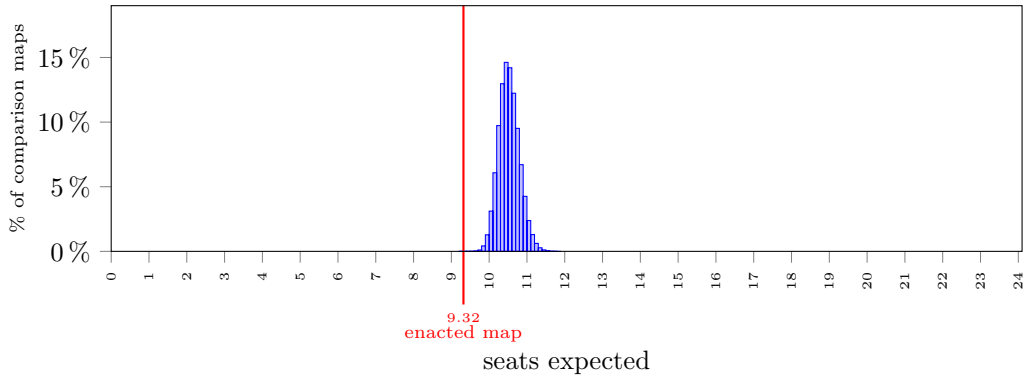
2018 Governor Election, 46.4% **TPVS for Democrats**



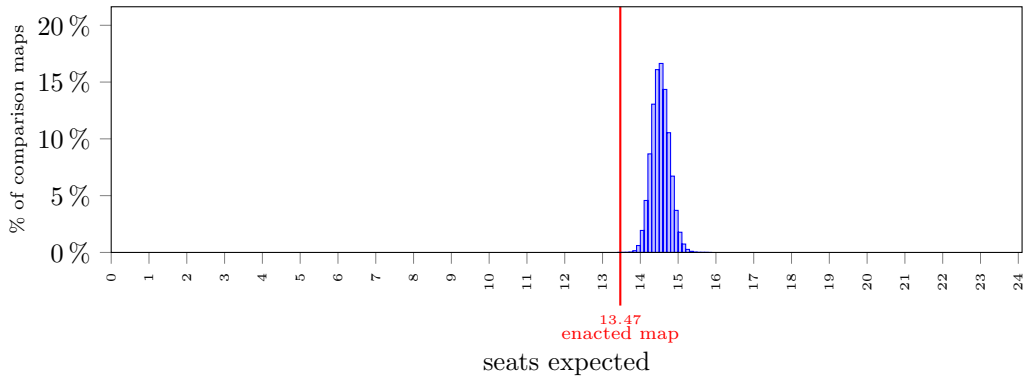
2016 Presidential Election, 50.2% **TPVS for Democrats**



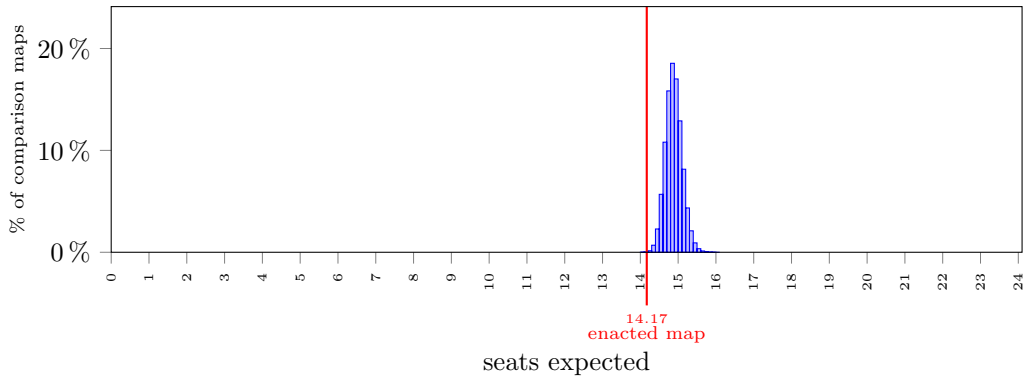
2016 Governor Election, 48.8% **TPVS for Democrats**



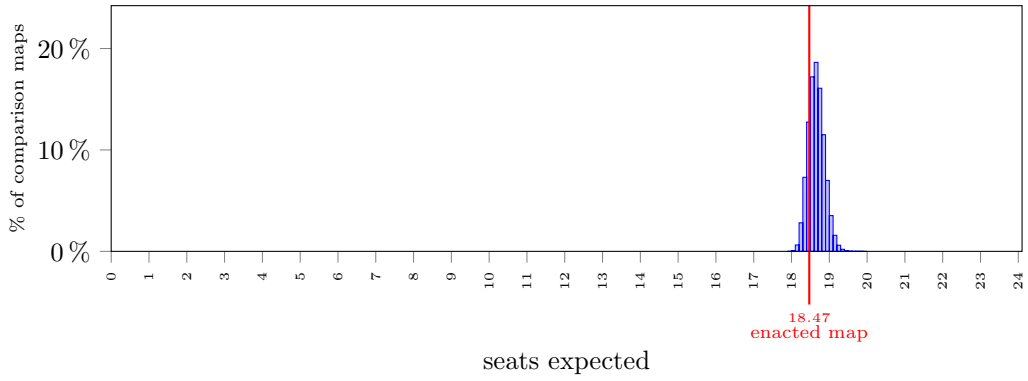
2014 Governor Election, 52.5% **TPVS for Democrats**



