IN THE SUPREME COURT OF WISCONSIN

No. 2021AP001450 OA

BILLIE JOHNSON, ERIC O'KEEFE, ED PERKINS and RONALD ZAHN,

Petitioners,

BLACK LEADERS ORGANIZING FOR COMMUNITIES, VOCES DE LA FRONTERA, LEAGUE OF WOMEN VOTERS OF WISCONSIN, CINDY FALLONA, LAUREN STEPHENSON, REBECCA ALWIN, CONGRESSMAN GLENN GROTHMAN, CONGRESSMAN MIKE GALLAGHER, CONGRESSMAN BRYAN STEIL, CONGRESSMAN TOM TIFFANY, CONGRESSMAN SCOTT FITZGERALD, LISA HUNTER, JACOB ZABEL, JENNIFER OH, JOHN PERSA, GERALDINE SCHERTZ, KATHLEEN QUALHEIM, GARY KRENZ, SARAH J. HAMILTON, STEPHEN JOSEPH WRIGHT, JEAN-LUC THIFFEAULT, and SOMESH JHA,

Intervenors-Petitioners,

v.

WISCONSIN ELECTIONS COMMISSION, MARGE BOSTELMANN in her official capacity as a member of the Wisconsin Elections Commission, JULIE GLANCEY in her official capacity as a member of the Wisconsin Elections Commission, ANN JACOBS in her official capacity as a member of the Wisconsin Elections Commission, DEAN KNUDSON in his official capacity as a member of the Wisconsin Elections Commission, ROBERT SPINDELL, JR. in his official capacity as a member of the Wisconsin Elections Commission and MARK THOMSEN in his official capacity as a member of the Wisconsin Elections Commission,

Respondents,

THE WISCONSIN LEGISLATURE, GOVERNOR TONY EVERS, in his official capacity, and JANET BEWLEY SENATE DEMOCRATIC MINORITY LEADER, on behalf of the Senate Democratic Caucus,

Intervenors-Respondents.

EXPERT REPORT OF DR. MOON DUCHIN ON BEHALF OF INTERVENORS-PETITIONERS CITIZEN MATHEMATICIANS AND SCIENTISTS

Comparison of Congressional and Legislative Districting Plans in Wisconsin

Moon Duchin
Professor of Mathematics, Tufts University
Senior Fellow, Tisch College of Civic Life

December 15, 2021

1 Qualifications

I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University. I hold a Ph.D. and an M.S in Mathematics from the University of Chicago, as well as an A.B. in Mathematics and Women's Studies from Harvard University.

My general research areas are geometry, topology, dynamics, and applications of mathematics and computing to the study of elections, voting, and civil rights. My redistricting-related work has been published in venues such as the Election Law Journal, Political Analysis, Foundations of Data Science, the Notices of the American Mathematical Society, Statistics and Public Policy, the Virginia Policy Review, the Harvard Data Science Review, Foundations of Responsible Computing, and the Yale Law Journal Forum. Although I have submitted reports and sworn affidavits, I have not testified as an expert at deposition or trial in any proceeding during the last 4 years.

My research has had continuous grant support from the National Science Foundation since 2009, including a CAREER grant from 2013–2018 and a Convergence Accelerator grant from 2019–2021 entitled "Network Science of Census Data." I am currently on the editorial board of the journals Advances in Mathematics and the Harvard Data Science Review. I was elected a Fellow of the American Mathematical Society in 2017 and was named a Radcliffe Fellow and a Guggenheim Fellow in 2018. A current copy of my full CV is attached as Appendix A to this report.

My research group, the MGGG Redistricting Lab (based at Tisch College), provided data support for the People's Maps Commission for which we were compensated with $22,000, through a contract dated June 30–October 31, 2021. As part of that support, we launched and hosted an online portal for members of the public to submit written testimony, draw demonstrative districting plans, and map their neighborhoods and communities of interest. The portal and the submission gallery can be viewed at `portal.wisconsin-mapping.org`.

2 Assignment

I have been retained by counsel for a group of Wisconsin voters ("Citizen Mathematicians and Scientists") who have intervened in this proceeding. I have been asked to provide an analysis of the Congressional, Senate, and Assembly plans put forward by the Citizen Mathematicians and Scientists, which I will refer to as `MathSci-CD` (Congressional), `MathSci-SD` (state Senate), and `MathSci-AD` (state Assembly). More specifically, I have been asked to provide a comparative analysis of plans offered by the Citizen Mathematicians and Scientists and plans enacted in Wisconsin in 2011, which I will refer to as `Previous-CD` (Congressional), `Previous-SD` (state Senate), and `Previous-AD` (state Assembly). As another point of comparison,I will also discuss the plans passed by the Wisconsin Legislature and vetoed by the Governor in November 2021 (`SB622-CD`, `SB621-SD`, `SB621-AD`).

The list of criteria discussed below—population equality, Voting Rights Act compliance, contiguity, nesting, respect for political boundaries, compactness, least change, respect for communities of interest, and the lag between Senate elections caused by staggered terms—was provided to me by the attorneys representing the Citizen Mathematicians and Scientists.

In performing my analysis, I consulted data products released by the Census Bureau as well as the Wisconsin Legislative Technology Services Bureau; the court order of November 30 and other supporting legal materials; and several articles connected to topics in this report. A complete list of the documents and data that I considered in connection with this matter is attached as Appendix B to this report.[1]

All of the quantitative work described in this report was performed by me with the support of research assistants working under my direct supervision. I am compensated at a rate of $400/hour, and my research assistants are compensated at a rate of $50-100 per hour. Compensation for the work that I and my assistants perform is not dependent on the conclusions that we reach or the outcome of the litigation.

3 Background

With the release of the 2020 Decennial Census, Wisconsin maintained its Congressional apportionment of 8 seats in the U.S. House of Representatives, based on a population of nearly 5.9 million.[2] At the state level, Wisconsin has a nested legislative map with 33 Senate districts and 99 Assembly districts. This means that the (rounded) ideal populations of Congressional, Senate, and Assembly districts are 736,715, 178,598, and 59,533, respectively. All observers agree that the enacted districts from the previous Census cycle are now unacceptably malapportioned, and many parties are now attempting to propose a new set of maps that closely balances the new Census figures while adhering to the various rules and priorities of Wisconsin redistricting.

As I understand the Wisconsin Supreme Court's order of Nov. 30, the Court will consider maps presented by various parties to the the litigation only if they comply with all applicable legal requirements and reflect a "least-change approach" with respect to the maps enacted in 2011 (described further in §5.3). Among maps that satisfy federal and state requirements and contend on least change, the court will look to traditional redistricting criteria like preserving communities of interest.

[1] For the quantitative elements of this report, I and my research assistants used publicly available software, including Python, QGIS, and GeoPandas, along with a publicly maintained codebase that can be accessed through GitHub in repositories such as [8] and [9].

[2] To be precise, the apportionment is based on a resident and overseas count of 5,897,473, while redistricting is done to balance the slightly smaller resident population of 5,893,718.

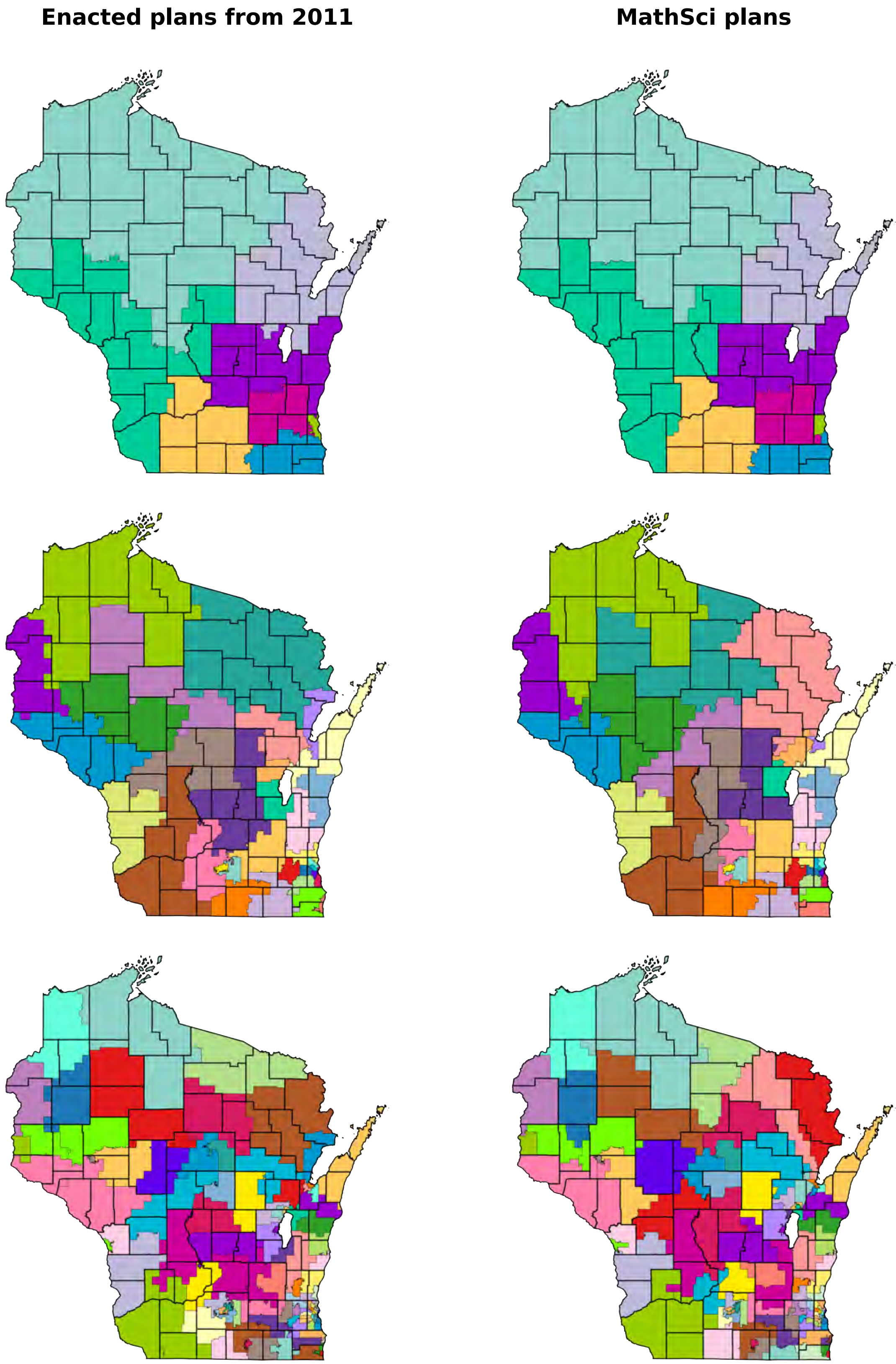


Figure 1: Side-by-side comparison of the Enacted plans from 2011 with the MathSci maps that are the focus of this report.

