EXHIBIT A

Expert Report for *Harkenrider et al. v. Hochul*

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March 9, 2022

I was asked by legal counsel in this case to examine the 2022 New York Senate and Congressional district plans. In particular, I was asked to examine the potential politically partisan impact of the newly enacted plans. In making my findings, I have applied standard statistical methods, which I regularly employ in my research and which have been published in peer-reviewed journals, to historical election returns and demographic data in New York.

A summary of my report and basic findings is as follows:

- Using historical election data, I find that the enacted 2022 Senate plan shows no statistically significant partisan bias in favor of either party.

- Using historical election data, I find that the enacted 2022 Congressional plan shows no statistically significant partisan bias in favor of either party.

In the next section of the report I review my qualifications. In Section 2, I discuss how to quantify and statistically estimate the partisan impact of electoral maps. Section 3 discusses the statistical model used to estimate partisan fairness. Section 4 provides an analysis of partisan bias for the enacted 2022 Senate map. Section 5 provides an analysis of partisan bias for the enacted 2022 Congressional map.

1   Qualifications

I am currently the Kay Sugahara Professor of Social Sciences and Statistics at the California Institute of Technology (Caltech). I previously served for seven years as the Chair of the Division of the Humanities and Social Sciences at Caltech (which is akin to being a dean at other universities). Further, I was also formerly on the faculty at the University of Chicago and a visiting professor at the University of Konstanz (Germany). A complete copy of my curriculum vitae is in Attachment 1 to this report.

I received my Bachelor of Science degree from the Massachusetts Institute of Technology and my Masters of Arts and Doctor of Philosophy degrees, both in political science, from the University of California, San Diego. I did post-doctoral work at Harvard University and the Harvard-MIT Data Center. I am an elected fellow of both the American Academy of Arts and Sciences and an inaugural fellow of the Society for Political Methodology. I am a former fellow of the Center for Advanced Study in the Behavioral Sciences.

I have written numerous articles published in the leading journals as set forth in my curriculum vitae. I am currently a Deputy Editor for Social Sciences of *Science Advances*, the open access journal of the American Association for the Advancement of Science. I previously served as co-editor of *Political Analysis*, the journal of the Society for Political Methodology, and I was a co-founding editor of the Political Science network (a collection of on-line journals). I have also previously served on the editorial boards of *Electoral Studies*, *Political Research Quarterly* and the

American Journal of Political Science. I have frequently served as a referee of manuscripts for most of the major journals in my fields of research and the National Science Foundation.

I have done extensive research on American elections and on statistical methods for analyzing social science data. I am a member of the Caltech/MIT Voting Technology Project, serving as the co-director of the project from October 1, 2005 to September 30, 2010.

Over the past two decades, I have been involved in numerous elections cases for both Democratic and Republican clients involving the federal Voting Rights Act, partisan gerrymandering, the evaluation of voting systems, or the statistical evaluation of electoral data. I have testified or consulted in court cases in both state and Federal courts in the states of Arizona, California, Florida, Georgia, Indiana, Illinois, Maryland, Michigan, Missouri, New Hampshire, New Mexico, Nevada, North Carolina, Ohio, Oregon, Oklahoma, Texas, Virginia, and Washington. In particular, I was an expert for the plaintiffs in the Florida litigation regarding its 2012 Congressional map and for the defendants in the Oregon litigation regarding its 2021 Congressional map, both of which focused on questions of partisan fairness of enacted legislative maps. I used the same methods as in this case.

My rate for expert witness work in this case is $600.00 per hour.

2   Measuring Partisan Impact of Redistricting Plans

A central concern about any redistricting plan is how it affects the translation of votes into seats. In particular, we would like to know whether a particular electoral map (or other feature of the electoral system) is politically fair. The concept of political fairness has been extensively studied in the political science literature. The most commonly accepted standard for fairness of voting in a legislature is statewide partisan symmetry (see Katz, King, and Rosenblatt 2020 and see Grofman and King 2007 for a historical review). The symmetry standard requires that parties with the same level of voter support be treated equally by the electoral system. In more concrete terms, the symmetry standard requires that each party should receive the same fraction of legislative seats for the same percentage of the vote.

This definition of political fairness can be straight-forwardly implemented and measured with electoral data using the idea of a seats-votes curve, which first appeared in the academic literature more then half of a century ago (see Kendall and Stuart 1950). A seats-votes curve is a simple mapping, stating for a given party's vote share what fraction of the seats they will receive.

Partisan symmetry requires that the seats-votes curves be the same for all political parties contesting an election. For example, if one party is able to translate 55% of the vote into 65% of the seats, then it would be symmetric (or fair) for the other party, if it were to receive 55% of the vote, to also receive 65% of the seats.

Political scientists define *partisan bias* as the deviation from partisan symmetry.[1] For example,

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[1]For early estimates of partisan bias in electoral systems see Tufte (1973) and Grofman (1983). For a review of

if the Republicans receive 5% more seats than is fair under a redistricting plan, than the plan has a bias of -5 percentage points. If the bias were reversed, so that the Democrats received 5% more seats than was fair, the partisan bias in the plan would be 5 percentage points.[2]

2.1 Distinguishing Symmetry (Partisan Fairness) from Proportionality

It is important to note that the concept of partisan symmetry as a definition of fairness does not appeal to any notion of proportionality. Proportional representation requires that a party's share of the seats should be roughly equal to their share of the vote in the election. Nor does partisan symmetry require that the two parties equally split the available number of seats. Because most electoral systems in the United States are single-member districts that are winner-take-all, in practice they normally give a "bonus" of varying sizes (above proportionality) in seats to the party that wins a majority of the votes across a state. In general, if a given party's average vote share is well above 50%, then it is likely that they will win well more than 50% of the seats. This is just a mechanical, or automatic, feature of single-member district electoral systems (see, for example, Powell and Vanberg 2000).

It is possible in a state where one party is getting well over half the votes, say 65% or 70%, that they win all the seats. This would happen, for example, if every district perfectly mirrored the partisan composition of the state. Because the partisan makeup of a state is rarely if ever evenly distributed, even a dominant political party typically is unlikely to sweep 100% of the seats. But it is a popular misconception that a party with 65% of the statewide vote is likely to win 65% of the seats. Because of the winner-take-all nature of the single member district system, a party with 65% of the statewide vote would be expected to win far more than 65% of the seats, though typically less than 100% of the seats.

On the other hand, a purely proportional system is one in which a one percent increase in the votes for a party leads to a one percent increase in seats for that party. In the United States, a one percent increase in votes for a party normally leads to a two to three percent increase in seats. Under the symmetry standard, there is nothing necessarily unfair about one party winning a greater proportion of seats than the other (see King and Browning 1987:1254–1259).

Partisan symmetry only requires that the electoral playing field be level for both parties. For example, it is not necessarily unfair for the Democrats to win 80% of the seats with 65% of the statewide vote, as long as the same opportunity is available to the Republicans. This notion of fairness is highly consistent with the American system of democratic representation.

A second criterion for evaluating a redistricting plan that comes from a seats-votes curve is *responsiveness*. Responsiveness measures how much an increase in a party's average district

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the literature, see King and Browning (1987) and Grofman and King (2007) and for an application using the concept in Congressional elections, see Cox and Katz (1999).