2012 Presidential Election, 52.8% **TPVS for Democrats**



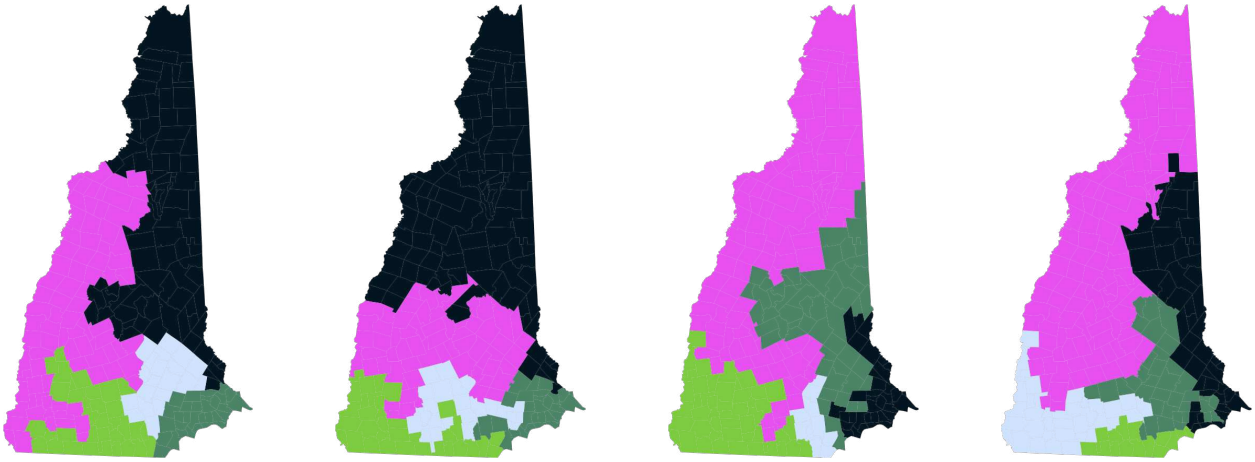
2012 Governor Election, 56.23% **TPVS for Democrats**



7 Analysis: Executive Council

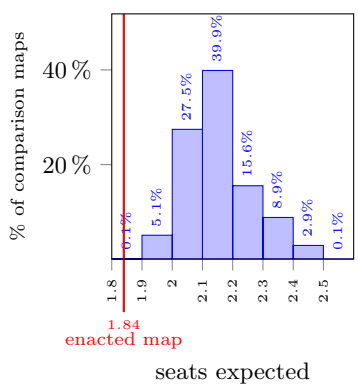
7.1 Main Analysis

Comparison map examples



Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99925%	9	99.9993%	17	99.9999985%	25	99.99957%
2	99.99948%	10	99.9996%	18	99.99983%	26	99.9997%
3	99.99947%	11	99.99978%	19	99.99972%	27	99.99989%
4	99.99928%	12	99.99973%	20	99.99973%	28	99.99984%
5	99.99963%	13	99.99983%	21	99.999929%	29	99.99975%
6	99.99984%	14	99.99918%	22	99.999961%	30	99.9997%
7	99.9983%	15	99.99998%	23	99.999969%	31	99.99971%
8	99.99925%	16	99.999981%	24	99.99944%	32	99.9987%

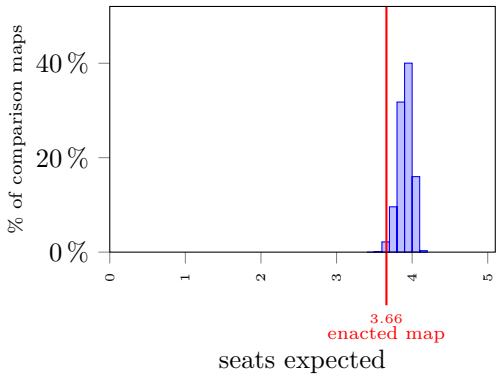


- **First level analysis:** In *every* run, the districting was in the most partisan 0.0016% of districtings (in other words, 99.9984% were less partisan, in *every* run).
- **Second level analysis:** My theorems imply that the enacted districting is among the most optimized-for-partisanship 0.04% of all alternative districtings of New Hampshire satisfying my districting criteria (in other words, 99.96% are less optimized-for-partisanship), measured by their ε-fragility for $\varepsilon = 0.0015\%$.

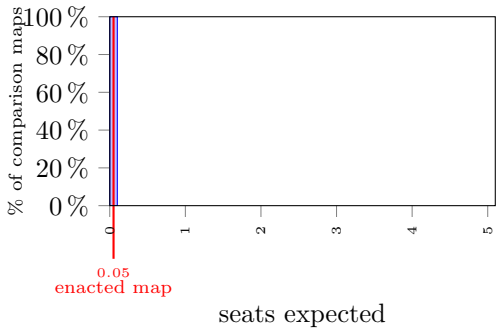
7.2 Results for individual elections

Below we show histogram results for individual elections. In each case we indicate the 2-party vote share of the Democrat in the election. We see that the map builds in structural advantages for Republicans which allows them to outperform the vast majority of comparison maps through the lens of close elections.

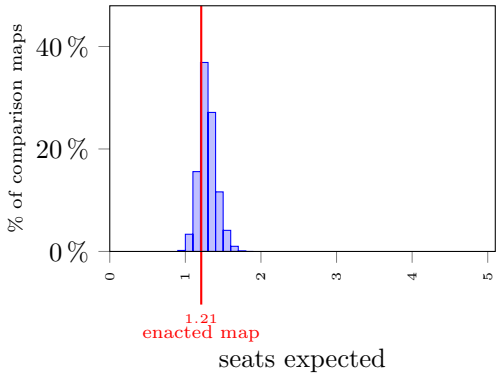
2020 Presidential Election, 53.7% **TPVS for Democrats**



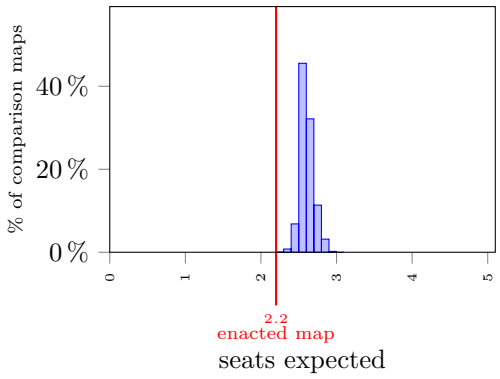
2020 Governor Election, 33.9% **TPVS for Democrats**