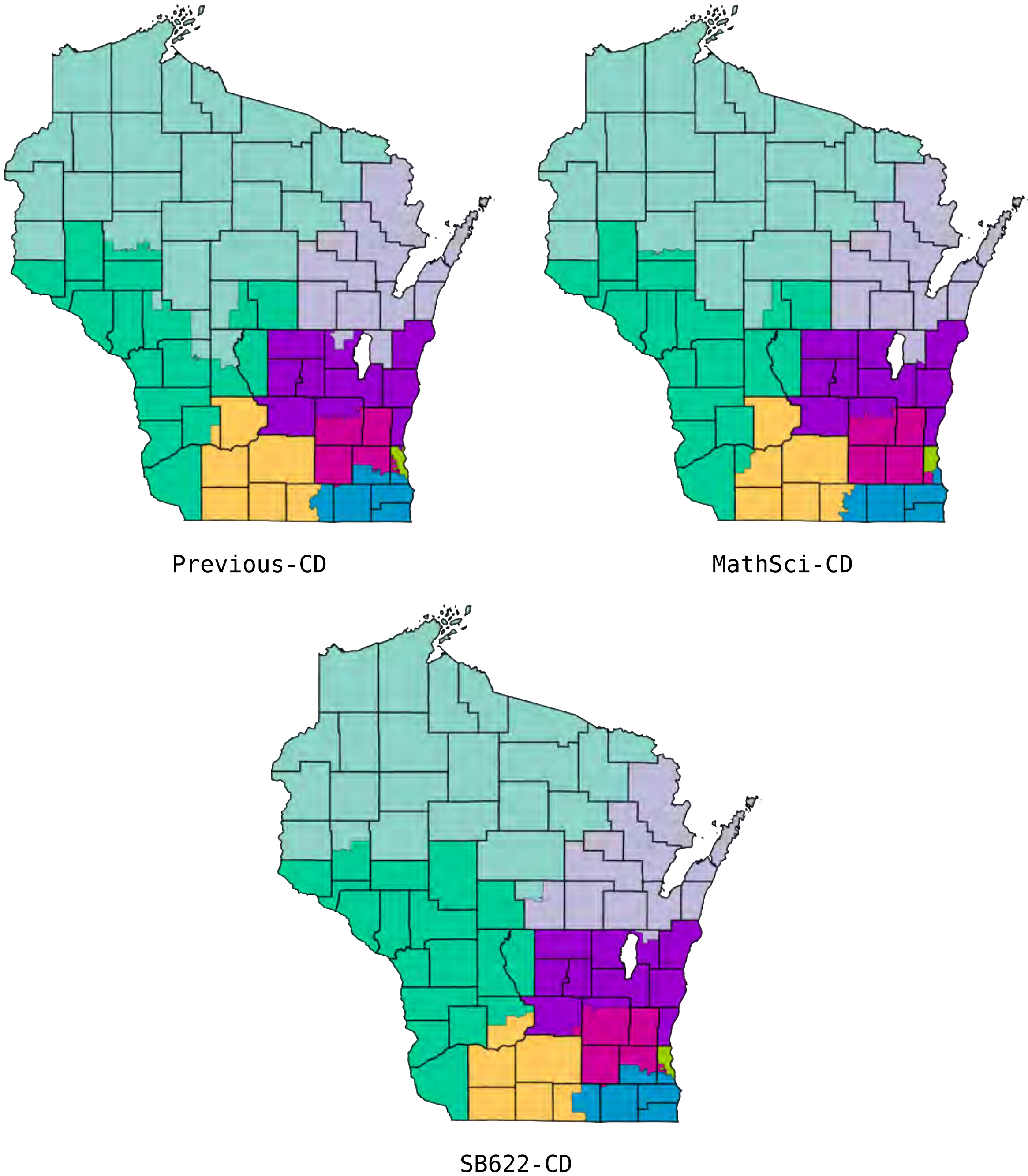
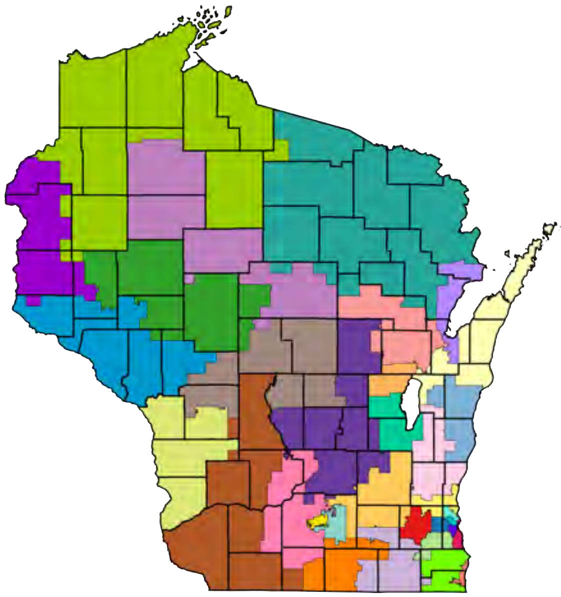
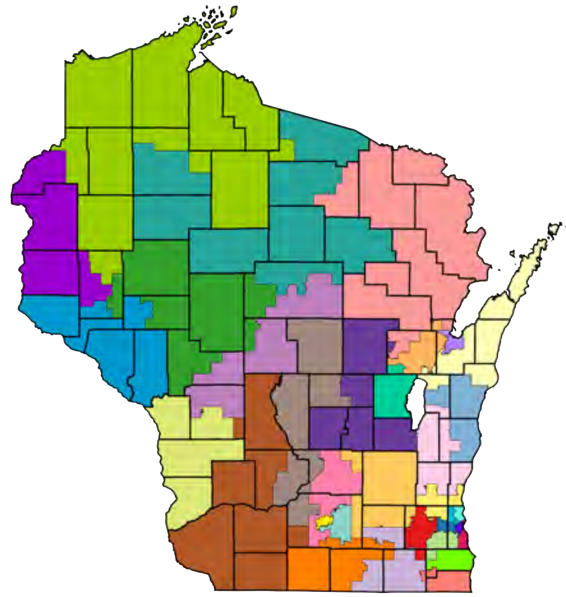


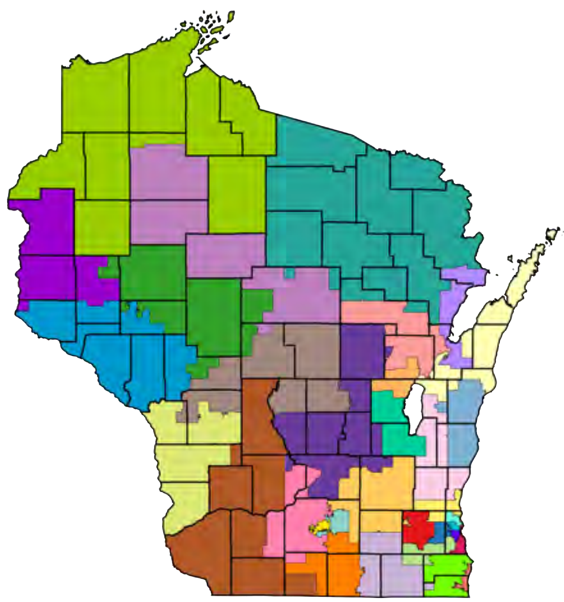
Figure 2: Congressional plans.



Previous-SD



MathSci-SD



SB621-SD

Figure 3: Senate plans.

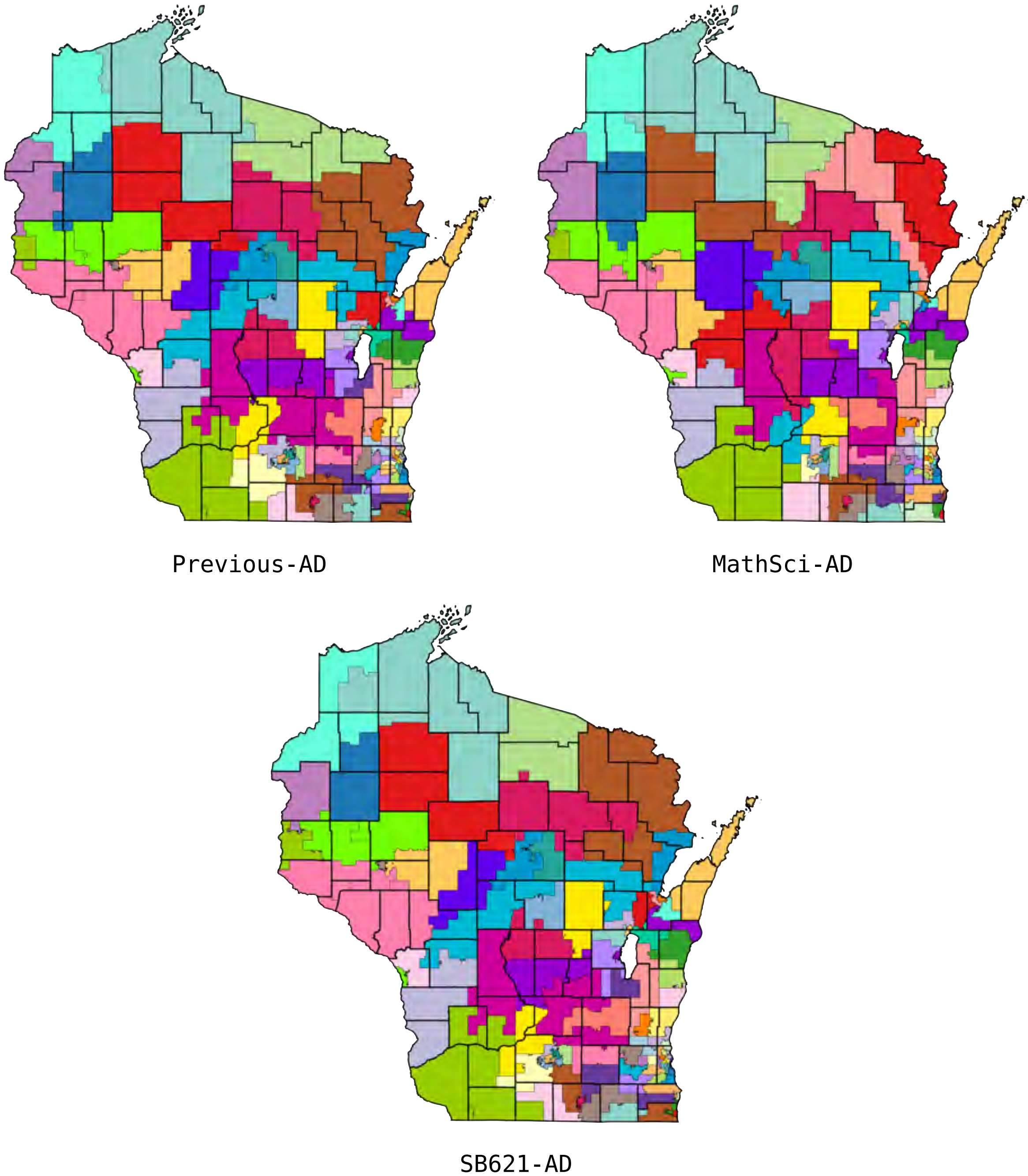


Figure 4: Assembly plans.