[2]The sign of partisan bias is only a convention. A plan becomes more fair as its bias gets closer to zero.

vote share increases its seat share.[3] For example, a responsiveness of say 2.6 means that a 1% increase in average vote share causes the party's expected seat share to rise by 2.6%. Unlike partisan symmetry, there is not an obviously "fair" or optimal amount of responsiveness for a redistricting plan. The larger the responsiveness of a given plan, the more sensitive the seat allocation is to changes in citizens' voting behavior. However, extreme amounts of responsiveness might be undesirable because it could lead to political instability, with very frequent changes in representatives for districts. It is the case, however, that smaller values of responsiveness typically correspond to redistricting plans designed to protect current incumbent legislators.[4]

2.2 Measuring Partisan Symmetry

Below I will discuss how to directly estimate partisan bias, responsiveness, as well as the entire seats-votes curve for a proposed redistricting map. It is somewhat involved and requires predicting counter-factual election results.

However, recently there have been several new measures of partisan symmetry proposed in the academic literature, such as the efficiency gap (Stephanopoulos and McGhee 2015), the mean-median test (Wang 2016), and declination (Warrington 2018). These newer measures are claimed to be simpler and more intuitive measures of partisan fairness. Unfortunately, while some of them measure some aspects of the seats-votes curve, Katz, King, and Rosenblatt (2020) show mathematically that none of them are accurate or complete measures of partisan symmetry. Therefore, they are not reliable measures of the partisan fairness of a proposed electoral map. Nonetheless, for the completeness of my analysis, in the sections below I calculate the efficiency gap for the enacted congressional and Senate maps.

2.3 Example of Redistricting Plans that Have Partisan Bias

In order to see how a redistricting plan can both produce partisan bias and affect responsiveness, consider a simple example of drawing a plan for a state with 1000 voters who need to be allocated to 10 equal size districts. A voter can be a supporter of either the Democratic or Republican Party — i.e., they are more likely to vote for a candidate of their preferred party. We will assume that the number of supporters statewide are equal at 500 for both parties. In order to make the drawing of different plans easy, we will assume that we can group the voters into districts according to their political preference. Table 1 gives four possible plans that have very different consequences for both partisan bias and responsiveness.

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[3]A bit more formally it is the derivative of the seats-votes curve.

[4]This happens because the best way to protect current incumbents is to pack likely Democratic voters into districts held by Democratic incumbents and pack likely Republican voters into Republican held districts. This means it would take a very large swing in votes toward one of the parties in a future election to dramatically alter the seat distribution between the parties. See Cox and Katz (2002) for a complete argument.

Table 1: Example of Redistricting Impact on Partisan Bias and Responsiveness

Plan	Description	Partisan Bias	Responsiveness
1	10 Districts with 50 Democrats and 50 Republicans	None	Very High
2	5 Districts with 75 Democrats and 25 Republicans and 5 Districts with 25 Democrats and 75 Republicans	None	Low
3	8 Districts with 40 Democrats and 60 Republicans and 2 Districts with 90 Democrats and 10 Republicans	Large Republican	Moderate
4	8 Districts with 60 Democrats and 40 Republicans and 2 Districts with 10 Democrats and 90 Republicans	Large Democratic	Moderate

Plan 1 creates 10 identical districts with 50 Democrats and 50 Republicans each. That is, each of the districts is a microcosm of the political divisions within the state. In terms of partisan symmetry, clearly this plan is fair since neither party is advantaged by how the districts are drawn. If there were a swing toward the Democrats in an election held under this plan — perhaps because there was a popular Democratic presidential candidate also running on the ballot, causing some Republican voters to vote for Democratic House candidates — they would likely win every district. Similarly, if there were a swing toward the Republican Party, the Republicans would likely win all the seats. For this reason, this plan has maximal responsiveness. It is as close to a winner-take-all election as is possible for a district-based system. A very small change in average district votes would lead to large changes in seat allocation. In fact, this plan highlights the recipe to maximize responsiveness of a plan: make as many of the districts highly competitive with expected vote shares near 50% as possible.

Plan 2 consists of 5 districts with 75 Democrats and 25 Republicans and five districts that are the mirror image of the first set with 75 Republicans and 25 Democrats. Plan 2 looks a good deal different from Plan 1, but it is also fair to the two parties, producing zero partisan bias. Unless vote swings are very large in either direction, we would expect the Democrats to win the first five districts and the Republicans to win the second five. That is, for most average district votes, each party gets about five seats, so the plan is symmetric. However, it is this stability that causes the responsiveness of this plan to be very low. Large numbers of voters would have to vote differently in order to change the election outcomes in any of the districts. This plan can be thought of as a stylized incumbent protecting plan: the first set of districts is designed to make the Democrat incumbents in them likely to win re-election and the second set are the Republican counterparts.

Plan 3 and 4 are actually the same plan, but with the roles of the two parties reversed. They were constructed using the standard recipe to maximize partisan bias in favor of one of the parties: Party A packs as many of the other Party B's supporters in as few districts as possible (creating inefficiently safe districts), while Party A spreads its own supporters across as many districts as

possible (creating winnable but not inefficiently safe districts). Plan 3 is a Republican gerrymander whereas Plan 4 is a Democratic one.

Consider Plan 3 with 8 districts that have 60 Republicans and 40 Democrats each and the two remaining districts have 90 Democrats and only 10 Republicans each. Clearly, except under the most unusual of circumstances, the Democratic candidates would likely win the last two districts. However, unless there were very large vote swings towards them, it is unlikely the Democrats would win many of the other eight districts. This is not the case for the Republicans. While they will never win the last two highly Democratic districts, they are likely to always win a significant number of the other eight. Thus, the map treats the two parties differently and will therefore display partisan bias. Responsiveness for these plans, however, would likely fall somewhere between the high levels seen in Plan 1 and the low levels in Plan 2. The last two districts display very little responsiveness, but the other eight districts, while not as competitive as the Plan 1 districts, are more competitive than the ones in Plan 2.

In order to actually calculate numerical estimates of partisan bias and responsiveness, we would need more information than is provided in Table 1. We would need to know the expected vote share in each of the districts (which is clearly strongly correlated to the number of partisans in the districts in our example), as well as the amount of variability we would expect to see around this mean in a given election. Given these two quantities, we could calculate the probability that a party will win each seat and therefore the seats-votes curve.

3 Method for Estimating Partisan Bias and Responsiveness of Plans

The methodology I will use to estimate the partisan bias and responsiveness of the 2022 enacted New York Senate and Congressional plans was originally developed by Andrew Gelman and Gary King and published in a leading peer-reviewed scholarly journal (Gelman and King 1994).[5] The procedure is based on regression analysis — the most widely used statistical method in the social sciences. The details of the statistical procedure can be found in Gelman and King's original article. The procedure consists of two parts.

First, using historical elections results, we generate a statistical forecasting model from a regression of New York Senate or Congressional Democratic district vote share (the independent variable) on the following set of predictors: the average vote share that the Democrats received in statewide races in the district, an incumbency indicator[6], and the fraction of the district that is Black, Asian, and Hispanic/Latino. That is, the forecasting model tells us our best estimate (or prediction) for the expected Democratic Senate or Congressional vote in a district with a given set

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[5] Their procedure has been actively studied and extended since its original publication. See, for example, Katz and King (1999) which extends the basic model to the case of more than two parties and Katz, King, and Rosenblatt (2020) that validates the use of "uniform partisan swing" that is used to estimate, for example, future election results.