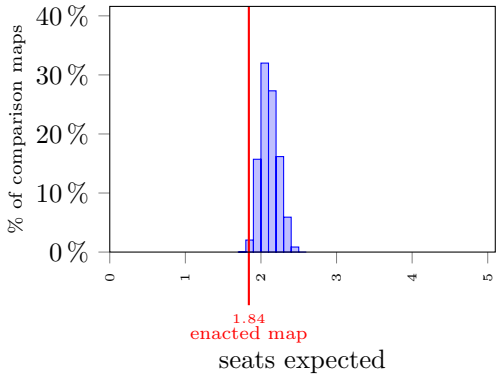
2018 Governor Election, 46.4% **TPVS for Democrats**



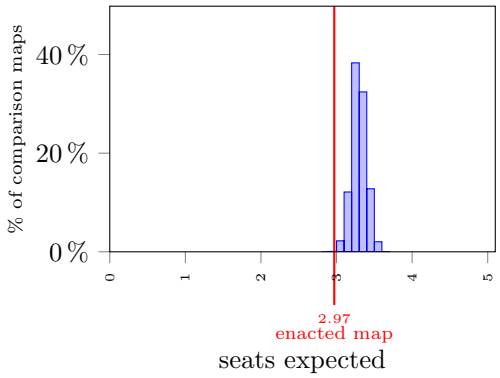
2016 Presidential Election, 50.2% **TPVS for Democrats**



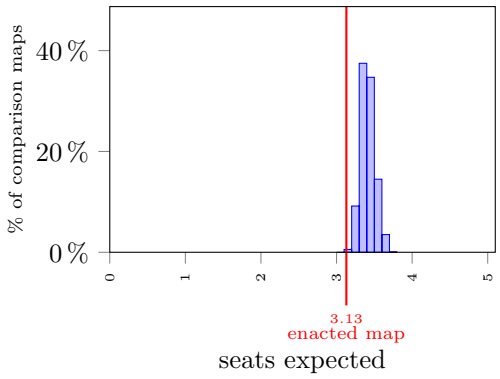
2016 Governor Election, 48.8% **TPVS for Democrats**



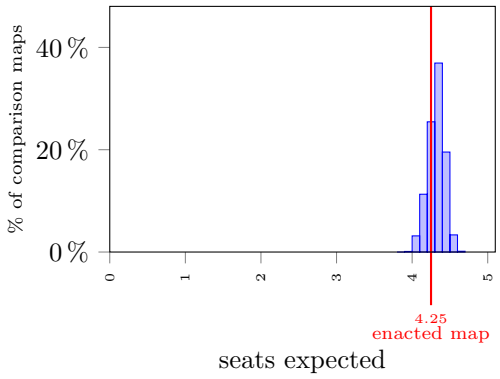
2014 Governor Election, 52.5% **TPVS for Democrats**



2012 Presidential Election, 52.8% **TPVS for Democrats**



2012 Governor Election, 56.23% **TPVS for Democrats**



Appendix A Robustness checks

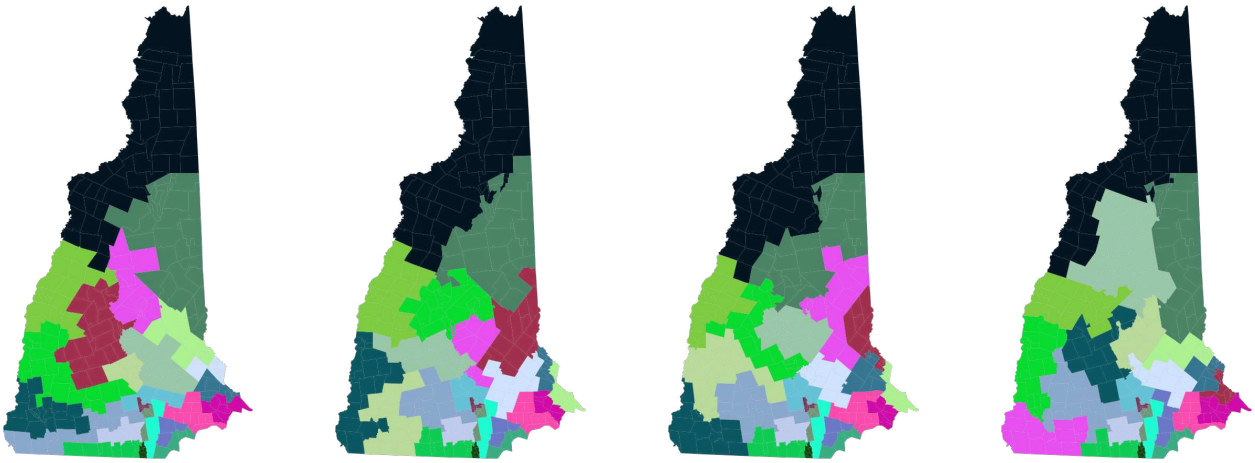
In this section we show robustness of our analysis to compactness and population thresholds.

A.1 Compactness thresholds

For each of the two enacted plans we have analyzed, we show alternative runs here with 0% and 10% thresholds, respectively, for the compactness of comparison districtings.