4 Executive summary

My analysis shows that the maps proposed by the Citizen Mathematicians and Scientists (which I will sometimes abbreviate as the MathSci maps or the MathSci plans) perform very well across the redistricting criteria that I have reviewed, including several metrics of least change that are grounded in the language of the November 30 court order.

My analysis shows that the MathSci maps track the enacted maps from 2011 closely, at all three levels, and perform well on measures of least change. The departures from those benchmark maps secure significantly improved compliance with the applicable legal requirements, as I understand them, compared to the 2011 maps.

The MathSci plans also compare highly favorably to redistricting plans passed by the Wisconsin Legislature and subsequently vetoed by the Wisconsin Governor. For example, the MathSci Congressional map is more compact, splits fewer counties, and splits fewer municipalities than the vetoed Congressional plan. It does so while maintaining strong least-change resemblance to the previous enacted plan: the MathSci Congressional districts have perfect marks on direct overlap and county overlap, as is also true for the vetoed plan; they have a close visual correspondence and are on average within a closer distance to the benchmark districts; and have nearly as little population displacement as the vetoed plan. (These least change metrics are discussed in more detail below.)

The proposed state legislative maps from the Citizen Mathematicians and Scientists also perform well relative to the vetoed state legislative maps. They are more compact and split fewer counties than both the 2011 enacted maps and the vetoed maps, following redistricting criteria that I understand to be required under Wisconsin law. And though these properties are in tension with the equalization of population, the MathSci legislative maps still achieve tighter population balance than do the vetoed plans.

Because the Citizen Mathematicians and Scientists have found ways to simultaneously improve on the properties of their maps described above, it is not surprising that their proposals must make greater changes from the benchmarks than the vetoed maps do. However, as for the Congressional plan, the MathSci legislative maps perform well on several key metrics of least change presented here.

5 Redistricting criteria

5.1 Federal requirements

Population equality. In Wisconsin, the interpretation of *One Person, One Vote* for Congressional districts requires as close to mathematical equality as is possible, with respect to the total population in the Decennial Census release. This was recognized in the court order of November 30, 2021.

As I understand it, mathematical equality is also the goal for legislative districts, but with greater allowance for inequalities caused by efforts to pursue legitimate state objectives, including legal requirements established by the Wisconsin State Constitution. In Wisconsin, legislative districts must conventionally have a top-to-bottom population deviation of no more than 2%, and over the last three redistricting cycles, the highest top-to-bottom deviation present in any legislative map was 1.6%.

Voting Rights Act compliance. The Voting Rights Act of 1965 (VRA) is federal law and is applicable at all levels of Wisconsin redistricting, as recognized in the Nov. 30 court order. In Wisconsin today, two minority groups—Black voters and Hispanic voters—are sufficiently numerous and geographically compact to constitute the majority of voting-age population in a single-member Assembly district. Indeed, people of color collectively make up 19.1% of Wisconsinites, or more than one in six residents.[3]

I analyzed the full set of statewide general elections from the previous Census cycle, as well as three primary elections: LG18P, the 2018 Lieutenant Governor contest in which Mandela Barnes won the primary and was subsequently elected in the general; Gov18P, the 2018 Governor's race in which Mahlon Mitchell unsuccessfully challenged Tony Evers in the primary; and SoS18P, the Secretary of State contest in which Arvina Martin lost to incumbent Douglas LaFollette. Barnes and Mitchell are Black candidates, while Martin is Native American.[4]

By running the leading methods of racially polarized voting analysis on the general elections from the last cycle, I confirmed that Black and Hispanic voters always prefer the Democratic candidate in statewide races, while White voters prefer the Republican in every contest except Tammy Baldwin's U.S. Senate race from 2018 (where White support for Baldwin is estimated at 51.1%). Black support for the Democratic candidate is estimated at over 91% in all 14 races; Hispanic support is always estimated over 66%, and usually between 70% and 80%.

In the primaries, Barnes and Mitchell are clear candidates of choice for Black voters, with each receiving roughly an estimated 80% share of the Black vote between the top two candidates. Both Black and Hispanic voters are roughly evenly split in support between Martin and LaFollette. Hispanic voters are similarly split between Barnes and his opponent, Kober, but support Mitchell at higher levels than White voters do.

This makes the Mitchell/Evers contest particularly informative for VRA purposes. Since the combination of geography and polarization makes the designation of an effective district clearest for Black voters, I will focus on that group below. A district is said to be *effective for Black voters* if several conditions are met: the has a substantial concentration of Black voting age population, the outcome in most general elections favors the the Black candidate of choice, and Mandela Barnes and Mahlon Mitchell perform strongly in their respective 2018 Democratic primary elections.

[3] In terms of voting age population, or VAP, Black adults make up 6.42%, Hispanic adults make up 6.16%, Asian and Pacific Islander (API) adults make up 3.29% of voting age population and American Indian/Native American adults make up 2.16%. In terms of residential location, Black Wisconsinites are the most concentrated, while Hispanic, API, and American Indian groups are more geographically dispersed.

[4] The Barnes-Kober and Martin-LaFollette primaries had only two candidates. In the primary for Governor, there were seven candidates receiving at least 25,000 votes; Mitchell and Evers were the top two finishers. I am regarding Mitchell-Evers head-to-head by recording each one's share of the two-way vote.

5.2 Wisconsin constitutional requirements

The Wisconsin constitution requires Assembly districts "to be bounded by county, precinct, town, or ward lines, that consist of contiguous territory, and be in as compact form as practicable." [3, Article IV Sec. 4] As I understand it, these criteria—respect for political boundaries, contiguity, and compactness of district shapes—are constitutionally required considerations for all legislative districts and are traditionally held to be significant for Congressional districts as well.

Contiguity. This is a checkbox (yes/no) consideration required by the Wisconsin constitution for state legislative districts and is a traditional requirement for Congressional districts. As the court put it in its Nov. 30 order, "[i]f annexation by municipalities creates a municipal 'island,' however, the district containing detached portions of the municipality is legally contiguous even if the area around the island is part of a different district."

Based on my understanding of this discussion of how Wisconsin courts apply contiguity, all districts in the MathSci plans qualify as contiguous.

Political boundaries. Wisconsin appears to place significant weight on the integrity of certain political subdivisions. In its Nov. 30 order, the Court cited a provision of the Wisconsin constitution (mentioned above) establishing that assembly districts are required to be "bounded by county, precinct, town, or ward lines." In light of the nesting requirement also established by the Wisconsin constitution and discussed below, the requirement to bound districts by county, precinct, town or ward lines also applies to Senate districts. I also understand that, although there is no constitutional obligation to preserve subdivisions like municipalities, Wisconsin courts have traditionally evaluated the number of municipalities that are divided by redistricting plans. I will measure this by recording the number of units whose population is split, and the number of pieces into which that population is divided.

To identify municipalities, I used the County Subdivision geography, described by the Bureau in their Geographic Areas Reference Manual and obtained from their Block Assignment Files [4, 5]. This has 1851 municipalities, but quite a few cross over into multiple counties. If we use the county identifier plus county subdivision identifier, then there are 1922 municipality-parts, each contained completely within a single county. Of these, 12 are water units, leaving 1910 municipalities on which I base the analysis below.

Compactness. The Wisconsin constitution specifically requires that "districts... be in as compact form as practicable." The redistricting literature contains dozens of specific metrics or scores for the measurement of compactness; the Nov. 30 court order does not specify any particular metrics for use in this case, so I have considered three leading alternatives.

I understand that federal courts drawing maps in Wisconsin have previously used "smallest circle" (also known as Reock) and "perimeter to area" (also known as Polsby-Popper) measures. Polsby-Popper is the name given in this setting to a metric from ancient mathematics: the isoperimetric ratio comparing a region's area to its perimeter via the formula $4\pi A/P^2$. Higher scores are considered more compact, with circles uniquely achieving the optimum score of 1. Political scientist Ernest Reock created a different score based on the premise that circles are ideal: it is computed as the ratio of a region's area to that of its circumcircle, where the circumcircle is defined as the smallest circle inside which the region can be circumscribed. Polsby-Popper is thought to be relevant as a measure of how erratically the geographical boundaries divide the districts, but this sometimes penalizes districts for natural features like coastlines of bays and rivers. Both of these scores depend on the planar contours of a district and this means that they are sensitive to the precise choice of shapefile, map projection, and cartographic resolution [10, 11].

Recently, some mathematicians (including myself) have argued for using discrete compactness scores, taking into account the units of Census geography from which the district is built.

The most commonly cited discrete score for districts is the *cut edges* score, which is a plan-wide score that counts how many adjacent pairs of geographical units receive different district assignments. In other words, cut edges measures the "scissors complexity" of the districting plan: how much work would have to be done to separate the districts from each other? Plans with a very intricate boundary would require many separations; thus, lower scores are better. Relative to contour-based scores, this discrete score controls for factors like coastline and other natural boundaries.

To compute all three compactness scores, I proceeded as follows: first, the Census Bureau block shapefile for Wisconsin contains 203,059 blocks, some of which are entirely water. Both the vetoed plans and the MathSci plans contain assignments for 202,510 blocks, omitting the water blocks. I clipped out water by restricting to these assigned blocks for all visualizations and calculations in this report. For instance, the cut edges score counts how many edges are cut out of the 429,400 edges connecting the 202,510 blocks that are assigned to some district in the plans under discussion. For Polsby-Popper and Reock, I have dissolved the districts into individual polygons and use the EPSG:32616 map projection centered in Wisconsin.

Nesting. Three-to-one nesting of Assembly districts into each Senate district is a requirement in the Wisconsin constitution. By convention, Assembly Districts 1-2-3 nest in Senate District 1, AD4-5-6 nest in SD2, and so on. The MathSci legislative plans and the vetoed legislative plans have been confirmed to achieve perfect nesting.

5.3 Least change

The Nov. 30 court order specifies that the Court should "make only necessary modifications to accord with the legal requirements" and avoid "treading further than necessary to remedy [the existing maps'] current legal deficiencies." The lead opinion in the Nov. 30 order leaves the question of measurement open, which provides room for interpretation. The concurring opinion elaborates on this by indicating that a limited approach will "alter[] district boundaries only as needed to comply with legal requirements."

There are two displacement metrics that give natural ways to measure least change: first, the *population displacement*, which is the number (or share) of people who are reassigned, measured by totaling the 2020 population in census blocks that are in a different district in a new map relative to the benchmark (2011 enacted) map. The second is *area displacement*, measuring the share of the state's land area that is reassigned. Though the population displacement is more directly relevant to voting behavior, the area displacement is more closely aligned with the language in the court order that is keyed to minimal alteration of district boundaries.