[6] This allows the outcomes to vary if there is Democratic, Republican, or no incumbent running in the election in the district.

of the predictors — e.g., Average statewide vote of 58%, without an incumbent running, in a district that has no Blacks, Asians, or Hispanics. We also get an estimate of how variable elections are over time.[7]

The average vote share that the Democrats received in statewide races is used purely as a measure of the partisan composition of the district, thus when the election happened is not particularly important. The regression on the historical election will calibrate how this is translated into a forecast of votes in the New York Senate or Congressional elections. That is, we do not want to assume that a one point increase in this statewide average corresponds to exactly a one point increase in Congressional vote share. Also, this fails to account for the variability that occurs between elections that is also captured by the regression model. Similarly, an incumbency indicator is included because we know that incumbents tend to do better than non-incumbents. Therefore, we want to control for this in making our prediction. The demographics are used as predictors just to further aide in predicting Congressional district vote.

In order to make the statistical model more robust, we jointly estimate the New York Senate and Congressional elections, as well as those for the New York Assembly. This partial pooling allows us to improve the precision of our estimates and is a common technique in statistics.[8] It is also, for example, the strategy that the non-partisan PlanScore.org uses to analyze proposed redistricting plans.[9]

Now that we have the forecasting model, we can evaluate a particular redistricting map. A plan is just a set of hypothetical districts with new values of these observable predictors, much like the examples in Table 1. For each plan, we can calculate the expected vote shares and variability for the districts in the plan. We can, therefore, calculate the probability a seat would be won by the Democratic candidate or determine what would happen as the vote share for the Democratic candidate increased on average in every district. This allows us to trace out the seats-votes curve using the stochastic uniform swing assumption and hence estimate both partisan bias and responsiveness (see Gelman and King 1994).

Since our forecasting model is a statistical approximation, it has inherent uncertainty captured by associated standard errors — for example, the expected Democratic vote share in a particular district may be 45%, plus or minus 3%. This estimation uncertainty will filter through to our estimates of partisan bias and responsiveness. However, we will be able to use standard statistical procedures to test if estimates are different from some value after we control for this estimation uncertainty.

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[7] The full model also controls from systematic unobserved characteristics.

[8] For a text book treatment of partial pooling, also called hierarchical modeling, see Gelman and Hill (2007).

[9] See a discussion of their methodology at: https://planscore.campaignlegal.org/models/data/2021B/

4   Partisan Impact of 2022 New York Senate map

Recall from Section 2 that a plan is fair if it treats the two parties symmetrically in terms of translating votes into seats. A plan is biased if it deviates from this partisan symmetry. If Democrats and Republicans (say in different election years) receive the same average vote share statewide, but the Republican win 5% more of the seats in their election, then the plan is biased towards the Republicans. For convenience in presenting results, I will use positive numbers for pro-Democratic biases and negative numbers for pro-Republican biases.

Table 2: Estimated District Results for enacted 2022 New York Senate Plan

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
1	49.8	8.4	49.2
2	44.0	8.7	24.0
3	56.4	8.7	76.8
4	42.0	8.6	17.4
5	54.1	8.7	68.2
6	55.8	8.8	74.6
7	57.2	8.8	79.4
8	54.1	8.7	68.8
9	54.4	8.4	70.1
10	72.0	8.7	99.4
11	67.5	8.9	97.0
12	73.1	8.9	99.6
13	79.0	8.7	100
14	79.9	8.8	100
15	61.9	8.5	92.2
16	65.3	8.8	96.0
17	71.8	8.6	99.5
18	78.4	8.8	100
19	74.8	8.9	99.8
20	77.5	8.7	99.9
21	77.9	8.9	99.9
22	66.4	8.7	97.4
23	65.9	8.6	96.7

Table 2 – *Continued from previous page*

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
24	40.9	8.7	14.3
25	78.5	8.7	100
26	43.3	8.8	22.5
27	69.0	8.8	98.4
28	68.8	8.9	98.6
29	75.1	8.6	99.8
30	74.4	8.8	99.7
31	83.3	8.7	100
32	79.6	8.7	99.9
33	79.4	8.8	100
34	83.3	8.8	100
35	83.1	8.7	100
36	69.0	8.8	98.2
37	65.4	8.7	96.0
38	80.2	8.7	100
39	59.9	8.9	86.4
40	54.2	8.7	68.6
41	53.4	8.6	64.8
42	53.5	8.8	65.1
43	46.0	8.6	31.9
44	43.3	8.7	20.8
45	55.2	8.7	72.3
46	50.4	8.5	52.3
47	41.8	8.6	17.3
48	49.4	8.8	47.1
49	38.7	8.6	9.6
50	40.3	8.8	13.1
51	39.6	8.6	11.0
52	50.2	8.6	51.3
53	51.2	8.6	55.6
54	39.6	8.7	11.6
55	51.5	8.7	58.1

Table 2 – *Continued from previous page*

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
56	54.6	8.5	70.5
57	55.3	8.6	73.1
58	36.3	8.7	5.4
59	37.8	8.6	7.7
60	52.8	8.7	62.2
61	43.0	8.6	20.8
62	39.2	8.7	11.0
63	60.7	8.7	88.6

Using the forecasting model described above, we can begin our analysis of the enacted 2022 New York Senate map. The first output of this analysis is predicted (or expected) Democratic vote share and the probability that a Democratic candidate wins each district. These can be seen in Table 2. As with all the subsequent analysis, I assume that no incumbents (of either party) contest a particular election. This is because in future elections held using the Senate map, we do not know which incumbents will run in each district. Further, the map partially determines which incumbents will run in future elections in each district.[10] For example, a newly drawn district that is highly favorable to the Republicans is likely to have Republican incumbents in future elections.

The first column of the table identifies the Senate district. The second column of the table tells us the expected vote share of the Democratic candidate in the district. The best way to think about this expected value is to consider observing many elections run with this map. If we averaged across all these hypothetical elections, say in district 3, then the average Democratic vote share would be 56.4% (or an average of 43.6% for the Republicans). Of course, there is wide variability in election outcomes from year to year, and the third column gives us a measure of this variability, the standard deviation of the expected vote. That is, in our large set of hypothetical elections, the result would vary from year to year, but about 95% of the time the Democratic vote share in district 3 should fall between 38.7% and 72.7%. This is because the 95% confidence interval for the expected vote is the estimate plus or minus twice its standard deviation. In this example, the upper bound is 56.4 + 2 x 8.7 = 73.8 and the lower bound is 56.4 - 2 x 8.7 = 39.0. The fourth column summarizes the first two by giving us the probability that the Democrat wins the district. In district 3, we see that the Democrat should win the election with a probability around 77% (or the Republican wins with probability 23%). This means over our large set of hypothetical elections in

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[10]Technically, incumbency is an endogenous consequence of the electoral map implemented.

district 3, the Democrats would win about 77% percent of the time. To be concrete, if we observed 100 elections in this map, we should expect to see the Democrats win about 77 times.