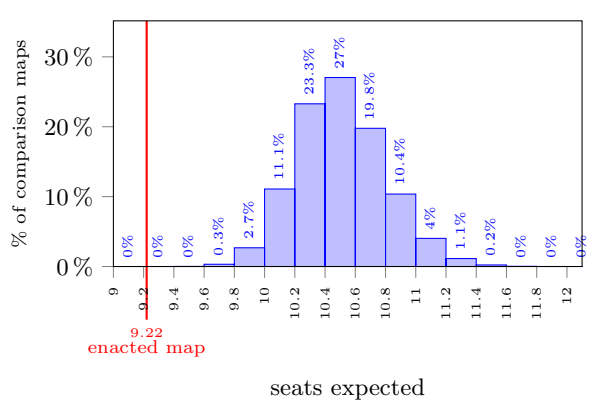
A.1.1 Senate, 0% threshold

Comparison map examples



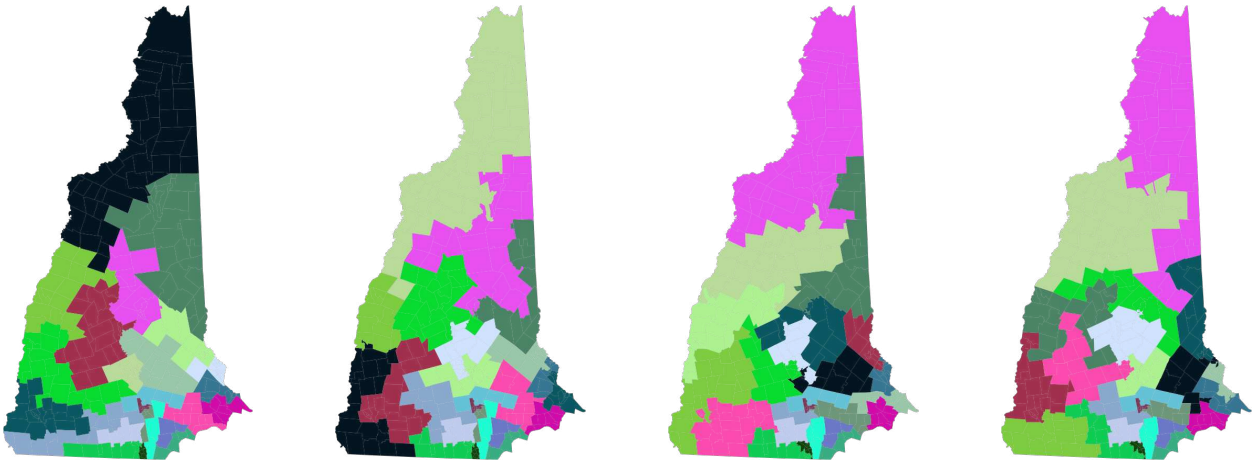
Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99969%	9	99.999961%	17	99.999949%	25	99.99976%
2	99.99957%	10	99.99904%	18	99.99982%	26	99.99977%
3	99.99969%	11	99.99949%	19	99.99981%	27	99.99984%
4	99.99987%	12	99.99986%	20	99.999928%	28	99.99912%
5	99.999994%	13	99.999922%	21	99.99979%	29	99.99987%
6	99.99988%	14	99.999943%	22	99.9995%	30	99.999931%
7	99.99972%	15	99.99984%	23	99.99982%	31	99.99979%
8	99.99948%	16	99.99985%	24	99.99974%	32	99.99985%



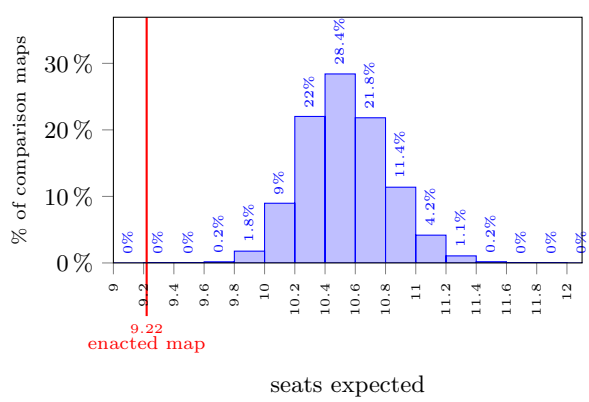
A.1.2 Senate, 10% threshold

Comparison map examples



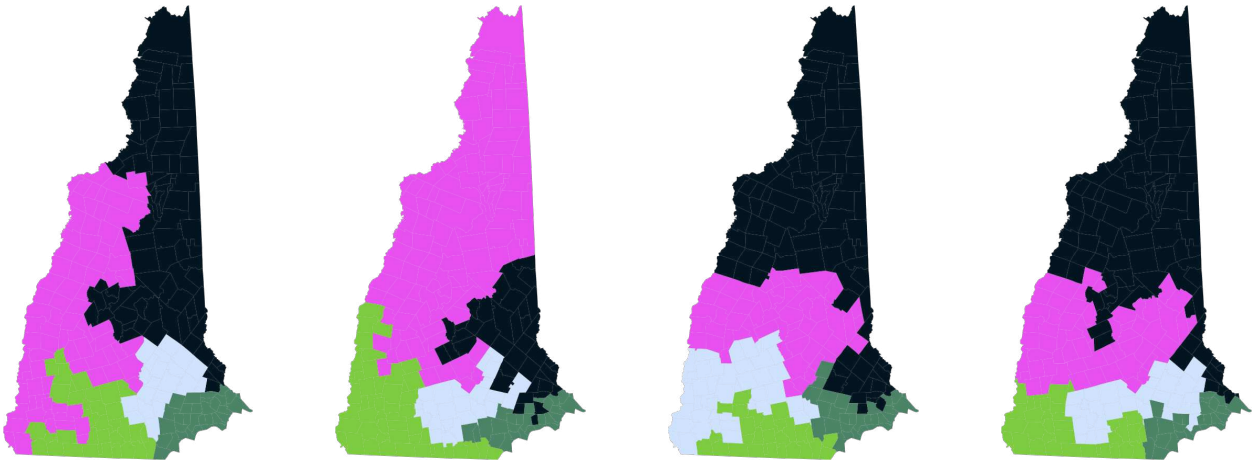
Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99925%	9	99.9997%	17	99.999949%	25	99.999988%
2	99.99968%	10	99.99971%	18	99.999985%	26	99.99911%
3	99.99981%	11	99.9988%	19	99.999%	27	99.999913%
4	99.99974%	12	99.999961%	20	99.999928%	28	99.99978%
5	99.999994%	13	99.99916%	21	99.99983%	29	99.999916%
6	99.99982%	14	99.999943%	22	99.99926%	30	99.999931%
7	99.99928%	15	99.999982%	23	99.99978%	31	99.99979%
8	99.99991%	16	99.999907%	24	99.999982%	32	99.99971%