A third metric relevant to least change, arguably the most closely aligned to the court order's language, is what I will call the *buffer distance*. This asks how much the boundaries of District 1 in the enacted maps would have to be buffered in order to contain all of District 1 in a new proposed map, and so on for the other districts.[5] So, for instance, if District 1 has a buffer distance of 10 miles from the old plan to the new, then the entirety of the new district lies within 10 miles of the old.

Finally, the concurring opinion also notes, in fn13, that there is precedent for counting how many districts "consisted of some or all of the same counties as the parallel predecessor districts." This suggests two more measurements that are helpfully related to least-change: an *direct overlap* count of how many districts in a new plan contain at least one populated block in common with their corresponding district in the old plan, and a more permissive *county overlap* count of how many districts in a new plan touch at least one county in common with their corresponding district in the old plan.[6]

[5] In the field of mathematics called metric geometry, this is the *asymmetric Hausdorff distance* between two bodies in the plane.

[6] County overlap is more permissive because two districts that touch opposite sides of a county will score a point

5.4 Traditional districting principles

In the course of its discussion of least change, the concurring opinion in the Nov. 30 court order maintains that other redistricting criteria may be considered, once maps have met their federal and state law requirements and give equally compelling arguments that they respect the principle of least change. This leaves the door open to any other traditional redistricting criteria. Two were mentioned explicitly: communities of interest and Senate term staggering.

Respect for communities of interest. The concurring opinion calls the deference to communities of interest, or COIs, a "universally recognized redistricting criterion." COIs are geographical areas whose residents have a strong set of shared interests, whether they are social, cultural, economic, or reflect other common policy concerns. In Wisconsin, members of the public submitted 1191 mapped areas to the portal of the People's Maps Commission through the end of August 2021. Earlier this Fall, my research group synthesized these into 36 geographic areas using standard clustering techniques from data science [6]—from Cluster C1 (Whitewater) through Cluster C36 (Suburban Appleton). Notably, five of the clusters, C4-C8, describe predominantly African-American neighborhoods on the north side of Milwaukee. The full set of COI clusters is overviewed in Appendix C.

The clustering was done through a measurement of geographical overlap: one person's painted area of interest is compared to another person's area; the difference between the two is measured with a distance very similar to the buffer distance described above. Then the 1191 areas are grouped into 36 clusters by maximizing the overlaps of areas within a cluster relative to the overlap of areas in different clusters. (Once you have pairwise distances, there are many classical methods in statistics to do just this sort of grouping task.)

If a COI cluster is smaller than the population of a district, the best way for a plan to respect the cluster would be for it to be wholly or mostly contained in a district. Conversely, a large cluster could be respected by having a district contained within its boundaries—for instance, if the residents of that cluster lie along a lakeshore and have common interests in tourism and recreation (as in Cluster C20, the Lake Superior shore community described in Appendix C), then a district inside that cluster would be well served by a representative who takes those interests to heart. Accordingly, the COI preservation score is built as follows: fix a threshold T between zero and one, which I have reported below ranging from 80% to 95%. Consider a cluster to be preserved by a plan if either some district has T share of its population in the cluster or conversely at least T share of the cluster's population is in a single district. Figures 5,6,7 illustrate this.

Full source and replication materials for the COI cluster scoring can be found at [6].

Senate staggering. Finally, a footnote in the concurring opinion mentions the possible goal of "minimizing the number of voters who must wait six years between voting for their state senator." State Senate terms are four years, with odd-numbered districts due to face their next election in 2022 and even-numbered districts due in 2024. The staggering consideration says that we should seek to keep down the number of voters who lived in odd-numbered districts under the previous Senate plan (so were expecting to vote on their State Senator in 2022), but are now reassigned to even-numbered districts (which forces them to wait until 2024).

for county overlap, but such a configuration will not necessarily imply direct overlap.

6 Comparison of Congressional maps

I will give a brief quantitative review of the criteria discussed above, comparing the 2011 enacted Congressional map to the newly vetoed plan and the MathSci plan. First, we confirm that the MathSci plan and the vetoed plan both secure the minimum possible (one person) top-to-bottom deviation.

Population deviation

	maximum positive deviation	maximum negative deviation	top-to-bottom
`Previous-CD`	52,681	−41,320	94,001 (12.8%)
`SB622-CD`	0	−1	1 (.000136%)
`MathSci-CD`	0	−1	1 (.000136%)

Table 1: Population deviation. The ten-year-old plan is unsurprisingly significantly malapportioned, but the new plans are one-person balanced.

Next, we compare districts that are effective from the point of view of Black voters in particular. All three plans have one district (CD4) in which BVAP is over 31%, all 15 general elections would have been won by the Black candidate of choice, and both preferred primary candidates prevail. (Mandela Barnes receives over 80% vote share and Mahlon Mitchell outpolls Tony Evers.)

Minority opportunity-to-elect

	effective districts for Black voters
`Previous-CD`	1
`SB622-CD`	1
`MathSci-CD`	1

Table 2: All plans have one district that offers Black voters the opportunity to elect a candidate of choice.

Next, we consider respect for political boundaries, finding that the MathSci Congressional plan is significantly better than the others on this fundamental redistricting principle.

Political boundaries

	county splits (out of 72)	county pieces	municipality splits (out of 1910)	municipality pieces	ward splits (out of 7136)
`Previous-CD`	12	27	35	70	0*
`SB622-CD`	10	22	24	48	48
`MathSci-CD`	7	15	13	27	8

Table 3: Comparing the plans' conformance to political boundaries. *Note: the `Previous-CD` plan is built from different geometries: 2010 wards (themselves made from 2010 census blocks) rather than 2020 geometries. This creates some very slight discrepancies, but the previous plan comes very close to perfect nesting with the new wards.

The MathSci Congressional plans are the most compact by all measures.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
Previous-CD	4293	0.209	0.440
SB622-CD	3410	0.280	0.456
MathSci-CD	3228	0.305	0.464

Table 4: Comparing compactness scores via one discrete and two contour-based metrics.

In least-change terms, the MathSci Congressional plan has nearly as little population displacement and substantially less area displacement than the vetoed plan. Buffer distance also favors the MathSci plan. Both new plans have perfect scores on direct overlap and county overlap.

Least change

	population displacement	area displacement	average buffer distance	direct overlap	county overlap
Previous-CD	–	–	–	–	–
SB622-CD	384,456 (6.5%)	462.5	11.5 miles	8/8	8/8
MathSci-CD	500,785 (8.5%)	150.4	5.1 miles	8/8	8/8

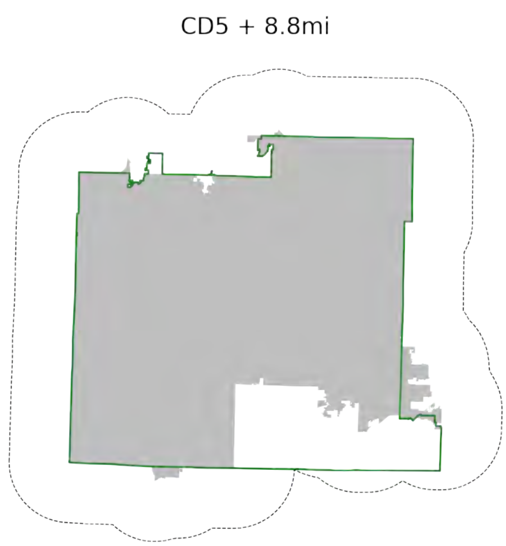


Table 5: Least change, shown with displacement metrics, buffer distance, and overlap counts. The figure illustrates the buffer distance from CD5 in the 2011 enacted plan (shaded) to the MathSci CD5 (green outline). The dotted boundary shows an 8.8-mile buffer around the old CD5, which is just enough to contain the new district. In this instance, besides being close in distance, the new district is more compact.

Finally, we consider how well the plans preserve the 36 COI clusters derived from over a thousand public submissions. Recall that the COI preservation score is obtained by setting an inclusion threshold, such as $T = 0.85$, and asking how many of the 36 clusters are either 85% contained within a single district or, in the case of very large clusters, have some district 85% contained within the cluster.

Figure 5 shows that the MathSci plan respects COI clusters substantially better overall than the vetoed plan. For instance, at the 85% threshold, the MathSci plan preserves 23 out of 36 clusters, while the vetoed plan preserves 21. `MathSci-CD` maintains an advantage of 2-4 clusters for most of the choices of threshold in this range.

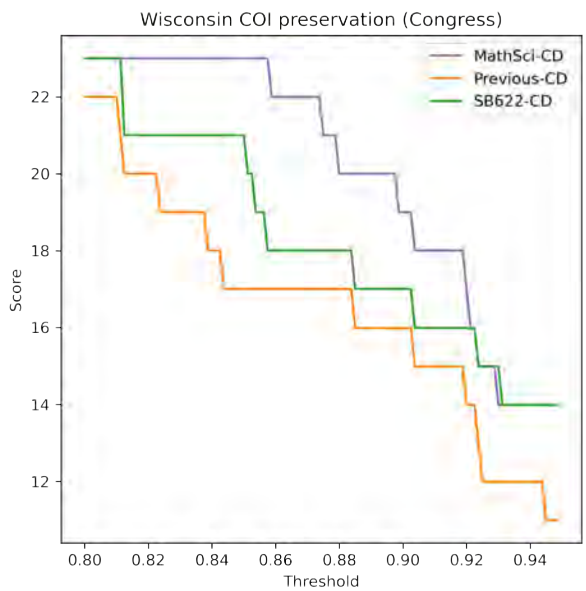


Figure 5: Trace plot of COI preservation for various thresholds of inclusion. A COI cluster is counted as preserved if the cluster is mostly within a single district or some district is mostly within the cluster. At 0%, all 36 clusters would be honored; as the threshold rises, the number of preserved clusters drops.

7 Comparison of Senate maps

Moving to the Senate maps, we start by observing that the MathSci plans have a top-to-bottom deviation of just 895 people, giving tighter balance than the vetoed plan.

Population deviation

	maximum positive deviation	maximum negative deviation	top-to-bottom
Previous-SD	22,874	−16,529	39,403 (22.1%)
SB621-SD	520	−506	1026 (0.57%)
MathSci-SD	428	−467	895 (0.50%)

Table 6: Population deviation. The ten-year-old plan is unsurprisingly significantly malapportioned.

In VRA terms, all three plans have two Senate districts (SD4 and SD6) that are majority-Black, go to the Black candidate of choice in every general election, have Mandela Barnes with over 83% of the vote in his primary election, and have Mitchell receiving more votes than Evers.

Minority opportunity-to-elect

	effective districts for Black voters
Previous-SD	2
SB621-SD	2
MathSci-SD	2

Table 7: All plans have two districts that offer Black voters the opportunity to elect a candidate of choice.

On both measures of county integrity, the MathSci Senate plan is better than the enacted plan and the vetoed plan. It is significantly better on municipality integrity than the enacted plan, and comparable to the vetoed plan.