Given the district results presented in Table 2, we can vary the election results to trace out the seats-votes curve via uniform swing. Suppose, for example, the observed election saw the Democrats win on average 63% of the Senate vote, then we could add 1% to each district to see which seats the Democrats would win had they had an average vote share of 64%. Similarly, we could add 2% to see what would have happened if they had won 65% of the vote and so forth. Similarly, we can subtract from each district to see what happens at lower average vote shares.
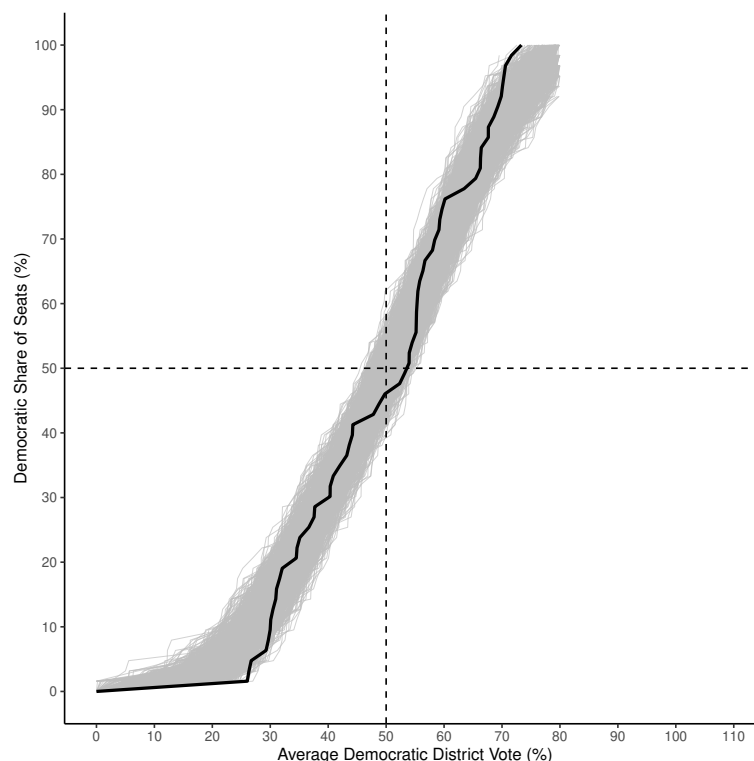

Figure 1: *Estimated Seats-Votes Curve for the 2022 Enacted New York Senate Map. The dark curve is based on the median district vote forecasts. The light gray curves are based on 500 draws of possible observed district vote shares from the model to represent statistical uncertainty.*

The full estimated seats-votes curve is presented in Figure 1. The dark line represents the curve estimated from the median estimated vote shares given in Table 2, column 2. This is our best estimate. The light gray lines are other draws that are consistent with the statistical forecasting model to give a sense of the variability in the estimated seats-votes curve. The curve looks relatively symmetric, including when we account for uncertainty.

Once we have traced out the seats-votes curve for the New York Senate map, we can directly calculate the partisan bias and responsiveness of the plan to statistically test for partisan fairness.

Figure 2 presents the estimates of the partisan bias of the enacted plan. Bias was estimated for five regions of vote shares: [49%, 51%], [51%, 55%], [55%, 60%], [61%, 65%], and [65%, 70%]. Recall that partisan bias compares the seat shares of the two parties for the same vote share. Thus, we need to specify the vote shares to estimate partisan bias at a given vote share on the seats-votes curve. To improve the statistical precision (i.e. make the confidence intervals smaller), we will average a range of possible vote shares. The regions were chosen to include plausible values for Democratic vote share that we may see in future elections. For example, in statewide elections over the last decade in New York, Democrats have averaged well over 60% of the vote.
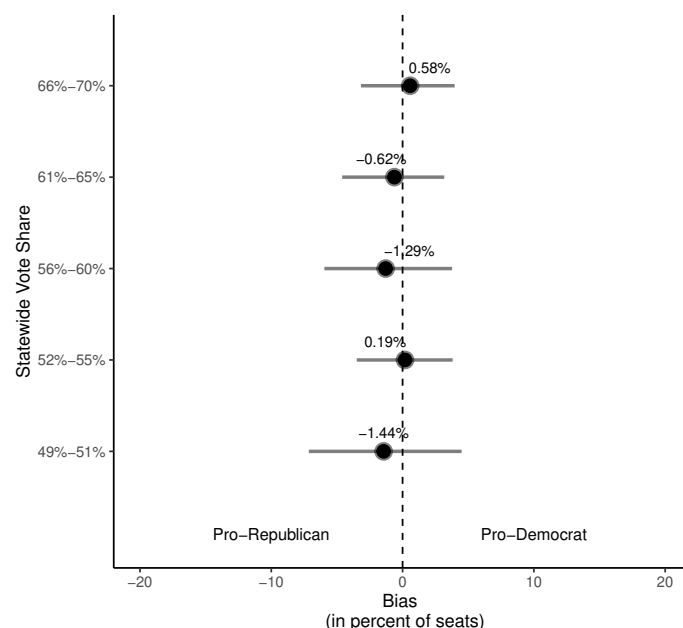

Figure 2: *Estimated Partisan Bias of the 2022 Enacted New York Senate Map. Positive values are pro-Democratic bias and negative values are pro-Republican bias.*

The center dot in the figure gives the point estimate of the partisan bias. The numerical estimate of the bias is denoted above the dot. As we can see for vote shares between 49% to 51%, as well as from 56% to 60%, and 61% to 65%, the point estimates of partisan bias are pro-Republican, but relatively small in magnitude. In the other ranges, the bias estimates are pro-Democratic, but also relatively small.

Given that these are statistical estimates, there is some inherent uncertainty in the estimates. This is captured in Figure 2 by the gray lines through each estimate. Technically, these lines constitute the "95% confidence interval" for the estimates. Given that these confidence intervals all cross the dotted line marking zero bias, we can say that the Senate plan shows no statistically

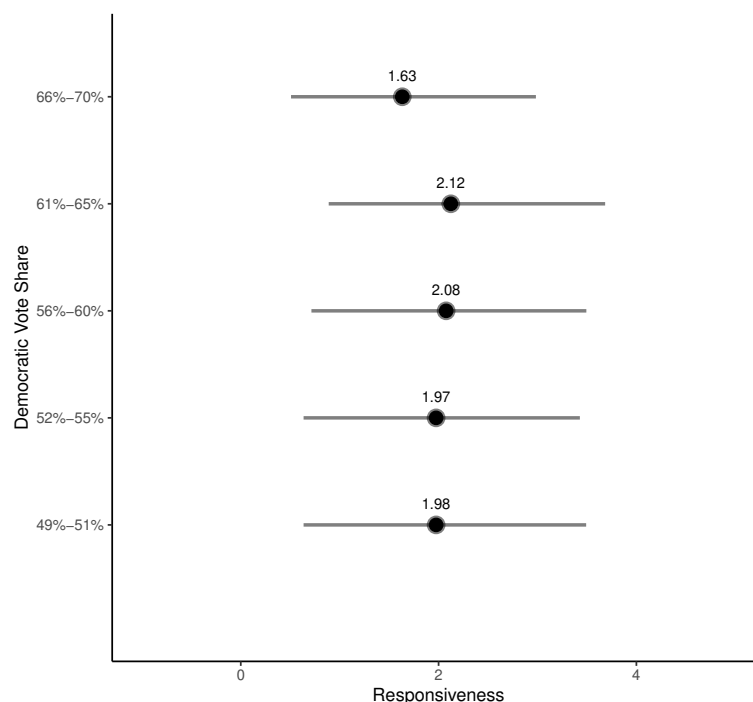significant partisan bias in favor of either party.[11]

Figure 3: *Estimated Responsiveness of the 2022 Enacted Senate Map*

Figure 3 presents the estimates of the responsiveness of the 2022 enacted New York Senate map. As with the previous figure, the dots represent our best estimate of responsiveness and the gray lines give the "95% confidence interval." The estimated responsiveness across all regions are similar at around 2. In other words, this means that if the average vote share to a party increased by 1 percentage point, then we would see their seat share increase by about 2 percentage points. These values are not out of the ordinary for district based electoral systems.[12]

Overall, the Democrats are expected to win 43.1 of the 63 seats, or about 69% of them, assuming there were no incumbents running, in the new map. Again since this is a statistical estimate the 95% confidence interval is from a low of 37 seats to a high of 49. This estimate, as discussed before, should be thought of as a long term average over many elections conducted with the map.