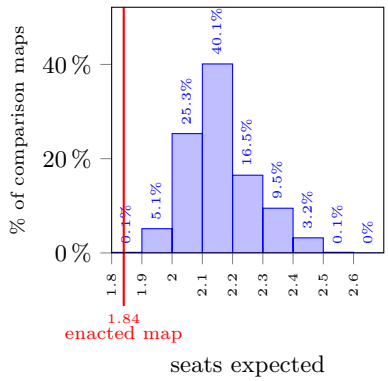
A.1.3 Executive Council, 0% threshold

Comparison map examples



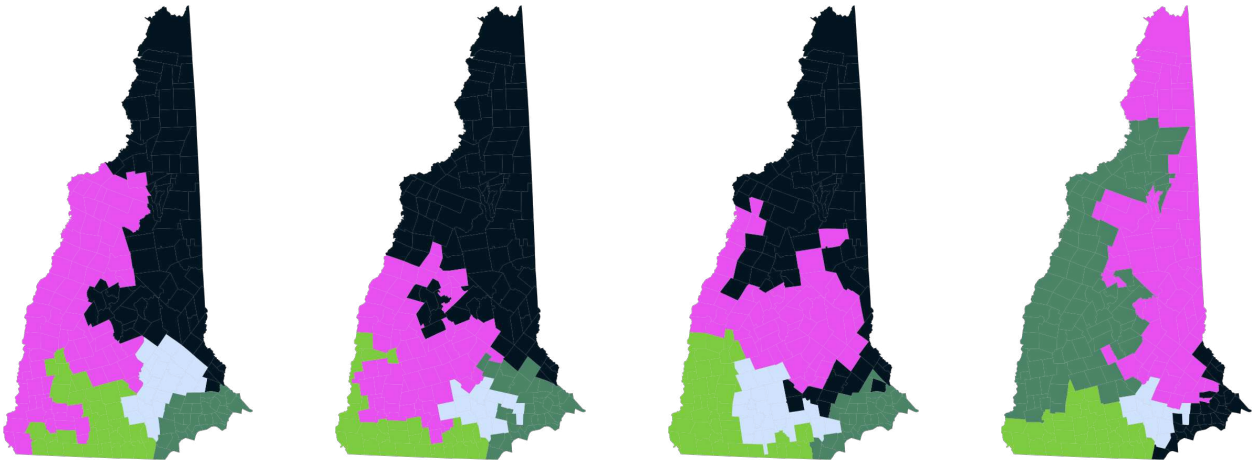
Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99968%	9	99.99977%	17	99.99977%	25	99.99962%
2	99.99975%	10	99.99981%	18	99.99979%	26	99.99978%
3	99.99983%	11	99.99988%	19	99.9988%	27	99.99968%
4	99.99977%	12	99.99945%	20	99.99981%	28	99.99987%
5	99.999942%	13	99.999946%	21	99.99973%	29	99.9988%
6	99.99923%	14	99.9992%	22	99.9998%	30	99.99928%
7	99.99977%	15	99.99937%	23	99.999984%	31	99.99968%
8	99.99977%	16	99.99977%	24	99.9981%	32	99.99981%



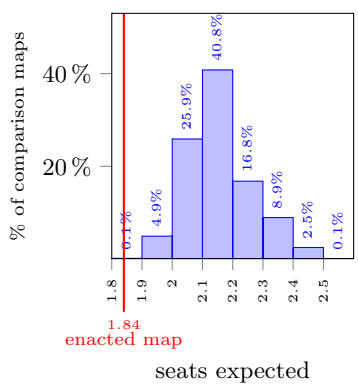
A.1.4 Executive Council, 10% threshold

Comparison map examples



Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99904%	9	99.9999921%	17	99.99939%	25	99.99933%
2	99.99968%	10	99.99961%	18	99.99979%	26	99.99961%
3	99.99956%	11	99.99965%	19	99.99986%	27	99.99936%
4	99.99929%	12	99.99988%	20	99.99968%	28	99.99989%
5	99.999932%	13	99.99935%	21	99.99908%	29	99.99934%
6	99.99969%	14	99.999931%	22	99.99983%	30	99.99978%
7	99.9986%	15	99.999982%	23	99.99927%	31	99.99925%
8	99.99969%	16	99.999938%	24	99.9995%	32	99.9984%