Political boundaries

	county splits (out of 72)	county pieces	municipality splits (out of 1910)	municipality pieces	ward splits (out of 7136)
Previous-SD	46	130	84	180	0
SB621-SD	42	115	28	62	0
MathSci-SD	28	86	31	69	0

Table 8: Comparing the plans’ conformance to political boundaries.

For all three ways of measuring district compactness, the vetoed Senate plan is comparable to the 2011 enacted plan while the MathSci map is as good or better.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
Previous-SD	10,928	0.230	0.402
SB621-SD	10,785	0.224	0.395
MathSci-SD	9754	0.260	0.402

Table 9: Comparing compactness scores via one discrete and two contour-based metrics.

On least change, the MathSci Senate plan has higher displacement than the vetoed plan, but has perfect overlap scores. The buffer distance also tells a story of restraint: the districts in `MathSci-SD` fall fully within a 17-mile buffer around their counterparts in the enacted plan, on average.

Least change

	population displacement	area displacement	average buffer distance	direct overlap	county overlap
Previous-SD	–	–	–	–	–
SB621-SD	459,061 (7.8%)	357.4	6.5 miles	33/33	33/33
MathSci-SD	1,513,824 (25.7%)	1470.6	17.0 miles	33/33	33/33

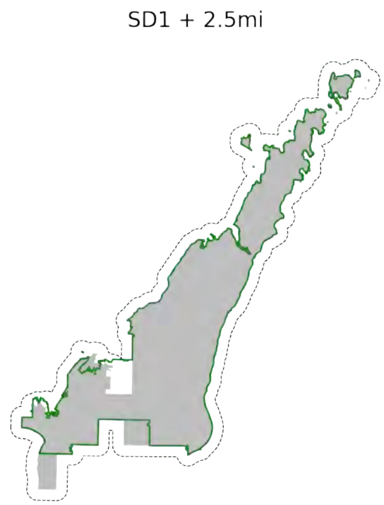


Table 10: Least change, shown with displacement metrics, buffer distance, and overlap counts. The figure illustrates the buffer distance from SD1 in the 2011 enacted plan (shaded) to the MathSci SD1 (green outline). The dotted boundary shows an 2.5-mile buffer around the old SD1, which is just enough to contain the new district. In this instance, besides being close in distance, the new district is more compact.

As the inclusion threshold varies, the MathSci Senate plan typically has one additional COI cluster preserved than can be found in the vetoed plan.

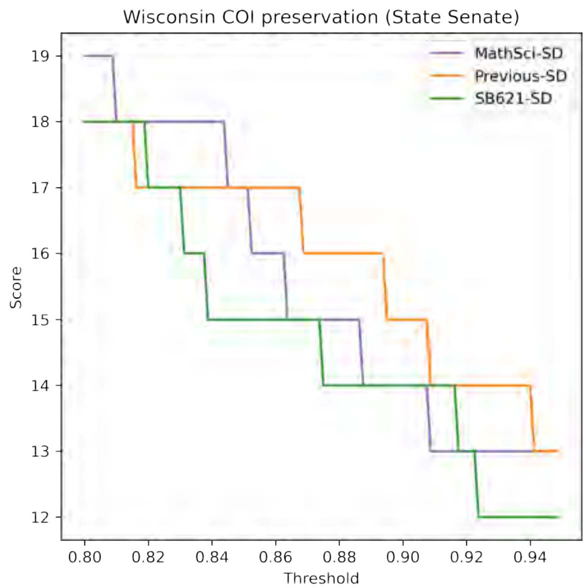


Figure 6: Trace plot of COI preservation for various thresholds of inclusion. A COI cluster is counted as preserved if the cluster is mostly within a single district or some district is mostly within the cluster. At 0%, all 36 clusters would be honored; as the threshold rises, the number of preserved clusters drops.

The Senate districts present the unique problem of disruption in the timing of their four-year cycle caused by shifting population from odd-numbered to even-numbered districts, as explained above. In the MathSci Senate plan, nearly 93% of the population will have an opportunity to vote on their State Senator at the usual four-year interval, or sooner.

Senate staggering

	odd-to-even
Previous-SD	–
SB621-SD	138,753 (2.35%)
MathSci-SD	422,492 (7.17%)

Table 11: Comparison of Senate staggering.

8 Comparison of Assembly maps

The MathSci Assembly map is even more tightly balanced than the vetoed plan, with a top-to-bottom deviation of just 438 people, under 3/4 of one percent of ideal district size.

Population deviation

	maximum positive deviation	maximum negative deviation	top-to-bottom
`Previous-AD`	12,183	−6905	19,088 (32.1%)
`SB621-AD`	231	−221	456 (0.77%)
`MathSci-AD`	220	−218	438 (0.74%)

Table 12: Population deviation. The ten-year-old plan is unsurprisingly significantly malapportioned.

In all plans, the same six Assembly districts—AD10,11,12,16,17,18—offer Black voters the opportunity to elect a candidate of choice. But the MathSci plan has an additional effective district, AD7.

Minority opportunity-to-elect

	effective districts for Black voters
`Previous-AD`	6
`SB621-AD`	6
`MathSci-AD`	7

Table 13: All plans have at least six effective Assembly districts; the MathSci plan has seven.

- `Previous-AD`: six districts over 55% BVAP, 15 out of 15 general elections, Barnes over 82%, Mitchell over Evers. The BVAP drops off to 27.3% in the seventh-highest district.
- `SB621-AD`: the same six districts have BVAP over 47%, 15 out of 15 generals, Barnes over 81%, and Mitchell over Evers. Then BVAP drops off to 25.3%.
- `MathSci-AD`: in those six districts, BVAP is over 36%, 15 out of 15 generals, Barnes is over 80%, and Mitchell outpolls Evers. But in addition, AD7 has 40.9% BVAP, a general election sweep, Barnes at 87.5%, and Mitchell at 68.8%.

Next, on splits, the MathSci Assembly plan is solidly the best of the three on county integrity. It splits significantly fewer municipalities than the enacted Assembly plan, though not as few as the vetoed plan.

Political boundaries

	county splits (out of 72)	municipality county pieces	municipality splits (out of 1910)	pieces	ward splits (out of 7136)
`Previous-AD`	58	229	126	296	0
`SB621-AD`	53	212	48	125	0
`MathSci-AD`	40	175	70	176	0

Table 14: Comparing the plans' conformance to political boundaries.

The vetoed plans are actually less compact than the enacted map from ten years ago. The MathSci Assembly plans are the most compact by all measures.

Compactness

	block cut edges (lower is better)	average Polsby-Popper (higher is better)	average Reock (higher is better)
`Previous-AD`	18,994	0.260	0.390
`SB621-AD`	19,196	0.243	0.379
`MathSci-AD`	17,781	0.282	0.406

Table 15: Comparing compactness scores via one discrete and two contour-based metrics.

The MathSci Assembly plan has higher displacement than the vetoed plan, but has strong overlap scores and an average buffer distance of 13 miles.

Least change

	population displacement	area displacement	average buffer distance	direct overlap	county overlap
`Previous-AD`	–	–	–	–	–
`SB621-AD`	933,604 (15.8%)	829.6	6.0 miles	99/99	99/99
`MathSci-AD`	2,299,625 (39.0%)	1947.9	13.0 miles	85/99	87/99

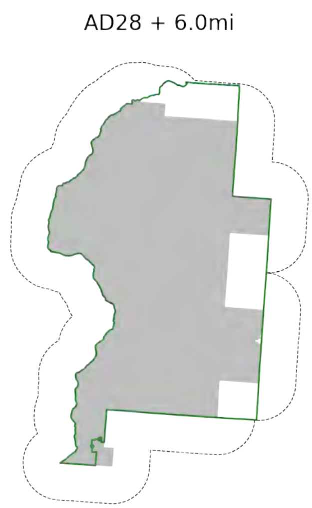


Table 16: Least change, shown with displacement metrics, buffer distance, and overlap counts. The figure illustrates the buffer distance from AD28 in the 2011 enacted plan (shaded) to the MathSci AD28 (green outline). The dotted boundary shows an 6-mile buffer around the old AD28, which is just enough to contain the new district. In this instance, besides being close in distance, the new district is more compact.

Finally, the MathSci Assembly plan preserves COI clusters at a level comparable to the vetoed plan.

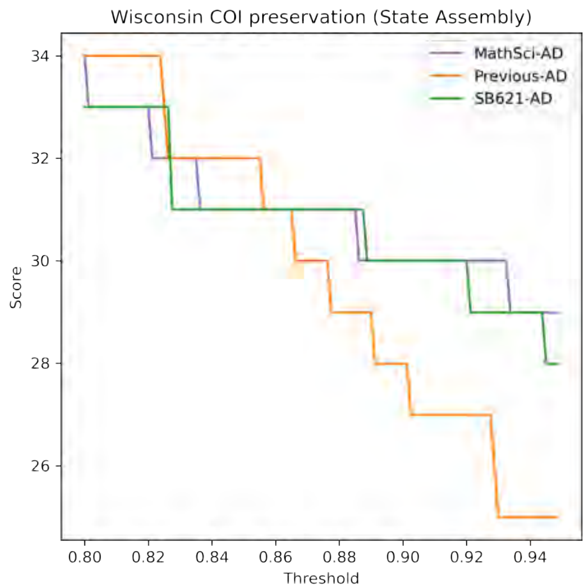


Figure 7: Trace plot of COI preservation for various thresholds of inclusion. A COI cluster is counted as preserved if the cluster is mostly within a single district or some district is mostly within the cluster. At 0%, all 36 clusters would be honored; as the threshold rises, the number of preserved clusters drops.

References

[1] Wisconsin Supreme Court, Order of November 30, 2021.

[2] Legislative Technology Services Bureau, Open Data Page. ata-ltsb.opendata.arcgis.com

[3] Legislative Reference Bureau, Wisconsin Constitution. docs.legis.wisconsin.gov/constitution/wi_unannotated

[4] U.S. Census Bureau, County Subdivisions. www2.census.gov/geo/pdfs/reference/GARM/Ch8GARM.pdf

[5] U.S. Census Bureau, Block Assignment Files. www.census.gov/geographies/reference-files/time-series/geo/block-assignment-files.html

[6] MGGG Redistricting Lab, Communities of Interest Data Products. github.com/mggg/coi-products

[7] MGGG Redistricting Lab, Wisconsin Communities of Interest Report. mggg.org/publications/Wisconsin-COI.pdf

[8] MGGG Redistricting Lab, GerryChain Python Package. github.com/mggg/gerrychain

[9] MGGG Redistricting Lab, MAUP GeoSpatial Python Package. github.com/mggg/maup

[10] Assaf Bar-Natan, Elle Najt, and Zachary Schutzmann, *The gerrymandering jumble: Map projections permute districts' compactness scores*. Cartography and Geographic Information Science, Volume 47, Issue 4, 2020, 321–335.

[11] Richard Barnes and Justin Solomon, *Gerrymandering and Compactness: Implementation Flexibility and Abuse.* Political Analysis, Volume 29, Issue 4, October 2021, 448–466.