As mentioned above, Katz, King, and Rosenblatt (2020) show mathematically that partisan bias is the only complete and accurate measure of partisan fairness of an electoral map. However, there are two other commonly used measures of partisan fairness used in litigation, the mean-median test (Wang 2016) and the efficiency gap (Stephanopoulos and McGhee 2015). The mean-median test, as noted by Wang (2016), is not appropriate in a state like New York where a single party is

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[11]Formally, we can not reject the null hypothesis that the bias is zero at conventional significance levels.
[12]See Kendall and Stuart (1950).

dominant and statewide vote shares are far from 50%.

For completeness of my analysis, I will calculate the efficiency gap, even though it is not a reliable measure of partisan fairness. We can plug in our point estimates of the forecasted district votes found in Table 2 as our estimate of how votes should be distributed in the new Senate map. This results in an efficiency gap of -0.5%.[13] Thus, we see that the efficiency gap is small in magnitude and shows that the Republicans are slightly more efficient at converting their votes into seats in the enacted New York Senate map.

4.1   Partisan Symmetry Analysis under Alternative Assumption about Incumbents

Table 3: Estimated District Results for enacted 2022 New York Senate Plan with incumbents

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
1	46.5	8.5	33.9
2	40.9	8.6	14.6
3	56.4	8.6	77.3
4	42.2	8.6	18.2
5	57.3	8.6	80.3
6	58.8	8.7	84.6
7	60.5	8.5	89.6
8	57.0	8.7	78.9
9	54.3	8.6	68.8
10	75.2	8.8	99.8
11	70.5	9.1	98.8
12	76.3	8.6	99.8
13	82.2	8.8	100
14	82.9	8.9	100
15	64.8	8.8	95.4
16	68.6	8.7	98.5
17	71.6	8.8	99.4
18	81.3	8.6	100
19	77.8	8.9	99.9
20	80.4	8.9	100

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[13]Given that efficiency gap was not developed as part of a complete statistical model, there is no way to estimate its statistical uncertainty. This is yet another reason why it is not a reliable estimate of partisan fairness.

Table 3 – *Continued from previous page*

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
21	80.8	8.9	100
22	69.4	8.8	98.4
23	66.0	8.8	96.7
24	38.0	8.8	8.8
25	81.7	8.7	100
26	46.2	8.7	33.3
27	69.3	8.8	98.3
28	71.6	8.9	99.4
29	78.1	8.8	99.9
30	77.4	8.8	99.9
31	86.5	8.6	100
32	82.8	8.7	100
33	82.7	8.8	100
34	86.2	8.7	100
35	86.2	8.8	100
36	69.0	8.6	98.4
37	68.4	8.6	98.8
38	83.3	9.0	100
39	63.0	8.3	93.9
40	57.1	8.7	79.7
41	56.5	8.7	77.8
42	56.4	8.5	77.4
43	42.8	8.6	21
44	40.2	8.5	12.7
45	58.4	8.5	83.8
46	53.6	8.6	65.2
47	38.6	8.7	9.7
48	52.8	8.5	62.8
49	35.7	8.7	4.7
50	40.5	8.7	13.9
51	36.7	8.6	6.1
52	53.4	8.5	66.2

Table 3 – *Continued from previous page*

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
53	51.2	8.7	56.1
54	36.3	8.6	5.9
55	54.7	8.6	70.5
56	57.1	8.7	79.6
57	58.1	8.6	83.2
58	33.2	8.7	2.5
59	34.8	8.7	4.4
60	56.2	8.5	76.5
61	40.0	8.8	13.4
62	36.2	8.6	5.3
63	60.5	8.7	88.9

As I previously noted, political scientists typically estimate the seats-votes curves of a redistricting plan assuming that no incumbents run. Of course, we know incumbents will likely run in future elections, it is just that these decisions to run or not by a particular incumbent are partially caused by the district map, and they will vary over time. However, as a robustness check, I re-ran the analysis assuming all incumbents are running in their successor districts except for those who have already announced, as of the date of this report, that they will not seek re-election.[14] This corresponds to Republican incumbents in districts 1, 2, 24, 43, 44, 46, 47, 49, 51, 54, 58, 59, 61, and 62; open seats in districts 3, 4, 9, 17, 23, 27, 36, 50, 53, and 63; and Democratic incumbents in all other districts.

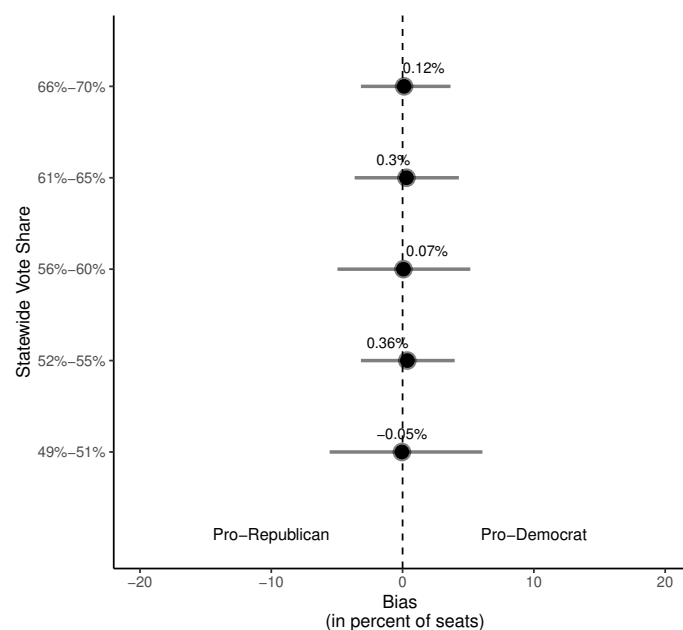[14]This scenario was provided to me by Counsel in this case.