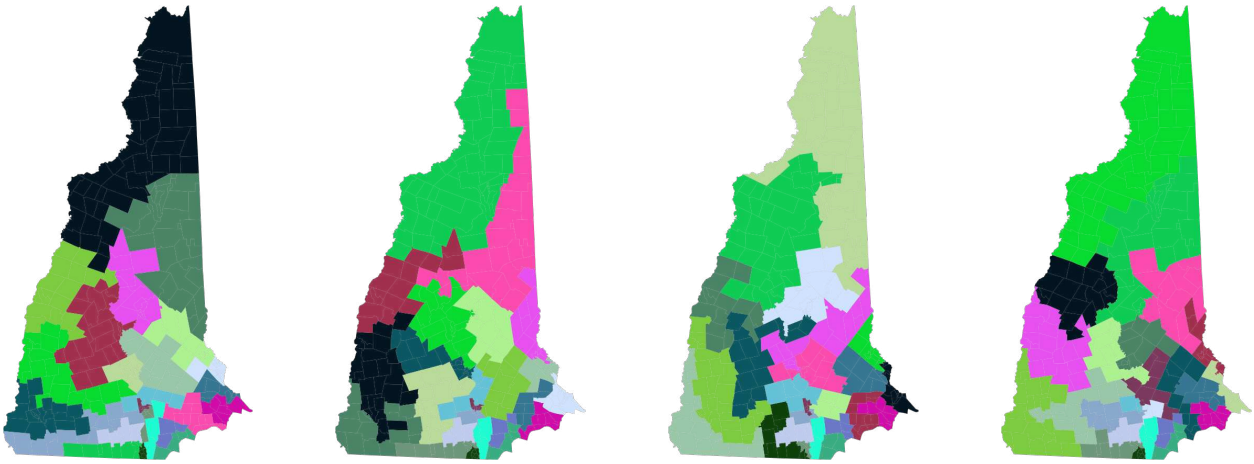


A.2 Population Thresholds

In our main analysis, we use constraints on population deviation from ideal which are just barely permissive enough to ensure the enacted map itself satisfies the constraints we impose. Here we show runs where a population deviation twice as large is allowed.

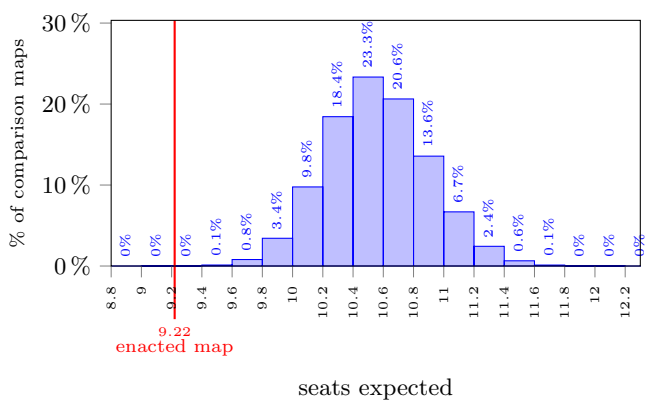
A.2.1 Senate, 10% threshold

Comparison map examples



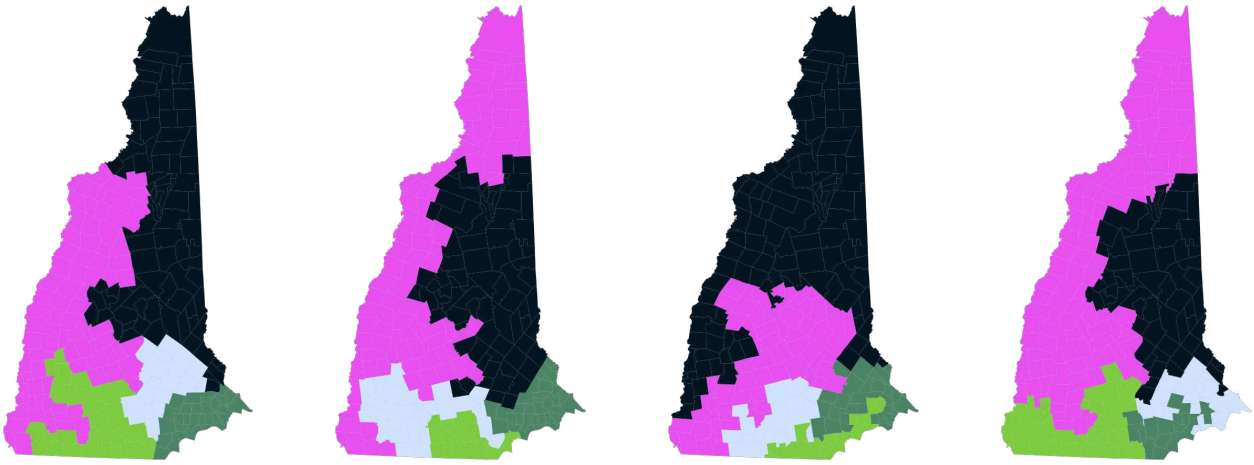
Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.99956%	9	99.99903%	17	99.99954%	25	99.99971%
2	99.99959%	10	99.9993%	18	99.99921%	26	99.9961%
3	99.99954%	11	99.9975%	19	99.9998%	27	99.99983%
4	99.9966%	12	99.99918%	20	99.99941%	28	99.99952%
5	99.9939%	13	99.9995%	21	99.9964%	29	99.9978%
6	99.9983%	14	99.9977%	22	99.9958%	30	99.99988%
7	99.9992%	15	99.999982%	23	99.9985%	31	99.999991%
8	99.9989%	16	99.998%	24	99.99985%	32	99.99911%



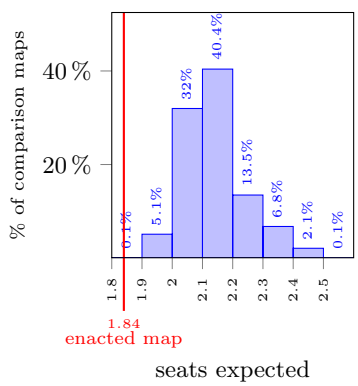
A.2.2 Executive Council, 2% threshold

Comparison map examples



Results

Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan	Run	Percentage of comparison maps less partisan than enacted plan
1	99.999931%	9	99.99925%	17	99.99951%	25	99.99958%
2	99.99949%	10	99.9998%	18	99.999976%	26	99.99966%
3	99.99945%	11	99.99964%	19	99.99979%	27	99.99976%
4	99.99953%	12	99.99947%	20	99.99987%	28	99.99951%
5	99.9988%	13	99.99972%	21	99.99935%	29	99.99967%
6	99.999987%	14	99.99971%	22	99.99968%	30	99.999934%
7	99.99986%	15	99.99951%	23	99.99979%	31	99.999976%
8	99.99968%	16	99.99964%	24	99.99946%	32	99.99962%



Appendix B Multimoves / Precinct splits

As discussed in Section 5 my algorithm can be set to allow multiple changes to a map to occur in one step, when this is necessary to produce a sufficiently rich set of comparison maps.