[12] Ernest C. Reock, Jr., *A Note: Measuring Compactness as a Requirement of Legislative Apportionment.* Midwest Journal of Political Science, Vol. 5, No. 1 (Feb., 1961), 70–74.

I declare under penalty of perjury of the United States that the foregoing is true and correct to the best of my knowledge and understanding.

Dated: December 15, 2021

Moon Duchin

Appendix A: CV of Moon Duchin

Moon Duchin

moon.duchin@tufts.edu - mduchin.math.tufts.edu
Mathematics STS Tisch College of Civic Life | Tufts University

Education

University of Chicago MS 1999, PhD 2005
Mathematics
Advisor: Alex Eskin Dissertation: *Geodesics track random walks in Teichmüller space*

Harvard University BA 1998
Mathematics and Women's Studies

Appointments

Tufts University
Professor of Mathematics 2021—
Assistant Professor, Associate Professor 2011–2021

Director | Program in Science, Technology, & Society 2015–2021
(on leave 2018–2019)

Principal Investigator | MGGG Redistricting Lab 2017

Senior Fellow | Tisch College of Civic Life 2017

University of Michigan
Assistant Professor (postdoctoral) 2008–2011

University of California, Davis
NSF VIGRE Postdoctoral Fellow 2005– 2008

Research Interests

Data science for civil rights, computation and governance, elections, geometry and redistricting.
Science, technology, and society, science policy, technology and law.
Random walks and Markov chains, random groups, random constructions in geometry.
Large-scale geometry, metric geometry, isoperimetric inequalities.
Geometric group theory, growth of groups, nilpotent groups, dynamics of group actions.
Geometric topology, hyperbolicity, Teichmüller theory.

Awards & Distinctions

Research Professor - MSRI Program in Analysis and Geometry of Random Spaces Spring 2022
Guggenheim Fellow 2018
Radcliffe Fellow - Evelyn Green Davis Fellowship 2018–2019
Fellow of the American Mathematical Society elected 2017
NSF C-ACCEL (PI) - Harnessing the Data Revolution: Network science of Census data 2019– 2020
NSF grants (PI) - CAREER grant and three standard Topology grants 2009– 2022
Professor of the Year, Tufts Math Society 2012–2013
AAUW Dissertation Fellowship 2004–2005
NSF Graduate Fellowship 1998–2002
Lawrence and Josephine Graves Prize for Excellence in Teaching (U Chicago) 2002
Robert Fletcher Rogers Prize (Harvard Mathematics) 1995–1996

Mathematics Publications & Preprints

The (homological) persistence of gerrymandering
Foundations of Data Science, online first. (with Thomas Needham and Thomas Weighill)

You can hear the shape of a billiard table: Symbolic dynamics and rigidity for flat surfaces
Commentarii Mathematici Helvetici, to appear. arXiv:1804.05690
(with Viveka Erlandsson, Christopher Leininger, and Chandrika Sadanand)

Conjugation curvature for Cayley graphs
Journal of Topology and Analysis, online first. (with Assaf Bar-Natan and Robert Kropholler)

A reversible recombination chain for graph partitions
Preprint. (with Sarah Cannon, Dana Randall, and Parker Rule)

Recombination: A family of Markov chains for redistricting
Harvard Data Science Review. Issue 3.1, Winter 2021. online. (with Daryl DeFord and Justin Solomon)

Census TopDown: The impact of differential privacy on redistricting
2nd Symposium on Foundations of Responsible Computing (FORC 2021), 5:1–5:22. online.
(with Aloni Cohen, JN Matthews, and Bhushan Suwal)

Stars at infinity in Teichmüller space
Geometriae Dedicata, Volume 213, 531–545 (2021). (with Nate Fisher) arXiv:2004.04321

Random walks and redistricting: New applications of Markov chain Monte Carlo
(with Daryl DeFord) For edited volume, Political Geometry. Under contract with Birkhäuser.

Mathematics of nested districts: The case of Alaska
Statistics and Public Policy. Vol 7, No 1 (2020), 39–51. (w/ Sophia Caldera, Daryl DeFord, Sam Gutekunst, & Cara Nix)

A computational approach to measuring vote elasticity and competitiveness
Statistics and Public Policy. Vol 7, No 1 (2020), 69–86. (with Daryl DeFord and Justin Solomon)

The Heisenberg group is pan-rational
Advances in Mathematics 346 (2019), 219–263. (with Michael Shapiro)

Random nilpotent groups I
IMRN, Vol 2018, Issue 7 (2018), 1921–1953. (with Matthew Cordes, Yen Duong, Meng-Che Ho, and Ayla Sánchez)

Hyperbolic groups
chapter in O ice Hours with a Geometric Group Theorist, eds. M.Clay,D.Margalit, Princeton U Press (2017), 177–203.

Counting in groups: Fine asymptotic geometry
Notices of the American Mathematical Society 63, No. 8 (2016), 871–874.

A sharper threshold for random groups at density one-half
Groups, Geometry, and Dynamics 10, No. 3 (2016), 985–1005.
(with Katarzyna Jankiewicz, Shelby Kilmer, Samuel Lelièvre, John M. Mackay, and Ayla Sánchez)

Equations in nilpotent groups
Proceedings of the American Mathematical Society 143 (2015), 4723–4731. (with Hao Liang and Michael Shapiro)

Statistical hyperbolicity in Teichmüller space
Geometric and Functional Analysis, Volume 24, Issue 3 (2014), 748–795. (with Howard Masur and Spencer Dowdall)

Fine asymptotic geometry of the Heisenberg group
Indiana University Mathematics Journal 63 No. 3 (2014), 885–916. (with Christopher Mooney)

Pushing fillings in right-angled Artin groups

Journal of the LMS, Vol 87, Issue 3 (2013), 663–688. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert
Young)

Spheres in the curve complex
In the Tradition of Ahlfors and Bers VI, Contemp. Math. 590 (2013), 1–8. (with Howard Masur and Spencer Dowdall)

The sprawl conjecture for convex bodies
Experimental Mathematics, Volume 22, Issue 2 (2013), 113–122. (with Samuel Lelièvre and Christopher Mooney)

Filling loops at infniity in the mapping class group
Michigan Math. J., Vol 61, Issue 4 (2012), 867–874. (with Aaron Abrams, Noel Brady, Pallavi Dani, and Robert Young)

The geometry of spheres in free abelian groups
Geometriae Dedicata, Volume 161, Issue 1 (2012), 169–187. (with Samuel Lelièvre and Christopher Mooney)

Statistical hyperbolicity in groups
Algebraic and Geometric Topology 12 (2012) 1–18. (with Samuel Lelièvre and Christopher Mooney)

Length spectra and degeneration of falt metrics
Inventiones Mathematicae, Volume 182, Issue 2 (2010), 231–277. (with Christopher Leininger and Kasra Raf)i

Divergence of geodesics in Teichmüller space and the mapping class group
Geometric and Functional Analysis, Volume 19, Issue 3 (2009), 722–742. (with Kasra Rafi)

Curvature, stretchiness, and dynamics
In the Tradition of Ahlfors and Bers IV, Contemp. Math. 432 (2007), 19–30.

Geodesics track random walks in Teichmüller space
PhD Dissertation, University of Chicago 2005.

Science, Technology, Law, and Policy Publications & Preprints

Models, Race, and the Law
Yale Law Journal Forum, Vol. 130 (March 2021). Available online. (with Doug Spencer)

Computational Redistricting and the Voting Rights Act
Election Law Journal, Available online. (with Amariah Becker, Dara Gold, and Sam Hirsch)

Discrete geometry for electoral geography
Preprint. (with Bridget Eileen Tenner) arXiv:1808.05860

Implementing partisan symmetry: Problems and paradoxes
Political Analysis, to appear. (with Daryl DeFord, Natasha Dhamankar, Mackenzie McPike, Gabe Schoenbach, and Ki-Wan Sim) arXiv:2008:06930

Clustering propensity: A mathematical framework for measuring segregation
Preprint. (with Emilia Alvarez, Everett Meike, and Marshall Mueller; appendix by Tyler Piazza)

Locating the representational baseline: Republicans in Massachusetts
Election Law Journal, Volume 18, Number 4, 2019, 388–401.
(with Taissa Gladkova, Eugene Henninger-Voss, Ben Klingensmith, Heather Newman, and Hannah Wheelen)

Redistricting reform in Virginia: Districting criteria in context
Virginia Policy Review, Volume XII, Issue II, Spring 2019, 120–146. (with Daryl DeFord)

Geometry v. Gerrymandering
The Best Writing on Mathematics 2019, ed. Mircea Pitici. Princeton University Press.

reprinted from ScientifciAmerican, November 2018, 48–53.

Gerrymandering metrics: How to measure? What's the baseline?
Bulletin of the American Academy for Arts and Sciences, Vol. LXII, No. 2 (Winter 2018), 54–58.

Rebooting the mathematics of gerrymandering: How can geometry track with our political values?
The Conversation (online magazine), October 2017. (with Peter Levine)

A formula goes to court: Partisan gerrymandering and the e iciency gap
Notices of the American Mathematical Society 64 No. 9 (2017), 1020–1024. (with Mira Bernstein)

International mobility and U.S. mathematics
Notices of the American Mathematical Society 64, No. 7 (2017), 682–683.

Graduate Advising in Mathematics

Nate Fisher (PhD 2021), Sunrose Shrestha (PhD 2020), Ayla Sánchez (PhD 2017),
Kevin Buckles (PhD 2015), Mai Mansouri (MS 2014)

Outside committee member for Chris Coscia (PhD 2020), Dartmouth College

Postdoctoral Advising in Mathematics

Principal supervisor Thomas Weighill (2019–2020)
Co-supervisor Daryl DeFord (MIT 2018–2020), Rob Kropholler (2017–2020), Hao Liang (2013–2016)

Teaching

Courses Developed or Customized

Mathematics of Social Choice | sites.tufts.edu/socialchoice
Voting theory, impossibility theorems, redistricting, theory of representative democracy, metrics of fairness.

History of Mathematics | sites.tufts.edu/histmath
Social history of mathematics, organized around episodes from antiquity to present. Themes include materials and technologies of creation and dissemination, axioms, authority, credibility, and professionalization. In-depth treatment of mathematical content from numeration to cardinal arithmetic to Galois theory.

Reading Lab: Mathematical Models in Social Context | sites.tufts.edu/models
One hr/wk discussion seminar of short but close reading on topics in mathematical modeling, including history of psychometrics; algorithmic bias; philosophy of statistics; problems of model explanation and interpretation.

Geometric Literacy
Module-based graduate topics course. Modules have included: p-adic numbers, hyperbolic geometry, nilpotent geometry, Lie groups, convex geometry and analysis, the complex of curves, ergodic theory, the Gauss circle problem.