Figure 4: *Estimated Partisan Bias of the Enacted New York Senate Map with Incumbents. Positive values are pro-Democratic bias and negative values are pro-Republican bias.*

The analysis proceeds directly as above's analysis without incumbent. The district estimates are presented in Table 3. The results are qualitatively similar to the scenario without any incumbents running, because the estimated impact of an incumbent is about 3 percentage points (with a 95% confidence interval of 2.85 to 3.25). That is, a Democratic incumbent on the ballot increases the vote share by about 3 percentage points.
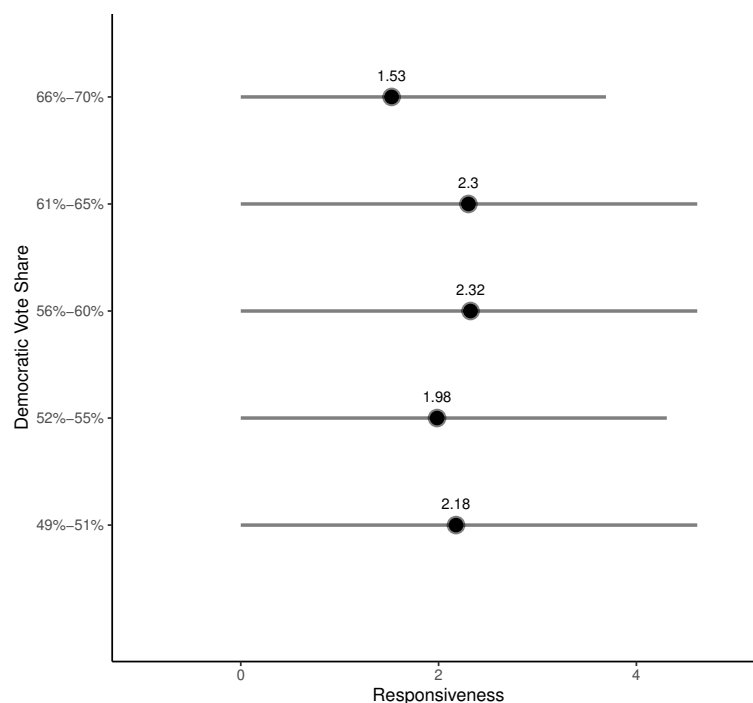
Figure 5: *Estimated Responsiveness of the Enacted New York Senate Map with Incumbents*

And once again we can calculate partisan bias for the map assuming this set of incumbents run. These results are presented in Figure 4. The results are qualitatively similar to the case without incumbents running. However, the point estimates do differ, but not in a statistically significant manner. We see again that in some regions there is a small bias in favor of Republicans and in others a small bias in favor of Democrats. More importantly, all the confidence intervals cross zero. Therefore, we can say that the Senate plan shows no statistically significant partisan bias in favor of either party with this given configuration of incumbents assumed to be running.[15]

The responsiveness estimates are presented in Figure 5. As with the bias estimates, the estimates do not qualitatively differ from the scenario without any incumbents running.

Again we can plug in the district vote estimates in the Senate map under this configuration of incumbents from Table 3 to calculate the efficiency gap. This results in an efficiency gap of -1.3%. This is a small, pro-Republican advantage in vote efficiency.

Overall, the Democrats are expected to win 44.3 of the 63 seats, or about 70% of them, assuming this particular configuration of incumbents running. Again since this is a statistical estimate the 95% confidence interval is from a low of 39 seats to a high of 49. This estimate, as discussed before, should be thought of as a long term average over many elections conducted with the map with this particular configuration of incumbents running.

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[15]Formally, we can not reject the null hypothesis that the bias is zero at conventional significance levels.

5 Partisan Impact of 2022 Congressional map

The analysis of the partisan fairness of the 2022 enacted Congressional map proceeds in exactly the same manner as my analysis of the 2022 enacted Senate map presented above.

Table 4: Estimated District Results for enacted 2022 Congressional Plan

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
1	54.9	8.5	72.2
2	45.1	8.7	28.2
3	56.4	8.7	76.2
4	55.8	8.6	74.5
5	76.0	8.7	99.8
6	67.7	9.0	97.6
7	77.3	8.8	99.9
8	72.6	8.8	99.5
9	72.9	8.4	99.8
10	72.0	8.7	99.5
11	58.0	8.6	82.0
12	72.5	9.0	99.5
13	82.5	8.6	100
14	75.5	8.8	100
15	82.4	8.7	100
16	65.0	8.6	96.0
17	55.6	8.6	74.4
18	51.1	8.8	55.2
19	49.0	8.8	45.6
20	51.3	8.6	55.6
21	39.8	8.8	12.4
22	51.9	8.7	58.1
23	39.1	8.7	10.5
24	38.8	8.7	9.8
25	53.3	8.6	65.2
26	55.6	8.7	73.7

Using the same forecasting model described above, we can begin our analysis of the enacted

2022 New York Congressional map. The first output of this analysis is a summary of each district with its expected Democratic vote share, expected variability in the Democratic vote share over time, and the estimated probability that a Democratic candidate wins the district. These can be seen in Table 4. As with all the previous Senate analysis, I assume that no incumbents (of either party) contest a particular election.
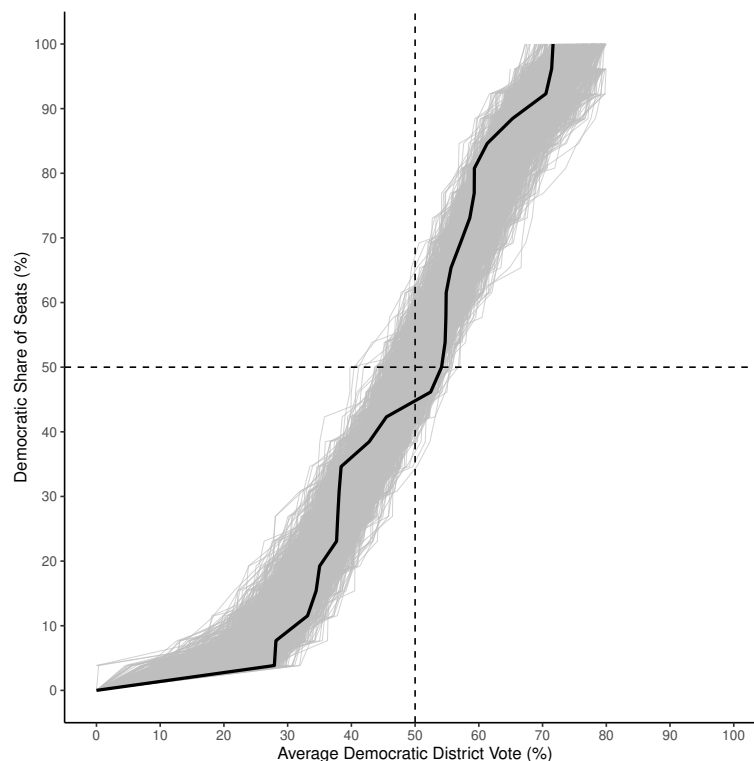

Figure 6: *Estimated Seats-Votes Curve for the Enacted Congressional Map. The dark curve is based on the median district vote forecasts. The light gray curves are based on 500 draws of possible observed district vote shares from the model to represent statistical uncertainty.*

Given the district results presented in Table 4, we can vary the election results to trace out the seats-votes curve via uniform swing to estimate the seats-votes curve. The full estimated seats-votes curve for the Congressional map is presented in Figure 6. The curve looks relatively symmetric, including when we account for uncertainty.