Here I describe details of this technique so that technical experts can understand how precisely our method works. These details are not necessary to understand the basic mechanics of the method, which are simply that:

- Multiple changes may be made to a map in a single step,

- The result of the changes must always be a valid comparison map, in the sense that it complies with the districting criteria we consider in our report, and
- Our implementation of multiple moves does not bias the algorithm to any map or family of maps.

For technical experts: these multiple moves can be implemented with a Metropolis-Hastings approach. In particular, a score function based on the deviation of an invalid map from the compactness and population thresholds can be defined. *The score function is set to be equal for all maps satisfying the districting criteria.* With this choice, a uniform stationary distribution can be constructed on the space of maps satisfying the districting criteria. The Metropolis-Hastings chain will occasionally leave the feasible region of the map-space for some number of steps before returning to the feasible region. The collection of steps made outside the feasible region can be performed in a single step, to give a single multi-move which transforms one valid map into another valid map.

Appendix C Theorems

The second level analyses in my report are calculated using the theorems from [CFMP]; in particular, Theorem 1.5 from that manuscript suffices for all of my second-level findings here.

In plain language, that theorem says that if I conduct m runs, and observe that in every run the enacted plan is in the bottom ε fraction of comparison maps, then I can conclude that the enacted plan is among the most carefully crafted α fraction of *all* maps satisfying the districting criteria (not just those encountered by the algorithm), measured by their ε-fragility, at a statistical significance calculated with the formula

$$p = \left(\frac{2\varepsilon}{\alpha}\right)^{m/2}.$$

In this report, I frequently have $m = 32$ runs and choose α to simply be 3 times as big as ε. In this case, we see that we can conclude that the enacted plan is among the most carefully crafted 3ε of all maps, at a statistical significance of

$$p = \left(\frac{2}{3}\right)^{16} \approx .0015 < .002.$$

Note that, for example, if we used instead a threshold of $\alpha = 4\varepsilon$, this would give significance of

$$p = \left(\frac{2}{4}\right)^{16} \approx .000015,$$

and taking a threshold of $\alpha = 6\varepsilon$ would give

$$p = \left(\frac{2}{6}\right)^{16} \approx .00000002,$$

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Assistant Professor, 2013–2017

Postdoctoral **Courant Institute, New York, NY**

NSF Postdoctoral Fellow, 2010–2013

Education **Rutgers University, New Brunswick, NJ**

Ph.D., May 2010.
Advisor: József Beck
Thesis: "Graphs, games and geometry".

Budapest Semesters in Mathematics, Budapest, Hungary: 2004–2005

University of Chicago: 2001–2004.
BA in Mathematics, with Honors.

Grants, Fellowships, and Awards

National Academy of Science Kavli Fellow
NSF Grant DMS-1700365 (2017–2020)
Sloan Fellowship (2016–2018)
NSF Grant DMS-1363136 (2014–2017)
NSF Postdoctoral Research Fellowship (2010–2013)
Torrey Fellow (Rutgers, 2005–2007)

Publications

Separating effect from significance in Markov chain tests, with M. Chikina, A. Frieze, J. Mattingly.
in *Statistics and Public Policy* **7** (2020) 101-114.

On random multi-dimensional assignment problems, with A. Frieze, T. Tkocz
in *Discrete Applied Mathematics*, **287** 1-9 (2020).

Stability of patterns in the Abelian sandpile, with C. Smart.
Annales Henri Poincaré **21**, pages 1383–1399 (2020).

A note on the localization number of random graphs: diameter two case, with A. Frieze, A. Dudek.
Discrete Applied Mathematics **254** (2019), 107-112.

On the rank of a random binary matrix, with A. Frieze, C. Cooper.
Proceedings of SODA 2019.

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Constraining the clustering transition for colorings of sparse random graphs, with M. Anastos, A. Frieze.
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Assessing significance in a Markov chain without mixing, with M. Chikina and A. Frieze.
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Looking for vertex number one, with A. Frieze.
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Apollonian structure in the Abelian Sandpile, with L. Levine and C. Smart.
GAFA **26** (2016) 306-336.

Separating subadditive Euclidean functionals, with A. Frieze.
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Walker-Breaker games, with L. Espig, A. Frieze, and M. Krivelevich.
SIAM J. Discrete Math. **29** (2015).

The topology of competitively constructed graphs, with A. Frieze.
The Electronic Journal of Combinatorics **21** #P2.26.

Convergence of the Abelian Sandpile, with Charles K. Smart.
Duke Mathematical Journal **162** (2013) 627–642.

An extension of the Moser-Tardos algorithmic Local Lemma.
SIAM J. Discrete Math. **28** (2014)

Critical graphs without triangles: an optimum density construction.
Combinatorica **33** (2013) 495–512

The Lefthanded Local Lemma characterizes chordal dependency graphs.
Random Structures & Algorithms **41** (2012) 546–556

Highly nonrepetitive sequences: winning strategies from the Local Lemma.
Random Structures & Algorithms **38** (2011)

Sets resilient to erosion.
Advances in Geometry **11** (2011) 201–224.

The Hales-Jewett number is exponential, with J. Beck and S. Vijay.
Analytic Number Theory: Essays in Honour of Klaus Roth (Eds: W.W.L. Chen, W.T. Gowers, H. Halberstam, W.M. Schmidt, R.C. Vaughan), Cambridge University Press 2009.

A finite goal set in the plane which is not a winner.
Discrete Mathematics **308** (2008) 6546–6551.

Distance Sequences in Locally Infinite Vertex-Transitive Digraphs.
Combinatorica **26** (2006) 577–585.

Preprints

Extremal Collections of k-Uniform Vectors, with J. Briggs
Submitted. Preprint at `https://arxiv.org/abs/1801.09609`.