Markov Chains (graduate topics course)
Teichmüller Theory (graduate topics course)
Fuchsian Groups (graduate topics course)
Continued Fractions and Geometric Coding (undergraduate topics course)
Mathematics for Elementary School Teachers

Standard Courses

Discrete Mathematics, Calculus I-II-III, Intro to Proofs, Linear Algebra, Complex Analysis, Differential Geometry, Abstract Algebra, Graduate Real Analysis, Mathematical Modeling and Computation

Weekly Seminars Organized
- Geometric Group Theory and Topology
- Science, Technology, and Society Lunch Seminar

Selected Talks and Lectures

Distinguished Plenary Lecture June 2021
75th Anniversary Meeting of Canadian Mathematical Society, Ottawa, Ontario *online (COVID)*

BMC/BAMC Public Lecture April 2021
Joint British Mathematics/Applied Mathematics Colloquium, Glasgow, Scotland *online (COVID)*

AMS Einstein Public Lecture in Mathematics [March 2020]
Southeastern Sectional Meeting of the AMS, Charlottesville, VA *postponed*

Gerald and Judith Porter Public Lecture
AMS-MAA-SIAM, Joint Mathematics Meetings, San Diego, CA January 2018

Mathematical Association of America Distinguished Lecture
MAA Carriage House, Washington, DC October 2016

American Mathematical Society Invited Address
AMS Eastern Sectional Meeting, Brunswick, ME September 2016

Named University Lectures
- Parsons Lecture | UNC Asheville October 2020
- Loeb Lectures in Mathematics | Washington University in St. Louis [March 2020]
- Math, Stats, CS, and Society | Macalester College October 2019
- MRC Public Lecture | Stanford University May 2019
- Freedman Memorial Colloquium | Boston University March 2019
- Julian Clancy Frazier Colloquium Lecture | U.S. Naval Academy January 2019
- Barnett Lecture | University of Cincinnati October 2018
- School of Science Colloquium Series | The College of New Jersey March 2018
- Kieval Lecture | Cornell University February 2018
- G. Milton Wing Lectures | University of Rochester October 2017
- Norman Johnson Lecture | Wheaton College September 2017
- Dan E. Christie Lecture | Bowdoin College September 2017

Math/Computer Science Department Colloquia
- Reed College Dec 2020
- Georgetown (CS) Sept 2020
- Santa Fe Institute July 2020
- UC Berkeley Sept 2018
- Brandeis-Harvard-MIT-NEU Mar 2018
- Northwestern University Oct 2017
- University of Illinois Sept 2017
- University of Utah Aug 2017
- Wesleyan Dec 2016
- Worcester Polytechnic Inst. Dec 2016
- Université de Neuchâtel Jun 2016
- Brandeis University Mar 2016
- Swarthmore College Oct 2015
- Bowling Green May 2015
- City College of New York Feb 2015
- Indiana University Nov 2014
- the Technion Oct 2014

- Wisconsin–Madison Sept 2014
- Stony Brook March 2013

Minicourses

- Integer programming and combinatorial optimization (two talks) | Georgia Tech May 2021
- Workshop in geometric topology (main speaker, three talks) | Provo, UT June 2017
- Growth in groups (two talks) | MSRI, Berkeley, CA August 2016
- Hyperbolicity in Teichmüller space (three talks) | Université de Grenoble May 2016
- Counting and growth (four talks) | IAS Women's Program, Princeton May 2016
- Nilpotent groups (three talks) | Seoul National University October 2014
- Sub-Finsler geometry of nilpotent groups (five talks) | Galatasaray Univ., Istanbul April 2014

Science, Technology, and Society

- The Mathematics of Accountability | Sawyer Seminar, Anthropology, Johns Hopkins February 2020
- STS Circle | Harvard Kennedy School of Government September 2019
- Data, Classification, and Everyday Life Symposium | Rutgers Center for Cultural Analysis January 2019
- Science Studies Colloquium | UC San Diego January 2019
- Arthur Miller Lecture on Science and Ethics | MIT Program in Science, Tech, and Society November 2018

Data Science, Computer Science, Quantitative Social Science

- Data Science for Social Good Workshop (DS4SG) | Georgia Tech (virtual) November 2020
- Privacy Tools Project Retreat | Harvard (virtual) May 2020
- Women in Data Science Conference | Microso Research New England March 2020
- Quantitative Research Methods Workshop | Yale Center for the Study of American Politics February 2020
- Societal Concerns in Algorithms and Data Analysis | Weizmann Institute December 2018
- Quantitative Collaborative | University of Virginia March 2018
- Quantitative Social Science | Dartmouth College September 2017
- Data for Black Lives Conference | MIT November 2017

Political Science, Geography, Law, Democracy, Fairness

- The Long 19th Amendment: Women, Voting, and American Democracy | Radcliffe Institute Nov–Dec 2020
- "The New Math" for Civil Rights | Social Justice Speaker Series, Davidson College November 2020
- Math, Law, and Racial Fairness | Justice Speaker Series, University of South Carolina November 2020
- Voting Rights Conference | Northeastern Public Interest Law Program September 2020
- Political Analysis Workshop | Indiana University November 2019
- Program in Public Law Panel | Duke Law School October 2019
- Redistricting 2021 Seminar | University of Chicago Institute of Politics May 2019
- Geography of Redistricting Conference Keynote | Harvard Center for Geographic Analysis May 2019
- Political Analytics Conference | Harvard University November 2018
- Cyber Security, Law, and Society Alliance | Boston University September 2018
- Clough Center for the Study of Constitutional Democracy | Boston College November 2017
- Tech/Law Colloquium Series | Cornell Tech November 2017
- Constitution Day Lecture | Rockefeller Center for Public Policy, Dartmouth College September 2017

Editorial Boards

Harvard Data Science Review
Associate Editor
since 2019

Advances in Mathematics
Member, Editorial Board since 2018

Selected Professional and Public Service

Amicus Brief of Mathematicians, Law Professors, and Students 2019
principal co-authors: Guy-Uriel Charles and Moon Duchin

Supreme Court of the United States, in Rucho v. Common Cause - cited in dissent

Committee on Science Policy 2020–2023

American Mathematical Society

Program Committee 2020–2021
Symposium on Foundations of Responsible Computing

Presenter on Public Mapping, Statistical Modeling 2019, 2020
National Conference of State Legislatures

Committee on the Human Rights of Mathematicians 2016–2019
American Mathematical Society

Committee on The Future of Voting: Accessible, Reliable, Verifaible Technology 2017–2018
National Academies of Science, Engineering, and Medicine

Visiting Positions and Residential Fellowships

Visiting Professor Department of Mathematics Fall 2021
Boston College | Chestnut Hill, MA

Fellow Radcliffe Institute for Advanced Study 2018–19
Harvard University | Cambridge, MA

Member Center of Mathematical Sciences and Applications 2018–19
Harvard University | Cambridge, MA

Visitor Microso Research Lab 2018–19
MSR New England | Cambridge, MA

Research Member Geometric Group Theory program Fall 2016
Mathematical Sciences Research Institute | Berkeley, CA

Research Member Random Walks and Asymptotic Geometry of Groups program Spring 2014
Institut Henri Poincaré | Paris, France

Research Member Low-dimensional Topology, Geometry, and Dynamics program Fall 2013
Institute for Computational and Experimental Research in Mathematics | Providence, RI

Research Member Geometric and Analytic Aspects of Group Theory program May 2012
Institut Mittag-Leffler | Stockholm, Sweden

Research Member Quantitative Geometry program Fall 2011
Mathematical Sciences Research Institute | Berkeley, CA

Postdoctoral Fellow Teichmüller “project blanc” Spring 2009
Agence Nationale de la Recherche (Collège de France) | Paris, France

Appendix B: Materials

A detailed list of materials consulted includes the following.

- Data products published by the Census Bureau, including the PL94-171 Decennial Census release, the 2015-19 American Community Survey, and the ACS Special Tabulation from the same 5-year period. The County Subdivisions dataset was used to extract block assignments to municipalities. TIGER/Line shapefiles were used to pair demographics with geography.
- Files defining the State's new enacted districts, and defining the updated 2020 wards, from the Legislative Technology Services Bureau [2]. Files defining the districts in the MathSci and vetoed plans were provided to me by the attorneys representing the Citizen Mathematicians and Scientists.
- The Nov. 30 court order [1] and the Wisconsin Constitution [3].
- Peer-reviewed articles including those by Bar-Natan–Najt–Schutzmann [10], Barnes–Solomon [11], and Reock [12].

Appendix C: Clustering of Communities of Interest

The following brief descriptions summarize the 36 geoclusters synthesized from mapped public comment submissions [6, 7].

Cluster C1 — **Whitewater.** (98 submissions) Primarily small towns and rural areas along I-94 corridor. Many Madison, Milwaukee, and Janesville commuters traveling for employment, hospitals, and goods and services. Concerns about public schools and preserving school district boundaries in redistricting. Shared recreational parks and trails. Includes lake and river communities. Area said to lack diversity. UW-Whitewater and Madison Area Technical College – Watertown and Fort Atkinson campuses have significant student population and provide local services. This cluster thematically split into subcluster 1-1, themed on the environment and recreational activities in the area, and subcluster 1-2, largely citing rural interests, concerns with schools and school districts.

Cluster C2 — **Walworth County.** (37 submissions) Small townships. Concerns about K-12 education. Shared services and shopping centers. Emphasis on preserving school district boundaries and municipalities in redistricting.

Cluster C3 — **Beloit-Janesville and Delavan-Darien area.** (34 submissions) Municipalities share health and public services, shopping centers, recreational spaces, and community events, largely centered in Janesville and Delavan. Emphasis on importance of school district boundaries. Home to Beloit College and Blackhawk Technical College. Pockets of diversity with large Hispanic population. Historical presence of KKK in Janesville highlighted by one submitter.

Cluster C4 — **Brown Deer.** (34 submissions) Diverse community with Black, Hispanic, Asian, White, Native American, Puerto Rican, and Hmong residents. Shared safety concerns that reference crime, reckless driving, policing, and pollution. Submissions describe infrastructure needs in affordable housing, parks, transportation, potholes, sidewalks, and roads. Schools, grocery stores, restaurants are important community spaces.

Cluster C5 — **Wauwatosa.** (11 submissions) Suburb with many young families and Milwaukee commuters. Shared values around diversity, education, green space, walkability, and historic preservation. Includes areas in West Milwaukee with predominantly African American population; concerns about infrastructure and housing insecurity.

Cluster C6 — **North Milwaukee.** (17 submissions) Cohesive Black neighborhoods, with notable Hmong community. Key concerns included road repair, violence (guns and policing), theft, and affordable housing.

Cluster C7 — **Sherman Park.** (72 submissions) Diverse neighborhoods with concerns about crime, poor infrastructure, and sanitation. Many submissions cite major stores, community centers, and churches as landmarks. Frequent reiteration that more activities are needed to occupy youth in the area. This cluster semantically split into C7-1 and C7-2. Submissions in C7-1 focused on the issues of the local economy while C7-2 has a discussion of vulnerable populations and neighborhood maintenance. Diversity was a strong theme in both.