Once we have traced out the seats-votes curve for the Congressional map, we can directly calculate the partisan bias and responsiveness of the plan to statistically test for partisan fairness. Figure 7 presents the estimates of the partisan bias of the enacted Congressional plan over several regions of possible vote shares.
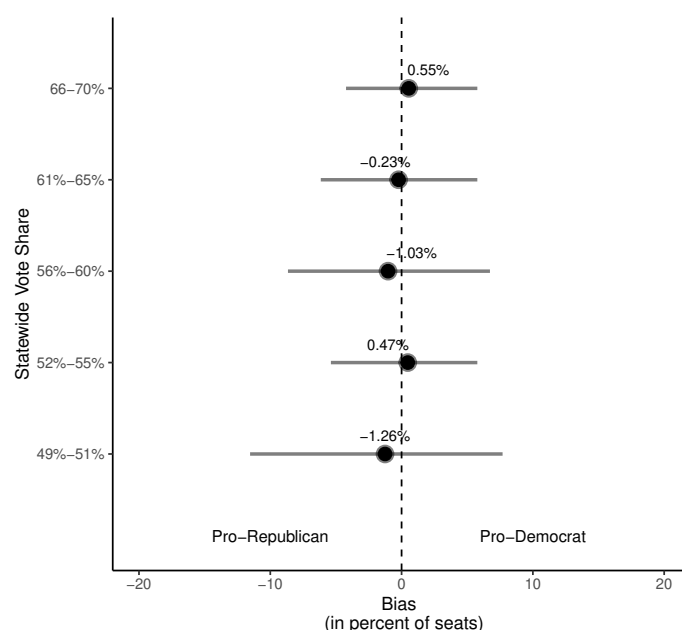
Figure 7: *Estimated Partisan Bias of the Enacted Congressional Map. Positive values are pro-Democratic bias and negative values are pro-Republican bias.*

As before, the center dot in the figure gives the point estimate of the partisan bias. The numerical estimate of the bias is denoted above the dot. As we can see for vote shares between 49% to 51%, as well as from 56% to 60%, and 61% to 65%, the point estimates of partisan bias are pro-Republican, but relatively small in magnitude. In the other ranges, the bias estimates are pro-Democratic, but also relatively small. Given that these confidence lines for all of these estimates all cross the dotted line marking zero bias, we can say that the Congressional plan shows no statistically significant partisan bias in favor of either party.[16]

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[16] Formally, we can not reject the null hypothesis that the bias is zero at conventional significance levels.
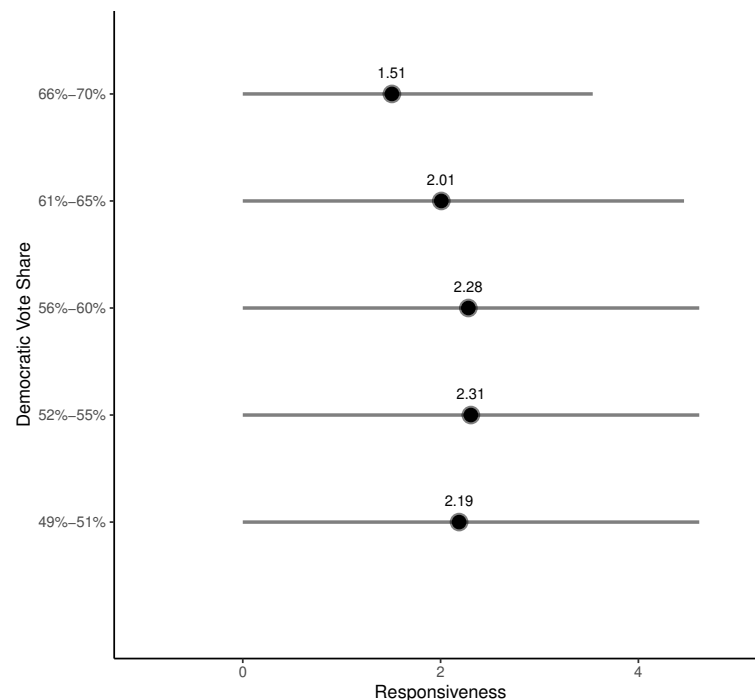
Figure 8: *Estimated Responsiveness of the Enacted Congressional Map*

Figure 8 presents the estimates of the responsiveness of the 2022 enacted Congressional map. As with the previous figure, the dots represent our best estimate of responsiveness and the gray lines give the "95% confidence interval." The estimated responsiveness across all regions are similar at around 2.

Overall, the Democrats are expected to win 18.7 of the 26 Congressional seats, or about 72% of them, assuming there were no incumbents running. Again since this is a statistical estimate the 95% confidence interval is from a low of 15 seats to a high of 22. This estimate, as discussed before, should be thought of as a long term average over many elections conducted with the map.

As before we can plug in the district vote share estimates in Table 4 to calculate the efficiency gap of the Congressional map, even though this is not a reliable estimate of partisan fairness. This results in an efficiency gap of -1.3%. Thus, the Republicans' distribution of votes is slightly more efficient than the Democrats'.

5.1 Partisan Symmetry Analysis under Alternative Assumption about Incumbents

Table 5: Estimated District Results for enacted 2022 Congressional Plan with incumbents

District	Predicted Democratic Vote (%)	Vote Standard Deviation	Prob. Democrat Wins (%)
1	54.9	8.8	70.8
2	42.1	8.6	17.6
3	56.2	8.7	76.6
4	55.9	8.7	75.6
5	79.3	8.8	100
6	70.6	8.8	98.9
7	80.8	8.5	99.9
8	75.3	8.6	99.9
9	75.7	8.7	99.7
10	75.1	8.7	99.8
11	54.9	8.9	70.8
12	75.6	8.8	99.8
13	85.5	8.9	100
14	78.4	8.6	100
15	85.1	8.7	100
16	68.1	8.7	98.4
17	58.5	8.7	83.9
18	54.0	8.8	68
19	52.3	8.8	60.4
20	54.1	8.5	68.2
21	36.9	8.7	6.7
22	51.9	8.7	58.7
23	36.0	8.6	5.0
24	36.1	8.6	5.5
25	56.4	8.6	77.0
26	58.5	8.6	84.4

As I previously noted in the analysis of the Senate map, political scientists typically estimate the seats-votes curves of a redistricting plan assuming that no incumbents run. As a robustness check, I re-ran the analysis assuming all incumbents are running in their successor districts except

for those who have already announced, as of the date of this report, that they will not seek re-election.[17] This corresponds to Democratic incumbents in districts 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 25, and 26; Republican incumbents in districts 2, 11, 21, 23, and 24; and open seats in districts 1, 3, 4, and 22.
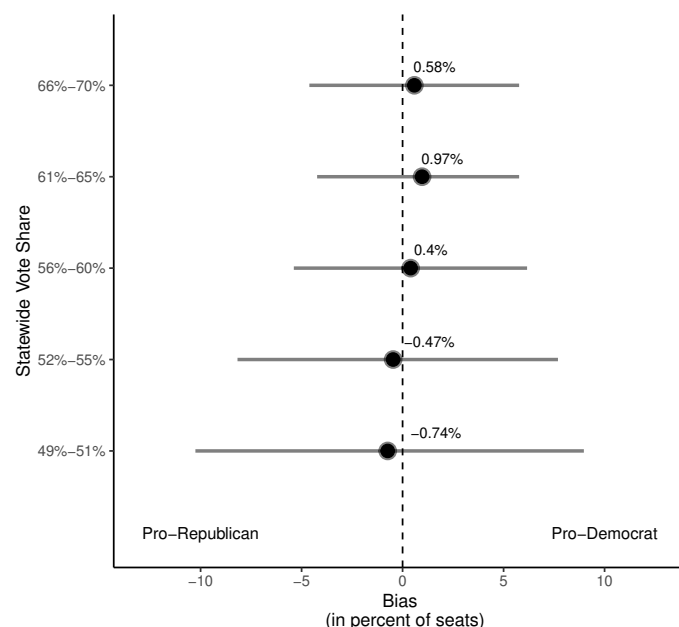

Figure 9: *Estimated Partisan Bias of the Enacted Congressional Map with Incumbents.* Positive values are pro-Democratic bias and negative values are pro-Republican bias.