A partisan districting protocol with provably nonpartisan outcomes, with A. D. Procaccia, D. Yu.
Submitted. Preprint at `https://arxiv.org/abs/1710.08781`.

On the cover time of the emerging giant, with A. Frieze, T. Tkocz
Submitted. Preprint at `https://arxiv.org/abs/1808.09608`.

Minors of a random binary matroid, with C. Cooper and A. Frieze.
Submitted. Preprint at `https://arxiv.org/abs/1612.02084`

Scalefree hardness of average-case Euclidean TSP approximation, with A. Frieze.
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Coverage in Popular Media

The amazing, autotuning sandpile. Article in *Nautilus* magazine (2015) by J. Ellenberg. `http://nautil.us/issue/23/dominoes/the-amazing-autotuning-sandpile.`

Das Wahnsinnsamt, Sandhäufchen und Apollonische Dreiecke. Article in *Spektrum der Wissenschaft* (German-language edition of Scientific American) by C. Pöppe, pages 67–71, August 2015.

Piling on and on and on.... Article and podcast interview by M. Breen, at `http://www.ams.org/samplings/mathmoments/mm117-sandpiles-podcast` (2015).

On sandpiles. Coverage in *AMS Math in the media* column, by Allyn Jackson (April 2015).

Math and the gerrymander. Coverage in *AMS Math in the media* column, by Tony Phillips (March 2017).

Study: Math proves Pennsylvania's congressional districts 'almost certainly' gerrymandered. Story in *Philly Voice* by Daniel Craig (March 1, 2017).

Where Allegheny County's Harrisburg delegation stands on redistricting reform. Story in *The Incline* by Sarah Anne Hughes (March 23, 2017).

Groups sue Pennsylvania over congressional district map, citing gerrymandering. Story

in the *Pittsburgh Post-Gazette* by Chris Potter (June 15, 2017).

Cake-cutting game theory trick could stop gerrymandering. Story in the *New Scientist* by Timothy Revell (November 1, 2017).

Top 100 Science stories of 2017 (**#17: fighting politics with math**). *Discover Magazine*, by Stephen Ornes, January-February 2018 issue.

TALKS

Social Science Applications Forum, Center of Mathematical Sciences and Applications, Harvard University, March 11, 2019.

The Statistical and Applied Mathematical Sciences Institute at Duke University, October 8 2018.

[*Colloquium*] University of Wisconsin, February 9 2018.

[*Colloquium*] University of Toronto, January 17 2018.

[*Colloquium*] Duke University, January 11 2018.

SIAM annual meeting, Random Structures mini-session, April 20, 2017.

Ohio State Discrete Math Seminar, April 20, 2017, at Ohio State University.

Atlanta Lecture Series in Combinatorics and Graph Theory XVIII, October 22-23, 2016, at Emory University.

Princeton Discrete Mathematics Seminar, October 13, 2016, at Princeton University.

[*Conference*]*STOC 2016*, June 19 2016, in Boston, MA.

Princeton Discrete Mathematics Seminar, March 10, 2016, at Princeton University.

University of Chicago Theory Seminar, October 20, 2015 at the University of Chicago.

CMU CS Theory Seminar, May 14, 2015.

[*Colloquium*] University of Geneva, March 5 2015.

Ohio State Discrete Math Seminar, November 6 2014, at Ohio State University.

[*Conference*] *SIAM DM14, Special Session on Combinatorics and Statistical Mechanics*, June 18 2014 in Minneapolis, MN (2 talks)

Princeton Discrete Math Seminar, March 13 2014, at Princeton University.

AIM workshop: Generalizations of chip-firing and the critical group, July 2013 at AIM.

[*Conference*] *Special Session on Combinatorics and Classical Integrability* at the AMS Spring Eastern Sectional Meeting, April, 2013 at Boston College.

[*Colloquium*] University of Illinois at Urbana-Champaign, January 30, 2013.

[*Colloquium*] CMU, January 16, 2013.

[*Colloquium*] University of Illinois at Chicago, December 5, 2012.

Cornell Workshop on Sandpiles and Number Theory, October 2012 at Cornell University in Ithaca, NY.

MIT Combinatorics Seminar, April 27, 2012, at MIT.

UPenn seminar on Combinatorics and Probability, February 21, 2012, at the University of Pennsylvania.

Rutgers Discrete Math Seminar, February 7, 2012, at Rutgers University in New Brunswick.

Princeton Discrete Math Seminar, September 27, 2012 at Princeton University.

Probabilistic Combinatorics Mini-symposium of SIAM DM12, June 19, 2012 in Halifax, Nova Scotia.

[*Conference*] *the 15th conference on Random Structures & Algorithms*, May 24, 2012, at Emory University.

Columbia Discrete Math Seminar, February 14, 2012 at Columbia University in New York, NY.

Rutgers Discrete Math Seminar, February 1, 2011 at Rutgers University in New Brunswick.

New York Number Theory Seminar, November 4, 2010.

Columbia Discrete Math Seminar, October 27, 2009 at Columbia University in New York, NY.

Princeton Discrete Math Seminar, October 22, 2009 at Princeton University.

The 14th International Conference on Random Structures and Algorithms, in Poznań, Poland, August 2009.

[*Conference*] *Special Session on Probabilistic and Extremal Combinatorics*, at the 2009 AMS Spring Sectional Meeting, UIUC in Urbana-Champaign, IL.

Rutgers Discrete Mathematics Seminar, April 28 at Rutgers University in New Brunswick.

[*Conference*] *National AMS meeting*, in Washington, DC, January 2009.

Rutgers Experimental Mathematics Seminar, February 7 at Rutgers University in New Brunswick.

Princeton Discrete Mathematics Seminar, in December 2007 at Princeton University.

[*Conference*] *Workshop on Extremal Combinatorics*, Alfred Renyi Institute of Mathematics, in Budapest, Hungary, June 2007.