Cluster C8 — **Washington Park.** (101 submissions) This area is centered around Washington Park in Northside Milwaukee. Common emphasis on diversity, violence, and infrastructure. This cluster thematically split into three subclusters. Cluster C8-1 has more focus on the student population from Marquette University and infrastructure concerns in the area, while cluster C8-2 cites importance of local commerce, as well as environmental issues (littering and illegal dumping) and vulnerable populations. Within cluster C8-3 there were significant shared concerns about violent crime and affordable housing.

Cluster C9 — **Greater Milwaukee County.** (11 submissions) Concerns about Milwaukee Public Schools and equitable access to green space, healthy food, and affordable housing.

Cluster C10 — **Milwaukee Northshore.** (103 submissions) Diverse communities including notable Black neighborhoods in North Milwaukee, edging into suburbs like Glendale and Whitefish Bay. Areas come together for local events, and submitters identify with school districts. Several commenters desire to keep students of University of Wisconsin - Milwaukee in the same district.

Cluster C11 — **Waukesha-New Berlin.** (16 submissions) Mix of city/urban and rural areas that share concern for public services and local parks. Many submitters cite needs for infrastructure development and maintenance (parks, streets, schools). Strong concern for school district funding. Includes student population from UW-Waukesha, Waukesha County Technical College, and Carroll College. AAPI community in Waukesha.

Cluster C12 — **Germantown-Menomonee Falls.** (15 submissions) Recreation activities and natural areas important for community engagement. Value on public schools. Travel between Menomonee Falls, Germantown, and West Bend for work, recreation, and shopping.

Cluster C13 — **Kenosha-Racine.** (7 submissions) Small cluster citing shared interests in hunting, fishing, and recreation. Environmental and economic development concerns.

Cluster C14 — **Kenosha.** (89 submissions) Communities in and around Kenosha, including Racine and Mt. Pleasant. Needs include healthcare services for underserved communities. Food insecurity is a concern. Shared recreational activities and unified school district. Diverse community but some describe divisions. Expansion of public transportation is important. Subsidized and affordable housing is available but more is wanted. Erosion and environmental concerns over Lake Michigan. This cluster splits into C14-1, whose top-cited themes include food insecurity, racial/ethnic diversity, and class differences, ranging from affluent to underserved neighborhoods. C14-2 entries focus on individual neighborhoods' identity and businesses, as well as themes of recreation and tourism in the area. Geographically, these clusters significantly overlap.

Cluster C15 — **Southwest Milwaukee.** (25 submissions) Edge of Milwaukee, including suburbs like Greenfield. Diverse area including Filipino residents and AAPI/Desi communities with origins cited from India, Pakistan, Iraq, Palestine, Afghanistan, Bangladesh, and Burma. Firefighters, police, teachers. Some areas have high property values.

Cluster C16 — **Bay View-Near Southside.** (39 submissions) Young families. Predominantly White with noted Hispanic and African American communities in Kosciuszko neighborhood and South Milwaukee. Mix of middle- and working class. Issues about potholes, traffic, and traffic safety. Communities on the shore were concerned with the environment and conservation of Lake Michigan.

Cluster C17 — **West Allis.** (23 submissions) Good public transit is cited. Shopping, dining, and entertainment. Young Professionals. Bringing back manufacturing jobs is concern. Noted Asian American communities, including Rohingya refugee population. Concerns about public safety and affordable housing.

Cluster C18 — **Eau Claire.** (36 submissions) Cluster includes Eau Claire and some surrounding rural/commuter areas with distinct concerns (such as needing better internet service). Affordable housing needs and homelessness are often referenced throughout the cluster. Large Asian immigrant population, including Hmong; some cite language barriers. Submitters note diversity within AAPI population. University of Wisconsin - Eau Claire is key part of the surrounding community, impacting housing, employment, and infrastructure.

Cluster C19 — **Western Wisconsin river communities.** (90 submissions) Cluster includes rural farming communities and small university cities. Recreation is centered around rivers. Strong local culture and businesses; significant student populations from University of Wisconsin - Stout and University of Wisconsin - River Falls. Some worry about the quality of education in schools. Environmental concerns are articulated regarding sustainability of agricultural practices and conservation of the St. Croix River. Some submitters call out the area's increasing pace of urban development. Subcluster C19-1 is more focused on recreation, small local businesses, and concerns about student population, while C19-2 tends to cite agricultural and environmental issues. Geographically, there is significant overlap between these subclusters.

Cluster C20 — **Lake Superior areas.** (20 submissions) This subcluster includes the Lake Superior coast (and the Apostle Islands) and a segment of the shared border with Minnesota. Several cite importance of preservation and conservation of Lake Superior and surrounding wetlands; recreation and economy (shipping, tourism) also depend on the lake. Important communities include the Red Cliff Band of Ojibwe, the Anishinabeg, the people of Odaawaa-Zaaga'iganiing, and the Lac Courte Oreilles Tribe.

Cluster C21 — **Western Wisconsin.** (35 submissions) Small towns and rural communities in Polk and surrounding counties. Service hubs in Rice Lake, St. Croix Falls, Osceola, and New Richmond. Twin Cities commuters. Important industries are agriculture and tourism, including campgrounds, resorts, and gaming enterprises. Region connected to technical colleges and UW-River Falls School of Agriculture. Retirement community along St. Croix Falls River. St. Croix Band of Ojibwe Chippewa stretches across Polk and Barren Counties. Some cite resources that address and serve the Native population, including Unity School and the recreational and hunting area in the Barrens.

Cluster C22 — **Northern Driftless Area.** (26 submissions) Cluster includes small farming communities stretching across southwestern Wisconsin, as well as a small urban hub in La Crosse. Mix of family farms and CAFOs (concentrated animal feeding operations). Student populations from UW-La Crosse, Viterbo University, and Western Technical College. Specific communities include a significant middle class and low-income Hmoob community; Amish community in Readstown; Ho-Chunk People of the Sacred Voice in Jackson County. Shared concerns about clean groundwater and safe drinking water, particularly on French Island. Concerns about environmental conservation, preserving recreational space, flooding along Kickapoo River, agricultural pollution and runoff from large CAFOs. Tourism industry. Needs include access to grocery stores in rural areas and affordable housing in La Crosse.

Cluster C23 — **Greater Baraboo.** (4 submissions) Small cluster including some Madison commuters and a student population at UW-Baraboo/Sauk County.

Cluster C24 — **Iowa County area.** (26 submissions) Rural community concerned about broadband, healthcare, water quality, and environmental protection of Driftless area and the fragile karst terrain. Strong county identity with emphasis on preserving Iowa County and surrounding local school districts in redistricting. Shared services, including Uplands Hills Hospital. Rich agricultural land with family farms. Attractions include Taliesin (home of architect Frank Lloyd Wright), "award-winning Uplands Cheese," and recreational areas.

Cluster C25 — **Madison suburbs.** (35 submissions) Emphasis on preserving school district boundaries, with the Oregon, Mt. Horreb, and Wisconsin Heights School Districts named. School districts serve multiple municipalities. Shared recreational parks and trails. Madison commuters.

Cluster C26 — **Greater Madison.** (46 submissions) Diverse population with young families and college students from flagship University of Wisconsin campus. Middle- and low-income residents. Shared concerns about affordable housing, gentrification, and water quality of the surrounding lakes. Desire for improved public transportation routes. Many cite value of local small businesses; one submitter noted that businesses are being priced out of the area.

Cluster C27 — **Wausau.** (16 submissions) Wausau is the economic and healthcare services hub in this area of Central Wisconsin, including Marathon County. Large Hmoob community. Home to institutions of higher education, including UW-Wausau and Northcentral Technical College. One submitter notes that nearby UW-Stevens Point now has responsibility for managing

UW-Wausau and UW-Marshfield campuses, which suggests that keeping them together in redistricting would be sensible. Outdoor interests include Rib Mountain State Park and Sunnyvale Park Lake.

Cluster C28 — **Northeast indigenous communities.** (7 submissions) Rural areas with indigenous communities including Menominee Nation, Stockbridge-Munsee Band of Mohicans, and Forest County Potawatomi. Shared concern for conservation, water quality, and pollution prevention. Needs included increased investment in road infrastructure and broadband service.

Cluster C29 — **Northwoods rural economy.** (8 submissions) Rural communities with reliance on tourism. Seasonal economy. Other employment opportunities include paper industry, manufacturing, trucking, and farming. Juvenile corrections institution is noted. Nicolet Area Technical College in Rhinelander has campuses and outreach centers across the area.

Cluster C30 — **Central Wisconsin.** (23 submissions) Rural and farming communities with service hubs in Stevens Point, Marshfield, and Wisconsin Rapids. Residents care about water quality and conservation areas. Emphasis on preserving school district boundaries, particularly the Marshfield School District, and linking Juneau and Adams Counties for their shared environmental interests. Identified employers include healthcare, government, Land O' Lakes, and the Ho-Chunk Nation. Reliance on tourism. Amish and Mennonite communities in Clark County. Student population at UW-Stevens Point.

Cluster C31 — **West Bend.** (7 submissions) This small cluster cites shared interests in schools and local businesses and gathering spots. Home to UW-Washington County, UW-Fond du Lac, and Moraine Park Technical College.

Cluster C32 — **Northern Door County school districts.** (14 submissions) Families strongly identify with local schools and school district boundaries. Reliance on farming and tourism. Mix of part- and full-time residents.

Cluster C33 — **Manitowoc and Shore area.** (21 submissions) UW-Manitowoc, UW-Sheboygan, Silver Lake College, and Concordia University are mentioned as focal points of community engagement. Environmental conservation is important; water resources regarding Lake Michigan, including clean water and wetlands. This cluster is spread out between lakeshore areas north of Milwaukee (such as Sheboygan) extending to the greater Manitowoc area.

Cluster C34 — **Green Bay area.** (21 submissions) City of Green Bay and surrounding suburbs and farming communities with shared recreational activities and cultural events. The Packers are a point of shared identity. Green Bay Public Schools serve many adjacent municipalities. Growing Hispanic community. Student population at UW-Green Bay, Northeast Wisconsin Technical College, and Saint Norbert College with commuting concerns. One submitter cites the "checkerboarding" of the Oneida Nation reservation by loss of land. Concerns around prison malapportionment due to Green Bay Correctional Institute.

Cluster C35 — **Oshkosh college zone.** (7 submissions) UW-Oshkosh community with satellite campuses at UW-Fond du Lac and UW-Fox Valley. Partnerships with Prairie Art Center and Fox Valley Technical College. Concerns about food deserts. Moraine Park Technical College in Fond du Lac with additional campuses in Beaver Dam and West Bend.

Cluster C36 — **Suburban Appleton.** (17 submissions) City of Appleton and surrounding suburbs. Appleton School District serves the greater area. Appleton and the town of Grand Chute are described as particularly intertwined. Significant student populations at Fox Valley Technical College and Lawrence University. Emphasis on preserving school district boundaries, Appleton-Grand Chute area, and uniting Appleton and suburbs in one State Senate and Congressional district.