The district estimates are presented in Table 5. The results are qualitatively similar to the scenario without any incumbents running, because the estimated impact of an incumbent is about 3 percentage points (with a 95% confidence interval of 2.85 to 3.25). That is, a Democratic incumbent on the ballot increases the vote share by about 3 percentage points.

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[17]The source for these are: https://ballotpedia.org/List_of_U.S._Congress_incumbents_who_are_not_running_for_re-election_in_2022
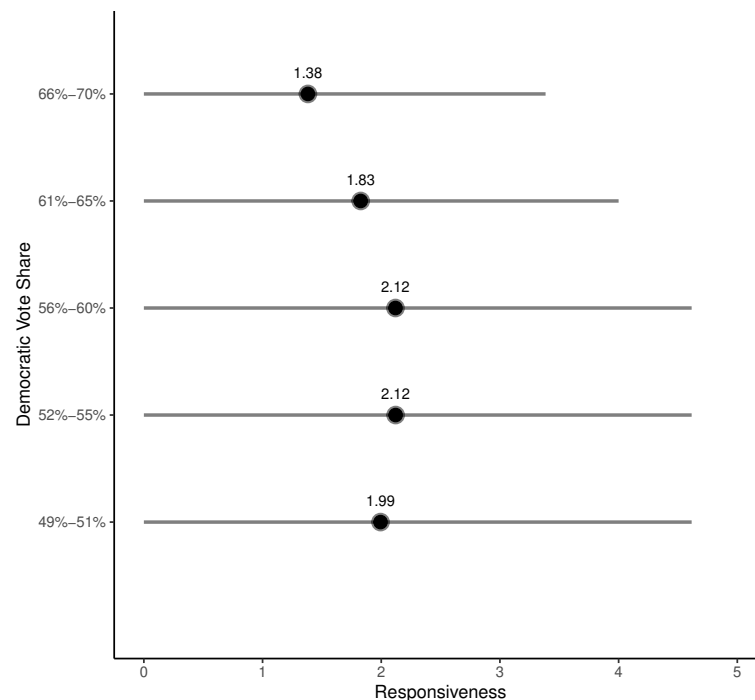
Figure 10: *Estimated Responsiveness of the Enacted Congressional Map with Incumbents*

And once again we can calculate partisan bias for the map assuming this set of incumbents run. These results are presented in Figure 9. The results are qualitatively similar to the case without incumbents running. However, the point estimates do differ, but not in a statistically significant manner. We see again that in some regions there is a small bias in favor of Republicans and in others a small bias in favor of Democrats. More importantly, all the confidence intervals cross zero. Therefore, we can say that the Congressional plan shows no statistically significant partisan bias in favor of either party with this given configuration of incumbents assumed to be running.[18]

The responsiveness estimates are presented in Figure 10. As with the bias estimates, the estimates do not qualitatively differ from the scenario without any incumbents running.

Overall, the Democrats are expected to win 19.3 of the 26 seats, or about 74% of them, assuming this particular configuration of incumbents running. Again since this is a statistical estimate the 95% confidence interval is from a low of 16 seats to a high of 22. This estimate, as discussed before, should be thought of as a long term average over many elections conducted with the map.

Again we can plug in the district vote estimates in the Congressional map under this configuration of incumbents from Table 5 to calculate the efficiency gap. This results in an efficiency gap of -0.5%. This is a very small, pro-Republican advantage in efficiency.

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[18]Formally, we can not reject the null hypothesis that the bias is zero at conventional significance levels.

5.2 Expected Seat Share

As discussed above, the Democrats are expected to win 18.9 of the 26 seats, or about 72% of them, assuming all open seats with around 65% of the average statewide vote share. If incumbents run as in the scenario described in the previous subsection, they do slightly better, netting 74% of the seats. This is clearly not proportional since the Democrats are getting more seats than their statewide vote share. This is expected since single member district systems give a bonus to the majority party. However, as the analysis of the estimated seats-votes curve shows, if the Republicans were to win around 65% of the statewide vote share, they too would be expected to win around 19 Congressional seats.
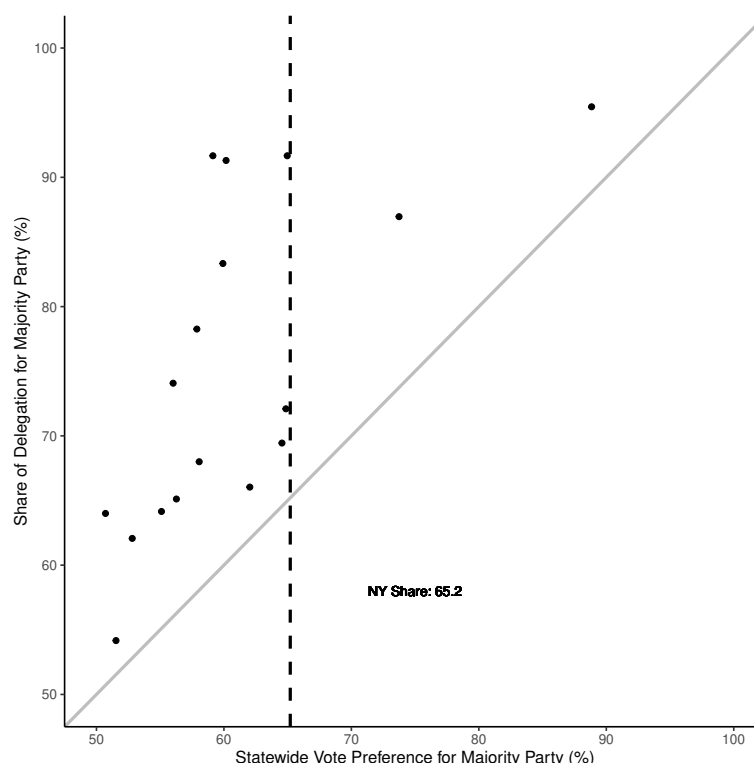

Figure 11: *Scatter plot of Majority Party Congressional Seat Shares versus their Average Statewide Vote Share from states with at least 20 Congressional seats from 1972 to 2020. The horizontal dashed line corresponds to the average statewide vote share in New York in the last decade.*

To give some historical context to an expected seat share for Democrats of 18.9 assuming no incumbents run, we can look at historical election results of larger states with 20 or more Congressional seats from 1972 to 2020 excluding New York.[19] The cutoff of 1972 was chosen because this

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[19]The states in the analysis for at least part of the time period are California, Florida, Illinois, Ohio, Pennsylvania, Texas. States might be included or excluded after reappointment caused by Census changes the size of their delegation.

was the first post-Census redistricting cycle that was subject to the U.S. Supreme Court's ruling *Reynolds v. Sims* (377 U.S. 533) that required equal sized districts for Congress.

This analysis is presented in Figure 11. This presents a scatter plot of the majority party's seat shares versus their average statewide district vote shares for the states with large Congressional delegations.

The non-proportionality of the single member district used to elect members of Congress is immediately apparent in this Figure. For every observation the majority party's seat share is above the diagonal line. This means that the majority party is receiving a larger seat share than their average statewide vote share.[20] Further, New York does not seem out of line with election results from other larger states. The average statewide vote share in New York is approximately 65.2% over the last decade, one of the highest of all state elections represented in the Figure, and they are expected to win about 72% or so of the seats. Some other state majority parties are winning this share of the seats with substantially smaller average statewide vote shares.

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[20] The same holds true if we use average Congressional district vote share.

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